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## CAN PRODUCTIVE GOVERNMENT SPENDING BE THE ENGINE OF LONG- RUN GROWTH WHEN LABOR SUPPLY IS ENDOGENOUS?

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# CAN PRODUCTIVE GOVERNMENT SPENDING BE THE ENGINE OF LONG-RUN GROWTH WHEN LABOR SUPPLY IS ENDOGENOUS?

## Abstract

We reexamine the properties of optimal fiscal policy and their implications for implementable capital accumulation. The setup is a standard endogenous growth model with public production services, augmented by elastic labor supply. We show that, when a benevolent government chooses a distorting income tax rate to finance public production services by taking into account the competitive decentralized equilibrium, public production services can no longer play their traditional role as an engine of long-run endogenous growth. This follows from a simple combination of Ramsey second-best fiscal policy and endogenous labor/leisure choices.

JEL Classification: E62, H21, O41.

Keywords: second-best policy, elastic labor supply, endogenous growth.

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## I. INTRODUCTION

It is known that when taxes are distortionary and government spending is unproductive, government involvement is bad for capital accumulation. By contrast, when government spending is productive, policymakers face a well-defined tradeoff: public production services can be the engine of perpetual economic growth, but they have to be financed by distortionary taxes. Then, the challenge is to identify the optimal tax rate and the associated optimal level of government spending. Since Barro's [1990] influential paper,<sup>1</sup> this has become one of the most active research areas (see, among others, Jones, Manuelli and Rossi [1993], Barro and Sala-i-Martin [1995], Glomm and Ravikumar [1994, 1997], Benhabib, Rustichini and Velasco [1996], Kneller, Bleaney and Gemmell [1999] and Turnovsky [2000a, b]).

This paper extends this analysis by endogenizing labor/leisure choices. In all other respects, our model is a standard Barro-type model of endogenous growth and optimal fiscal policy. Namely, a model in which a benevolent government chooses a distorting income tax rate to finance public production services by taking into account the competitive decentralized equilibrium. That is, we solve for Ramsey second-best fiscal policy.<sup>2</sup> We show that endogenizing labor/leisure decisions changes the results drastically: public production services can no longer generate perpetual long-run growth. Therefore, the result that public production services are capable of generating long-term growth is not robust.

We wish to emphasize that despite the influence and popularity of the Barro-type model of endogenous growth and optimal fiscal policy, there have been no versions of this model that combine second-best policy and elastic labor supply. Turnovsky [2000a] has studied optimal fiscal policy in a similar model with elastic labor supply, but he focuses on fiscal policies that can replicate the first-best outcome obtained by the central planner.

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<sup>1</sup> Early models of capital accumulation, in which public capital is a factor of private production, also include Shell [1967] and Arrow and Kurz [1970]. Barro's model is a variant of the AK model (see Rebelo [1991]) because it results in a linear production function. See e.g. Jones and Manuelli [1997] for a survey of different classes of endogenous growth models.

<sup>2</sup> By Ramsey second-best fiscal policy, we mean that the government's objective is to find the optimal fiscal policy (in our case, the income tax rate and the associated level of public production services) that achieves maximal consumer utility and induces the competitive allocation of resources. See Lucas and Stokey [1983] and Lucas [1990]. Other well-known applications of the Ramsey approach to optimal fiscal policy include Chamley [1986], Stiglitz [1987], Zhu [1992], Jones, Manuelli and Rossi [1993], Chari, Christiano and Kehoe [1994], Judd [1999], etc.

Our model has five distinct features. First, it is a one-sector endogenous growth model with public production services. Second, the government uses distortionary income taxes to finance its expenditures. Third, we include labor/leisure as a choice variable so that labor supply is endogenous. Fourth, the government is able to commit itself to future policies. Fifth, the optimal fiscal policy (i.e. income tax and public services) is chosen by a benevolent government subject to the decentralized competitive equilibrium. In other words, when the government chooses its optimal fiscal policy, it takes into account the optimal behavior of private agents (who have acted competitively by taking prices, tax policy and public services as given), the economy's constraints, and market-clearing price determination. As we said above, this is a typical Ramsey second-best policy.

Our main results are as follows. We first show that in a competitive decentralized equilibrium, for any feasible economic policy, the relation between long-run growth and the income tax rate is inverse U-shaped. That is, as in most models in this literature, the growth rate increases and then decreases with the distortionary tax rate. The critical tax rate, that ensures long-run growth, depends mainly on the productivity of public capital services and the rate of intertemporal substitution for labor.

We then endogenize fiscal policy. Our results imply that, while the short-run growth rate can be positive, the long-run growth rate is zero. That is, productive government spending, financed by optimally chosen distortionary taxes, can implement positive capital accumulation only in the short run. It cannot implement positive capital accumulation in the long run. Specifically, the Ramsey tax and spending policy cannot stimulate the rate of long-run growth, although they can influence the levels of long-run output, consumption and employment. This resembles the neo-classical model. Therefore, the growth implications of Ramsey second-best fiscal policy are very different from the implications that have been drawn from similar studies that assume that labor supply is inelastically supplied.

The intuition is as follows. When labor supply is elastic and endogenously chosen, any increase in output requires more labor input, as well as more private capital and public production services (factors are complementary to each other). In this case, since leisure enters preferences, higher economic activity exerts *ceteris paribus* a negative effect on households' welfare. At the same time, in a decentralized setup, individual agents have not internalized the positive effects of public production services (this is basically a coordination

failure problem); this results in a wedge between the social and the private rate of capital return, and hence leads to inefficiently low economic growth. Under these circumstances, to get the right quantity of public services and so increase the growth rate, the government has to resort to higher income taxes. In general equilibrium, and with Ramsey second-best taxation, the positive growth effect from government spending is counter-balanced by the negative growth effects from distorting taxes and the disutility from work effort. The two latter adverse effects exactly offset the former positive effect and, eventually, long-run growth is zero.

By contrast, when fiscal policy is chosen by a social planner, or when labor is inelastically supplied, public production services can generate long-run growth.<sup>3</sup> Therefore, public production services are not capable of playing their traditional role as an engine of long-run growth, when two conditions are present: First, policymakers seek to guide the decentralized economy. Specifically, in our setup, Ramsey second-best policy is also chosen to close the wedge between social and private rates of return arising from decentralized private behavior. Second, labor/leisure is optimally chosen. This basically means that higher economic activity comes at the cost of less leisure.

The rest of the paper is as follows. Section II presents the economy. Section III characterizes the competitive decentralized equilibrium, for any fiscal policy. Section IV solves for Ramsey second-best policies and studies their implications. Section V discusses conclusions and extensions. An Appendix contains technical details.

## **II. THE ECONOMY**

This section sets up a closed economy with a private sector and a government sector. We will keep the model as simple as possible so as to make our results directly comparable to those of the literature. The private sector consists of a representative household and a representative firm, who both act competitively. The household consumes, supplies labor elastically and rents out its assets to the firm. The firm produces output by choosing private inputs (capital and labor) and taking advantage of public production

services. The government taxes the firm's output to finance public production services.<sup>4</sup> There is no uncertainty, time-horizons are infinite and time is continuous. Economic agents are endowed with perfect foresight.

This section will solve for a decentralized competitive equilibrium, given economic policy.

### ***The Problem of the Representative Household***

The household maximizes intertemporal utility:

$$(1) \quad \int_0^{\infty} u(c, L) e^{-\rho t} dt ,$$

where  $c$  is private consumption,  $L$  is labor services and the parameter  $\rho > 0$  is the rate of time preference. The instantaneous utility function  $u(c, L)$  is increasing in  $c$  and decreasing in  $L$ ; is twice continuously differentiable and concave in  $(c, L)$ ; and satisfies a constant elasticity of intertemporal substitution and the Inada conditions. For simplicity, we assume that  $u(c, L)$  is additively separable in  $c$  and  $L$ , and takes the functional form:<sup>5</sup>

$$(2) \quad u(c, L) = \log c - \frac{1}{1+\eta} L^{1+\eta} .$$

where,  $\eta \geq 1$ . That is, the elasticity of intertemporal substitution for  $c$  is 1 and that for labor is  $-\eta$ .

The household saves in the form of assets, denoted by  $a$ , so that it receives interest income  $ra$ , where  $r$  is the market asset return. The household also supplies elastically its labor services  $L$ , so that wage income is  $wL$ , where  $w$  is the market wage rate. It also receives net dividends  $d$  from ownership of firms. Thus, the household's budget constraint is:

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<sup>3</sup> Our results are therefore consistent with Turnovsky [2000a], who gets long-run growth under "first-best" policy making. They are also consistent with Park and Philippopoulos [2000], who get long-run growth under fixed labor supply.

<sup>4</sup> Our qualitative results do not change if we use income taxes on households or capital taxes on firms. This is because the model is a variant of the AK-model at aggregate level (see below). In general, output taxes are less distortionary than capital taxes and thus satisfy the production efficiency principle: taxes should be levied on the final good, not intermediate inputs.

<sup>5</sup> This instantaneous utility function is commonly used in a growing economy. As in Benhabib and Farmer [1994], Benhabib and Perli [1994], Guo and Lansing [1999] and Milesi-Ferretti and Roubini [1998], with a Cobb-Douglas technology, a logarithmic utility function of consumption is the only formulation of preferences that is consistent with constant labor supply in a growing economy.

$$(3) \quad \dot{c} + \dot{a} = ra + wL + d,$$

where a dot over a variable denotes time derivative and the initial stock of assets,  $a_0$ , is given.

The household acts competitively by taking prices as given. The necessary conditions are equation (3) above, as well as the familiar conditions:

$$(4) \quad \dot{c} = c[r - \rho];$$

$$(5) \quad L = \left[ \frac{w}{c} \right]^{-1}.$$

The necessary conditions (3), (4), and (5) are completed with the addition of the transversality condition  $\lim_{t \rightarrow \infty} \left( \frac{1}{c} a e^{-\rho t} \right) = 0$ . A unique solution exists given the assumed utility function.

### ***The Problem of the Representative Firm***

Firms choose private capital  $k$  and labor  $L$ , but they take public production services  $g$  as given. The production function is increasing and twice continuously differentiable in  $(k, L, g)$ .<sup>6</sup> It also satisfies the Inada condition for  $(k, L, g)$ . Public production services  $g$  are assumed to be non-exclusive and thereby the aggregate production function exhibits overall increasing returns to scale in the three factors. Specifically, the firm's production function is:

$$(6) \quad y = A g^{1-\alpha} k^\beta L^{1-\alpha-\beta},$$

where  $A > 0$ ,  $0 < \alpha < 1$  and  $\alpha + \beta \leq 1$  (the condition  $\alpha + \beta \leq 1$  is needed for existence of a solution to the firm's problem). Following Rebelo [1991], this formulation permits persistent capital accumulation in the long run.

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<sup>6</sup> Following Barro [1990], Benhabib et al. [1996], Turnovsky [2000a] and many others, we assume that it is the flow of public services that provides production externalities rather than the stock of public capital. On the other hand, Futagami et al. [1993], Glomm and Ravikumar [1994] and Turnovsky [2000b] use the stock of public capital, while Baxter and King [1993] and Lansing [1999] do the same in calibrated RBC models. It would be interesting to see whether our main results change if we use stocks.

The government taxes the firm's output at a rate  $0 < \tau < 1$  in each time period. The representative firm acts competitively by taking prices, policy instruments and public services as given. It maximizes profits  $\pi$  given by:

$$(7) \quad \pi = (1 - \tau)y - rk - wL.$$

The familiar first-order conditions for  $k$  and  $L$  are respectively:

$$(8) \quad r = (1 - \tau)Ag^{1-\alpha} k^{\alpha-1} L^{1-\alpha};$$

$$(9) \quad w = (1 - \tau)Ag^{1-\alpha} k^{\alpha} L^{-\alpha}.$$

### ***The Government's Budget Constraint***

We assume, for simplicity, that the government balances its budget at each point of time. Then, by using (6), the government's budget constraint is:

$$(10) \quad g = \tau y = A^{\frac{1}{1-\alpha}} k^{\frac{\alpha}{1-\alpha}} L^{\frac{1-\alpha}{1-\alpha}}.$$

## **III. COMPETITIVE EQUILIBRIUM ALLOCATIONS**

We will now characterize the Decentralized Competitive Equilibrium (DCE), for any feasible fiscal policy. With endogenous government spending, fiscal policy can be fully summarized by the path of income tax rates,  $\tau_t$ .

Using (10) into (6), the economy-wide output in a DCE is:

$$(11) \quad y = A^{\frac{1}{1-\alpha}} k^{\frac{\alpha}{1-\alpha}} L^{\frac{1-\alpha}{1-\alpha}}.$$

so that, at aggregate level, output is linear in private capital. Hence, this is a variant of the AK-model augmented with endogenous labor supply.<sup>7</sup>

Using (10) into (8), the return to capital in a DCE is:

$$(12) \quad r = A^{\frac{1}{1-\alpha}} (1 - \tau)^{\frac{1-\alpha}{1-\alpha}} L^{\frac{1-\alpha}{1-\alpha}}.$$

This is the return that drives private consumption/saving decisions in a DCE. This return differs from the social one, which follows from (11) and is  $R = A^{\frac{1}{1-\alpha}} L^{\frac{1-\alpha}{1-\alpha}}$ . Notice that  $r < R$ . That is, as is typically the case in models with production externalities, when private

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<sup>7</sup> If we use capital taxes, the linear AK technology is preserved even with endogenous labor supply.



agents do not internalize the externalities offered by public production services, the decentralized rate of capital return, and hence the rate of economic growth, are inefficiently low.

Working similarly, (9) and (10) give the wage rate in a DCE:

$$(13) \quad w = A^{\frac{1}{\alpha}} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} kL^{-\frac{1}{\alpha}}.$$

Thus, the firm's realized profit is in a DCE:

$$(14) \quad \pi = (1 - \alpha - b) A^{\frac{1}{\alpha}} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} kL^{-\frac{1}{\alpha}}.$$

Then, using (12), (13) and (14), as well as  $a = k$  and  $d = \pi$ , into (3), (4) and (5), the system of dynamic equations for  $k, c, L$  in a DCE is:

$$(15a) \quad \dot{k} = \frac{\Delta(\alpha)}{k} kL^{-\frac{1}{\alpha}} - c;$$

$$(15b) \quad \dot{c} = c \left[ \Delta(\alpha) L^{-\frac{1}{\alpha}} - \rho \right];$$

$$(15c) \quad L = \left[ -\frac{k}{c} \Delta(\alpha) \right]^{\frac{1}{1-\alpha}};$$

where  $\Delta(\alpha) \equiv A^{\frac{1}{\alpha}} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} > 0$ .<sup>8</sup>

Therefore, we have solved for a Decentralized Competitive Equilibrium (DCE), for any feasible economic policy as summarized by the income tax rate,  $\alpha$ . In this equilibrium:

(a) private decisions are optimal; (b) all constraints are satisfied; (c) all markets clear; (d) the

transversality condition  $\left[ \Delta(\alpha) L^{-\frac{1}{\alpha}} - \rho \right] < \rho$  is satisfied.<sup>9</sup> This DCE is summarized by

equations (15a), (15b) and (15c).

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<sup>8</sup> It is important for the properties of Ramsey policy in the next section to note that due to externalities (i.e.  $0 < \alpha < 1$ ), we have the realized, or social, return to capital,  $R = \alpha^{-1} \Delta(\alpha)$ , in the resource constraint (15a), but the perceived, or private, return to capital,  $r = \Delta(\alpha)$ , in the Euler equation (15b). In other words, there is a wedge between the rate of capital return that determines the stream of income in (15a) and the rate of capital return that drives consumption/saving decisions in (15b). Of course, without externalities,  $r$  and  $R$  coincide.

<sup>9</sup> Boundedness of lifetime utility also satisfies the transversality condition.

### *Decentralized Competitive Equilibrium in the long run*

It will be useful for what follows in the next section, to study the properties of the DCE in the long run. Since public production services are expected to generate long-term growth, we focus on Balanced Growth Path (BGP) solutions. That is, solutions on which: (a) consumption and capital can grow at the same rate; (b) labor supply is constant. Then, the conditions for non-negative long-run growth can be summarized as follows:

**Proposition 1:** *In a Decentralized Competitive Economy given the tax rate  $\tau$ , consumption and capital can grow without a finite limit in the long run, and thereby the long-run growth rate is non-negative, provided that the exogenous tax rate satisfies:*

$$(16) \quad \tau \in \Theta \equiv \left\{ \tau \in [0,1] \mid \rho^{-\frac{1}{1+\alpha}} \leq \Delta(\tau) \leq A^{\frac{1}{2}} (1-\tau)^{\frac{1-\alpha}{2}} \right\}.$$

**Proof:** Recall that  $\Delta(\tau) \equiv A^{\frac{1}{2}} (1-\tau)^{\frac{1-\alpha}{2}} > 0$ . For the upper bound,  $\Delta(\tau)$  has a maximum at  $\tau = 1-\alpha$ . For the lower bound of  $\Delta(\tau)$ , we combine (15a) and (15c) so that we have  $\dot{k} = \frac{\Delta(\tau)}{k} L^{-\alpha} [1 - L^{-(1+\alpha)}]$ . Hence, the condition  $L \geq \frac{1}{1+\alpha}$  ensures non-negative

growth. On the other hand, by (15b),  $L > \left[ \frac{\rho}{\Delta(\tau)} \right]^{-\frac{1}{\alpha}}$  is required for strictly positive growth.

Therefore, when the exogenous tax rate satisfies  $\Delta(\tau) > \rho^{-\frac{1}{1+\alpha}}$ , the economy can grow at a strictly positive rate in the long run; when  $\Delta(\tau) = \rho^{-\frac{1}{1+\alpha}}$ , the economy ceases to grow in the long run.

Note that equations (15a)-(15c) and Proposition 1 above imply that there is a non-monotonic, inverse U-shaped, relation between the income tax rate and the growth rate, where the maximum is at  $\tau = 1-\alpha$ . This is a well-known result in this literature: for relatively low tax rates, the growth rate increases with the tax rate, but for relatively high tax

rates, the growth rate decreases with the tax rate.<sup>10</sup> Also, note that the critical tax rate, that ensures long-run growth, depends mainly on the productivity of public capital services and the rate of intertemporal substitution for labor.<sup>11</sup>

We are now ready to endogenize economic policy, . By choosing , the government will attempt to internalize the existing externalities and also collect tax revenues to finance the optimal provision of public services.

#### IV. OPTIMAL (RAMSEY) ECONOMIC POLICY

We assume that the government chooses income taxes, , by acting as a benevolent planner that plays Stackelberg *vis-a-vis* the private sector. In particular, the government takes into account the DCE summarized by equations (15a), (15b) and (15c). This means that the government will also take into account the private agents' response to its tax policy. We assume commitment technologies on behalf of the government, so that decisions are made once-and-for-all and become an open-loop equilibrium. All this means that the government will find the optimal implementable competitive decentralized allocation that maximizes the utility of the representative consumer. Thus, this is a Ramsey second-best policy problem.

Formally, the government chooses the path of to maximize (1)-(2) subject to (15a), (15b) and (15c). The current-value Hamiltonian,  $H(c, k, L, \lambda_c, \lambda_k)$ , of this problem is:

$$(17) \quad H \equiv \log c - \frac{1}{1+\theta} L^{1+\theta} + \lambda_c \left[ \Delta(\tau) L^{-\theta} - \rho \right] + \lambda_k \left[ 1 - L^{-(1+\theta)} \right] \left[ \frac{\Delta(\tau)}{k} k L^{-\theta} \right],$$

where  $\lambda_c$  and  $\lambda_k$  are dynamic multipliers associated respectively with (15a) and (15b).

The necessary conditions with respect to  $\tau, L, c, \lambda_c, k$  and  $\lambda_k$  are given by (18a), (18b), (18c), (18d), (18e), (18f) respectively, and the transversality condition in (18g):<sup>12</sup>

<sup>10</sup> See e.g. Barro and Sala-i-Martin [1995, chapter 4].

<sup>11</sup> This follows from the property of  $\Theta$  and the continuity of  $\Delta(\tau)$  in  $\tau \in (0, 1)$ . In particular, the lower bound of  $\Delta(\tau)$  is increasing in  $\Theta$  as the elasticity  $\frac{1}{1+\theta}$  increases.

<sup>12</sup> The government directly chooses quantity allocations (i.e. consumption, labor and capital), as well as economic policy instruments. This is as in e.g. Lucas and Stokey [1983], Chamley [1986], etc. An

$$(18a) \quad \frac{\dot{c}}{c} + \frac{1}{c} \frac{\dot{k}}{k} - \frac{k}{c} L^{-(1+\alpha)} = 0;$$

$$(18b) \quad -\dot{L} + \Delta(L) L^{-1} \left[ c + k \left[ \frac{1}{c} + \left( 1 + \frac{\alpha}{c} \right) L^{-(1+\alpha)} \right] \right] = 0;$$

$$(18c) \quad \dot{c} = \rho - \frac{1}{c} \left[ \Delta(L) L^{-1} - \rho \right];$$

$$(18d) \quad \dot{L} = c \left[ \Delta(L) L^{-1} - \rho \right];$$

$$(18e) \quad \dot{k} = \rho - k \left[ 1 - L^{-(1+\alpha)} \right] \left[ \frac{\Delta(L)}{c} L^{-1} \right];$$

$$(18f) \quad \dot{k} = \frac{\Delta(L)}{c} k L^{-1} - c;$$

$$(18g) \quad \Delta(L) L^{-1} - \rho < \rho. ^{13}$$

Since the utility function and the constraints are continuous and bounded, and the utility function is strictly concave in the controls  $(c, L)$  and the constraints are linear in  $c$  and  $k$  and strictly concave in  $L$  and  $\tau$ , a Ramsey fiscal policy and an implementable optimal resource allocation exists. Further, since  $\bar{H}(c, k, L, \tau, \rho) \equiv \max_{c, k} H(c, k, L, \tau, \rho)$  is concave in  $c, k$  and  $L$  for given  $(\tau, \rho)$ , the necessary conditions, (18a)-(18g), are also sufficient for optimality.<sup>14</sup> Therefore, collecting arguments, we have:

**Proposition 2:** *Under the assumptions on the utility and production functions, there exists a Ramsey income taxation and an associated level of public production services, which implement a decentralized competitive allocation.*

Observe that equations (18a)-(18f) constitute a system of six equations in  $c, k, L, \tau, \rho$ . Following usual practice, we will reduce the dimensionality of this system

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alternative way of formulating the Ramsey policy problem would be to assume that the government chooses economic policy instruments only to maximize the consumer's indirect utility function.

<sup>13</sup> This condition guarantees that lifetime utility is bounded.

<sup>14</sup> This is based on Arrow's sufficiency theorem in the optimal control theory.

to facilitate analytical tractability. Define the consumption-to-capital ratio as  $z \equiv \frac{c}{k}$  and the auxiliary variable  $\bar{c} \equiv c - c^*$ . Then, by taking logarithms on both sides of (18a), differentiating with respect to time, and using (18b), (18c), (18d), (18e) and (18f), we get after some algebra:<sup>15</sup>

$$(19a) \quad \dot{z} = \left[ \left[ 1 - \frac{1}{\sigma} \right] \Delta(\bar{c}) L^{-1} + z - \rho \right] z;$$

$$(19b) \quad \dot{\bar{c}} = \rho - 1;$$

$$(19c) \quad \dot{\bar{c}} = \frac{\Delta(\bar{c})}{\Delta(\bar{c}^*)} \left[ \left[ 1 - \frac{1}{\sigma} \right] \Delta(\bar{c}) L^{-1} + z - \rho - \frac{1 + \sigma}{1 + \sigma} \left[ \frac{1}{L^{1+\sigma}} - 1 \right] \frac{1}{L} \right];$$

$$\text{where } \Delta(\bar{c}) = \frac{(1 - \frac{1}{\sigma}) \Delta(\bar{c})}{(1 - \frac{1}{\sigma})} \text{ and } L = \left[ -\frac{\Delta(\bar{c})}{z} \right]^{\frac{1}{1 + \sigma}}.$$

Therefore, the original six-dimensional system (18a)-(18f) in  $c, k, L, \tau, \bar{c}, \bar{k}$  has been reduced to the three-dimensional system (19a)-(19c) in  $z, \bar{c}, \bar{k}$ . The dynamics of the latter are equivalent to those of the former. The next subsection will study the properties of (19a)-(19c) in the steady state.

### ***Steady State***

This subsection analyzes the steady state of the Ramsey problem. A Balanced Growth Path (BGP) of (19a)-(19c) is defined to be a steady state in which: (a) The consumption-to-capital ratio,  $z \equiv \frac{c}{k}$ , is constant. Thus,  $\dot{z} \equiv 0$  in (19a). This means that  $c$  and  $k$  can grow at a common constant rate. (b) The tax rate,  $\tau$ , is constant. Thus,  $\dot{\tau} \equiv 0$  in (19c). (c) The auxiliary variable  $\bar{c}$  is constant. Thus,  $\dot{\bar{c}} \equiv 0$  in (19b).<sup>16</sup>

Then, we have:

<sup>15</sup> Details are available upon request.

<sup>16</sup> This steady state definition is consistent with the analysis of the DCE in the long run at the end of previous section.

**Proposition 3:** *Given Proposition 2: (a) With endogenous labor supply, public production services cannot generate long-run endogenous growth as in the standard Barro-type model. That is, with endogenous labor supply and Ramsey second-best fiscal policy, the economy cannot lie on its strictly positive balanced growth path. (b) The zero long-run growth rate is supported by multiple (two) second-best output tax rates.*

**Proof:** Let us denote the steady state values of  $(z, \tilde{c}, \tilde{k})$  by  $(\tilde{z}, \tilde{c}, \tilde{k})$ . To solve for  $(\tilde{z}, \tilde{c}, \tilde{k})$ , we start with (19a). Setting  $\dot{z} \equiv 0$ , (19a) implies that the long-run consumption-to-capital ratio is:

$$(20) \quad \tilde{z} = \rho - \left[1 - \frac{1}{\rho}\right] \Delta(\tilde{c}) L^{-1}.$$

which is always positive since  $\Delta(\tilde{c})$  is positive.

We continue with (19b). Setting  $\dot{\tilde{c}} \equiv 0$ , we simply get:

$$(21) \quad \tilde{c} = \frac{1}{\rho}.$$

Finally, consider (19c). Setting  $\dot{\tilde{k}} \equiv 0$  and using (20) and (21), we get:

$$(22) \quad \tilde{k} = \frac{1}{1+\rho},$$

Combining (20)-(22), it follows that  $\tilde{z} = \frac{\rho}{1+\rho}$  and  $\Delta(\tilde{c}) = \rho^{-\frac{1}{1+\rho}}$ . Then, from

(18d) and (18f), it follows  $\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{\dot{\tilde{k}}}{\tilde{k}} = 0$  in the steady state.

Finally, since  $\Delta(\tilde{c}) \equiv A^{-1} (1 - \tilde{c})^{\frac{1}{1+\rho}}$  is an inverse U-shaped function, there are two values of  $\tilde{c}$  satisfying  $\Delta(\tilde{c}) = \rho^{-\frac{1}{1+\rho}}$ .<sup>17</sup>

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<sup>17</sup> In this class of models, the optimal long-run capital tax rate is positive even in equilibria with commitment. This is simply because government spending is productive. When government spending is not productive, the optimal long-run tax rate on capital is zero (see e.g. Chamley [1986] and Judd [1985]).

Interestingly, unlike the case with exogenous policy (see Proposition 1 above), this result does not depend on the size of elasticity  $\frac{1}{\sigma}$  of intertemporal substitution. That is, as long as labor supply is endogenously determined, the Ramsey second-best fiscal policies lead to zero long-run growth regardless of the degree of intertemporal substitution for labor. However, the elasticity of intertemporal substitution for labor does affect the levels of output, consumption and labor (see e.g. equation (22)). These properties resemble those of a neo-classical growth model.

As we have explained in some detail in the Introduction, the intuition is clear. When labor supply is elastic and endogenously chosen, any increase in output requires more labor input, as well as more private capital and public production services. In this case, since leisure enters preferences, higher economic activity exerts ceteris paribus a negative effect on households' welfare. At the same time, in a decentralized setup, private agents have not internalized the positive externalities of public production services, and hence there is a wedge between the social and the private rate of capital return. To get the right quantity of public services, and hence increase the growth rate, the government has to resort to higher income taxes. In general equilibrium, the positive growth effect from government spending is fully offset by the negative growth effects from distorting taxes and the disutility from work effort.

To understand our result further, we also study the “social planner’s” problem. This is defined to be the benchmark case in which a social planner chooses fiscal policy (i.e. the income tax rate and the associated level of public services) subject to the economy’s constraints only. Thus, the crucial difference from the case above is that now optimizing policymakers do not face a wedge between social and private rates of return arising from decentralized private behavior [compare (15a) and (15b) in a DCE above]. Then, the following lemma shows that the growth rate can be strictly positive (under certain parameter values) even if labor supply is endogenously chosen. Thus, we have:

***Lemma 1:*** *Consider the case in which distortionary taxes are chosen by a social planner. Then, (a) Public production services can generate long-run endogenous growth even with endogenous labor supply. This happens when the parameter values*

satisfy  $0 < \left[ \frac{1}{\rho} \right] \left[ A(1-\tau)^{(1-\sigma)} \right]^{1+\sigma} < 2\rho$ . (b) There is a unique long-run

income tax rate, which is as in the standard Barro-type model, i.e.  $\tau = 1 - \frac{1}{2\rho}$ .

**Proof:** See Appendix.

Therefore, in the social planner's case, the tax rate can be designed in such a way that the distortions from savings and labor supply are completely eradicated. Also, in this case, maximizing growth rate is equivalent to welfare maximization. Moreover, unlike the second-best case, the social planner's tax rate is unique and constant over time; thus, there are no transitional dynamics. Therefore, this benchmark case recovers the properties of the basic Barro-type AK model. Note that Turnovsky [2000b] gets similar results, when he shows that "first-best" policy can yield positive long-run growth even if labor/leisure decisions are endogenously chosen.

Therefore, the assumption of Ramsey second-best fiscal policy is crucial to our result. Obviously, the assumption of elastic labor supply is equally crucial. For instance, Park and Philippopoulos [2000] have shown that under Ramsey second-best fiscal policy as in the present paper, public production services are capable of generating long run growth if labor is inelastically supplied. As was explained above, this is because inelastic labor supply does not introduce additional adverse welfare effects and can hence permit public services to play their growth-enhancing role.

## V. CONCLUSIONS AND EXTENSIONS

This paper has presented a standard endogenous growth model to reexamine the role of Ramsey (second-best) fiscal policy in the growth process. Fiscal policy took the form of public production services financed by output taxes. We showed that the long-run growth rate is zero once the labor supply becomes elastic. The result that public production services cannot play their traditional role as an engine of endogenous growth is somewhat surprising. Nevertheless, it happens when: (a) there is a wedge between social and private



rates of return arising from decentralized private behavior; (b) higher economic activity can come only at the cost of less leisure.

Note that our result is consistent with the general consensus in the growth literature. Namely, it is widely accepted that many things may fundamentally change when labor supply becomes endogenous and leisure enters preferences (see e.g. Jones and Manuelli [1997], de Hek [1998] and Turnovsky [2000a, b] who explicitly recognize the importance of labor supply endogeneity).<sup>18</sup>

We close the paper with three possible extensions. First, we could use the stock of public capital instead of the flow of public productive services. Second, it is interesting to consider the case in which the quality of labor/leisure can be improved.<sup>19</sup> Our feeling is that in this case the implementable growth rate could become positive. Third, we could add human capital, as in Lucas [1988], so that labor/leisure decisions are also affected by human capital accumulation.

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<sup>18</sup> Also, Benhabib and Perli [1994], Benhabib and Farmer [1994], Ladron-de-Guevara et al. [1999] and Ortigueira [2000] have shown indeterminacy in endogenous growth models with elastic labor supply. Cazzavillan [1996], Schmitt-Grohe and Uribe [1997] and de Hek [1998] have studied stability of a competitive equilibrium in models with government spending and leisure choices. In a neoclassical growth model, Lansing [1999] has examined the relation between Ramsey (second-best) redistributive taxation and long-run growth.

<sup>19</sup> Milesi-Ferretti and Roubini [1998] have considered optimal taxation with various leisure activities including the quality in leisure time or home-production.

## APPENDIX

**Proof of Lemma 1:** To solve the social planner's problem, we assume that the government chooses  $c, k, L, g$  and  $\tau$  to maximize (1)-(2) subject to: (a) the economy's resource constraint,  $\dot{k} = y - c - g$ , where  $y = Ag^{1-\alpha} k^\alpha L$ ; (b) its own budget constraint,  $g = \tau y$ .

By working as in the decentralized case above, the necessary conditions are

$$\dot{c} = c \left[ \frac{y}{k} - \rho \right], \quad \dot{k} = (1 - \tau)y - c, \quad L^{1+\alpha} = \frac{y}{c} \quad \text{and the transversality condition}$$

$$\dot{c} = c \left[ \frac{y}{k} - \rho \right] < \rho.$$

By manipulation of the above equations, it follows that the efficient output tax rate is  $\tau = 1 - \alpha$  over time. Also, we can easily get a condition which is sufficient for endogenous growth. To do so, let denote the BGP values of  $(c, k, L, \tau)$  by  $(\hat{c}, \hat{k}, \hat{L}, \hat{\tau})$ . Then, we have

$$\hat{z} \equiv \frac{\hat{c}}{\hat{k}} = \rho; \quad \text{and} \quad \hat{L} = \left[ \rho^{-1} A^{\frac{1}{1-\alpha}} (1 - \tau)^{\frac{1-\alpha}{1-\alpha}} \right]^{\frac{1}{1+\alpha}}.$$

Then, the efficient common rate at which long-run consumption and capital grow,  $\dot{c} = \dot{k} = c \left[ \frac{y}{k} - \rho \right] > 0$ , is positive if

$$\left[ \left[ \frac{c}{\rho} \right] \left[ A(1 - \tau)^{\alpha(1-\alpha)} \right]^{\frac{1}{1+\alpha}} - \rho > 0 \right]. \quad \text{This long-run growth rate is unique.}$$

## REFERENCES

- Arrow K. and M. Kurz [1970]: *Public Investment, the Rate of Return and Optimal Fiscal Policy*, Jones Hopkins University Press.
- Barro R. [1990]: Government spending in a simple model of economic growth, *Journal of Political Economy* 98, S103-S125.
- Barro R. and X. Sala-i-Martin [1995]: *Economic Growth*. McGraw Hill, New York.
- Baxter M. and R. King [1993]: Fiscal policy in general equilibrium, *American Economic Review* 83, 315-334.
- Benhabib J. and R. Farmer [1994]: Indeterminacy and increasing returns, *Journal of Economic Theory* 63, 19-41.
- Benhabib J. and R. Perli [1994]: Uniqueness and indeterminacy: on the dynamics of endogenous growth, *Journal of Economic Theory* 63, 113-142.
- Benhabib J., A. Rustichini and A. Velasco [1996]: Public capital and optimal taxes without commitment, Working Paper no. 96-19, *C.V. Starr Center*, New York University.
- Cazzavillan G. [1996]: Public spending, endogenous growth and endogenous fluctuations, *Journal of Economic Theory* 71, 394-415.
- Chamley C. [1986]: Optimal taxation of capital income in general equilibrium with infinite lives, *Econometrica* 54, 607-622.
- Chari V. V., L. Christiano, and P. Kehoe [1994]: Optimal fiscal policy in a business cycles model, *Journal of Political Economy* 102, 617-652.
- de Hek P. [1998]: An aggregative model of capital accumulation with leisure-dependent utility, *Journal of Economic Dynamics and Control* 23, 255-276.
- Futagami K., Y. Murata, and A. Shibata [1993]: Dynamic Analysis of an endogenous growth model with public capital, *Scandinavian Journal of Economics* 95, 607-625.
- Glomm G. and B. Ravikumar [1994]: Public investment in infrastructure in a simple growth model, *Journal of Economic Dynamics and Control* 18, 1173-1188.
- Glomm G. and B. Ravikumar [1997]: Productive government expenditures and long-run growth, *Journal of Economic Dynamics and Control* 21, 183-204.
- Guo J.T. and K.J. Lansing [1999]: Fiscal policy, increasing returns and endogenous fluctuations, Working Paper, no. 99-08, *Federal Reserve Bank of San Francisco*.

- Jones L. and R. Manuelli [1997]: The sources of growth, *Journal of Economic Dynamics and Control* 21, 75-114.
- Jones L., R. Manuelli and P. Rossi [1993]: Optimal taxation in models of endogenous growth, *Journal of Political Economy* 101, 485-517.
- Jones L., R. Manuelli and P. Rossi [1997]: On the optimal taxation of capital income, *Journal of Economic Theory* 73, 93-117.
- Judd K. [1985]: The welfare cost of factor taxation in a perfect foresight model, *Journal of Public Economics* 28, 59-83.
- Judd K. [1999]: Optimal taxation and spending in general competitive growth models, *Journal of Public Economics* 71, 1-26.
- Kneller R., M. Bleaney and N. Gemmell [1999]: Fiscal policy and growth: evidence from OECD countries, *Journal of Public Economics* 74, 171-190.
- Ladron-de-Guevara A., S. Ortigueira and M. Santos [1999]: A two-sector model of endogenous growth with leisure, *Review of Economic Studies*, 66, 609-631.
- Lansing K.J. [1998]: Optimal fiscal policy in a business cycles model with public capital, *Canadian Journal of Economics* 31, 337-364.
- Lansing K.J. [1999]: Optimal redistributive capital taxation in a neoclassical growth model, *Journal of Public Economics* 73, 423-453.
- Lucas R. [1988]: On the mechanics of development planning, *Journal of Monetary Economics* 22, 3-42.
- Lucas R. [1990]: Supply-side Economics: an analytical review, *Oxford Economic Papers* 42, 293-316.
- Lucas R. and N. Stokey [1983]: Optimal monetary and fiscal policy in an economy without capital, *Journal of Monetary Economics* 12, 55-93.
- Milesi-Ferretti G. and N. Roubini [1998]: On the taxation of human and physical capital in models of endogenous growth, *Journal of Public Economics* 70, 237-254.
- Ortigueira S. [1999]: A dynamic analysis of an endogenous growth model with leisure, *Economic Journal* 109, 43-62.
- Park H. and A. Philippopoulos [2000]: On the dynamics of growth and fiscal policy with redistributive transfers, *Working Paper*, Forthcoming in *Journal of Public Economics*.

- Rebelo S. [1991]: Long-run policy analysis and long-run growth, *Journal of Political Economy* 99, 500-521.
- Schmitt-Grohe S. and M. Uribe [1997]: Balanced budget rules, distortionary taxes and aggregate instability, *Journal of Political Economy* 105, 976-1000.
- Stiglitz J. [1987]: Pareto efficient and optimal taxation and the new welfare economics, in *Handbook of Public Economics*, vol. II, edited by A. Auerbach and M. Feldstein. North-Holland, Amsterdam.
- Shell K. [1967]: A model of incentive activity and capital accumulation, in *Essays on the Theory of Optimal Economic Growth*, edited by K. Shell. The MIT Press.
- Turnovsky S. [2000a]: Fiscal policy, elastic labor supply and endogenous growth, *Journal of Monetary Economics* 45, 185-210.
- Turnovsky S. [2000b]: The transitional dynamics of fiscal policy: long-run capital accumulation and growth, Mimeo, *University of Washington at Seattle*.
- Zhu X. [1992]: Optimal fiscal policy in a stochastic growth model, *Journal of Economic Theory* 58, 250-289.