A joint Initiative of Ludwig-Maximilians-Universität and Ifo Institute for Economic Research

CESifo Working Papers

ON INTERGENERATIONAL RISK SHARING WITHIN SOCIAL SECURITY SCHEMES

Andreas Wagener*

CESifo Working Paper No. 499

June 2001

Presented at CESifo Workshop on Public Pensions, May 2001

CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de

* Financial support by Deutsche Forschungsgemeinschaft (DFG) is gratefully acknowledged. I am indebted to Friedrich Breyer, Georges Casamatta, and Rüdiger Pethig for helpful comments.

CESifo *Working Paper No. 499*
June 2001

ON INTERGENERATIONAL RISK SHARING WITHIN SOCIAL SECURITY SCHEMES

Abstract

One of the main reasons to include pay-as-you-go (PAYG) schemes in multi-pillared pension systems is that they may entail beneficial risk-sharing and diversification features However, depending on the "pension formula" these features vary significantly for different types of PAYG schemes. We derive individually most-preferred PAYG rules (represented by a risk-sharing parameter) for young and old members of a society. These preferences depend among others on the correlation between the risks of PAYG scheme and funded schemes and on the trust in the durability of the pension rule. We find that the generations' interests with respect to the optimal PAYG policy need not necessarily clash, in particular not if future economic conditions are expected to be similar to today's. We discuss the implications of these findings for the political economy of multi-pillar pension systems.

Keywords: Social security, intergenerational risk sharing, pay-as-you-go pensions, majority voting.

JEL Classification: H55, D78.

Andreas Wagener
Universität Siegen
VWL IV, FB 5
Hölderlinstr. 3
57068 Siegen
Germany
Wagener@vwl.wiwi.uni-siegen.de

On Intergenerational Risk Sharing Within Social Security Schemes

1 Introduction

It is now widely acknowledged that pay-as-you-go (PAYG) social security schemes exhibit potentially beneficial effects in reallocating risks inter- and intragenerationally. These effects include risk-sharing (Merton, 1983; Enders and Lapan, 1993; Gale, 1991; Richter, 1993) and diversification features (Hauenschild, 1999; Dutta et al., 2000). Both effects may provide a rationale for including PAYG schemes into the pension mix although they yield lower (expected) rates of return than other forms of old-age provisions. More generally, risk sharing and diversification provide the major rationale for employing a multi-pillared pension system at all.
It is also well-known that various forms of PAYG schemes differ considerably in their risk allocation features; the "pension formula" is of crucial importance here (**?**Lindbeck, 2000; Thøgersen, 1998). This becomes most easily visible from the defining property of a PAYG scheme that, in every period, current pension payments are financed out of current contributions. Assume that the economy is subject to stochastic shocks which also affect the pension scheme (say, productivity shocks such that labour incomes vary randomly). Then there are, in principle, two ways of keeping the budget balance of the PAYG scheme intact: by adjusting benefits or by adjusting contributions (or, of course, by a combination thereof). From an *ex ante*-perspective these two policies change the perception of the PAYG scheme considerably:

- If it is the policy of the PAYG scheme to keep the contribution rate constant (fixed contribution [FC] scheme) then pensions depend on the future development of the economy and future economic risks have to a considerable extent to be borne by pensioners. Investing in the "PAYG asset" is thus risky. Also the link between own contributions (during the working periods of one's life) and pension benefits (during retirement) becomes rather weak. However, seen from an *ex ante* perspective the exposure to a risky PAYG pension is not unambiguously negative since it may contribute to the diversification of the risks of old-age consumption as a whole (when other stochastic sources of old-age income are available).

- Now suppose that it is the PAYG policy to keep benefits constant (say, in terms of the replacement ratiom measured in real terms). We then speak of a FR (fixed-replacement) scheme. In such a scheme contributions to the PAYG scheme during working age create a riskless entitlement to a pension of a certain size. The scheme thus entails a closer link between contributions and pensions. The pension as such is non-stochastic and the PAYG scheme thus provides insurance against risks of old-age consumption. Shifting away from the elderly the risks of adverse changes in the economic environment (as far as they affect the pension scheme) means entirely loading them upon the contributors. However, the contributors (ie., the younger generations) themselves might not find it very attractive to alone bear the exposure to these risks; they would rather like to see the risk shared with their parents. Conversely, if their parents' earnings are low, risk sharing in effect implies sharing the prospects for a brighter future – and selfish youngsters might not agree to that.

Pure FC and FR schemes have recently been discussed by Thøgersen (1998) and, in a more policy-oriented way, by Lindbeck (2000). In this paper we extend the analysis to allow for convex combinations of FR and FC schemes – which will be represented by a policy parameter $\gamma \in [0,1]$ that measures the degree of intergenerational risk sharing inherent in the PAYG pension policy. As mixtures of pure FC and FR schemes, "intermediate" policies $0 < \gamma < 1$ assemble a rather complex mix of risk sharing and diversification features.

Our particular interest here is in the preferences of individuals over the different PAYG pension mixes (formally, the shape of expected indirect utility as a function of γ). We demonstrate how these preferences vary with the position in the life cycle and with the economic situation at the date when preferences are solicited. A main finding is that, for a rather wide range of economic environments, the most preferred value of γ for both older and younger individuals lies in the interior (and not at the extremes) of the unit interval. Ie., in that range everybody prefers a mixed PAYG scheme to a pure FC or FR scheme. It is particularly interesting to observe how individual preferences towards risk sharing within the PAYG scheme vary with the stochastic properties of the returns on savings which may be interpreted as the second, funded element of a diversified, multi-pillar pension scheme. Roughly, if the returns to the two pillars are positively (negatively) correlated, individuals wish to have a PAYG mix with an intermediate value of γ if their expectations on the future wage rate (which determines the rate of return in the PAYG scheme) are rather optimistic (pessimistic). Furthermore, the preferences of the younger generations for intergenerational risk sharing are also affected by the expectations the young hold on the "durability" of the scheme, ie., they depend on whether the young expect the pension formula to be changed again within their lifetime.

Soliciting the preferences of individuals for intergenerational risk sharing and diversification in PAYG schemes is necessary, or at least helpful, for designing socially optimal PAYG schemes (although we do not touch this point here). Moreover, knowledge of individual preferences may help to explain the actual performance of PAYG schemes and policies if one regards them as the outcome of some political process into which individual (voter) preferences enter as inputs. In so far our findings may provide an explanation for, or are at least consistent with, the empirical observation that real-world pension schemes typically are of a mixed, not of a pure type.

Our motivation to investigate a continuum of mixed PAYG pension policies and individual preferences over that continuum is threefold.

- First, mixed schemes are the empirically dominant form of pension schemes. Pure FC or FR PAYG schemes do not exist in reality. Pension formulae and actual policies typically combine elements of both schemes, often in a way that is difficult to disentangle.A piece of evidence is given in Figure 1 which depicts the contribution and replacement ratios[1] for the German statutory PAYG scheme (*Gesetzliche Rentenversicherung*, GRV) over the last decades.

  Figure 1 goes here.

  With a FR scheme the upper curves should be constant in Figure 1, while for a FC scheme the lower curve should be flat. Since obviously neither of the curves is constant, the German PAYG scheme is not of a pure type: Both pensioners and contributors face variations in the pension parameters that affect them.[2]

  Furthermore, many aspects of the recent policy debate on averting the old-age crisis in PAYG pension schemes can also be thought of in terms of FC/FR-mixtures. E.g., one of the guidelines for the recent German pension reform act[3] is to keep the contribution rate stable, rather than – as it has since long been the paradigm of the German pension policy – the consumption level of the elderly. Given the demographic forecasts for the next decades, this shift towards a FC-policy almost certainly implies a cut in future pensions. From a risk perspective, it also sends the message that pensions might become more volatile with respect to the future ups and downs of the economy. The tendency towards FC schemes is, however, not unabated. The widespread call for actuarially fairer PAYG schemes not

[1] Strictly speaking, the GRV statistics do not know any indicator called replacement ratio. The three upper curves in Figure 1 depict annual data for the pension level of a retiree who had average earnings during his working period, as a percentage of the average earnings of the currently working. This may *cum grano salis* be interpreted as the replacement ratio for the average earner at the beginning of his retirement period. Since pensions in the GRV are adapted annually, the replacement ratio of retirees (ie., their pension relative to their own previous earnings) varies annually, too. In the stylized OLG framework of this paper we necessarily have to abstract from this complexity. Similarly, we will treat the contribution rate as constant over the whole working period of an individual – which evidently is not the case from the lower curve in Figure 1.

[2] A superficial view on Figure 1 might suggest that the contribution rate of the German PAYG scheme (the bottom line) was rather constant over time. However, as already pointed out by Werding (1998) in a different context, the graph of Figure 1 is highly misleading here. In fact, the bottom line exhibits much greater volatility than the three upper plots; the coefficient of variation for the contribution rate is 0.12, while it only ranges between 0.064 and 0.07 for the top lines.

[3] While at the date of writing it is not yet entirely clear what the final version of this so-called *Altersvermögensgesetz* will look like in detail, the basic guidelines can be expected to remain unchanged in the course of parliamentary negotiations.

only involves less intragenerational redistribution, but by linking pensions more closely to contributions also means a shift towards a FR scheme.

At least partially, the distinction whether a PAYG scheme is more of the FC or the FR type is determined by the provisions for adjusting pensions to changes in the environment. In a number of countries, PAYG pensions adjust according to the price index, while in others they are linked to changes in current (net) wages. Finally, not few countries adopt mixtures of these procedures (for details see VDR, 1999; ISSA, 2000). From an *ex ante* perspective and measured in real terms, price-index related adjustment rules come close to a FR scheme: At the beginning of the retirement period the pension is fixed in real terms, and retirees will not participate in the ups and downs of the business cycle. This is different when pensions are – as it *de facto* happens in a FC scheme – indexed to current wages and, thus, are stochastic in real terms. The widely applied mixtures of price and wage indexation can (in our stylized framework) be depicted by intermediate values of γ.

- A second motivation for considering mixtures of FC and FR schemes comes from the interpretation of a PAYG scheme as an asset: At the price of contributions returns in the form of pensions can be earned (see, e.g., Gale, 1991). Pure FR and FC schemes then constitute two specific payment streams with different risk/return patterns. As an implication of the basic diversification theorems from finance theory individuals will often prefer a mixed portfolio of (PAYG) assets to "keeping all their eggs in one basket", ie., they might prefer intermediate values of γ to the polar cases. One might call this *intra-PAYG diversification* – as contrasted to the usual diversification of multi-pillared pension schemes into funded and non-funded pillars. One of the aims of this paper is to elucidate the preferences of individuals for intra-PAYG diversification or different degrees of intergenerational risk-sharing under various circumstances.

- Third, one might wish to explain the fluctuations observed in Figure 1 and similar graphs for other countries – or why none of the lines there is flat. As shown in Wagener (1999) pure FR and FC PAYG schemes are Pareto non-comparable from an *ex ante* perspective: Neither is unambiguously Pareto superior to the other.[4] This already might provide a normative rationalization of using a mixed scheme. Yet, non-comparability of the extreme cases does not provide any guarantee that one can single out a specific FC/FR-mix as the best one – and we will not try this here. Our focus is on individual preferences

[4]Three remarks on this: (i) Thøgersen (1998) has shown that FC schemes dominate FR schemes. This result is, however, due to the very special and debatable setting in that paper, and cannot be generalized. (ii) If the set of policy options only consists of the pure FC and the pure FR scheme, both choices are efficient since neither is dominated by the other. (iii) From an *ex post*-perspective, FR schemes dominate FC schemes with equal expected implicit rate of return.

over risk-sharing in PAYG schemes as an input to political decisions. While the policy process is generally not considered to be guided by Paretian welfare criteria alone, in a welfarist understanding collective choices (should) emerge from the aggregation of individual preferences by some mechanism (majority voting, say). One of the goals of this paper is therefore to show that individual preferences for intergenerational risk-sharing in PAYG schemes are such that the volatile curves in Figure 1 can indeed be explained as the outcome of politically aggregating these preferences.

This paper is organized as follows: Section 2 presents the framework of our analysis. The main Sections 3 and 4 derive the individual preferences over different PAYG-mixes. Preferences depend on the position in the life cycle (old/young) and on the perception of how long the pension policy will remain unchanged (which is only relevant for the young). The penultimate Section 5 discusses implications of our findings for policy-making on PAYG schemes. The final Section 6 concludes.

2 A Simple Model

We employ a model of a simple 2-OLG economy under uncertainty. We assume that all prices (respectively, their probability distributions) are exogenous, e.g., because the economy is a small, open one. Generation t ($t = 0, 1, \ldots$) consists of $N \in \mathbb{N}$ identical individuals each of whom lives for the two periods t (working age) and $t+1$ (old age).[5] During working age each individual inelastically supplies one unit of labour, while during old-age all individuals are retired and do not work. Members of generation t derive utility from consumption $\tilde{c}_t^1$ during youth and consumption $\tilde{c}_t^2$ in old-age which are both random from the viewpoint of generation t's date of birth. We indicate random variables by a tilde; the respective variable without a tilde denotes a realization. We write s_t, B_t, and P_t to denote, respectively, generation t's per-capita savings, their social security contribution and their pensions (payable at date $t+1$). The variables w_t and R_t denote, respectively, the wage and the interest rate prevailing in t. The budget constraints for the two periods in the life of a member of generation t are:

$$\tilde{c}_t^1 = \tilde{w}_t - \tilde{B}_t - \tilde{s}_t \quad \text{and} \quad \tilde{c}_t^2 = \tilde{R}_{t+1} \cdot \tilde{s}_t + \tilde{P}_t. \tag{1}$$

Individual preferences with respect to the distribution of consumption over time and states are assumed to be representable by an additively separable von-Neumann/Morgenstern index

$$U_t = \mathbf{E}[u(\tilde{c}_t^1) + \hat{u}(\tilde{c}_t^2)], \tag{2}$$

[5]The assumption that N is constant over time is inessential in all that follows. One could easily incorporate non-stochastic population change rates in the analysis. It should be noted, however, that including stochastic population growth rates may affect the analysis.

where $\mathbf{E}$ denotes the expectations operator. We will make the stochastics explicit presently. All individuals are identical. They are supposed to be risk averse and to have decreasing absolute risk aversion (DARA). Their utility functions $\nu = u, \hat{u}$ thus possess the usual properties: They are smooth, strictly increasing and strictly concave ($\nu'(c) > 0 > \nu''(c)$ for all c) and satisfy

$$\frac{\partial}{\partial c}\left(-\frac{\nu''(c)}{\nu'(c)}\right) < 0 \quad \text{for all} \quad c > 0. \tag{3}$$

Define

$$\mathcal{R}_{\nu}(c) := -c \cdot \frac{\nu''(c)}{\nu'(c)} \tag{4}$$

as the Arrow-Pratt measure of relative aversion for ν. We will have to resort to this concept if we want to obtain at informative comparative static results in what follows. $\mathcal{R}$ has been used by Rothschild and Stiglitz (1971), Merton (1982, p. 613) and in particular by Cheng et al. (1987) to obtain at unique characterizations of optimal reactions to (price) changes in stochastic environments. Roughly speaking, $\mathcal{R}(c)$ measures the proportion of the actuarial value of a risky prospect that the agent is willing to pay as a risk premium. It also measures the response of this risk premium to an increase in the scale of a risk.[6]

Stochastics take their origin in price (ie., wage and interest rate) uncertainties. We assume that the wage rates at different points in time are independently distributed (no autocorrelation). Furthermore, the interest factor in t and the wage rate in any other period are stochastically independent. The interest and the wage rate at the same point in time may, however, be dependent random variables. Hence, for all $t \neq \tau$:

$$\mathrm{Cov}(\tilde{w}_t, \tilde{w}_\tau) = \mathrm{Cov}(\tilde{w}_t, \tilde{R}_\tau) = 0.$$

We will use the notation $\mathbf{E}_t$ to indicate the expectation for random variables x_t with time index t. As serial stochastic dependence is excluded there is no need to specify information sets upon which the expectations are formed. We will denote the support of interest rates by $\mathbf{R}$ and that of wage rates by $\mathbf{W}$; both $\mathbf{R}$ and $\mathbf{W}$ are assumed to be intervals of the positive real line. There is no loss in the generality of the insights of this paper to assume that $\mathbf{R}$ and $\mathbf{W}$ are time-independent.

We only consider pay-as-you-go (PAYG) pension schemes. Hence, for all t:

$$\tilde{B}_{t+1} = \tilde{P}_t \quad \text{a.s.} \tag{5}$$

[6] Given a stochastic variable $\tilde{x}$, define ρ as the proportion of the expected returns $\mathbf{E}\tilde{x}$ that an agent is willing to give up to get rid of the risk: $\nu((1-\rho)\mathbf{E}\tilde{x}) = \mathbf{E}\nu(\tilde{x})$. Then, by the standard argument (Pratt, 1964; Kimball, 1990) we get for small risks: $\rho = 0.5\mathcal{R}_{\nu}(\mathbf{E}\tilde{x}) \cdot \dfrac{\mathrm{Var}(\tilde{x})}{\mathbf{E}^2\tilde{x}}$. $\mathcal{R}$ thus is a proxy for the proportion of $\mathbf{E}\tilde{x}$ that the agent is willing to give up as a risk premium. Further $\mathcal{R}$ gives (twice) the response of ρ to a scale shift of the distribution of $\tilde{x}$.

Pensions and contributions to the PAYG scheme are assumed to be wage-related. Precisely, the pension for a member of generation t (to be received in $t+1$) is given by:

$$\tilde{P}_t = \beta \cdot [(1-\gamma_t)\tilde{w}_t + \gamma_t \tilde{w}_{t+1}] \tag{6}$$

with $0 < \beta < 1$ and $0 \leq \gamma_t \leq 1$ for all t. We assume that β is constant throughout.[7] To clarify eq. (6) we discuss its extreme cases (Thøgersen, 1998):

- $\gamma_t = 0$: The PAYG pension of generation t is a constant share of their earnings during working age: $\tilde{P}_t = \beta \tilde{w}_t$. Thus β represents the replacement rate, ie. the ratio between the pension and the pensioner's previous earnings. From (5) we obtain that the contributions levied on those working must amount to $B_{t+1} = \beta w_t$; clearly they are a function of their elder's past earnings.

- $\gamma_t = 1$: The pension of generation t is a constant share of their children's gross earnings, which maybe understood as (a proxy for) the living standard prevailing among the working people in $t+1$: $\tilde{P}_t = \beta \tilde{w}_{t+1}$. Consequently, the contribution to the PAYG scheme amounts to $\tilde{B}_{t+1} = \beta \tilde{w}_{t+1}$; it is a constant share of current earnings such that β now may be interpreted as a contribution rate.

If $\gamma_t = 0$ for all t, then the PAYG scheme has a fixed replacement rate (for short: FR scheme) while for $\gamma = 1$ it works with a fixed contribution rate (FC scheme). For intermediate values $0 < \gamma_t < 1$ pensions P_t and contributions B_{t+1} in $t+1$ are (proportional to) a convex combination of today's and last period's wage rates $\tilde{w}_{t+1}$ and $\tilde{w}_t$. In what follows we will often interpret γ as a measure for intergenerational risk-sharing: It measures the extent to which pensions are affected by current wage shocks or, equivalently, the degree to which young individuals can shift their income risk to their parents. Note that if γ is fixed at a value other than zero or one over several periods then neither the replacement rate nor the contribution rate are constant. For generation t, the (implicit) contribution and the replacement rate can be calculated from B_t/w_t and P_t/w_t, respectively.

Individuals take all parameters, prices, and distributions as given. The pension scheme is not *a priori* fixed, but emerges as the result of a political process. Since we consider β as given the policy problem is one-dimensional and decisions are to be made only on the values of γ_t, ie., on the degrees of intergenerational risk-sharing in the course of time. Such decisions are made (at most) in every model period. The decision on γ_t, which directly affects the pension payments

[7] Papers that endogenize β are, to name but a few, Browning (1975), Hu (1982), Sjoblom (1985) or Casamatta et al. (2000). In the latter paper the pension rule looks similar to (6); it contains a parameter α (called Bismarckian factor) which measures the degree to which the pension scheme is redistributive.

of generation t and the contribution payments for generation $t+1$, is made at date $t+1$, but still before the wage and interest rates $\tilde{w}_{t+1}$ and $\tilde{R}_{t+1}$ have been realized.[8]

As an ingredient to political decision making in welfarist social choice approaches (which include the democratic ones) we need information on the citizens' most preferred values of the policy parameters. To gather this information is the aim of the following sections.

3 The Preferred Degree of Risk Sharing for the Retirees

Suppose we are at the end of period t. Generation t will retire at the beginning of period $t+1$ and afterwards Nature will draw the interest factor and the wage rate for that period. As a result of his past decision, each individual close to retirement holds some non-negative amount of savings s which is now beyond his control. Shortly before date $t+1$ members of generation t are thus only interested in

$$\mathbf{E}_{t+1}\hat{u}(\tilde{R}_{t+1}s+\tilde{P}_t)=\int_{\mathbf{R}}\int_{\mathbf{W}}\hat{u}(\tilde{R}_{t+1}s+\beta\cdot[(1-\gamma_t)w_t+\gamma_t\tilde{w}_{t+1}])f(\tilde{R}_{t+1},\tilde{w}_{t+1})\,\mathrm{d}\tilde{w}_{t+1}\mathrm{d}\tilde{R}_{t+1}. \quad (7)$$

where f is the bivariate density function of $\tilde{R}_{t+1}$ and $\tilde{w}_{t+1}$. Define

$$\gamma_t^o(w_t) := \arg\max_{0\leq\gamma\leq 1}\mathbf{E}_{t+1}\hat{u}(\tilde{R}_{t+1}s+\beta\cdot[(1-\gamma_t)w_t+\gamma_t\tilde{w}_{t+1}]) \quad (8)$$

as the most preferred degree of risk-sharing for generation t in its old age. By definition, γ_t^o is a function of generation t's previous earnings.

As (7) is still a double integral some of the more interesting properties of $\gamma_t^o(w_t)$ will depend on the nature of stochastic dependence between the interest factor $\tilde{R}_{t+1}$ and the future wage rate $\tilde{w}_{t+1}$. To capture these effects we employ a refined concept of stochastic dependence due to Lehmann (1966) which was used in the economics literature by Cheng and Magill (1985) and Magill and Nermuth (1986): Two random variables $\tilde{R}_{t+1}$ and $\tilde{w}_{t+1}$ are said to be *positively (negatively) dependent* if for all $(a,b)\in\mathbb{R}^2$

$$\mathrm{Prob}[\tilde{R}_{t+1}\leq a,\tilde{w}_{t+1}\leq b] \quad \geq (\leq) \quad \mathrm{Prob}[\tilde{R}_{t+1}\leq a]\cdot\mathrm{Prob}[\tilde{w}_{t+1}\leq b] \quad (9)$$

with strict inequality for some $(a,b)\in\mathbb{R}^2$. If there is equality of the two sides above for all $(a,b)\in\mathbb{R}^2$, then $\tilde{R}_{t+1}$ and $\tilde{w}_{t+1}$ are *independent.* Positively dependent random variables tend to "hang together".[9] It can be shown that, if $\tilde{R}_{t+1}$ and $\tilde{w}_{t+1}$ are positively (negatively) dependent and if g is a decreasing function, then $g(\tilde{R}_{t+1})$ and $\tilde{w}_{t+1}$ are negatively (positively) dependent (Lehmann, 1966, Lemma 1(iii)). We now formulate

[8]Otherwise, it would not make sense to speak of risk-sharing here. Rather we would end up with bang-bang solutions for the generations' most preferred values of γ_t. *Mutatis mutandis*, all results obtained for political choices on β could be immediately applied here such that our analysis would not have to offer anything new.

[9]Lehmann (1966) uses the term positive quadrant dependence rather than positive dependence only. The terminology here follows Magill and Nermuth (1986). An asymmetric, but more intuitive formulation of (9) can

Fact 1 *Suppose that $\hat{u}$ satisfies $\mathcal{R}_{\hat{u}}(c) \leq 1$ for all c.*

1. *There exist $\underline{w}_t$ and $\bar{w}_t$ with $\underline{w}_t < \bar{w}_t$ such that*

$$\gamma_t^o(w_t) \begin{cases} = 1 & \text{if} \quad w_t \leq \underline{w}_t \\ \in (0,1) & \text{if} \quad \underline{w}_t < w_t < \bar{w}_t \\ = 0 & \text{if} \quad w_t \geq \bar{w}_t. \end{cases}$$

Here, $\underline{w}_t < \mathbf{E}_{t+1}\tilde{w}_{t+1}$. Further, $\bar{w}_t < (>)[=]\mathbf{E}_{t+1}\tilde{w}_{t+1}$ if $\tilde{R}_{t+1}$ and $\tilde{w}_{t+1}$ are positively (negatively) dependent [independent].

2. *The function $\gamma_t^o(w_t)$ is strictly decreasing in w_t for all $\underline{w}_t < w_t < \bar{w}_t$.*

Proof: Differentiating (7) with respect to γ yields:

$$F(\gamma, w_t) := \beta \mathbf{E}_{t+1}\Big((\tilde{w}_{t+1} - w_t)\hat{u}'(\tilde{R}_{t+1}s + \beta \cdot [(1-\gamma_t)w_t + \gamma_t \tilde{w}_{t+1}])\Big). \qquad (10)$$

In an interior solution for $\gamma_t^o(w_t)$ we have $F(\gamma_t^o(w_t), w_t) = 0$.

- Consider the case that $\gamma = 0$. Then $F(0, w_t) = \beta \mathbf{E}_{t+1}\Big((\tilde{w}_{t+1} - w_t)g(\tilde{R}_{t+1})\Big)$ where we set $g(\tilde{R}_{t+1}) := \hat{u}'(\tilde{R}_{t+1}s + \beta w_t)$. The function g is decreasing since we assumed that savings are positive: $g'(\tilde{R}_{t+1}) = s\hat{u}'' < 0$. Hence, g and w_{t+1} are negatively (positively) dependent [independent] whenever $\tilde{R}_{t+1}$ and $\tilde{w}_{t+1}$ are positively (negatively) dependent [independent]. From Lehmann (1966, Lemma 3) it is known that if two random variables are positively (negatively) dependent [independent], their covariance is positive (negative) [zero]. Applying this result, we obtain

$$\beta^{-1}F(0, w_t) = \mathbf{E}_{t+1}((\tilde{w}_{t+1} - w_t)g(\tilde{R}_{t+1})) \; > (<)[=] \; \mathbf{E}_{t+1}(\tilde{w}_{t+1} - w_t) \cdot \mathbf{E}_{t+1}g(\tilde{R}_{t+1}) \qquad (11)$$

  if $\tilde{w}_{t+1}$ and $\tilde{R}_{t+1}$ are negatively (positively) dependent [independent]. If $\gamma_t^o = 0$ is supposed to hold (ie., the retirees' most preferred value of γ_t is zero) we must have $0 = F(0, w)$ and thus, due to $g > 0$,

$$w_t \; > (<)[=] \; \mathbf{E}_{t+1}\tilde{w}_{t+1}$$

  if $\tilde{w}_{t+1}$ and $\tilde{R}_{t+1}$ are negatively (positively) dependent [independent].

- If γ_t^o is an interior most-preferred value we have $F(\gamma_t^o, w_t) = 0$. By the implicit-function theorem,

$$\frac{\mathrm{d}\gamma_t^o(w_t)}{\mathrm{d}w_t} = -\frac{F_{w_t}(\gamma, w_t)}{F_\gamma(\gamma, w_t)}.$$

be obtained by using conditional probabilities. Then (9) is equivalent to the following set of conditions:

$$\text{Prob}[\tilde{R}_{t+1} \leq a | \tilde{w}_{t+1} \leq b] \; \geq \; (\leq) \, \text{Prob}[\tilde{R}_{t+1} \leq a] \quad \text{and} \quad \text{Prob}[\tilde{w}_{t+1} \leq a | \tilde{R}_{t+1} \leq b] \; \geq \; (\leq) \, \text{Prob}[\tilde{w}_{t+1} \leq a].$$

Here, $F_\gamma = \beta \mathbf{E}_{t+1}((\tilde{w}_{t+1} - w_t)^2 \hat{u}'') < 0$. Hence, γ_t^o is decreasing in w_t if and only if $F_{w_t} < 0$. Calculate that

$$\begin{aligned} F_{w_t}(\gamma_t, w_t) &= \beta \mathbf{E}_{t+1} \left(-\hat{u}'(\tilde{c}_t^2) + (\tilde{w}_{t+1} - w_t)\beta(1-\gamma)\hat{u}''(\tilde{c}_t^2) \right) \qquad (12) \\ &= \beta \int_{\mathbf{R}} \int_W \left(-\hat{u}'(\tilde{c}_t^2) + (\tilde{w}_{t+1} - w_t)\beta(1-\gamma)\hat{u}''(\tilde{c}_t^2) \right) f(\tilde{R}_{t+1}, \tilde{w}_{t+1}) \, \mathrm{d}\tilde{w}_{t+1} \mathrm{d}\tilde{R}_{t+1}. \end{aligned}$$

As $u' > 0$ this is clearly negative for $\gamma = 1$. However, one can show that under DARA the second term is strictly positive for $\gamma < 1$ (see the Appendix) and thus tends to offset the first. Yet verify that

$$\begin{aligned} F_{w_t}(\gamma_t, w_t) &< \beta \mathbf{E}_{t+1} \left(-\hat{u}'(\tilde{c}_t^2) - [\beta(1-\gamma)w_t + \beta\gamma\tilde{w}_{t+1}] \, \hat{u}''(\tilde{c}_t^2) \right) \\ &< \beta \mathbf{E}_{t+1} \left(-\hat{u}'(\tilde{c}_t^2) - \tilde{c}_t^2 \hat{u}''(\tilde{c}_t^2) \right) \\ &= \beta \mathbf{E}_{t+1} \left(-\hat{u}'(\tilde{c}_t^2) \cdot \left[1 - \mathcal{R}_{\hat{u}}(\tilde{c}_t^2) \right] \right) \end{aligned}$$

which is negative if $\mathcal{R}_{\hat{u}} \leq 1$ for all $\tilde{c}_t^2$.

Define the wage level $\bar{w}_t$ as that (smallest) one where $\gamma_t^o = 0$; formally $\bar{w}_t$ satisfies $\gamma_t^o(\bar{w}_t) = 0 = F(\gamma_t^o(\bar{w}_t), \bar{w}_t)$. Further set $\underline{w}_t$ to be that (largest) value of w_t such that $\gamma_t^0 = 1$; formally $\underline{w}_t$ solves $\gamma_t^o(\underline{w}_t) = 1$ and $F(\gamma_t^o(\underline{w}_t), \underline{w}_t) = 0$. Given that F_{w_t} is strictly decreasing in w_t, both $\bar{w}_t$ and $\underline{w}_t$ are unique and $\bar{w}_t > \underline{w}_t$. Fact 1 then readily follows. ■

Fact 1 states that the most preferred value of γ_t for members of generation t shortly prior to their retirement is a (weakly) decreasing function of their earnings during working age. Moreover, there is some earnings level $\bar{w}_t$ beyond which these individuals prefer γ_t to be zero which corresponds to a FR scheme. Finally there exists a lower wage level $\underline{w}_t$ below which retirees always prefer γ_t to be one – which corresponds to the FC scheme.
At first glance these results seem intuitive: A positive value of γ_t means that generation t's pensions depend on the yet uncertain future wage rate $\tilde{w}_{t+1}$. Risk-averse individuals will find that unattractive *per se*, and they will even more do so the higher is their own wage level upon which their pension would otherwise be contingent. However, there are two obstacles that stand against this line of intuitive reasoning – and that in turn explain why Fact 1 looks a bit more complicated.

First, the argument that risk-averse members of generation t will find it *generally* unattractive to make their pension contingent on uncertain future incomes needs an important qualification. The argument entails that risk-averse retirees would never accept any lottery about their pension whose expected value is smaller or even below the certain pension they could ensure for $\gamma_t = 0$. This intuition is flawed; it overlooks that retirees already face some risk for their old-age consumption because the returns on their savings are stochastic. As Fact 1 states, this might

shift the limit value $\bar{w}_t$ (above which a positive value of γ_t is never welcome) *below* the expected value of future wages $\mathbf{E}_{t+1}\tilde{w}_{t+1}$. This will always be the case when future wages and the interest rate are negatively dependent. To understand this, recall that for $\gamma_t = 0$ the riskiness of old-age consumption is solely due to the return risk on savings while positive levels of γ_t add to this another risk via wages $\tilde{w}_{t+1}$. Such an additional risk will be welcome if it entails diversification possibilities – which happens if and only if the new stochastic returns are negatively correlated. Then generation t is willing to accept some additional risk even if it goes along with a loss in terms of expected return compared to the "safe return" they would earn for $\gamma_t = 0$. Of course, if future interest and wage rates are positively dependent an increase in γ_t will increase the exposure to the interest rate risk without offering any opportunities for diversification. Hence, retirees will at most accept such a higher risk exposure if they are compensated by a higher return, ie. for $\mathbf{E}_{t+1}\tilde{w}_{t+1} > w_t$.
Fact 1 reflects the potential diversification possibilities of multi-pillar pension schemes which recently have been elaborated by Hauenschild (1999), Dutta et al. (2000) or Wagener (1999). Diversification possibilities generally exist if the rates of return for two assets are negatively dependent; they imply that risk-averse individuals are also willing to include the low-return asset in their portfolios. This message is readily transferred to the present analysis: A negative correlation between wages and interest rates can make the exposure to additional risks of old-age consumption worthwile.

The second obstacle against a first-order intuition behind Fact 1 is related to the observation that, the higher their own wage rate w_t has been the more reluctant will members of generation t be to accept higher values of γ_t. Eq. (12) identifies two effects of an increase in w_t on γ_t^o that point into opposite directions. The first one (captured in $-\hat{u}'$) is always negative and may be termed a substitution effect: An increase in w_t makes the lottery over $\tilde{w}_{t+1}$ induced by γ_t more costly and thus less attractive. The second effect (captured by the expression containing $\hat{u}''$) may be termed the DARA effect: Higher wages w_t induce a higher propensity to risk-taking. This effect is unambiguously positive (as shown in the Appendix) and thus works counter the substitution effect. The assumption that relative risk aversion $\mathcal{R}_{\hat{u}}$ is less than one then amounts to preventing the DARA effect from dominating the substitution effect.
The assumption that relative risk aversion falls below unity is often made. In particular, Cheng et al. (1987) identify it as being equivalent to a series of "plausible" comparative statics results in various settings under uncertainty, ranging from portfolio choices over liquidity preferences to optimal firm behaviour with price uncertainty. In our framework $\mathcal{R}_{\hat{u}} \leq 1$ is merely a sufficient, not a necessary condition for Fact 1 to hold.[10]

[10](i) The necessary condition $F_{w_t} < 0$ in (12) does not seem to have any accessible equivalent in terms of preferences and/or underlying stochastics. (ii) A slightly weaker sufficient condition than $\mathcal{R}_{\hat{u}} \leq 1$ can be obtained

4 The Preferred Degree of Risk Sharing for the Young Generation

We now turn to the *ex ante* most preferred value of γ_t for generation $t+1$. The parameter γ_t (at least) determines that generation's contributions to the PAYG scheme. As before, we assume that political decisions on γ_t take place after the revelation of w_t, but before Nature draws $\tilde{w}_{t+1}$. *Ex ante*, the expected lifetime utility of members of generation $t+1$ amounts to:

$$U_{t+1} = \mathbf{E}_{t+1} \max_{s_{t+1}} \left\{ u(\tilde{w}_{t+1} - \beta((1-\gamma_t)w_t + \gamma_t \tilde{w}_{t+1}) - s_{t+1}) + \mathbf{E}_{t+2}\hat{u}(\tilde{R}_{t+2}s_{t+1} + \tilde{P}_{t+1}) \right\}, \quad (13)$$

where we take into account that generation $t+1$ makes its savings decision after $\tilde{w}_{t+1}$ is known. For their old age in $t+2$, generation $t+1$ must form expectations not only about interest and wage rates $\tilde{R}_{t+2}$ and $\tilde{w}_{t+2}$, but also about the pension parameter γ_{t+1} that will prevail then. For our purpose (namely, for retrieving generation $t+1$'s preferences for γ_t) effects on γ_{t+1} matter only in so far as they are caused by variations in γ_t. To capture these effects we assume that individuals perceive the future value γ_{t+1} to depend (in a deterministic way) on γ_t and, possibly, on the (realized value of the) future wage rate $\tilde{w}_{t+1}$:[11]

$$\gamma_{t+1} = \Gamma(\gamma_t, \tilde{w}_{t+1}). \quad (14)$$

(Of course, γ_{t+1} may also depend on other arguments; they do, however, not play any role for the analysis to follow).

In period $t+1$, the members of generation $t+1$ choose savings s as to maximize

$$u(w_{t+1} - B_{t+1} - s) + \mathbf{E}_{t+2}\hat{u}(\tilde{R}_{t+2}s + \tilde{P}_{t+1}) \quad (15)$$

Observe in (15) that the wage rate w_{t+1} and the pension contribution B_{t+1}, which is determined by γ_t, are (at the point in time in question) already given and non-stochastic. Furthermore, the pension parameter γ_{t+1} is known. Nevertheless the second-period expectation is a double integral since stochastics extend over the interest factor $\tilde{R}_{t+2}$ and the wage rate $\tilde{w}_{t+2}$, with the later affecting the pension payment $\tilde{P}_{t+1}$. Optimal savings are characterized by the FOC that the expected marginal utilities of consumption have to be equalized across time:

$$-u'(c^1_{t+1}) + \mathbf{E}_{t+2}(\tilde{R}_{t+2}\hat{u}'(c^2_{t+1})) = 0. \quad (16)$$

by help of the concept of *partial* relative risk aversion (Menezes and Hanson, 1970; Cheng et al., 1987). Given a utility function $\nu(c)$ where $c = Y + \tilde{x}$, partial relative risk aversion is defined by $\mathcal{P}_\nu(Y,\tilde{x}) := -\tilde{x}\cdot\nu''(Y+\tilde{x})/\nu'(Y+\tilde{x})$. The condition $\mathcal{R}_{\hat{u}} \leq 1$ in Fact 1 can then be replaced by $\mathcal{P}_{\hat{u}}(Y,\tilde{x}) \leq 1$ for all $Y = \tilde{R}_{t+1}s$ and $\tilde{x} = \beta(1-\gamma)w_t + \beta\gamma w_{t+1}$.

[11] An alternative approach would be to introduce stochastics over γ_{t+1} that are conditional on γ_t, ie., $H(x|\gamma_t) := \text{Prob}(\gamma_{t+1} \leq x|\gamma_t)$. Then changes in γ_t cause the stochastics of γ_{t+1} to change. The insights obtainable from such an approach seem, however, to be (even) more limited than with assumption (14).

For sake of later reference, it will be useful to consider the income effect on savings right here. Define first period-income of generation $t+1$ as

$$Y_{t+1} := \tilde{w}_{t+1} - \beta(\gamma_t \tilde{w}_{t+1} + (1-\gamma_t) w_t).$$

Implicit differentiation of (16) then yields that the marginal rate of savings is positive, but smaller than one:

$$s_Y := \frac{\partial s_{t+1}}{\partial Y_{t+1}} = \frac{u''(c^1_{t+1})}{u''(c^1_{t+1}) + \mathbf{E}_{t+2}(\tilde{R}^2_{t+2} \hat{u}''(c^2_{t+1}))} \in (0,1). \tag{17}$$

Generally, the assessment of γ_t by the younger generation can be expressed as:

$$\frac{\partial U_{t+1}}{\partial \gamma_t} = \mathbf{E}_{t+1} \left\{ \beta(w_t - \tilde{w}_{t+1}) u'(c^1_{t+1}) + \mathbf{E}_{t+2} \left(\frac{\partial \Gamma}{\partial \gamma_t} \cdot (\tilde{w}_{t+2} - \tilde{w}_{t+1}) \hat{u}'(c^2_{t+1}) \right) \right\}. \tag{18}$$

All effects of γ_t on savings cancel out in (18) by the envelope theorem.

4.1 Case I: Future Decisions are Independent of Today's

We start our analysis with the simplest case that today's decision on γ_t has (or is perceived to have) no effect on the value of γ_{t+1}. We denote by $\gamma_t^y(w_t)$ the most preferred value of γ_t for generation $t+1$ in that case; an interior value of $\gamma_t^y(w_t)$ thus solves (**??**) for $\partial\Gamma/\partial\gamma_t \equiv 0$. We obtain:

Fact 2 *Let γ_{t+1} be independent of γ_t. Suppose that $\mathcal{R}_u(c) \leq 1$ and that savings are convex in income: $s_{YY} \geq 0$.*

1. *There exist $\check{w}_t < \hat{w}_t < \mathbf{E}_{t+1}\tilde{w}_{t+1}$ such that*

$$\gamma_t^y(w_t) \begin{cases} = 0 & \text{if} \quad w_t \leq \check{w}_t \\ \in (0,1) & \text{if} \quad \check{w}_t < w_t < \hat{w}_t \\ = 1 & \text{if} \quad w_t \geq \hat{w}_t. \end{cases}$$

2. *The function $\gamma_t^y(w_t)$ is strictly increasing for $\check{w}_t < w_t < \hat{w}_t$.*

Proof: With γ_{t+1} independent of γ_t, the second term in (18) disappears. We thus only have to be concerned with the properties of $G(\gamma_t, w_t) := \mathbf{E}_{t+1}[(w_t - \tilde{w}_{t+1}) u'(c^1_{t+1})]$. If γ_t^y is an interior solution, we have $G(\gamma_t^y(w_t), w_t) = 0$.

- We first verify that $u'(c^1_{t+1})$ is a decreasing function of $\tilde{w}_{t+1}$:

$$\frac{\partial u'(c^1_{t+1})}{\partial \tilde{w}_{t+1}} = u''(c^1_{t+1}) \cdot \left((1-\beta\gamma_t) - \frac{\partial s_{t+1}}{\partial \tilde{w}_{t+1}} \right) \overset{(*)}{<} 0.$$

  Here the inequality (*) follows from (17): $\partial s_{t+1}/\partial \tilde{w}_{t+1} = (1-\beta\gamma_t) s_Y < (1-\beta\gamma_t)$.

Trivially, $(w_t - \tilde{w}_{t+1})$ is decreasing in $\tilde{w}_{t+1}$. Hence, we can apply CHEBYSHEV's Inequality to G to obtain that

$$G(\gamma_t, w_t) = \mathbf{E}_{t+1}[(w_t - \tilde{w}_{t+1})u'(c^1_{t+1})] > \mathbf{E}_{t+1}(w_t - \tilde{w}_{t+1}) \cdot \mathbf{E}_{t+1}u'(c^1_{t+1}).$$

Hence, for $w_t \geq \mathbf{E}_{t+1}\tilde{w}_{t+1}$ we have $G(\gamma_t, w_t) > 0$ for all γ. Thus, $\gamma^y_t = 1$ is the most preferred choice. In order for γ^y_t to be an interior solution we must have $0 = G(\gamma_t, w_t)$ or $w_t < \mathbf{E}_{t+1}\tilde{w}_{t+1}$.

- We now wish to show that $G(\gamma^y_t, w_t)$ is strictly increasing in w_t. Calculate:

$$G_{w_t} = \mathbf{E}_{t+1}\left[u'(c^1_{t+1}) - (1-\gamma_t)\beta(1-s_Y)(\tilde{w}_{t+1} - w_t)u''(c^1_{t+1})\right] \tag{19}$$

where we used (17). G_{w_t} comprises two effects with different signs: the first one being positive ($u' > 0$), the second one being unambiguously negative under the assumptions made in the proposition (see the Appendix). Yet,

$$\begin{aligned} G_{w_t} &> \mathbf{E}_{t+1}\left[u'(c^1_{t+1}) + (\tilde{w}_{t+1}(1-\beta\gamma_t) - \beta(1-\gamma_t)w_t)(1-s_Y)u''(c^1_{t+1})\right] \\ &\overset{(**)}{\geq} \mathbf{E}_{t+1}\left[u'(c^1_{t+1}) + (\tilde{w}_{t+1}(1-\beta\gamma_t) - \beta(1-\gamma_t)w_t)\frac{c^1_{t+1}}{Y_{t+1}}u''(c^1_{t+1})\right] \\ &= \mathbf{E}_{t+1}\left[u'(c^1_{t+1})(1-\mathcal{R}_u(c^1_{t+1}))\right] > 0. \end{aligned}$$

Here the first inequality follows from $u'' < 0$ and $\beta < 1$ while the final one comes from $\mathcal{R}_u < 1$. The equality in the third line holds by definition. The second inequality (**) comes from the convexity of the savings function or, equivalently, the concavity of c^1_{t+1} in Y_{t+1}:

$$1 - s_Y = \frac{\partial c^1_{t+1}}{\partial Y_{t+1}} \leq \frac{c^1_{t+1}}{Y_{t+1}}.$$

Now define the wage level $\hat{w}_t$ mentioned in the claim as that (smallest) one where $\gamma^y_t = 1$; formally $\hat{w}_t$ satisfies $\gamma^y_t(\hat{w}_t) = 1$ and $0 = G(\gamma^y_t(\hat{w}_t), \hat{w}_t)$. Further define $\check{w}_t$ as that (largest) level of w_t such that $G(0, w_t) \leq 0$. As $G_{w_t} < 0$, both $\hat{w}_t$ and $\check{w}_t$ are unique and $\check{w}_t < \hat{w}_t$. Fact 2 then readily follows from combining the two items above. ■

Fact 2 states that the young generation will only accept a value of γ_t below unity if their parents' earnings fall below a certain level $\hat{w}_t$ which itself is smaller than the expected value $\mathbf{E}_{t+1}\tilde{w}_{t+1}$ of generation $t+1$'s own earnings. The intuition is immediate: First, a large w_t means that the (non-stochastic) burden imposed on the young by a pension scheme with $\gamma_t < 1$ is the larger the lower is γ_t. Furthermore, the lower γ_t, the higher the exposure of generation $t+1$ to its own wage risk (recall that $\partial Y_{t+1}/\partial\tilde{w}_{t+1} = 1 - \beta\gamma_t$). Thus, by increasing γ_t the risk of the own income can be partially shifted to the elder generation whose high pension claims stemming from w_t will in turn be "devaluated" by higher values of γ_t. These two effects imply that $\gamma^y_t = 1$ always holds when w_t exceeds $\mathbf{E}_{t+1}w_{t+1}$. But even if w_t is up to a certain amount smaller than $\mathbf{E}_{t+1}w_{t+1}$, the most preferred value of γ_t for the young is one since there the risk reduction effect dominates the (now negative) income effect. This is simply an implication of risk aversion. If w_t is sufficiently smaller than $E_{t+1}\tilde{w}_{t+1}$ (precisely, if it is smaller than $\hat{w}_t$), the young will successively attach higher utility to the income effect (ie., leaving their parents with the low pension claim derived from w_t rather than having them participating in the – then relatively good – prospects of the own income) than to the risk shifting effect; the optimal value of γ_t^y now falls below unity. For small levels of w_t (below $\check{w}_t$) it may even reach zero.

Fact 2 further states that generation $t+1$'s desire to accede risks to their parents (ie., their preference for higher levels of γ_t) increases with their parents' earnings, at least in the range $(\check{w}_t, \hat{w}_t)$ where γ_t^y is an interior solution. This seems to be a standard implication of DARA: The larger w_t (and thus the larger is the inherited pension burden $\beta(1-\gamma_t)w_t$), the smaller is the disposable income of generatzion $t+1$ during working age. Given decreasing absolute risk aversion, this means that the reluctance to risk taking is greater and the willingness to expose oneself to (the own wage) risks is smaller the higher is w_t. Hence, the optimal level of γ_t^y increases.

While this property sounds natural, two caveats must be added. The first one is analogous to the one discussed in the previous section for the elder generation: An increase in w_t entails a (now) positive substitution and a negative DARA effect. The assumption $\mathcal{R}_u$ then ensures that the former dominates the latter. The second one involves the savings function: (Relative) Risk aversion is formulated in terms of consumption c_{t+1}^1, not in terms of income Y_{t+1}. The above reasoning is, however, based on an argument related to income, not on consumption (= disposable income minus saving). If we want the argument to be based on assumptions for $u(c^1)$ we must control for the fact that part of any reduction in incomes is compensated by a reduction in savings (see (17)). Via the convexity of the savings function we can ensure that the marginal reduction in consumption is always smaller than the average one. Hence, properties such as DARA or the magnitude of relative risk aversion $\mathcal{R}_u$ can also be related to disposable incomes, not only to consumption levels.

The convexity of the savings function or, equivalently, the concavity of the consumption function which we use in Fact 2 an often-made "Keynesian" assumption in macroeconomic models which also has empirical support (see Carroll and Kimball (1996) for further references). Of course, the curvature properties of savings or consumption functions are determined by the properties of the utility functions u and $\hat{u}$. Carroll and Kimball (1996, Theorem 1) provide a full characterization showing that the consumption function is concave (but not necessarily strictly so) if preferences belong to the HARA class – which is well compatible with relative risk aversion lying below unity. Convexity of s is a sufficient, but not a necessary assumption for the validity of Fact 2; if savings are concave in income (but not too much so) the youngs' most preferred level of γ_t

will still be increasing in their parents' earnings as long as $\mathcal{R}_u \ll 1$. Moreover, Fact 2 does not require that s_{t+1} be strictly convex (savings may be linear in income thus).

Figure 2 depicts the optimal values of γ_t from the perspectives of the young and the old generations, plotted as functions of w_t. The graphs can easily be derived from Facts 1 and 2.

Figure 2 goes here.

We will return to the implications of these preferences for political decision-making in Section 5.

4.2 Case II: No Revoting Opportunities

So far we have considered the case that the young assume that today's decision on γ_t has no implications for their own pension $\tilde{P}_{t+1}$. This assumption may seem appropriate, given the experience of generation $t+1$'s predecessors: Today's decision on γ_t involves that the value of γ_{t-1} (if such a value exists) has no significance for generation t's pensions $\tilde{P}_t$. If generation $t+1$ extrapolates this experience of its elders to their own old-age, the assumption $\partial\tilde{P}_{t+1}/\partial\gamma_t \equiv 0$ seems justified. One might, however, assume instead that today's decision γ_t is a "historical singularity" that will have validity forever or at least for the foreseeable future as it is relevant for those living today. Starting with the seminal paper by Browning (1975), this no-revoting assumption is made in many prominent models of voting on social security (Boadway and Wildasin, 1989; Casamatta et al., 2000; Veall, 1986, and others). In this section we therefore examine the most preferred risk-sharing parameter of the younger generation when they expect that today's decision to be still in effect when they will retire. Ie., they assume that

$$\gamma_{t+1} = \gamma_t \tag{20}$$

We will denote the most preferred value of the risk sharing parameter under assumption (20) by $\tilde{\gamma}_t^y = \tilde{\gamma}_t^y(w_t)$. Invoking (20), eq. (18) turns into:

$$\frac{\partial U_{t+1}}{\partial \gamma_t} = \beta \mathbf{E}_{t+1}\left\{(w_t - \tilde{w}_{t+1})u'(c_{t+1}^1) + \mathbf{E}_{t+2}\left((\tilde{w}_{t+2} - \tilde{w}_{t+1})\hat{u}'(\tilde{c}_{t+1}^2)\right)\right\}. \tag{21}$$

Hence, the assessment of γ_t by the young now differs from that in the previous section by the additional term

$$\Omega(\gamma_t) := \mathbf{E}_{t+1}\mathbf{E}_{t+2}\left\{(\tilde{w}_{t+2} - \tilde{w}_{t+1})\hat{u}'(\tilde{R}_{t+2}\tilde{s}_{t+1} + \beta(\gamma_t\tilde{w}_{t+2} + (1-\gamma_t)\tilde{w}_{t+1}))\right\}, \tag{22}$$

which contains stochastics for $\tilde{w}_{t+1}$, $\tilde{w}_{t+2}$ and $\tilde{R}_{t+2}$. Roughly speaking, if $\Omega > 0[< 0]$ then the function $\tilde{\gamma}_t^y(w_t)$ lies above [below] of $\gamma_t^y(w_t)$. The analysis of Ω is quite involved so that general results can hardly be obtained. Some partial results are available, however.

Fact 3 *1. Assume that $\tilde{w}_{t+2}$ and c^2_{t+1} are negatively dependent or independent. If $\mathbf{E}_{t+2}\tilde{w}_{t+2} \geq \mathbf{E}_{t+1}\tilde{w}_{t+1}$, then $\Omega(\gamma_t) > 0$.*

2. Assume that $\tilde{w}_{t+2}$ and c^2_{t+1} are positively dependent. Then $\Omega(\gamma_t) < 0$ if $\mathbf{E}_{t+2}\tilde{w}_{t+2} \leq \mathbf{E}_{t+1}\tilde{w}_{t+1}$ and if the covariance of $\tilde{w}_{t+1}$ and $\mathbf{E}_{t+2}\hat{u}'(c^2_{t+1}(\tilde{w}_{t+1}))$ is small.

3. (i) If $\tilde{w}_{t+2}$ and $\tilde{R}_{t+2}$ are independent, then $\tilde{w}_{t+2}$ and c^2_{t+1} are positively dependent for $\gamma_t > 0$ and independent for $\gamma_t = 0$.

(ii) If $\tilde{w}_{t+2}$ and $\tilde{R}_{t+2}$ are positively dependent, then $\tilde{w}_{t+2}$ and c^2_{t+1} are also positively dependent. The converse is not true.

Proof: Let $\gamma_t \in [0,1]$. Then:[12]

$$\begin{aligned}\Omega &= \mathbf{E}_{t+1}\mathbf{E}_{t+2}\left(\tilde{w}_{t+2}\hat{u}'(\tilde{c}^2_{t+1})\right) - \mathbf{E}_{t+1}\left(\tilde{w}_{t+1}\mathbf{E}_{t+2}\hat{u}'(\tilde{c}^2_{t+1})\right)\\ &(*)\ \ \mathbf{E}_{t+1}\left(\mathbf{E}_{t+2}\tilde{w}_{t+2}\mathbf{E}_{t+2}\hat{u}'(\tilde{c}^2_{t+1})\right) - \mathbf{E}_{t+1}\left(\tilde{w}_{t+1}\mathbf{E}_{t+2}\hat{u}'(\tilde{c}^2_{t+1})\right)\\ &= [\mathbf{E}_{t+2}\tilde{w}_{t+2} - \mathbf{E}_{t+1}\tilde{w}_{t+1}]\mathbf{E}_{t+1}\mathbf{E}_{t+2}\hat{u}'(\tilde{c}^2_{t+1}) - \mathrm{Cov}_{t+1}\left(\tilde{w}_{t+1}, \mathbf{E}_{t+2}\hat{u}'(c^2_{t+1})\right). \qquad (23)\end{aligned}$$

Here $\hat{u}'$ has to be evaluated at

$$c^2_{t+1} = \tilde{R}_{t+2}\tilde{s}_{t+1} + \beta((1-\gamma_t)\tilde{w}_{t+1} + \gamma_t\tilde{w}_{t+2}). \qquad (24)$$

The first equality in this chain is from Fubini's Theorem. The second line and its missing sign will be explained below. The third line emerges from the second by application of the covariance formula and collecting terms (note that from the viewpoint of $t+1$, the expectation $\mathbf{E}_{t+2}\hat{u}'$ is – as a function of $\tilde{w}_{t+1}$ – a random variable).

Verify that the covariance in the final line is negative. This follows from CHEBYSHEV's inequality since $\mathbf{E}_{t+2}\hat{u}'$ decreases with $\tilde{w}_{t+1}$. Namely, for all $\gamma_t \in [0,1]$:

$$\frac{\partial}{\partial\tilde{w}_{t+1}}\mathbf{E}_{t+2}\hat{u}'(c^2_{t+1}) = (1-\beta\gamma_t)s_Y\mathbf{E}_{t+2}(\tilde{R}_{t+2}\hat{u}''(c^2_{t+1})) + \beta(1-\gamma_t)\mathbf{E}\hat{u}''(c^2_{t+1}) < 0$$

Let us turn to the second line of (23) and its missing sign (*). Due to Lemma 3 in Lehmann (1966) this sign has to read $>$, $<$ or $=$ if, respectively, $\tilde{w}_{t+2}$ and c^2_{t+1} are negatively dependent, positively dependent or independent. (Note that $\hat{u}'$ is decreasing.) Using that the covariance in the final line has negative sign, the first two items in Fact 3 follow immediately.

Now recall (24). It is immediate that as wage rates are serially independent by assumption, any sort of dependence of c^2_{t+1} and $\tilde{w}_{t+2}$ can fully be attributed to dependence between $\tilde{w}_{t+2}$ and $\tilde{R}_{t+2}$. Furthermore, as c^2_{t+1} is a function of $\tilde{w}_{t+2}$ for all $\gamma_t > 0$, c^2_{t+1} and $\tilde{w}_{t+2}$ cannot be stochastically independent unless $\gamma_t = 0$ *and* $\tilde{R}_{t+2}$ and $\tilde{w}_{t+2}$ are independent.

[12]To avoid notational clutter we slightly abuse notation here when writing $\mathbf{E}_{t+2}$ both for the expectation over the (marginal) density of $\tilde{w}_{t+2}$ and over the joint density of $(\tilde{R}_{t+2}, \tilde{w}_{t+2})$. Whenever the expectations operator applies to arguments involving both $\tilde{R}_{t+2}$ and $\tilde{w}_{t+2}$ it should be thought of as a double integral, of course.

To show that (for $\gamma_t \neq 0$) independence of $\tilde{R}_{t+2}$ and $\tilde{w}_{t+2}$ implies positive dependence of c_{t+1}^2 and $\tilde{w}_{t+2}$ we combine Examples (iii) and (iv) in Lehmann (1966, p. 1139): If two random variables $\tilde{x}_1$ and $\tilde{x}_2$ are stochastically independent, then $\tilde{x}_1$ and $g(\tilde{x}_1 + \tilde{x}_2)$ are positively dependent whenever g is non-decreasing. For our case put $\tilde{x}_1 = \tilde{w}_{t+2}$, $\tilde{x}_2 = \tilde{R}_{t+2}$ and $g = c_{t+1}^2$ which is affine in $(\tilde{w}_{t+2}, \tilde{R}_{t+2})$. Similarly, one shows that positive dependence of $\tilde{R}_{t+2}$ and $\tilde{w}_{t+2}$ implies positive dependence of c_{t+1}^2 and $\tilde{w}_{t+2}$. ■

Fact 3 can best be understood by considering a young individual who believes that the γ_t chosen today will also have validity for his retirement age (and maybe beyond) as a person that combines the preferences of a young without such beliefs (Fact 2) and an individual close to retirement (Fact 1). The term Ω then captures the young individuals' anticipation of their old-age. It is closely related to the term F defined in (10). From Fact 1 we know that a pensioner who expects future wages to match today's will accept a positive level of γ if and only if the pension scheme offers diversification opportunities upon his savings. This will happen if future wages and interest rates are non-positively correlated. The first item of Fact 3 restates exactly this: If risk-sharing by the pension scheme contributes to diversification of his own old-age consumption (namely, if his consumption and his children's wage rate tend to vary inversely) a young individual will be ready to agree to a positive value of γ today – although he would not do so were there a revoting opportunity on γ.

Conversely, if increasing γ acerbates the risk exposure of old-age consumption (namely, if wages and interest rates in $t+2$ are positively dependent), then a young today is, *ceteris paribus*, more reluctant to accept positive values of γ than he would be under the assumptions of Fact 2 (ie., if γ were only to hold for a single period). This is stated in the second item of Fact 2. There is a counter-effect at work here, however, captured by the covariance term in (23).

The covariance term is – including the negative sign – always positive. It drives the young to accept higher values of γ_t, regardless of the stochastic relationships that prevail between the random variables. It may be interpreted as an intertemporal diversification effect.

For a special setting the following result compares the optimal values of γ_t^y for the case considered here and that of the previous section:

Fact 4 *Assume that* $\mathbf{E}_{t+1}\tilde{w}_{t+1} = \mathbf{E}_{t+2}\tilde{w}_{t+2}$, *that* $\tilde{w}_{t+2}$ *and* $\tilde{R}_{t+2}$ *are independent and suppose that* $\Omega(1) < 0$. *Let* $\breve{w}_t$ *and* $\hat{w}_t$ *be defined as in Fact 2. Then there exist* w_t' *and* w_t'' *with* $w_t' < \breve{w}_t < \hat{w}_t < w_t''$ *such that:*

$$\tilde{\gamma}_t^y(w_t) \begin{cases} = 0 & \text{if} \quad w_t \leq w_t' \\ \in (0,1) & \text{if} \quad w_t' < w_t < w_t'' \\ = 1 & \text{if} \quad w_t \geq w_t''. \end{cases}$$

The **proof** is straightforward from Fact 3. Figure 3 depicts the new situation.

Figure 3 goes here.

Compared to the most-preferred risk-sharing parameter γ_t^y of the previous section, the new curve $\tilde{\gamma}_t^y(w_t)$ is stretched and, on average, flattened. Contrary to Fact 2 we can no longer show that $\tilde{\gamma}_t^y$ is strictly monotonic in w_t without imposing rather restrictive conditions on preferences. Generally, there is thus no clear-cut relation between γ_t^y and $\tilde{\gamma}_t^y$. Hu (1982, Proposition 5) makes a related observation: In a model with voting on the contribution rate β an unambiguous relation between the most-preferred contribution rates with and without a revoting hypothesis does not exist.

Finally note that the overall assessment of γ_t under the no-revoting assumption (20) can be rewritten in terms of expected marginal utility during old-age only. Invoking (16) in (21) yields:

$$\frac{\partial U_{t+1}}{\partial \gamma_t} = \beta \mathbf{E}_{t+1} \mathbf{E}_{t+2} \left\{ \hat{u}(c_{t+1}^2) \left(-\tilde{R}_{t+2} w_t + (\tilde{R}_{t+2} - 1)\tilde{w}_{t+1} + \tilde{w}_{t+2} \right) \right\}. \tag{25}$$

This formulation explicitly shows how the PAYG pension scheme (with positive value of γ) affects consumption possibilities of a young member of generation $t+1$ via three different wage rates w_t, $\tilde{w}_{t+1}$ and $\tilde{w}_{t+2}$. The perspective is chosen to be date $t+2$ here, and the interest rate $\tilde{R}_{t+2} - 1$ allows to compare payments at different points in time. Eq. (25) once again highlights the difficulties in assessing changes in γ: It is a triple integral and encompasses stochastic dependencies even if all primitive random variables $\tilde{w}_\tau$ and $\tilde{R}_\tau$ are independent.

4.3 Case III: Dependent Choices

We now briefly consider the intermediate case that the decision on γ_t is thought to have implications for the future value γ_{t+1} that fall short of the no-revoting assumption of the previous section: $\gamma_{t+1} = \Gamma(\gamma_t, \tilde{w}_{t+1})$. Similar to the term Ω in (22) we can from (18) define a function

$$\hat{\Omega}(\gamma_t) := \mathbf{E}_{t+1} \left\{ \frac{\partial \Gamma}{\partial \gamma_t} \cdot \mathbf{E}_{t+2} \left((\tilde{w}_{t+2} - \tilde{w}_{t+1}) \hat{u}'(\tilde{R}_{t+2}\tilde{s}_{t+1} + \beta(\gamma_{t+1}\tilde{w}_{t+2} + (1-\gamma_{t+1})\tilde{w}_{t+1})) \right) \right\} \tag{26}$$

with the interpretation that, compared to the function γ_t^y defined in Fact 2, the youngs' most preferred value of γ_t weakly increases [decreases] if $\hat{\Omega}$ is negative [positive].

Let us first consider the case that γ_{t+1} is (perceived to be) independent of the realization of $\tilde{w}_{t+1}$; ie. $\partial\Gamma/\partial\tilde{w}_{t+1} \equiv 0$. Then $\hat{\Omega}$ equals

$$\hat{\Omega}(\gamma_t) = \frac{\partial \Gamma}{\partial \gamma_t} \cdot \mathbf{E}_{t+1} \mathbf{E}_{t+2} \left\{ (\tilde{w}_{t+2} - \tilde{w}_{t+1}) \hat{u}'(\tilde{R}_{t+2}\tilde{s}_{t+1} + \beta(\gamma_{t+1}\tilde{w}_{t+2} + (1-\gamma_{t+1})\tilde{w}_{t+1})) \right\}. \tag{27}$$

Here the double expectation is very similar to the original version of Ω. In particular, its sign can be analyzed with the same reasoning as in Fact 3. If individuals believe that γ_{t+1} increases with γ_t, then Facts 3 and 4 fully go through (with Ω being replaced by $\hat{\Omega}$). Ie., the range of w_t where the young individuals' most preferred value of γ_t is an intermediate one becomes larger as compared to the independence case of Fact 2; cf. Figure 3.

The assumption $\partial\Gamma/\partial\gamma_t > 0$ seems plausible in the sense that, to the extent that it prevails in the current period, γ_t also serves as a signal reflecting the preferences and outcomes of future decision processes. However, since the function Γ only reflects subjective beliefs the converse opinion might also be held, ie. individuals might believe that future values of γ vary negatively with today's values. Then all inequality signs referring to Ω in Fact 3 have to be reversed for $\hat{\Omega}$. In turn, Fact 4 has to be adapted, too: The function $\tilde{\gamma}_t^y(w_t)$ on average gets steeper (rather than flatter) than the original function $\gamma_t^y(w_t)$.
A crucial assumption in the reasoning above is that γ_{t+1} is held to be independent of $\tilde{w}_{t+1}$. Individuals in generation $t+1$ might, however, also believe that their wage income w_{t+1} systematically impacts on future decisions on γ. Unfortunately, allowing for non-zero partials of γ_{t+1} with respect to $\tilde{w}_{t+1}$ renders (26) entirely inaccessible. One easily verifies that the sign of $\hat{\Omega}$ not only depends on the signs and magnitudes of the first-order partials of Γ, but also those of the cross-partial $\partial^2\Gamma/(\partial\gamma_t\partial\tilde{w}_{t+1})$. As we are not able to justify any assumption on this second-order derivative, we refrain from a detailed analysis of that case.[13]

5 Implications for the Political Economy of Risk Sharing

Political decisions on γ (or any other issue) should — at least in a welfarist understanding — somehow account for the citizens' preferences. How this precisely happens depends on the social choice mechanisms at work. E.g. in the case of majority voting the median voter determines the outcome of the political process. In a 2-OLG model without intra-cohort differences the representative agent of the more populous generation is the median voter. In the outset of this paper we assumed that all generations are of equal size. In so far, a unique median voter equilibrium does not exist in our framework. However, it is an easy exercise to incorporate different population sizes into the analysis. As long as population changes are deterministic and fully foreseen, none of Facts 1 through 4 is affected. Hence in a model with different sizes of neighboring generations a unique majority equilibrium for voting on $\gamma_t(w_t)$ exists for every wage rate w_t. It has the property that if the young outweigh the old, political decisions on γ_t will follow the γ^y-curve (characterized in Facts 2 and 4) and the γ^o-curve otherwise (see Fact 1). These curves represent the (single-peaked) preferences of individuals in different positions in the life cycle. None of these preferences is of the bang-bang type: Between the ranges where the most preferred value of γ is an extreme one, there always exists an interval where intermediate

[13] In his analysis of voting over the contribution rate in social security schemes, Hu (1982) also uses an hypothesis on how individuals believe that future decisions will be affected by today's and by other primitives of his model. To obtain informative results, Hu also has to make assumptions on the signs and magnitudes of the first and the mixed partials of this belief function. In particular, he assumes that today's decisions positively affect future ones – which would correspond to the assumption $\partial\Gamma/\partial\gamma_t > 0$ in our framework. The assumptions on the mixed partials in Hu (1982) seem, however, rather *ad hoc* and debatable.

values are most welcome. Ie., even if we assume that political decisions are based upon majority voting our 2-OLG model can explain intermediate values for γ – and thus fluctuations both in the replacement rate and in the contribution rate. Even if the political decision procedure involves that one generation outvotes the other, there are large ranges of w_t such that the decisive generation chooses to implement some intergenerational risk sharing ($\gamma_t \in (0,1)$) rather than entirely loading the pension risk upon one of the two generations ($\gamma_t \in \{0,1\}$).[14]

The median-voter model lacks attractiveness in that one typically does not observe that policy choices evolve through one generation outvoting and fully exploiting the other (Veall, 1986; Breyer and Craig, 1997). It might therefore be more insightful to view policy-making as search for a reconciliation of the interests of the groups in a society. Let us start with the case that the young believe that today's decision on γ_t has no implications beyond today's contributions and pensions: $\partial\Gamma/\partial\gamma_t = 0$. Figure 2 then exhibits that for large values of last period's wage rate w_t (precisely, for $w_t > \max\{\bar{w}_t, \hat{w}_t\}$) the generations' interests are at full clash: the young want the largest degree of risk-sharing $\gamma_t = 1$, while the old prefer not to have any risk-sharing at all: $\gamma_t = 0$. A similar clash of interests occurs at very low levels of w_t, but now the roles of the generations have been interchanged. Between these two "clash zones" there is a range where both generations desire some, but no full risk-sharing; here both γ_t^o and γ_t^y are positive, but smaller than 1.

In all these interpretations a "small" or "large" magnitude of w_t is related to the expected future wage rate $\mathbf{E}_{t+1}\tilde{w}_{t+1}$. Changing the perspective, we can thus state: If generations expect a slump in wage rates ($\mathbf{E}_{t+1}\tilde{w}_{t+1} < w_t$) or if they expect a splendid boom ($\mathbf{E}_{t+1}\tilde{w}_{t+1} \gg w_t$) then their conflict of interest will be most severe. If they expect a moderate increase in wage levels (w_t is smaller than $\mathbf{E}_{t+1}\tilde{w}_{t+1}$, but not too much) their interests are more in harmony: Both generations prefer some intermediate degree of risk-sharing (but not necessarily the same one).[15]

Now turn to the case that the younger generation assumes that the pension parameter γ will stay in effect for longer than just for period $t+1$. As Figure 3 demonstrates under the assumptions made in Fact 4, the realm for which the interests of the generations are diametrically opposed now gets smaller as compared to the previous case (at least it will never expand). Now it is even conceivable that both generations wish to have interior values of γ_t in case they expect that future wages fall below today's.

To summarize, harmony of interests with respect to risk-sharing by the pension scheme is greater

[14] Recently, Anderberg (1999) analysed how the mix of public and private old-age income insurance can be determined by majority voting. He finds that when private schemes suffer from adverse selection typically a mix rather than an exclusive reliance upon one of the two methods emerges in the majority voting equilibrium – which would happen without the adverse selection problem. A similar effect can be observed in our model: If the wage rate uncertainty is removed, only bang-bang solutioms can emerge in a median voter equilibrium.

[15] In the intersection of γ_t^y and γ_t^o in Figure 2 there is of course *perfect* coincidence of interests. However, this is a non-generic event.

in slightly optimistic, “normal” circumstances where future incomes are expected to be moderately higher than today’s. Strong conflicts arise if the economy is at the brink of a decline or of exceptional growth.

6 Concluding Remarks

This paper started with the observation that in real-world PAYG pension schemes neither the contribution nor the replacement rate are constant over time. One possible interpretation for this is that PAYG policies involve some positive, but varying degree of intergenerational risk-sharing. The aim of the paper then was to elucidate the preferences that risk-averse individuals in different positions in the life cycle and under various initial conditions (represented by the “historical” wage rate) develop for risk-sharing through the pension scheme. We did so in a two-pillar pension scheme where individuals have access to private old-age provisions in the form of savings. Note, however, that our two-pillar pension model in fact includes three assets: The parameter γ determines the composition of the first, PAYG pillar, ie., the extent to which contributions to the PAYG scheme go along with a riskless or a risky pension. The preferences for the parameter γ of the PAYG scheme is then influenced by the return risk of the second pillar (savings) – which should not come as a surprise if one views old-age provisions as a portfolio problem, but which nevertheless add to the complexity of correctly assessing PAYG schemes.
We did not enter into the question whether individuals would like to have a PAYG scheme at all; the parameter $\beta > 0$ was assumed to be exogenous and invariable. Many advocates of radical changes in social security schemes argue that due to their low rates of return PAYG schemes should be abolished. They would argue that this paper misses the really important point, namely eliciting preferences for $\beta = 0$ or $\beta > 0$. Given that savings in our model earn stochastic returns, however, there is scope for PAYG social security here, even for schemes with relatively low expected rates of return (Gale, 1991; Hauenschild, 1999; Wagener, 1999): Rational individuals will therefore wish to include PAYG schemes in their old-age income mix for reasons of diversification and insurance. The main question of this paper then was to which extent these two motives enter individual preferences. The most important finding was that under “normal”, slightly optimistic expectations for the future in fact *both* motives play a role from *all* individuals’ points of view. Aggregating these individual preferences in the political process then might explain PAYG pension formulae that leave both the contribution and the replacement rate subject to fluctuations.

Appendix

Further Analysis of Equation (12)

We want to show that, given DARA, the second term in (12) is positive for all $\gamma < 1$. We are done when we can demonstrate that $\mathbf{E}_{t+1}((\tilde{w}_{t+1} - \tilde{w}_t)\hat{u}''(\cdot)) > 0$. To see this, we expand the expression under the expectations operator by $\hat{u}'$:

$$\begin{aligned}\mathbf{E}_{t+1}((\tilde{w}_{t+1} - w_t)\hat{u}'') &= \mathbf{E}_{t+1}\left((\tilde{w}_{t+1} - w_t)\hat{u}'\frac{\hat{u}''}{\hat{u}'}\right) \\ &> \mathbf{E}_{t+1}((\tilde{w}_{t+1} - w_t)\hat{u}') \cdot \mathbf{E}_{t+1}\frac{\hat{u}''}{\hat{u}'} = 0. \end{aligned} \tag{28}$$

The inequality in the second line can be established by a non-monotone version of CHEBYSHEV's inequality, reported in Mitrinovic et al. (1993, p. 248).[16] First note that from the DARA-assumption $\hat{u}''/\hat{u}'$ is increasing in $\tilde{w}_{t+1}$. Next verify that the value of $(\tilde{w}_{t+1} - w_t)u'$ is positive if and only if $\tilde{w}_{t+1} > w_t$; in terms of $\tilde{w}_{t+1}$ the function $(\tilde{w}_{t+1} - w_t)\hat{u}'$ thus changes its sign once from negative to positive. Hence, for all x in the support of $\tilde{w}_{t+1}$,

$$\int^x (\tilde{w}_{t+1} - w_t)\hat{u}' f(\cdot, \tilde{w}_{t+1})\mathrm{d}\tilde{w}_{t+1} < \mathbf{E}_{t+1}\left((\tilde{w}_{t+1} - w_t)\hat{u}'\right) = 0,$$

where h is the (marginal) density of $\tilde{w}_{t+1}$. The final equality follows from (10), evaluated at γ_t^o. Clearly, $\int^x f(\cdot, \tilde{w}_{t+1})\mathrm{d}\tilde{w}_{t+1} > 0$ for all x in the support of $\tilde{w}_{t+1}$. Hence, CHEBYSHEV's Inequality as presented in footnote 16 applies and we get:

$$\mathbf{E}_{t+1}((\tilde{w}_{t+1} - w_t)\hat{u}'') > \mathbf{E}_{t+1}\left(\frac{\hat{u}''}{\hat{u}'}\right) \cdot \mathbf{E}_{t+1}((\tilde{w}_{t+1} - w_t)\hat{u}') = 0, \tag{29}$$

where the final equality comes from (10), evaluated at γ_t^o.

Further Analysis of Equation (19)

We want to show that, given DARA and convexity of savings, the second term in (19) is negative. If s_Y were independent of $\tilde{w}_{t+1}$ we could apply the same token as in the previous appendix. However, s_Y is generally not constant. Inspection of the proof in the previous appendix yet shows that the non-monotone version of CHEBYSHEV's Inequality (cf. footnote 16) remains

[16] Formally, this version reads: Let $h_1, h_2 : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}$ be integrable functions of some (random) variable θ and be h_1 increasing. Furthermore, let $f : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}_+$ be an integrable function. If, for all $x \in (\underline{\theta}, \bar{\theta})$,

$$\frac{\int_{\underline{\theta}}^x h_2(\theta)f(\theta)\mathrm{d}\theta}{\int_{\underline{\theta}}^x f(\theta)\mathrm{d}\theta} \leq \frac{\int_{\underline{\theta}}^{\bar{\theta}} h_2(\theta)f(\theta)\mathrm{d}\theta}{\int_{\underline{\theta}}^{\bar{\theta}} f(\theta)\mathrm{d}\theta},$$

then: $\int_{\underline{\theta}}^{\bar{\theta}} f(\theta)\mathrm{d}\theta \int_{\underline{\theta}}^{\bar{\theta}} h_1(\theta)h_2(\theta)f(\theta)\mathrm{d}\theta \geq \int_{\underline{\theta}}^{\bar{\theta}} h_1(\theta)f(\theta)\mathrm{d}\theta \int_{\underline{\theta}}^{\bar{\theta}} h_2(\theta)f(\theta)\mathrm{d}\theta$. In our case, we put $\theta \equiv \tilde{w}_{t+1}$, $h_1(\tilde{w}_{t+1}) := \hat{u}''/\hat{u}'$, $h_2(\tilde{w}_{t+1}) := (\tilde{w}_{t+1} - w_t)\hat{u}'$, and $f(\theta) \equiv f(\cdot, \tilde{w}_{t+1})$ as the marginal density of $\tilde{w}_{t+1}$.

applicable if $(1-s_Y)u''(c^1_{t+1})/u'(c^1_{t+1})$ is increasing in $\tilde{w}_{t+1}$. Given that u exhibits DARA, a sufficient (yet not necessary) condition for this to hold is that $s_{YY} \geq 0$:

$$\frac{\partial}{\partial \tilde{w}_{t+1}} \left((1-s_Y) \frac{u''(c^1_{t+1})}{u'(c^1_{t+1})} \right) = (1-\gamma_t \beta) \left[-s_{YY} \frac{u''(c^1_{t+1})}{u'(c^1_{t+1})} + (1-s_Y)^2 \frac{\partial}{\partial c^1_{t+1}} \left(\frac{u''(c^1_{t+1})}{u'(c^1_{t+1})} \right) \right] > 0$$

since u''/u' is negative and increasing in c^1_{t+1}.

References

Anderberg, D., 1999. Determining the mix of public and private provision of insurance by majority rule. European Journal of Political Economy 15, 417–440.

Boadway, R., Wildasin, D. E., 1989. A median voter model of social security. International Economic Review 30, 307–328.

Breyer, F., Craig, B., 1997. Voting on social security: Evidence from OECD countries. European Journal of Political Economy 13, 705–724.

Browning, E., 1975. Why the social insurance budget is too large in a democracy. Economic Inquiry 13, 373–388.

Carroll, C. D., Kimball, M. S., 1996. On the concavity of the consumption function. Econometrica 64, 981–992.

Casamatta, G., Cremer, H., Pestieau, P., 2000. The political economy of social security. Scandinavian Journal of Economics 102, 503–522.

Cheng, H., Magill, M. J., 1985. Futures markets, production, and diversification of risk. Journal of Mathematical Analysis and Applications 107, 331–355.

Cheng, H.-C., Magill, M. J., Shafer, W. J., 1987. Some results on comparative statics under uncertainty. International Economic Review 28, 493–507.

Dutta, J., Kapur, S., Orszag, J. M., 2000. A portfolio approach to the optimal funding of pensions. Economics Letters 69, 201–206.

Enders, W., Lapan, H. E., 1993. A model of first and second-best social security programs. In: Felderer, B. (Ed.), Public Pension Economics. (Journal of Economics, Supplementum 7). Springer, Wien/New York, pp. 65–90.

Gale, D., 1991. The efficient design of public debt. In: Dornbusch, R., Draghi, M. (Eds.), Public Debt Management: Theory and History. Cambridge University Press, Cambridge, pp. 14–51.

Hauenschild, N., 1999. Uncertain incomes, pay-as-you-go-systems, and diversification. Zeitschrift für Wirtschafts- und Sozialwissenschaften 119, 491–507.

Hu, S. C., 1982. Social security, majority-voting equilibrium and dynamic efficiency. International Economic Review 23, 269–287.

ISSA, 2000. Social Security Programs Throughout the World. International Social Security Association, `http://www.issa.int/`.

Kimball, M. S., 1990. Precautionary saving in the small and in the large. Econometrica 58, 53–73.

Lehmann, E., 1966. Some concepts of dependence. Annals of Mathematics and Statistics 37, 1137–1153.

Lindbeck, A., 2000. Pensions and contemporary socioeconomic change. NBER Working Paper W7770, National Bureau of Economic Research, Cambridge.

Magill, M., Nermuth, M., 1986. On the qualitative properties of futures market equilibrium. Journal of Economics 46, 233–252.

Menezes, C. F., Hanson, D., 1970. On the theory of risk aversion. International Economic Review 11, 481–487.

Merton, R. S., 1982. On the microeconomic theory of investment under uncertainty. In: Arrow, K. J., Intriligator, M. D. (Eds.), Handbook of Mathematical Economics. Vol. II. North-Holland, Amsterdam, pp. 601–669.

Merton, R. S., 1983. On the role of social security as a means for efficient risk sharing in an economy where human capital is not tradable. In: Bodie, Z., Shoven, J. B. (Eds.), Financial Aspects of the United States Pension System. The University of Chicago Press, Chicago, pp. 325–358.

Mitrinovic, D. S., Pecaric, J., Fink, A., 1993. Classical and New Inequalities in Analysis. Kluwer Academic Publishers, Dordrecht etc.

Pratt, J. W., 1964. Risk aversion in the small and in the large. Econometrica 32, 122–136.

Richter, W. F., 1993. Intergenerational risk sharing and social security in an economy with land. In: Felderer, B. (Ed.), Public Pension Economics. (Journal of Economics, Supplementum 7). Springer, Wien/New York, pp. 91–103.

Rothschild, M., Stiglitz, J. E., 1971. Increasing risk II: Its economic consequences. Journal of Economic Theory 3, 66–84.

Sjoblom, K., 1985. Voting for social security. Public Choice 45, 225–240.

Thøgersen, Ø., 1998. A note on intergenerational risk sharing and the design of pay-as-you-go pension programs. Journal of Population Economics 11, 373–378.

VDR, 1999. Rentenversicherung im internationalen Vergleich. DRV-Schriften, Vol. 15. Verband Deutscher Rentenversicherungsträger, Frankfurt/Main.

VDR, 2001. Rentenversicherung in Zeitreihen. Verband Deutscher Rentenversicherungsträger, Frankfurt/Main, URL: http://www.vdr.de.

Veall, M., 1986. Public pensions as optimal social contracts. Journal of Public Economics 31, 237–251.

Wagener, A., 1999. Pensions as a portfolio problem: Fixed contribution rates vs. fixed replacement rates. Discussion paper, Dept. of Economics, University of Siegen.

Werding, M., 1998. Zur Rekonstruktion des Generationenvertrages. Mohr Siebeck, Tübingen.

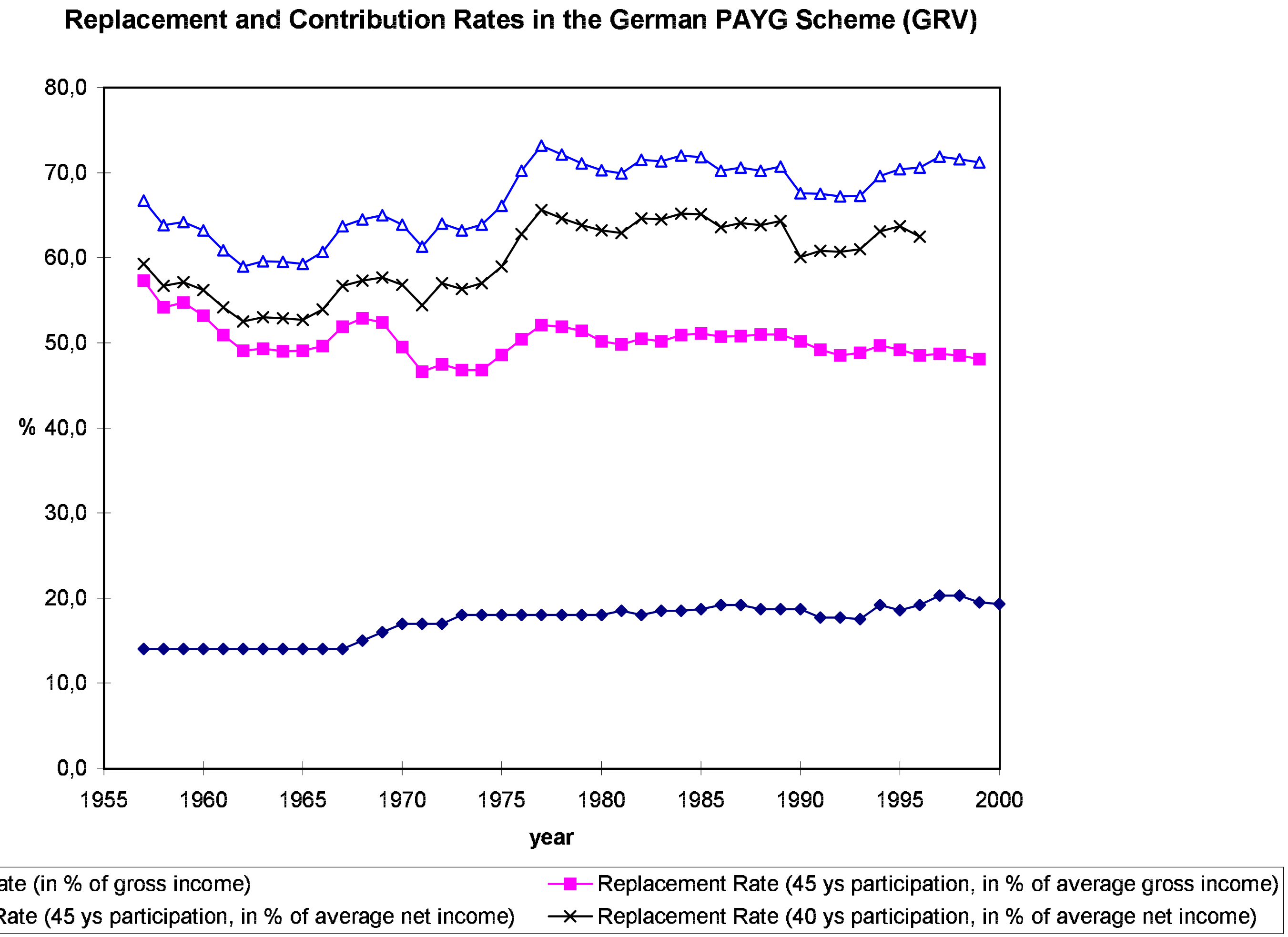
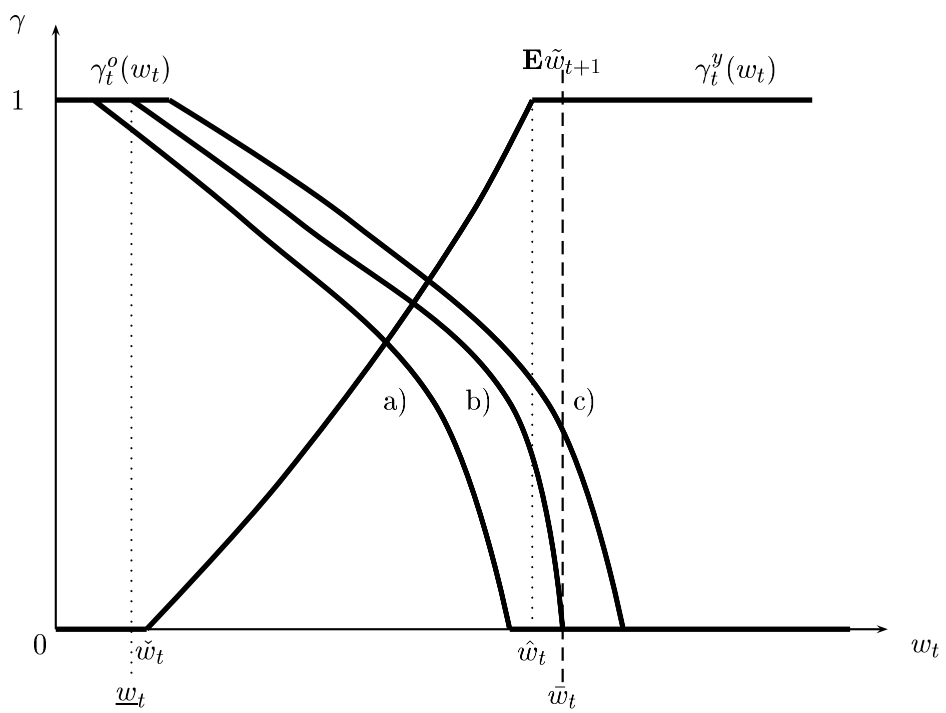


Figure 1: Contribution and Replacement Rates* of the GRV

Source: VDR (2001)

* For details see footnote 1.



$a)$, $b)$, $c)$: $\tilde{R}_{t+1}$ and $\tilde{w}_{t+1}$ are positively dependent, independent, negatively dependent (respectively)

Figure 2: Preferences for γ_t (independent decisions)

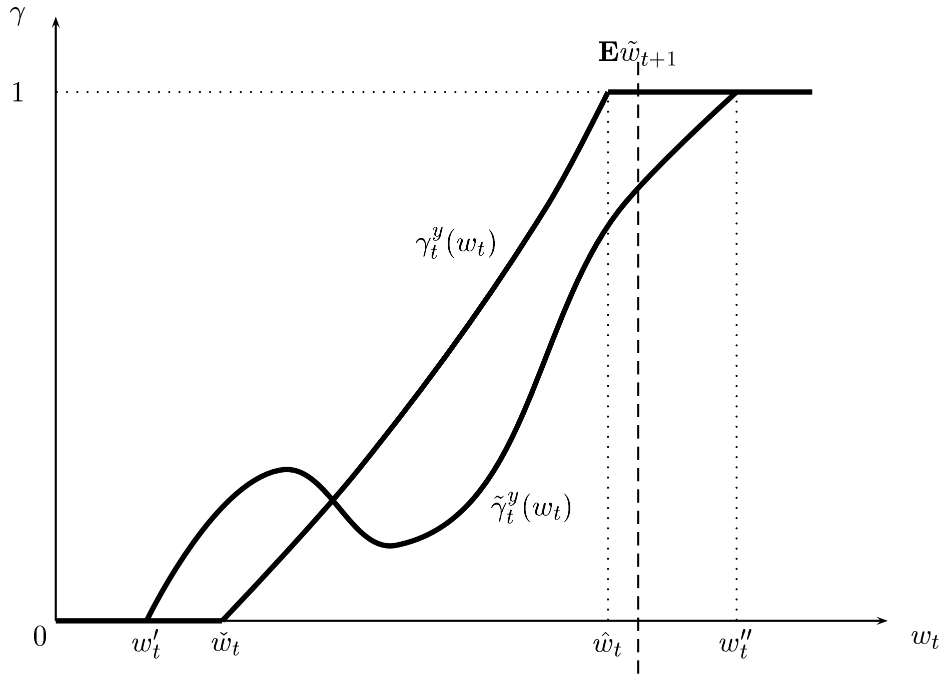


Figure 3: Preferences of the young: Independent decisions vs. No-Revoting