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CONTROLLING COMPLEXITY IN SPATIAL MODELLING

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Abstract

The present complexity approach is based on two assumptions: A1: measurability of deviations of outcomes with respect to reference values; A2 : extension of A1 to multi-set analysis. Complexity is then defined in terms of multi-set deviation compared to single-set ones; an interpretation is given in terms of information costs; examples show the relevance of the interpretation. As a useful by-product the explicit solution of the quadratic part of the discrete logistic – one of the examples – is derived; a set of p_{ij}-numbers is introduced, and a workable method for generating them exposed. Extensions are considered, in particular controllability. A further application is then proposed, namely to hypergraph conflict analysis, in particular conflict resolution. Many decisional conflicts at the spatial level can be axiomatised in this form; it is shown how the use of particular structures – in the mathematical sense of that word – of the problem allows of reducing greatly the degree of complexity of the problem, and hence the difficulty of finding a solution.

Keywords: Chaos, complexity, conflict, dynamics, hypergraphs, information

JEL Classification: R1

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1. Introduction.

The notion of complexity has, from its very nature, a large number of dimensions.

The first objective of the present paper is to try and start from an axiomatic set-up, and to derive an operational definition from there on. Applications – especially to (dynamical) regional science and to spatial conflict resolution - should show the relevance of the approach.

2. Complexity.

One will start with setting out the basic principles of the analysis.

2.1. Principles.

Consider a set O consisting of non-intersecting subsets O_i :

$$O \stackrel{\Delta}{=} \{ O_i \} , \quad i = 1,...,I \qquad (1)$$

Each of those subsets is structured by a certain number of relations, R_i, possibly overlapping between subsets :

$$S_i \stackrel{\Delta}{=} \{ O_i ; R_i \} \qquad (2)$$

Examples of R_is are : technological relations, economic relations, sociological relations, informational relations, topological relations,...

We now introduce :

Assumption 1 : each separate subset O_i can be modelled via its R_i such as to give outcomes on its elements, outcomes that can be characterised by a certain degree of deviation, δ_i, with respect to reference outcomes, e.g. the real world outcomes; a possible measure is the average residual variation coefficient, which, for a linear spatio-dynamic model with explanatory variables X_i, n in number, can be expressed as (Gajda, 1995) :

$$\delta_i \stackrel{\Delta}{=} (nT)^{-1} i' \hat{\mu}_i^{-1} \hat{\sigma}_i \{\Lambda_i^{-1}[X_i(X_i'X_i)^{-1}X_i'+I]\Lambda_i^{-1}{}'\}^{-1/2} i \qquad (3)$$

Here $\hat{\sigma}_i$ is a diagonal matrix of residual standard errors, T the length of the projection period, and Λ_i a suitable matrix of (temporal and/or spatial) lag coefficients; i is the unit column vector, and μ_I-hat a diagonal matrix of means used as "deflators" for the coefficient of variation.

One can now consider connecting different subsets with their respective relations, and this by introducing new "superrelations", this process generating the following structure:

$$S \overset{\Delta}{=} \{ O ; R_i ; R \} , \quad i = 1,...,I \tag{4}$$

We put forward :

Assumption 2 : the set O can be modelled in the same way as its subsets O_i; the outcomes can also be characterised by a global degree of deviation, δ, analogous to that introduced in assumption 1.

This allows of presenting the following :

Definition : there exists complexity iff :

$$\delta \gg \max_i \delta_i, \quad \forall i \tag{5}$$

Examples are numerous : the Krugman-Thurow (Krugman, 1996; Thurow, 1996) debate is one of them, chaotic behaviour is another; we will come back to this latter point in a more systematic way later on.

It should be remarked that by introducing probability or fuzziness structures one can refine the analysis; for instance (5) could be stated as :

$$\text{prob} \{ \delta \gg \max \delta_i , \forall i \} \geq \alpha \tag{6}$$

where α is a given probability threshold.

Another example is a combination of probability and fuzziness structures :

$$\text{prob} \{ y^p \varepsilon F(y) \} \leq \beta \tag{7}$$

meaning that the probability that projected values of y belong to the fuzzy environment of a given set of y's is equal to or smaller than β.

2.2. Interpretation.

In fact complexity has something to do with the cost of information, which is the unifying concept; we will illustrate this at the hand of some examples.

The Krugman-Thurow debate could be decided - at least theoretically, and partly in a practical way - by gathering sufficient information on the strategies of prominent players in the international competitive game.

Another example is that of the travelling salesman problem : the elementary sets are made up of at most only three nodes for which the solution is self-evident; connecting two or more of these subsets produces complexity in the NP-sense (it has been computed that for the 11 nodes case, one needs the mass of the universe to store the complete enumeration information...).

We now take the case of the "exact" random number generator :

$$r_i = \alpha\, r_{i-1} - \beta \;,\; 0 < r_i < 2, \forall i \qquad (8)$$

The separate terms of the right hand side can be modelled exactly (the first by $r_o(\alpha)^i$, the second by a constant); the result of "connecting" them by simple addition is unpredictable, except by iterative calculation, which represents the information cost.

A last example is that of the discrete logistic, often used in spatial analysis, written as :

$$x_i - x_{i-1} = \alpha\, x_{i-1} (1 - \beta\, x_{i-1}) \qquad (9a)$$

$$= \alpha\, x_{i-1} - \gamma\, x^2_{i-1} \qquad (9b)$$

Again the separate terms of the right hand member can be exactly modelled, the first one like the first one of equation (8), the second one as :

$$x_i = \gamma^{-1} \sum_{j=1}^{2^i} p_{ij} (\gamma\, x_0)^j \qquad (10)$$

where the p_{ij} are coefficients to be derived in section 2.3 hereafter. Connecting the linear and quadratic parts of (9) produces, as is well known, "complete" chaos starting from a critical value of α.

2.3. Deriving the quadratic part of (9).

Nothing is known to the author about an explicit solution of that quadratic part, so it is derived hereafter.

The equation is :

$$x_i - x_{i-1} = - \gamma x^2_{i-1} \qquad (11)$$

or

$$x_i = x_{i-1} (1 - \gamma x_{i-1}) \qquad (12)$$

The following properties hold :

1. $x_0 < 0 \Rightarrow 1 - \gamma x_i > 1$, $\forall i$, leading to a negative explosive process;

2. $x_0 = 0 \Rightarrow x_i = 0$, $\forall i$;

All properties hereafter assume $x_i > 0$:

3. $0 < 1 - \gamma x_0 < 1 \Rightarrow x_i \to 0$;

4. $1 - \gamma x_0 = 0 \Rightarrow x_i = 0$, $\forall i$;

5. $1 - \gamma x_0 < 0 \Rightarrow x_i \to -\infty$.

One sees that the process depends qualitatively on the initial values for given γ, whereas its linear analog only derives its sign from them.

The explicit solution to (11) or (12) is obtained from the recursive multiplication of :

$$x_i / x_{i-1} = 1 - \gamma x_{i-1} \qquad (13)$$

which gives :

$$x_i = x_0 (1 - \gamma x_0) (1 - \gamma x_1) .... (1 - \gamma x_{i-1}) \qquad (14)$$

Defining :

$$c \overset{\Delta}{=} \gamma x_0 \qquad (15)$$

one obtains again recursively :

$$\gamma x_1 = c (1 - c) \qquad (16)$$

$$\gamma x_2 = c (1 - c) [1 - c (1 - c)] \qquad (17a)$$

$$= c (1 - c) - c^2 (1 - c)^2 \qquad (17b)$$

which allows of computing, once more recursively, the p_{ij} coefficients of equation (10); here they are, for i = 1, 2, 3 and 4 (the derivation af the latter is detailed in the appendix, section 6) :

i = 1 : 1 , -1 ;

i = 2 : 1 , -2 , 2 , -1 ;

i = 3 : 1 , -3 , 6 , -9 , 10 , -8 , 4 , -1 ;

i = 4 : 1 , -4 , 12 , -30 , 64 , -118 , 188 , -258 , 302 , -298 , 244 , -162 , 84 , -32 , 8 , -1 .

One should notice the systematic sign alternation and additivity to zero.

The result, as equation (17b) clearly shows, is derived from an already computed part, and its square. The coefficients can be computed once and for all and stored, exactly like the eigenvalues of a transition matrix in linear difference equations.

2.4. Extensions.

Other aspects of (theoretical) spatial economics can be studied along the lines exposed above.

Will be mentioned only the case of bifurcations (see Paelinck, 1986; see also Hazewinkel et al., 1985) which do see to creep up regularly in spatial analysis; reference is made here to Kaashoek and Paelinck, 1994, 1996 and 1998, where potentialised partial differential equations are studied : due to the essential openness of spatio-economic units, a bifurcation parameter pops up quite naturally in the analytical solution, and simulations confirm its presence.
Recall that a partial differential equation in one space variable, x, and time, t, and for some function to be inferred, f(x,t), is a relation of the form :

$$h(x,t; f; f_x,f_t; f_{xx},f_{xt},f_{tt}; ...) = 0 \qquad (18)$$

where, in general, h is a given function of the variables x and t, of the function f, and of a finite number of its partial derivatives.

A potentialised version of the classical wave equation has the following form ;

$$\ddot{f}(x,t) = \alpha^2 \int_{-l}^{+l} w(x,\xi) f''(\xi,t) d\xi \qquad (19)$$

where $w(x,\xi)$ is a so-called "spatial discount function", its convolution with some variable representing the well-known "potential" in spatial economics.

A natural companion of spatial discounting is classical time-discounting; introducing both together gives rise to the type of chaotic "landscapes" reproduced by figure 1.

Despite this aspect of the solution, the latter is controllable, as shown in Kaashoek and Paelinck, 1998.

2.5. Conclusion.

Modern analysis of spatio-dynamic phenomena more often than not leads to complex dynamics, it being recalled that discrete dynamics is even richer in outcomes than continuous dynamics; the case of equations (9) and (10) illustrates this point. It is natural, then, to apply rigorous complexity analysis to get better insights into the relative positions of the problems encountered; the conflict resolution analysis to follow reinforces this conclusion.

3. Hypergraph conflict analysis.

Hypergraph Conflict Analysis (HCA) was introduced by Paelinck and Vossen, 1983. Since then a certain number of further general contributions have appeared (de Koster and Paelinck, 1984; Paelinck and Vossen, 1987; van Gastel, 1989; van Gastel and Paelinck, 1988, 1989, 1991a and b, 1992a), and also two studies on hypergraph conflict resolution (or reduction : de Koster and Paelinck, 1985, van Gastel and Paelinck, 1992b).

This section takes up the latter problem again, but in a different vein - the complexity one -, presenting some cases (binary, fuzzy, multidimensional) with their possible solutions, before coming to particular and general conclusions.

3.1.Principles.

Suppose there to be a number of agents, or groups of agents (A_i) confronted with a set of possible options (O_j), whatever the latter may be : in spatial analysis one might encounter infrastructural projects, regional development issues, and many other strategic choice problems; the agents could agree or disagree with some of the options, and this state of affairs can be set out in a table or matrix (C), table 1 reproducing a 3x3 binary (only full agreement or disagreement is hypothesised here) case.

TABLE 1 : matrix C

	Options	O_1	O_2	O_3
Agents				
A_1		1	0	0
A_2		0	1	0
A_3		0	0	1

The hypergraph nature of table 1 results from the fact that for each agent the agreeable options are a subset of the overall set of possible options.
Several measures can be proposed to show the “degree of conflict”. One, which will be noted δ, divides the minimum number of zeros taken over the columns of C by the number of agents (for table 1 this is 2/3). Another measure, noted τ, is the transversal number, defined as the cardinal of the minimal set of options on which all agents taken together agree; for table 1 this number is 3. A relative transversal number, τ^*, would divide τ by the cardinal of the set of potential options, so in the case of table 1, $\tau^* = 1$. It should be intuitively clear that δ and τ (or τ^*) are interrelated.

If $\delta = 0$, or alternatively, $\tau = 1$, there would be no conflict, as all agents agree on at least one option. Hypergraph conflict resolution aims at computing an optimal way of “turning over” agents, so as to drive δ down to zero and τ up to 1.

3.2. Hypergraph conflict resolution : binary case.

The core idea is that of minimising a (linear) “effort” function, taking into account the relative resistance a potential negotiator is supposed to meet with when trying to "turn over" some of the agents; that function looks as follows :

Δ

$$\text{Min } w'c = \varphi \qquad (20)$$

Vector w contains agent-option specific weights w_{ij}; it could be extended by interaction terms, but from what follows it will be clear that this would not change the further analysis and solution algorithm. c is a vector of agent-option specific actions to be undertaken, containing the same variable for all the zeros in each column of matrix C, so the weights in w are the sums of the column-relevant weights, $\Sigma_i w_{ij}$, $\forall$ j.

The conditions to be satisfied are than as follows :

$$i'c = 1 \qquad (21)$$

meaning that in one column the zeros should be replaced by ones, and :

$$\hat{c}c = c \qquad (22)$$

these being binary constraints. The analytical solution to (20)-(22) in terms of one of the c_js is then given by :

$$c_j = ½ + (2\mu_j)^{-1}(w_j - \lambda) \qquad (23)$$

in which $\lambda>0$ (dual variable pertaining to (21); strict positivity is due to the fact that one of the conditions (22) is redundant (but the value of λ depends on the redundancy chosen), and μ_j (one of the dual variables pertaining to (3)) is positive or negative according to the choice of λ and the optimal values of the c_js. In the case of table 1 groups to be "optimally" negotiated with are agents 1-2, 2-3, and 1-3; it is easily seen that, whatever the size of C, complete enumeration is algorithmically the implementation of (23), so the solution is clearly polynomial. The specific structure of the model allows for such a simple solution procedure (for other examples of such a case, see Paelinck, 1996; Paelinck and Kulkarni, 1999; Paelinck and Paelinck, 1998, 1999).

3.3. Fuzzy case.

Hypergraph conflict analysis has been extended to the fuzzy case, in which the binary entries of C have been replaced by entries over the closed interval [0,1]; in that case, it is also useful to consider agreement thresholds.

The approach of section 3 remains valid, under some slight modification of the objective function : the weights of function (20) are now to be multiplied by the distance that separates the actual agreement levels from their threshold values, at least for those entries where the former fall short of the latter; the solution is still equation (23) and the solution algorithm the one presented in section 3.2.

3.4. Fuzzy multidimensional case.

The solution of section 3.2 can again be easily extended in the same vein as was set out in section 3.3.

Indeed the only modification takes place in the objective function, in which the weights are now sets of weights (corresponding to the various dimensions of an option : economic, social, environmental,…), each of them to be applied to the distance separating actual agreement from the threshold value corresponding to a given dimension of an option.

In fact this suggests an alternative : changing the characteristics of certain dimensions of an option, so that all agents agree on the new option thus generated; practically this is the basis for finding a compromise, i.e. adding a new option to the initial set. It is known that such a move never increases a conflict, and might even reduce or solve it; however, an additional problem creeps up, that of the extra economic, social, environmental and other costs to be traded off against the “cost” of negotiating out one item of the initial set of available options.

3.5. Numerical examples.

We first treat a binary example, taken from table 1, table 2 showing the “degrees of effort”.

TABLE 2 : degrees of effort

	Options	O_1	O_2	O_3
Agents				
A_1			2	3
A_2		1		1
A_3		2	3	

Function (1) to be minimised under constraints (2) and (3) is then :

$$\varphi = 3\, c_1 + 5\, c_2 + 4\, c_3 \qquad (24)$$

the minimum being attained for $c_1 = 1$.

A fuzzy case, still based on table 1, is introduced by table 3, giving the threshold values for different agent-option combinations.

TABLE 3 : threshold values

	Options	O_1	O_2	O_3
Agents				
A_1			4	2
A_2		6		7
A_3		7	5	

Function (20) now becomes, for the same “degrees of effort” as in the binary case :

$$\varphi = 2\ c_1 + 2.3\ c_2 + .9\ c_3 \qquad (25)$$

which under constraints (21) and (22) is minimised for $c_3 = 1$.

Finally table 4 presents the threshold values for a two-dimensional case (economic and environmental aspects of a project, say), also derived from table 1, the “degrees of effort” being shown between parentheses.

TABLE 4 : two-dimensional threshold and effort values

	Options	O_1	O_2	O_3
Agents				
A_1			0.4(2) 0.2(1)	0.2(3) 0.4(5)
A_2		0.6(1) 0.4(4)		0.3(1) 0.6(4)
A_3		0.7(2) 0.5(3)	0.5(3) 0.1(1)	

Function (20) now becomes :

$$\varphi = 5.1\ c_1 + 2.7\ c_2 + 5.3\ c_3 \qquad (26)$$

which under conditions (21) and (22) is minimised for $c_2 = 1$.

3.6. Conclusion.

The practical problem that one faces is that of implementing the degrees of agreement and the threshold levels of the agents involved in a conflict.

The positive value of the approach just developed is that it clarifies the structure of the problem, reducing it to its bare essentials (for an example on the abortion problem in the Netherlands, see van Gastel, 1989, chapter 5) and showing ways to efficiently go about reducing partly or totally existing conflicts.

4. General conclusion.

The main analytical idea to be stressed in conclusion is the fact that special structural features of a complex problem represent a quantity of information that allows to reduce the complexity and so to lead to more feasible solutions of the problem. This has been especially illustrated by the choice problem which at its very start is a combinatorial one, but given the special features diagnosed, the solutions appeared to be much more easily accessible; other examples have been mentioned.

Complexity cannot be reduced to simplicity, but it can be approached in such a manner that the problem that embeds it becomes tractable, which is of the utmost importance for a whole range of spatial - regional and urban – questions.

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6. Appendix : a scheme for computing the p_{ij}s.

We will detail the case i = 4.

The linear part is the complete set of p_{ij}s for i = 3 (see section 3 above).

The quadratic part can be arranged in a square matrix which can be immediately derived from the linear part by using the rule for squaring a sum of terms; table 5 hereafter reproduces the result. Summing the matrix by south-west - north-east rows, changing signs (these totals have been reproduced in the margins of table 5) and adding to the linear part produces the result listed in section 3 for i = 4.

The procedure can easily be computerised.

Table 5.

-1	1	-6	12	-18	20	-16	8	-2
6		9	-36	54	-60	48	-24	6
-21			36	-108	120	-96	48	-12
54				81	-180	144	-72	18
-110					100	-160	80	-20
184						64	-64	16
-257							16	-8
302								1
		-298	244	-162	84	-32	8	-1

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