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ARE NASH TAX RATES TOO LOW OR TOO HIGH?
An Example of the Role of Economic Growth in Models with Public Goods

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ARE NASH TAX RATES TOO LOW OR TOO HIGH? AN EXAMPLE OF THE ROLE OF ECONOMIC GROWTH IN MODELS WITH PUBLIC GOODS

Abstract

We reconsider the conventional wisdom that, in the presence of public goods and distortionary taxation, Nash tax rates are inefficiently low due to free riding. We use a model in which the public good is natural resources. Specifically, a general equilibrium model of a world economy, in which there is long-term growth and world-wide environmental quality has public good features. We show that the type of the spillover effect from one country/player to another (and hence whether we under-tax, or over-tax, in a Nash equilibrium relative to a cooperative one) can be reversed when we introduce dynamics. In particular, the spillover effect changes from positive to negative once the same model allows for economic growth. This implies that, when the economy grows, Nash pollution tax rates are inefficiently high. This happens because in a growth model, medium- and long-run capital tax bases are elastic. In our AK growth model, the long-run effect on growth and tax bases more than outweighs any short-run free rider effects, and therefore Nash tax rates are too high.

Keywords: Public goods, externalities, economic growth

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I. INTRODUCTION

Recall two well-known results from the literature on public goods:[1] First, when decision-making is decentralized, tax rates - and the associated level of public good provision - decrease with the size of population. This is because when the number of participants increases, the incentive to free ride on the supply provided by others becomes stronger and so the willingness to pay taxes decreases. Then, the second result follows naturally: When we switch from decentralized (i.e. Nash) to centralized (i.e. cooperative) decision-making, tax rates increase. In other words, a coordinated increase in tax rates - and the associated higher level of public goods - is Pareto improving.

In terms of externalities in the game formulation, these results presuppose that the spillover effect from one agent to another is *positive*. That is, an increase in agents' j action leads to an external welfare benefit upon the remaining agent $i \neq j$. For instance, in the context of public goods, an increase in j's tax rate leads to higher tax revenues and higher public good provision for $i \neq j$. In turn, as is known from game theory, in the presence of positive spillovers, players' actions increase when we switch from Nash to cooperative equilibria (see e.g. Cooper and John [1988]).[2]

This paper shows that the type of the spillover effect (and hence whether we under-tax, or over-tax, in a Nash equilibrium relative to a cooperative one) can be reversed, when we introduce dynamics into a model with public goods. In particular, the nature of spillover effect *changes from positive to negative*, once the same model allows for economic growth. This implies that, contrary to conventional wisdom, Nash tax rates are inefficiently high in a dynamic setup. They also increase with the size of population. This happens because in growing economies, medium- and long-run capital tax bases are elastic.

[1] See e.g. Oakland [1985], Mueller [1989, chapter 2] and Mas-Colell et al. [1995, chapter 11].

[2] That is, in the presence of positive (resp. negative) spillover effects, players' actions increase (resp. decrease) when we switch from Nash to cooperative equilibria. This is because cooperation enables agents to internalize the positive (resp. negative) spillovers.

We focus on that category of public goods whose quality is damaged by economic activity, but can be improved by policy intervention.[3] That is, the government taxes the activity of agents who generate negative externalities, and then uses the collected tax revenues to maintain the public good. A classic example is the environment. We therefore use a model of endogenous growth, where the public good is environmental quality.[4] Also, since we want to compare non-cooperative and cooperative equilibria, we choose to work in the context of a world economy composed of a finite number of countries, which set their policies either by playing Nash or by coordinating vis-à-vis each other.[5]

The model is as follows. Consider an endogenous growth model of a world economy, where world-wide environmental quality is a public good. In each country, private agents consume, save in domestic capital[6] and produce goods by using a linear "AK" technology.[7] Production degrades environmental resources, but clean-up policy improves them. In particular, in each country, the government finances its clean-up policy by taxing domestic output. Then, world environmental resources (which enter the private agents' utility function as a public good) are the sum of national environmental

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[3] For various categories of public goods, which are generalizations of the pure public good concept, see e.g. Oakland [1985].

[4] Environmental quality (i.e. water, air and soil quality) has been one of the most popular examples of public goods, because of the obvious open-access character of natural resources (for growth and the environment, see e.g. John and Pecchenino [1994] and the survey paper by Kolstad and Krautkraemer [1993]). However, our results are more general. They can hold in a large class of growth models with public goods, whose provision is reduced by consumption or production externalities, but the government can contribute to their maintenance (e.g. environment, public facilities, roadways, national parks, health).

[5] Our setup is fairly general. For instance, we can use a closed economy, in which individuals act as consumers/produces and voters. In that framework, in a Nash equilibrium, voters choose their contribution to the provision of public good in a decentralized and voluntary way, while, in a cooperative equilibrium, a mechanism (e.g. the government) coordinates voters' actions. The results are conceptually the same.

[6] For simplicity, factors of production are internationally immobile.Thus, there is only one spillover effect from one player/country to another, and this arises from the public-good character of environmental quality. This allows us to focus on the main issue, which is the efficient management of public good provision. If factors were internationally mobile, players/countries would also compete with each other for mobile tax bases. This would add spillover effects, which would mitigate of exacerbate our results depending on their nature (i.e. whether these additional spillover effects are positive or negative). For instance, in the literature on international capital mobility and tax competition (see e.g. Keen and Kotsogiannis [2000] and the references cited there), the spillover effect is positive: an increase in country's j capital tax rate stimulates growth in country $i \neq j$. Then, tax rates are inefficiently low in a Nash equilibrium.

[7] We use the AK model of growth for simplicity. This model can generate long-term, endogenous growth. Also, it does not introduce any sources of market failure. On the other hand, it implies that the return to capital and hence the rate of growth are independent of the beginning-of-period capital stock. For the AK model, see e.g. Barro and Sala-i-Martin [1995, chapter 4].

resources. National governments are benevolent. They decide what pollution tax rate their citizens are going to pay for the public good and hence how much of the public good will be provided. In doing so, they either play Nash vis-à-vis each other, or they cooperate. We focus on symmetric equilibria in national policies. Within this framework, we solve for a long-run equilibrium in which the economy can grow at a constant positive rate without damaging the environment (this is known as sustainable development).

Here is our story. In a static setup, capital tax bases are exogenously given. Then, an increase in tax rates leads always to an increase in tax revenues and hence an increase in resources available for clean-up policy. This guarantees a *positive* spillover from one country/player to another. Namely, a rise in country's j tax rate leads to higher tax revenues and higher clean-up policy in j, and this implies an improvement in world-wide environmental quality and therefore an external benefit upon the remaining country $i \neq j$. Then, a positive spillover implies that the Nash tax rate is inefficiently low. Also, the tax rate decreases with the size of population. This is the standard free-rider result (i.e. there is too little environmental protection). When national governments coordinate, they choose higher tax rates relative to the case in which they do not coordinate.

In contrast, in a dynamic setup with economic growth, capital tax bases are not given. Now, although higher tax rates can lead to higher tax revenues and clean-up policy in the short-run, they discourage capital accumulation and economic growth in the medium- and long-run. In this case, the implications of economic growth generate a *negative* spillover from one country to another. Namely, a rise in country's j tax rate leads to lower economic growth and eventually smaller tax bases, lower tax revenues and lower clean-up policy in j, and this implies a worse world-wide environmental quality and therefore an external cost upon the remaining country $i \neq j$. In our AK model, in which the economy grows at a constant positive rate all the time, the long-run effect on growth and tax bases more than outweighs any short-run free-rider effects. The net spillover effect is negative and therefore the Nash tax rate is inefficiently high. It also increases with the size of population.

Consequently, in a dynamic setup with growth, the decentralized world economy will end up with too high tax rates (i.e. excessive environmental protection), relative to the case in which there is centralized decision-making. Intuitively, in the absence of coordination, players/countries do not internalize the harmful effect of their tax rates (and hence lower growth, smaller tax bases and lower clean-up policies at home) on the provision of the public good (here, world-wide environmental quality). They therefore set too high pollution tax rates. Lower tax rates are Pareto-improving, because they are good for both growth and the environment. This is because low taxes stimulate growth and generate tax bases broad enough to finance clean-up policies. The latter can improve the environment, despite the adverse effect of higher growth and pollution. Note that these results are consistent with the general view that the best way to e.g. improve the environment, lower poverty or promote equity on an enduring basis is to enhance prospects for sustained economic growth.

The rest of the paper is organized as follows. Section II presents the model and solves for a competitive equilibrium. Section III solves for uncoordinated policies. Section IV solves for coordinated policies. Section V solves a static version of the model. Section VI closes the paper. Mathematical proofs are in an Appendix.

II. COMPETITIVE EQUILIBRIUM

Consider a world economy composed of a finite number of countries, $i = 1, 2, ..., I$. Each country i is populated by a representative private agent and a national government.[8] We assume continuous time, infinite horizons and perfect foresight. This section solves for a world competitive equilibrium, given tax policy and the motion of public goods.

Private agents

The representative private agent in country i gets utility from consumption, c^i, and the average stock of natural resources across countries, $\overline{N}$. That is, $\overline{N} \equiv \frac{1}{I}\sum_{i=1}^{I} N^i$,

[8] Our results do not change if we use a decentralized setup with households and firms in each country. Working with a private agent saves on space.

where N^i denotes the stock of natural resources in country i.[9] Thus, the infinite-lived private agent in each country i maximizes intertemporal utility:

$$\int_0^{\infty} [u(c^i, \overline{N})] e^{-rt} dt \tag{1}$$

where the parameter $r > 0$ is the rate of time preference. The instantaneous utility function $u(.)$ is increasing and concave in its two arguments, and also satisfies the Inada conditions. For simplicity, we assume that $u(.)$ is additively separable and logarithmic:

$$u(c^i, \overline{N}) = \log c^i + n \log \overline{N} \tag{2}$$

where the parameter $n > 0$ is the weight given to environmental quality relative to private consumption.

In each country i, private agents produce goods by using an AK technology. Thus, in each i, output, y^i, is linear in capital, k^i:

$$y^i = Ak^i \tag{3}$$

where $A > 0$ is a parameter.

If $0 \leq q^i < 1$ is a proportional tax on output, the budget constraint of the representative private agent in country i is:

$$\dot{k}^i + c^i = (1 - q^i) Ak^i \tag{4}$$

where a dot over a variable denotes a time derivative. Initial capital stocks, k_0^i, are given.

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[9] Our results do not change if we use aggregate natural resources, $N \equiv \sum_{i=1}^{I} N^i$, as an argument in (1)-(2).

Private agents act competitively by taking tax policy, q^i, and the provision of public good, $\overline{N}$, as given. The latter is justified by the open-access and public-good features of the environment. The control variables are c^i and k^i, so that the first-order conditions for a maximum are equation (4) as well as the familiar Euler condition:

$$\dot{c}^i = [(1-q^i)A - r]c^i \tag{5}$$

Government budget constraint

Each national government i finances its environmental policy, g^i, by taxes on domestic output, $q^i y^i$. Thus, at any point of time, the budget is balanced:

$$g^i = q^i y^i \tag{6}$$

Natural resources and pollution

The motion of natural resources in each country i, N^i, is:

$$\dot{N}^i = dN^i - p^i + g^i \tag{7}$$

where the parameter $d \geq 0$ is the rate of regeneration of natural resources. That is, natural resources increase over time with natural regeneration, dN^i, and clean-up policy, g^i, but they decrease with pollution emission, p^i. Initial stocks of resources, N_0^i, are given.

Pollution, p^i, is a by-product of output produced, y^i.[10] Specifically, we assume:

[10] Our results do not change if pollution is also a by-product of consumption. On the other hand, this is different from the case of resource extraction, where natural resources are extracted from preserved natural environments to be used as inputs in production. See e.g. Kolstad and Krautkraemer [1993] for a more general model. Note that irrespectively of whether we treat the generation of pollution as a by-product of economic activity or as resource extraction, the main issue is the public-good character of the environment and its efficient management. Also, note that the assumption that pollution is a by-product of production is consistent with evidence that the world's largest emitters are the rich countries (US, Japan and Germany).

$$p^i = y^i \tag{8}$$

that is, for simplicity, one unit of output generates one unit of pollution. Equation (8) also implies that, in our model, output taxes act as pollution taxes.

Using (3), (6) and (8) into (7), the motion of natural resources in each country i becomes:

$$\dot{N}^i = \boldsymbol{d}N^i - (1-\boldsymbol{q}^i)Ak^i \tag{9}$$

We summarize this section. We have solved for a World Competitive Equilibrium (*WCE)*. In this equilibrium: (i) Private decisions maximize utility [this is summarized by (4) and (5)]. (ii) All constraints are satisfied and all markets clear [this is summarized by (9)]. The *WCE* holds for given initial conditions and any feasible economic policy. Since private agents have not internalized the effects of their decisions on the public good, the *WCE* is inefficient. This provides a rationale for government intervention. Therefore, the next two sections will endogenize economic policy by assuming that national governments are benevolent and play Stackelberg vis-à-vis private agents. In section III, they play Nash vis-à-vis each other, while in section IV they cooperate.

III. NON-COOPERATIVE ECONOMIC POLICIES

This section solves for a non-cooperative (Nash) game among benevolent national governments. Thus, each national government i maximizes the utility of its own private agent by taking as given the policies of other national governments $j \neq i$. In doing so, it plays Stackelberg vis-à-vis private agents.

Each national government i chooses $\boldsymbol{q}^i, c^i, k^i, N^i$ to maximize (1)-(2), subject to (4), (5) and (9), and by taking as given $\boldsymbol{q}^j, c^j, k^j, N^j$ where $j \neq i$. The current-value Hamiltonian of this problem, H^i, is:[11]

[11] Following most of the growth literature, we assume commitment technologies on behalf of Stackelberg governments. Thus, we do not study time inconsistency issues.

$$H^i \equiv \log c^i + n \log \frac{\sum N^i}{I} + l^i c^i\left[(1-q^i)A - r\right] + g^i\left[(1-q^i)Ak^i - c^i\right] +$$
$$+ m^i\left[dN^i - (1-q^i)Ak^i\right] \tag{10}$$

where l^i, g^i and m^i are dynamic multipliers associated with the constraints (5), (4) and (9) respectively. Thus, from the viewpoint of country i, l^i is the social value of private valuation of assets, g^i is the social value of its capital, and m^i is the social value of the country's natural resources.

Symmetric Nash Equilibrium

We focus on Symmetric Nash Equilibria (*SNE*) in policy strategies. At a *SNE*, strategies are symmetric *ex-post.* Thus, $x^i = x^j \equiv x$, where $i \neq j$ and $x \equiv (q, c, k, N, g, l, m)$.[12] Invoking symmetry into the first-order conditions for $q^i, c^i, l^i, g^i, k^i, m^i, N^i$, we get respectively (from now on, we omit the country-superscript i):

$$lc + gk = mk \tag{11a}$$

$$\dot{l} = rl - \frac{1}{c} - l\left[(1-q)A - r\right] + g \tag{11b}$$

$$\dot{c} = \left[(1-q)A - r\right]c \tag{11c}$$

$$\dot{k} = (1-q)Ak - c \tag{11d}$$

$$\dot{g} = rg - (1-q)Ag + (1-q)Am \tag{11e}$$

$$\dot{N} = dN - (1-q)Ak \tag{11f}$$

$$\dot{m} = rm - \frac{\hat{n}}{N} - dm \tag{11g}$$

[12] For the properties of symmetric (Nash and cooperative) equilibria, see e.g. Cooper and John [1988].

where $\hat{n} \equiv \frac{n}{I}$ denotes the "effective" weight given to environmental quality relative to private consumption (where I is the number of countries).

These necessary conditions are completed with the addition of a transversality condition that guarantees utility is bounded. A sufficient condition for this to hold is:

$$[(1-q)A - r] + d < r \tag{12}$$

so that the growth rate of consumption, $[(1-q)A - r]$, plus the rate of regeneration of natural resources, d, is less than the rate of time preference, r.

We now transform the variables to facilitate analytical tractability. Let us define $z \equiv \frac{c}{k}$, $y \equiv mk$ and $f \equiv mN$. Then, Appendix A shows that the dynamics of (11a)-(11g) are equivalent to the dynamics of (13a)-(13d) below:

$$\dot{z} = (z - r)z \tag{13a}$$

$$\dot{y} = \left[(1-q)A - z + r - \frac{\hat{n}}{f} - d\right]y \tag{13b}$$

$$\dot{f} = \left[r - \frac{\hat{n}}{f} - \frac{(1-q)Ay}{f}\right]f \tag{13c}$$

$$\left[z + d + \frac{\hat{n}}{f}\right]y = 1 \tag{13d}$$

where (13a)-(13d) constitute a system in z, y, f, q. Note that only (13a)-(13c) are dynamic, while (13d) is static. Thus, the dynamics of q are the dynamics of z, y, f.

To sum up, we have solved for a Symmetric Nash Equilibrium in pollution tax rates among national policymakers. This equilibrium is summarized by equations (13a)-(13d) and the terminal condition (12).

Long-run Symmetric Nash Equilibrium

This subsection solves for a long-run Symmetric Nash Equilibrium. We will study Sustainable Balanced Growth Paths *(SBGPs)*. That is, steady states in which consumption and capital can grow positively without damaging the environment.[13]

We look for a steady state of (13a)-(13d) in which $\dot{z} = \dot{y} = \dot{f} = 0$. Since $z \equiv \frac{c}{k}$, $y \equiv mk$ and $f \equiv mN$, conditions $\dot{z} = \dot{y} = \dot{f} = 0$ imply that natural resources grow at the same constant positive rate with capital and consumption.[14] Hence, this is a properly defined *SBGP*. Let us denote the steady state values of $(z,\ y,\ f,\ q)$ by $(\tilde{z},\ \tilde{y},\ \tilde{f},\ \tilde{q})$. Appendix B solves for $(\tilde{z},\ \tilde{y},\ \tilde{f},\ \tilde{q})$ and proves the existence and uniqueness of a long-run equilibrium. The main results are summarized in the following proposition:

PROPOSITION 1: Focusing on symmetric Nash equilibria in national policies, if the parameter values satisfy the conditions:

$$A > r + d \qquad (14a)$$

$$r > 2d \qquad (14b)$$

$$2\hat{n}d > r - d \qquad (14c)$$

there exists a unique well-defined long-run pollution tax rate denoted by $\tilde{q}^{nash}$. *This tax rate lies in the region* $0 < 1 - \frac{r+d}{A} < \tilde{q}^{nash} < 1 - \frac{r}{A} < 1$ *and is a solution to:*

$$\hat{n}\left[r + d - (1-\tilde{q})A\right]\left[r + (1-\tilde{q})A\right] = (1-\tilde{q})A\left[(1-\tilde{q})A - d\right] \qquad (15)$$

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[13] For *SBGPs*, see e.g. Aghion and Howitt [1998, chapter 5].

[14] This is as follows: (i) Since $z \equiv \frac{c}{k}$, $\dot{z} = 0$ implies that consumption, c, and capital, k, grow at the same positive rate. (ii) Since $y \equiv mk$, $\dot{y} = 0$ implies that the social valuation of natural resources, m, and capital, k, grow at equal, but opposite, rates. (iii) Since $f \equiv mN$, $\dot{f} = 0$ implies that social valuation of natural resources, m, and natural resources, N, grow at equal, but opposite, rates. Then, the combination of (i)-(iii) implies that consumption, capital and natural resources grow at the same constant positive rate.

This pollution tax rate, in turn, supports a unique well-defined steady state in which consumption, capital and natural resources grow at the same constant positive rate. Thus, the steady state is a Sustainable Balanced Growth Path (SBGP).

Proof: See Appendix B.[15]

(15) is an equation in $\tilde{q}$ only. Its total differentiation implies:[16]

$$\tilde{q} = q(\overset{-}{d}, \overset{-}{r}, \overset{+}{A}, \overset{-}{\hat{n}}) \tag{16}$$

We therefore have the following comparative static properties of the long-run pollution tax rate, $\tilde{q}$: (i) When natural resources can regenerate themselves (i.e. d is high), the need for environmental protection gets smaller. (ii) When we care about the future (i.e. r is low), we choose tougher environmental policy. (iii) When the productivity of private capital is high (i.e. A is high), we can afford higher tax rates. (iv) When private agents themselves value environmental quality (i.e. $\hat{n}$ is high), there is no big need for environmental policy.

In turn, the properties of the Sustainable Balanced Growth Path *(SBGP)* follow directly from the properties of the pollution tax rate, $\tilde{q}$.[17] That is, a lower (resp. higher)

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[15] Conditions (14a)-(14c) are jointly sufficient for a well-defined and unique long-run equilibrium to exist. Condition (14a) requires the productivity of private capital, A, to be higher than the rate of time preference, r, plus the regeneration rate of natural resources, d. This is a familiar condition for long-term growth, but we require a stronger condition than usually (see e.g. Barro and Sala-i-Martin [1995, p. 142]) because here richer economies must also devote resources to environmental quality. Condition (14b) guarantees that (12) holds and so the attainable utility is bounded (again, see e.g. Barro and Sala-i-Martin [1995, p. 142]). Condition (14c) states that: (i) Existence gets easier when the rate of regeneration of natural resources, d, is relatively high. (ii) Existence gets easier when we care about the future (i.e. r is low). (iii) Existence gets easier when economic agents value environmental quality (i.e. a high $\hat{n}$ helps existence). (iv) Since $\hat{n} \equiv \frac{n}{I}$, existence gets harder as the size of population, I, increases. This is a familiar result in the literature on public goods: as the size of population increases, any problems associated with decentralized (i.e. Nash) decision-making become worse (see also below).

[16] Signs above parameters give equilibrium properties.

[17] Equation (11c) implies that the properties of growth are opposite from those of the tax rate. Also, recall that consumption, capital and natural resources grow at the same constant positive rate in the long-run (see footnote 14 above).

pollution tax rate leads to a higher (resp. lower) rate of economic growth and improving (resp. deteriorating) environmental quality. Intuitively, lower tax rates lead to higher capital accumulation, higher economic growth and therefore larger tax bases, which lead to a greater ability to engage in clean-up policy.[18] Eventually, there is better environmental quality, despite the adverse effect of higher growth and pollution. This result is consistent with the main result of the literature on growth and the environment. Namely, economies that achieve a sustained growth path will ultimately be characterized by improving environmental quality (see e.g. John and Pecchenino [1994]).

The above are intuitive comparative static results. However, notice the effect of the number of countries, I on equilibrium outcomes. Since $\hat{n} \equiv \frac{n}{I}$, (16) implies that an increase in I leads ceteris paribus to an increase in the tax rate. Such a positive effect from the size of population on the Nash tax rate seems to be opposite from the standard one, which is traditionally negative (see the discussion in the Introduction). As we show below, this seemingly paradoxical result arises because the spillover effect is in fact negative. Specifically, the spillover effect changes from positive to negative, once we move from a static setup (which is what the literature traditionally uses) to a dynamic setup (which is what we use here). Then, a negative spillover effect can justify the positive effect of I on Nash tax rates.

We therefore move on to study cooperative equilibria. Before we do so, we have to check whether the above long-run equilibrium is dynamically stable. Appendix C shows that this is the case indeed. In particular, we show that there are no transitional dynamics and the economy jumps immediately to its long-run equilibrium, as it happens in the basic AK model.

[18] Recall that this is an AK model of growth. Thus, at any point of time, the return to capital and hence the rate of economic growth are independent of the beginning-of-period capital stock (see (5) above). In other words, capital is very elastic. In a more general model, where the state of the economy matters more, we would also have a short-run effect that works in an opposite direction from the long-run effect we have here. Specifically, in the short-run, tax bases (e.g. capital) are inelastic so that a lower tax rate leads to lower tax revenues and hence lower clean-up policy. That is, in the short run, lower tax rates on polluting firms' output can lead to worse environmental quality. Using such a more general model, John and Pecchenino [1994] show the possibility of multiple equilibria, some of which are characterized by high growth and high environmental quality, and some of which are characterized by low growth and bad environmental quality. In our simpler AK model, we get a unique equilibrium in which a tougher environmental policy, in the form of output taxes, is bad for both growth and the environment.

IV. COOPERATIVE ECONOMIC POLICIES

Now a hypothetical planner chooses jointly all q^i, c^i, k^i, N^i for $i = 1,2,...,I$ to maximize the sum of national utilities (1)-(2), subject to (4) and (5) for each i and the motion of world-wide natural resources. Taking the sum of (9) over all is , the motion of word-wide natural resources is $\sum \dot{N}^i = d\sum N^i - \sum (1-q^i)Ak^i$. This centralized problem is Pareto-optimal. The current-value Hamiltonian of this problem, H , is:

$$H \equiv \sum \log c^i + n\sum \log \frac{\sum N^i}{I} + \sum\ l^i c^i \left[(1-q^i)A - r\right] + \sum\ g^i \left[(1-q^i)Ak^i - c^i\right]$$
$$+ m\left[d\sum N^i - \sum (1-q^i)Ak^i\right] \qquad (17)$$

where l^i and g^i are new multipliers associated with (5) and (4) for each i, and m is the multiplier associated with the public good.

We work exactly as in the previous section. That is, we first derive the first-order conditions for $q^i, c^i, l^i, g^i, k^i, m, N^i$ and then focus on Symmetric Cooperative Equilibria (*SCE*) in national policies. Then, instead of (11a)-(11g), we get:

$$lc + gk = mk \qquad (18a)$$

$$\dot{l} = rl - \frac{1}{c} - l\left[(1-q)A - r\right] + g \qquad (18b)$$

$$\dot{c} = \left[(1-q)A - r\right]c \qquad (18c)$$

$$\dot{k} = (1-q)Ak - c \qquad (18d)$$

$$\dot{g} = rg - (1-q)Ag + (1-q)Am \qquad (18e)$$

$$\dot{N} = dN - (1-q)Ak \qquad (18f)$$

$$\dot{m} = rm - \frac{n}{N} - dm \qquad (18g)$$

(18a)-(18g) differ from (11a)-(11g) in that we now have n instead of $\hat{n} \equiv \frac{n}{I}$. This is how it should be. Namely, in a cooperative equilibrium where actions are coordinated and externalities are internalized, equilibrium outcomes are not affected by public-good type problems.

Then, we have in place of (13a)-(13d):

$$\dot{z} = (z - r)z \tag{19a}$$

$$\dot{y} = \left[(1-q)A - z + r - \frac{n}{f} - d\right]y \tag{19b}$$

$$\dot{f} = \left[r - \frac{n}{f} - \frac{(1-q)Ay}{f}\right]f \tag{19c}$$

$$\left[z + d + \frac{n}{f}\right]y = 1 \tag{19d}$$

Therefore, (19a)-(19d), together with the terminal condition, (12), summarize a Symmetric Cooperative Equilibrium in pollution tax rates among national policymakers.

Long-run Symmetric Cooperative Equilibrium

We again solve for a steady state in which $\dot{z} = \dot{y} = \dot{f} = 0$. Let $\left(\tilde{z},\ \tilde{y},\ \tilde{f},\ \tilde{q}\right)$ denote the new steady state values. Then, working exactly as in Appendix B, we have instead of Proposition 1:

PROPOSITION 2: Focusing on symmetric cooperative equilibria in national policies, if the parameter values satisfy the following conditions:

$$A > r + d \tag{20a}$$

$$r > 2d \tag{20b}$$

$$2nd > r - d \tag{20c}$$

there exists a unique well-defined long-run pollution tax rate denoted by $\tilde{q}^{coop}$*. This tax rate lies again in the region* $0<1-\frac{r+d}{A}<\tilde{q}^{coop}<1-\frac{r}{A}<1$ *and is a solution to:*

$$n\left[r+d-(1-\tilde{q})A\right]\left[r+(1-\tilde{q})A\right]=(1-\tilde{q})A\left[(1-\tilde{q})A-d\right] \qquad (21)$$

This pollution tax rate, in turn, supports a unique well-defined steady state in which consumption, capital and natural resources grow at the same constant positive rate. Thus, the steady state is a Sustainable Balanced Growth Path (SBGP).

Proof: As in Appendix B.[19]

By using the comparative static results in (16) and since $n>\hat{n}\equiv\frac{n}{I}$, inspection of (15) and (21) reveals that the pollution tax rate decreases when we switch from non-cooperation to cooperation, i.e. $\tilde{q}^{nash}>\tilde{q}^{coop}$. As we said above this is different from the standard result, and can happen only when the spillover effect (arising from the presence of public goods in the form of world-wide natural resources) from one country/player to another is negative. In fact, this is the case here.

To understand how the model works, we discuss first the static, special case (the proof of this case is in section V below). In a static setup, capital tax bases are exogenously given. Thus, an increase in tax rates leads always to an increase in tax revenues and an increase in resources available for clean-up policy. This implies a positive spillover effect from one country to another. That is, an increase in country's j tax rate leads to higher tax revenues and higher clean-up policy in j, and this is good for world-wide environmental quality. There is therefore an external welfare benefit upon country $i\neq j$. In turn, a positive spillover implies that there is too little action in a non-cooperative (Nash) equilibrium. This is the free-rider problem.

In contrast, in a dynamic setup with economic growth, tax bases are not given. Now, although higher tax rates can lead to higher tax revenues and higher clean-up policy

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[19] Note that since $n>\hat{n}\equiv\frac{n}{I}$, condition (20c) can hold much easier than its counterpart in the Nash equilibrium, (14c). Thus, the existence and uniqueness of an equilibrium get easier under coordination.

in the short-run, they discourage capital accumulation and economic growth. This eventually leads to smaller tax bases, lower tax revenues and lower clean-up policy. In terms of externalities, the implications of economic growth generate a negative spillover from one country to another. That is, an increase in country's j tax rate leads to lower capital accumulation, smaller tax bases, lower tax revenues and lower clean-up policy in j, and this is bad for world-wide environmental quality (despite the fall in growth and polluting activities). There is therefore an external welfare cost upon country $i \neq j$. In our AK model, the long-run effect on growth and tax bases more than outweighs any short-run effects.[20] Therefore, the net spillover effect is negative. In turn, a negative spillover implies that the Nash tax rate is inefficiently high, and this inefficiency increases with the size of population.

Therefore, in a dynamic setup with growth, the decentralized world economy will end up with too high pollution tax rates, relative to the case in which there is centralized decision-making. Without coordination, national policymakers do not internalize the harmful effect of their tax rates (and hence low economic growth, small tax bases and low clean-up policies) on the provision of world-wide environmental quality. They therefore set too high pollution tax rates. A switch to coordination leads to lower pollution tax rates, and this is good (i.e. efficient) for both growth and the environment. This is because low tax rates stimulate growth and generate tax bases broad enough to finance clean-up policies and improve the environment, despite the adverse effect of higher growth and pollution.

To complete the analysis, we study the static, special case.

V. FREE RIDING IN THE STATIC SPECIAL CASE

We now consider the special case without dynamics. Equation (4) is reduced to:

$$c^i = (1-q^i)Ak_0^i \tag{22}$$

[20] Of course, the AK is a special growth model. However, it allows us to show clearly how the results change when we move from a static setup (where capital tax bases are fully inelastic and so the free-rider effect is logically the dominant one) to a dynamic setup (where tax bases are very elastic and so the long-run effect on growth and tax bases is logically the dominant one).

and equation (9) can be written as:

$$N^i = (1+\boldsymbol{d})N_0^i - (1-\boldsymbol{q}^i)Ak_0^i \tag{23}$$

where k_0^i and N_0^i are given, initial values. Then, the problem is to maximize (2), where c^i and N^i are as in (22) and (23) respectively.

In a Nash game, each national government i chooses $\boldsymbol{q}^i$ to maximize:

$$\log(1-\boldsymbol{q}^i)Ak_0^i + \boldsymbol{n}\log\frac{\sum[(1+\boldsymbol{d})N_0^i - (1-\boldsymbol{q}^i)Ak_0^i]}{I} \tag{24}$$

so that, in a symmetric Nash equilibrium, the first-order condition gives:

$$\frac{N}{c} = \hat{\boldsymbol{n}} \tag{25}$$

which is a well-known condition (see e.g. Mueller [1989, chapter 2] and Mas-Colell et al. [1995, chapter 11]).[21]

Using (22) and (23) for c^i and N^i into (25) and totally differentiating, we get:

$$\boldsymbol{q} = \boldsymbol{q}(\overset{-}{\boldsymbol{d}}, \overset{+}{A}, \overset{+}{\hat{\boldsymbol{n}}}, \overset{+}{k_0}, \overset{-}{N_0}) \tag{26}$$

so that, since $\hat{\boldsymbol{n}} \equiv \frac{\boldsymbol{n}}{I}$, the Nash tax rate decreases with the number of participants, I.[22] This is a standard comparative static result (see e.g. Oakland [1985] and Mueller [1989, chapter 2]).

[21] Recall that here it is the average total quantity that enters the utility function (see equations (1)-(2) above). This is why (25) is not exactly the textbook condition. Thus, to get the latter, we should have $N \equiv \sum_{i=1}^{I} N^i$, instead of $\overline{N} \equiv \frac{1}{I}\sum_{i=1}^{I} N^i$, as an argument in (1)-(2).

[22] Note that the effects of $\boldsymbol{d}$ and A remain as in the dynamic case above (compare (26) with (16)).

In contrast, in a cooperative game, the planner chooses q^i to maximize the sum of utilities. Thus,

$$\sum \log(1-q^i)Ak_0^i + n\sum \log \frac{\sum[(1+d)N_0^i-(1-q^i)Ak_0^i]}{I} \qquad (27)$$

so that, in a symmetric cooperative equilibrium, the first-order condition gives:

$$\frac{N}{c} = n \qquad (28)$$

which is the familiar Samuelsonian condition for Pareto optimality.[23]

Using the comparative static properties in (26) and since $n > \hat{n} \equiv \frac{n}{I}$, inspection of (28) and (25) reveals that $\tilde{q}^{nash} < \tilde{q}^{coop}$. That is, in the static case, the decentralized world economy will end up with too low pollution tax rates. This is the free-rider result. A switch to coordination will lead to higher pollution tax rates, higher tax revenues, more clean-up and better environment.

VI. RELATED PAPERS, LIMITATIONS AND EXTENSIONS

We developed a model in which the environment has public good features, and then examined whether the conventional view that Nash tax rates are too low goes through in a model with economic growth. We showed that the type of spillover effect from one player to another arising from the public good character of the environment (and hence whether we under-tax or over-tax in a Nash equilibrium relative to a cooperative one) is reversed once the model allows for economic growth. This happens because medium- and long-run capital tax bases are elastic. We used a simple model to make the logic of our results as clear as possible.

[23] See footnote 21 above.

As far as we know, this point has not been made before. However, there are some papers that are related to ours. Keen and Kotsogiannis [2000] use a model of a federal economy to show that "vertical externalities" are likely to give Nash tax rates that are inefficiently high. This is because national policymakers do not internalize the pressure on federal expenditures they create, when they raise their own tax rates. However, their results are driven by *ex ante* different types of externalities (horizontal and vertical). In our paper, by contrast, the type of the same, single externality is reversed when the model becomes dynamic.

Glomm and Lagunoff [1999] show that, concerning the provision of public goods, whether voluntary (i.e. decentralized) or coercive (i.e. centralized) mechanisms prevail, depends crucially on whether the game is static or dynamic. This is because, in a dynamic setup, the accumulation process mitigates the problem of conflicting interests occurring in coercive communities and hence such communities become more attractive.

John and Pecchenino [1994] use a richer model than ours which shows (among other things) that short-lived agents may over-invest in environmental quality in a decentralized equilibrium. However, this is a natural consequence of their *OLG* model. As is known, *OLG* models lead to inefficiently low consumption and hence inefficiently low pollution (in their model, pollution is a by-product of consumption).

We feel that our work, as well as the above papers, are consistent with the general result that many properties, concerning the comparison between non-cooperative and cooperative outcomes, may change once the model allows for dynamics (recall, for instance, the well-known result that non-cooperative outcomes lose their appeal when the game becomes repeated). Here, we showed that short-run free-rider behavior can be reversed in the long-run, at which point tax bases become elastic.

How general are our results? We have experimented with variations of the model to see how robust our results are. For instance, consider the transition equation describing the motion of the public good (7). A more general specification could be $\dot{N}^i = \delta N^i - \beta p^i + \sigma g^i$, where δ is the rate of self-regeneration/degeneration of the public good, β is the rate of degradation of the public good as a result of economic activity and σ is a technology coefficient that translates resources into public goods (see also John and Pecchenino [1994]). In our model, we set $d > 0$ and $b = s = 1$. It would be interesting to specify ranges of parameter values under which the spillover effect can be reversed, and ranges under which the spillover effect remains as in the static model, once we allow for economic growth. However, we feel that here we have provided a non-trivial example, which shows the importance of economic growth for the type of spillover effect from one player to another. And we did this in a fairly standard setup. We therefore leave any possible extensions for future work.

APPENDICES

APPENDIX A: From equations (11a)-(11g) to equations (13a)-(13d)

Equations (11a)-(11g) constitute a seven-equation dynamic system in q, l, c, k, g, N, m. Taking logarithms on both sides of (11a) and differentiating with respect to time, we get:

$$\frac{\dot{l}c + l\dot{c} + \dot{g}k + g\dot{k}}{lc + gk} = \frac{\dot{m}k + m\dot{k}}{mk} \tag{A.1}$$

Substituting (11b), (11c), (11d), (11e) and (11g) for the rates of growth of l, c, k, g and m respectively into (A.1), we obtain:

$$1 = mc + dmk + \frac{\hat{n}k}{N} \tag{A.2}$$

If we define $z \equiv \frac{c}{k}$, (11c) and (11d) give (13a). If we also define $y \equiv mk$ and $f \equiv mN$, (11d), (11g) and (11f) give (13b) and (13c). Using these definitions into (A.2), we get (13d).

APPENDIX B: Proof of Proposition 1

Setting $\dot{z} = 0$, equation (13a) implies:

$$\tilde{z} = r \tag{B.1}$$

Setting $\dot{y} = 0$ and using (B.1), equation (13b) implies:

$$\tilde{f} = \frac{\hat{n}}{[(1-\tilde{q})A - d]} \tag{B.2}$$

Setting $\dot{f} = 0$ and using (B.1)-(B.2), equation (13c) implies:

$$\tilde{y} = \frac{\hat{n}[r + d - (1-\tilde{q})A]}{(1-\tilde{q})A[(1-\tilde{q})A - d]} \tag{B.3}$$

Then, using (B.1)-(B.3) into (13d), we get:

$$\hat{n}\left[r + d - (1-\tilde{q})A\right]\left[r + (1-\tilde{q})A\right] = (1-\tilde{q})A\left[(1-\tilde{q})A - d\right] \tag{B.4}$$

which is equation (15) in the text. Note that (B.4) is a quadratic equation in $\tilde{q}$ only. If we solve (B.4) for $\tilde{q}$, (B.2) and (B.3) respectively will give $\tilde{f}$ and $\tilde{y}$. So the main task is to solve (B.4). We work as follows: In the first step, we specify the region in which a well-defined solution (if any) should lie. In the second step, we establish the existence and uniqueness of such a solution.

Consider the first step. A well-defined solution requires: (i) $(1-\tilde{q})A - r > 0$, i.e. $\tilde{q} < 1 - \frac{r}{A}$. This is required for the economy to grow in the long-run. (ii) $(1-\tilde{q})A + d < 2r$, i.e. $1 - \frac{2r-d}{A} < \tilde{q}$. This is required for the transversality condition (12) to hold. (iii) $r + d - (1-\tilde{q})A > 0$, i.e. $1 - \frac{r+d}{A} < \tilde{q}$. This follows from inspection of (B.2)-(B.4) above. (iv) $(1-\tilde{q})A - d > 0$, i.e. $\tilde{q} < 1 - \frac{d}{A}$. Again, this follows from inspection of (B.2)-(B.4). (iv) $2(1-\tilde{q})A - d > 0$, i.e. $\tilde{q} < 1 - \frac{d}{2A}$. This is required for the left-hand side of (B.4) to be monotonically increasing in $\tilde{q}$ (see below why we need this). Now, if we combine (i)-(iv), and given the parameter restrictions in (14a) and (14b) in Proposition 1, the "binding" lower boundary for $\tilde{q}$ is $0 < 1 - \frac{r+d}{A}$,[24] and the "binding" upper boundary for $\tilde{q}$ is $1 - \frac{r}{A} < 1$.[25] Thus,

$$0 < 1 - \frac{r+d}{A} < \tilde{q} < 1 - \frac{r}{A} < 1 \qquad \text{(B.5)}$$

which gives the region in which a well-defined solution (if any) for $\tilde{q}$ should lie.

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[24] In particular, if we assume $r > 2d$ [which is condition (14b)], it follows $1 - \frac{2r-d}{A} < 1 - \frac{r+d}{A} < \tilde{q}$. That is, when $1 - \frac{r+d}{A} < \tilde{q}$, it also always holds $1 - \frac{2r-d}{A} < \tilde{q}$, so that the transversality condition is satisfied. In turn, we assume $A > r + d$ [which is condition (14a)] so that $1 - \frac{r+d}{A} > 0$. Therefore, the binding lower boundary is $1 - \frac{r+d}{A}$ which is positive.

Consider the second step. We study whether such a solution for $\tilde{q}$ actually exists and whether it is unique. Recall that $\tilde{q}$ solves (B.4). Define the left-hand side of (B.4) by $L(\tilde{q})$ and the right-hand side by $R(\tilde{q})$. Then, $L_q(\tilde{q}) > 0$ (see condition (iv) above) and $R_q(\tilde{q}) < 0$. Concerning the lower boundary, i.e. $1-\frac{r+d}{A}$, we have $L\left(1-\frac{r+d}{A}\right)=0$ which is always smaller than $R\left(1-\frac{r+d}{A}\right)>0$. Concerning the upper boundary, i.e. $1-\frac{r}{A}$, we have $L\left(1-\frac{r}{A}\right)> R\left(1-\frac{r}{A}\right)>0$, if the parameter values satisfy condition (14c) in the text. Since $L_q(\tilde{q}) > 0$ and $R_q(\tilde{q}) < 0$ monotonically, these values of $L(\tilde{q})$ and $R(\tilde{q})$ at the lower and upper boundaries mean that $L(\tilde{q})$ and $R(\tilde{q})$ intersect once, as it is shown in Figure 1 below.

Figure 1 here

Therefore, a unique, well-defined solution for $\tilde{q}$ exists. This in turn supports - via (B.2) and (B.3) - a unique well-defined solution for $\tilde{f}$ and $\tilde{y}$.

APPENDIX C: Dynamic Stability

We study stability properties around steady state. Linearizing (13a), (13b) and (13c) around the unique steady state implies that the local dynamics are approximated by the linear system:

$$\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} J_{zz} & J_{zy} & J_{zf} \\ J_{yz} & J_{yy} & J_{yf} \\ J_{fz} & J_{fy} & J_{ff} \end{bmatrix} \begin{bmatrix} z \\ y \\ f \end{bmatrix} \qquad \text{(C.1)}$$

where, the elements of the Jacobian matrix evaluated at the steady state are:

$$J_{zz} \equiv \frac{\partial \dot{z}}{\partial z} = r > 0,\; J_{zy} \equiv \frac{\partial \dot{z}}{\partial y} = 0,\; J_{zf} \equiv \frac{\partial \dot{z}}{\partial f} = 0,$$

[25] In particular, $r > 2d$ implies $1-\frac{r}{A} < 1-\frac{d}{A} < 1-\frac{d}{2A}$. Thus, the binding upper boundary is $1-\frac{r}{A}$.

$$J_{yz} \equiv \frac{\partial \dot{y}}{\partial z} = -\tilde{y} < 0, \; J_{yy} \equiv \frac{\partial \dot{y}}{\partial y} = 0, \; J_{yf} \equiv \frac{\partial \dot{y}}{\partial f} = \frac{\hat{n}\tilde{y}}{\tilde{f}^{2}} > 0,$$

$$J_{fz} = \frac{\partial \dot{f}}{\partial z} = 0, \; J_{fy} = \frac{\partial \dot{f}}{\partial y} = -(1-\tilde{q})A < 0, \; J_{ff} = \frac{\partial \dot{f}}{\partial f} = r > 0.$$

The determinant of the Jacobian matrix, denoted by $\det(J)$, is $\det(J) = r\hat{n}(1-\tilde{q})A\frac{\tilde{y}}{\tilde{f}^{2}}$, which is positive. Hence, there are two possibilities: Either there are three positive eigenvalues, or one positive and two negative eigenvalues. Since all three variables (z, y, f) are forward-looking ones, the former possibility (i.e. three positive roots) will give local determinacy (i.e. a unique path), while the latter possibility (i.e. one positive and two negative roots) will give local indeterminacy (i.e. multiple paths, each of which is consistent with the same initial condition and with convergence to the same unique steady state). We therefore examine the characteristic equation of the Jacobian matrix being evaluated at the steady state. This is:

$$e^{3} - 2re^{2} + \left[r^{2} + \frac{(1-\tilde{q})A\hat{n}\tilde{y}}{\tilde{f}^{2}} \right] e - \frac{r(1-\tilde{q})A\hat{n}\tilde{y}}{\tilde{f}^{2}} = 0 \qquad \text{(C.2)}$$

where e is an eigenvalue. Note that the coefficient on e^{2} is negative, the coefficient on e is positive and the constant term is negative. That is, there are three sign alterations in (C.2). We now use Descartes' Theorem (see Azariadis [1993]), which states that the number of positive roots cannot be higher than the number of sign alterations. Thus, we cannot have more than three positive roots. Next, define $e' \equiv -e$. In this case, there are no sign alterations in (C.2). Thus, we cannot have a negative root. Combining results, there are three positive roots. Therefore, we have local determinacy.

What does it mean? Without predetermined variables, determinacy means that the forward-looking variables jump immediately to take their long-run values and stay there (until the system is disturbed in some way). Thus, there are no transitional dynamics and the saddle-path solution is equivalent to the steady state. This is as in the basic AK model (see e.g. Barro and Sala-i-Martin [1995]).

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FIGURE 1