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WELFARE COSTS OF INFLATION IN A DYNAMIC ECONOMY WITH SEARCH UNEMPLOYMENT AND ENDOGENOUS GROWTH

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Abstract

Recent work on money and endogenous growth finds modest welfare costs of inflation. Furthermore, high inflation reduces the growth rate. We present a monetary endogenous growth model with labor market frictions in the form of search unemployment which is calibrated for the US economy. Interestingly, both employment and the growth rate may even increase with the rate of inflation depending on the elasticity of labor supply. Considering the transition dynamics following a change in the monetary policy, the optimal quarterly inflation rate is found to amount to approximately 3.5% in the benchmark case. A reduction of the inflation rate from its optimal value to zero results in a welfare loss equal to 0.3% of total consumption.

Keywords: Welfare costs of inflation, money demand, search unemployment, endogenous growth, transistion dynamics

JEL Classification: O42, E31, J64

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1 Introduction

The welfare costs of inflation have attracted attention in recent literature on endogenous growth. In these models, money is commonly introduced using a cash-in-advance (CIA) constraint. High inflation reduces the return from working as income earned in the previous period cannot be spent until the next period. As a consequence, households substitute leisure for labor and the growth rate declines. In Gomme (1993), endogenous growth arises through human capital accumulation as in Lucas (1988,1990). Gomme finds modest welfare costs of inflation. In particular, a 10% money growth rate (8.5% inflation rate) results in welfare costs of no more than 0.03% of income. Wu and Zhang (1998) analyze a monetary endogenous growth model of Romer (1986). They find significant welfare costs of inflation in the range from half to 5% points for an annual monetary growth of 10%. Different from Gomme (1993), the growth rate effect is important in their model.

We also analyze an endogenous growth model as suggested by Romer (1986). Money is introduced using a CIA constraint. In addition to existing work on monetary endogenous growth models, we introduce labor market frictions building on the work of Pissarides (1990). Our results are in contrast with the results from Gomme (1993) and Wu/Zhang (1998). In particular, money growth increases employment and the growth rate in our benchmark case. The optimal quarterly inflation rate amounts to 3.5% implying welfare gains of 0.3% of total consumtpion compared to the zero-inflation economy. Hence, the magnitude of the welfare effects is approximately equal to the one found by Wu/Zhang (1998), but of opposite sign.

Our surprising results are caused by the consideration of labor market frictions. Unemployment results from time-consuming and costly matching of vacancies with searching agents. Following a rise in the inflation rate, agents substitute search for leisure, similar to the leisure-labor substitution effect in the endogenous growth models of Gomme (1993) and Wu/Zhang (1998), who assume Walrasian labor markets instead. The reduction in search effort reduces the probability of firms to fill a job and tends to decrease employment. However, in our model, there is also an opposing employment-enhancing effect from a rise in the rate of inflation. High inflation also reduces the level of consumption (relative to the one of capital) as agents substitute real money balances and capital as in Tobin (1965). In our economy, wages result from decentralized Nash bargaining. Following a decline in consumption, the reservation wage of the households and, hence, the bargained wage decrease.[1] Consequently, firms increase their vacancies, which boosts employment. The net effect of a rise of inflation on employment is positive for our benchmark calibration of the model for the US economy.

In addition to existing literature on monetary endogenous growth, we will not only introduce labor market frictions in order to study the welfare costs of inflation, but we will also account for the transition dynamics following a change in the money growth rate. Recent quantitative evaluations of policy measures find a significant effect of transition dynamics on welfare. For example, Lucas (1990) analyzes the abolition of capital income taxes in an endogenous growth model with human capital accumulation. In steady state, the change in welfare amounts to a 3% consumption equivalent increase. As demonstrated by Grüner/Heer (2000), also considering the transition from the old to the new steady state reduces the welfare gain of such a policy to 1% of total consumption. Furthermore, in our model of search unemployment, the transition dynamics and hence welfare results may be different from the one in standard models with Walrasian labor markets. Typically, welfare is measured by life-time utility of the households which is a function of leisure and consumption. Following a policy change, consumption and leisure immediately adjust in standard endogenous growth models, ie they are jump variables. In particular, employment and consumption may even overshoot their new steady state values as shown by Grüner/Heer (2000). However, in the presence of labor market frictions, employment behaves fundamentally different and only adjusts gradually to the new steady state, ie employment is a sluggish variable. Consequently, the magnitude of instantaneous utility during transition differs significantly from the one in the new steady state and the analysis of balanced-growth path without any further consideration of the transition might even lead to the wrong policy conclusion.

Our model follows the search unemployment model of Shi/Wen (1999) with two extensions: first, we introduce money using a cash-in-advance constraint. And second, endogenous growth is introduced into the model with the help of a production technology which displays constant returns in (social) capital.

[1]To be more specific, the level of consumption and wages fall. On the balanced growth path, however, consumption and wage will grow at a higher rate.

The complete model is presented in section 2. The model is calibrated for the US economy using standard parameters from applied general equilibrium studies. Section 3 presents a balanced-growth analysis. As one major result, inflation is shown to possibly increase the growth rate in the presence of search unemployment and wage bargaining. In section 4, the welfare costs of inflation are analyzed explicitly accounting for the transition dynamics. As our second major result, the optimal inflation rate is shown to be well above zero for realistic values of labor market parameters. Section 5 concludes.

2 The model

The model introduces money demand and endogenous growth in the search model of Shi/Wen (1999). Three sectors can be distinguished: households, firms, and the monetary authority. The representative household maximizes his expected intertemporal utility subject to his budget constraint. Firms produce a consumption-investment good using capital and labor. Labor markets are subject to frictions as matching vacancies with searching workers is a time-consuming transaction. Wages are bargained and deviate from the marginal product of labor. Money is incorparated using a cash-in-advance constraint.

2.1 Households

Households are of measure one. The representative household consists of different members who pool their receipts. Agents either work, search for a job, or enjoy leisure. Let n denote the share of agents working, s the share of agents searching, and $1 - n - s$ denote the unemployed agents not actively searching for a job. n, s, and $1 - n - s$ can also be interpretated as the number of employed agents, the number of unemployed agents, and the agents who are out of the labor force. All variables are indexed by time, which is continuous. For convenience, we omit the time index. Furthermore, working n and searching s is assumed to cause disutility to the agents.

Households maximize their intertemporal utility

$$\int_0^{\infty} U(c,s,n)\, e^{-\rho t} dt = \int_0^{\infty} \left[\ln c - \beta \frac{(n+s))^{\eta}}{\eta} \right] e^{-\rho t} dt, \tag{1}$$

where c and ρ denote consumption and the discount rate of the household, respectively.

As a distinguishing feature of this economy, employment n changes only gradually:

$$\dot{n} = qs - \theta n, \tag{2}$$

where q denotes the probability of the searching agents to find a job. The individual agents take q as given, while it is determined endogenously in the labor market. Finding a job takes time. As a possible explanation, workers and firms are placed in different locations or the screening and dissemination of information is a time-consuming activity. Furthermore, workers are separated from their job at the rate θ, which is given exogenously.

Households face two constraints, a budget constraint and a cash-in-advance (CIA) constraint. They receive income from capital k, labor n, profits Ω, and real lump-sum transfers τ from the monetary authority. Real assets a consist of capital k and real money balances m and accumulate according to:

$$\dot{a} = \dot{k} + \dot{m} = wn + rk + \Omega + \tau - c - \pi m, \tag{3}$$

where w, r, and π denote the wage rate, the interest rate, and the inflation rate, respectively. The initial endowments at time zero k_0 and n_0 are given.

The consumption good c consists of ψc cash goods and $(1-\psi)c$ credit goods. Purchases of the cash good are subject to the CIA constraint:

$$\psi P c \leq P m, \tag{4}$$

where P denotes the price level with $\pi \equiv \frac{\dot{P}}{P}$. Since the analysis only considers the situation $\pi \geq 0 > -r$, equation (4) will always hold as an equality at an optimum.

In the case of an interior solution, $s, n, 1-n-s > 0$, the first-order conditions of the household are given by:

$$\frac{\dot{c}}{c} = r - \rho - \frac{\psi \pi}{1+\psi \pi} \frac{\dot{\pi}}{\pi}, \tag{5}$$

$$q\lambda_n = \beta\,(s+n)^{\eta-1}, \tag{6}$$

$$\dot{\lambda}_n = (\theta+\rho)\lambda_n + \beta\,(s+n)^{\eta-1} - \frac{w}{c}\frac{1}{1+\psi\pi}, \tag{7}$$

where λ_n denotes the current value shadow price of employment to the household. Equations (5) and (7) describe the dynamics of consumption c and the current value shadow price of employment, λ_n. According to equation (6), the marginal benefit of search, $q\lambda_n$, is equal to the marginal cost of search, $\beta(s+n)^{\eta-1}$.

2.2 The monetary authority

The economy-wide nominal money supply $M = Pm$ grows at the rate μ:

$$\frac{\dot{M}}{M} = \mu. \tag{8}$$

The seignorage obtained from money creation is paid to the households as a lump-sum transfer implying:

$$\tau = \mu m. \tag{9}$$

2.3 Firms

Firms are identical and of measure one. They use labor n and capital k in order to produce the consumption-investment good y with the technology $f(k, \bar{k}, n)$. The externality in aggregate capital accumulation $\bar{k}$ (which equals k in equilibrium) results in constant returns to capital as in Romer (1986):

$$y = f(k, \bar{k}, n) = Ak^{\alpha}n^{1-\alpha}\bar{k}^{1-\alpha}. \tag{10}$$

y can either be consumed by the households or accumulated.

Firms maximize discounted profits:

$$\int_0^{\infty} \Omega e^{-\int_0^t r(h)dh} dt, \tag{11}$$

where profits are given by:

$$\Omega = f(k, n, \bar{k}) - (r + \delta)k - wn - \kappa wv. \tag{12}$$

Firms take the interest rate r and the wage rate w as given. Capital depreciates at the rate δ. The cost of maintaining a number v of job vacancies is proportional to the wage rate w, the proportionality factor being denoted by κ. The vacancy costs κwv represent real costs of production for hiring, screening, or training of workers.

Workers separate from a job at rate θ. The individual firm takes the rate ϑ at which a vacancy is filled as given. The firm's employment evolves according to:

$$\dot{n} = \vartheta v - \theta n. \tag{13}$$

The maximization conditions of the firm are as follows:

$$f_k(k, n, \bar{k}) = r + \delta, \tag{14}$$

$$\kappa w = \lambda_F \vartheta, \tag{15}$$

$$\dot{\lambda}_F = (\theta + r)\lambda_F - \left(f_n(k, n, \bar{k}) - w\right), \tag{16}$$

where λ_F and $f_i(k, n, \bar{k})$ denote the current value shadow price of employment to the firm and the marginal product of i, $i = k, n$, respectively. According to the optimal decision on the capital stock k, equation (14), the marginal product of capital equals the interest rate r plus depreciation δ. In equation (15), the marginal cost of a vacancy v is equated to its marginal benefit. The dynamics of the current value shadow price of employment λ_F are described in equation (16).

2.4 Matching and wage determination

Labor markets are subject to frictions and are characterized by two-sided search. Time is needed in order to match vacancies with searching workers. Furthermore, there is an externality from searching and posting a vacancy. An increase in the number of searchers reduces the probability of other searching agents to find a job, while it increases the probability of firms to fill their vacancies. Vice versa, an additional vacancy increases the probability of a searching agent to get a job, while it reduces the probability of the other firms to fill a vacancy. Thus, a negative externality arises whenever the number of active agents increases on the same side of the market and a positive externality arises if the number of agents increases on the other side of the market.

In order to simplify notation, let v and s also denote the aggregate numbers of vacancies and searching agents, respectively. The number of aggregate matches L is an increasing function of both aggregate vacancies v and aggregate searching agents s. If no vacancy is posted ($v = 0$), no jobs can be matched, $L = 0$. Similarly, if agents are not searching for a job ($s = 0$), $L = 0$. In particular, the flow of job matches L is described by the following constant returns to scale technology:

$$L = L_0 v^{\gamma} s^{1-\gamma}, \quad L_0 > 0,\ 0 < \gamma < 1. \tag{17}$$

The job finding rate of the searching agents is given by $q = \frac{L}{s}$, while the firms fill a vacancy at rate $\vartheta = \frac{L}{v}$. In equilibrium, the number of agents finding a job is equal to the number of filled vacancies, $\vartheta v = qs$.

Wages result from decentralized bargaining between the firm and the marginal worker. Both the firm and the worker receive a rent from a successful match. More specifically, the wage is determined by Nash bargaining which maximizes a product of weighted surpluses of the household and the firm:

$$\max_{w} \left[f_n(k, n, \bar{k}) - w\right]^{1-\lambda} \left[w - \beta(s+n)^{\eta-1}c\right]^{\lambda}, \tag{18}$$

where λ measures the bargaining power of the workers. The term $\beta(s+n)^{\eta-1}c$ can be interpreted as the agent's reservation wage.[2]

The wage as resulting from the Nash bargaining problem is given by:

$$w = \lambda f_n(k, n, \bar{k}) + (1-\lambda)\beta(s+n)^{\eta-1}c. \tag{19}$$

The wage is lower than the marginal product of labor and exceeds the reservation wage of the household. Accordingly, both agents share a rent, the shares depending on the bargaining power λ.

[2]For a discussion of the wage determination see Shi/Wen (1999).

2.5 Stationary competitive search equilibrium

Definition. The competitive search equilibrium is a collection of decision rules $\{c, s, v, k\}$ and prices $\{w, r, \pi\}$ such that

1. Individual variables equal aggregate variables.
2. Households maximize their utility (1) subject to (2), (3), and (4).
3. Firms maximize profits (11) subject to (12) and (13).
4. Wages and interest rates are given by (19) and (14), respectively.
5. Assets accumulates according to (3).
6. Agents do not take into account the effect of their decisions on the matching rates q and ϑ: $\vartheta v = qs$.
7. Employment evolves according to (2).
8. Nominal money grows at the exogenous rate μ.
9. The goods market clear:
$$\dot{k} = f(k, n, \bar{k}) - \delta k - c - \kappa w v. \tag{20}$$
10. The externality in capital accumulation is equal to the aggregate capital stock, $\bar{k} = k$.

In the stationary equilibrium, our economy can be described with the help of the initial endowment of labor n_0 and capital k_0 and the following dynamic equations in the stationary variables $\tilde{c} \equiv \frac{c}{k}$, π, $\tilde{\lambda}_F \equiv \frac{\lambda_F}{k}$, n, and λ_n:

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = r - \rho - \frac{\psi\pi}{1+\psi\pi}\frac{\dot{\pi}}{\pi} - \left(A\bar{n}^{1-\alpha} - \delta - \tilde{c} - \kappa\tilde{w}v\right), \tag{21}$$

$$\frac{\dot{\pi}}{\pi} = \frac{\pi - \mu + r - \rho}{\frac{\psi\pi}{1+\psi\pi}}, \tag{22}$$

$$
\begin{aligned}
\frac{\dot{\tilde{\lambda}}_F}{\tilde{\lambda}_F} &= r + \theta - \frac{(1-\alpha)An^{-\alpha} - \tilde{w}}{\tilde{\lambda}_F} - \left(A\bar{n}^{1-\alpha} - \delta - \tilde{c} - \kappa\tilde{w}v\right), && (23)\\
\dot{n} &= qs - \vartheta v, && (24)\\
\dot{\lambda}_n &= (\rho+\theta)\lambda_n + \beta(s+n)^{\eta-1} - \frac{\tilde{w}}{\tilde{c}}\frac{1}{1+\psi\pi}, && (25)
\end{aligned}
$$

where the variables r, q, ϑ, s, $\tilde{w} \equiv \frac{w}{k}$, and v are given by the non-linear equations (6) and:

$$
\begin{aligned}
r &= \alpha An^{1-\alpha} - \delta, && (26)\\
\tilde{w} &= \lambda(1-\alpha)An^{-\alpha} + (1-\lambda)\beta(s+n)^{\eta-1}\tilde{c}, && (27)\\
\kappa\tilde{w} &= \tilde{\lambda}_F\vartheta, && (28)\\
\vartheta &= L_0\left(\frac{s}{v}\right)^{1-\gamma}, && (29)\\
q &= L_0\left(\frac{v}{s}\right)^{\gamma}. && (30)
\end{aligned}
$$

The local behavior of the model's dynamics is studied by Shi/Wen (1997) for the case of constant search intensity s. Given $s = s_0$, the steady state is proven to be locally stable if the intertemporal elasticity of substitution, $1/\sigma$, is sufficiently large. In the present model with endogenous search intensity s, the stability of the steady state cannot be shown analytically but only numerically for the specific calibration used.

2.6 Calibration

The effects of a change in the inflation rate (as resulting from a change in the growth rate of money supply) cannot be studied analytically but only numerically. For this reason, the model is calibrated in order to match characteristics of the US economy. If not mentioned otherwise, our parameters are taken from Shi/Wen (1999). The unit time length corresponds to one quarter.

The production elasticity of private capital is set equal to $\alpha = 0.25$. Capital depreciate at a rate of $\delta = 0.01$. The parameter A is calibrated with the help

Table 1

Calibration of parameter values for the US economy

Description	Function	Parameter
utility function	$U = \ln c - \beta (s+n)^{\eta}$	$\beta = 3.209$, $\eta = 3.5$
time preference	ρ	$\rho = 0.01$
production function	$y = Ak^{\alpha}n^{1-\alpha}\bar{k}^{1-\alpha}$	$A = 0.131$, $\alpha = 0.25$
depreciation	δ	$\delta = 0.01$
growth rate	$g = \dot{y}/y$	$g = 0.35\%$
vacancy costs	$\kappa v w$	$\kappa = 2.309$
matching function	$L = L_0 v^{\gamma} s^{1-\gamma}$	$\gamma = 0.6$, $L_0 = 1.0$
job separation rate	θ	$\theta = 0.05$
wage bargaining power	λ	$\lambda = 0.4$
money growth rate	μ	$\mu = 1.3\%$
CIA constraint	$\psi c \leq m$	$\psi = 0.84$

of equation (26). Output at time 0 is normalized to one. The steady-state quarterly growth rate of output is set equal to $g = 0.35\%$. The monetary parameters of the model are taken from Cooley/Hansen (1995). The quarterly growth of money supply rate is set equal to $\mu = 0.013$. The share of consumption which is subject to the cash-in-advance constraint is set equal to $\psi = 0.84$.

Following Shi/Wen (1999), the labor market parameters λ, γ, L_0, and θ are chosen as presented in table 1. The discount rate of the households is set equal to $\rho = 0.01$, η is calibrated in order to obtain a labor supply elasticity of $\epsilon = 1/(\eta - 1) = 0.4$. Furthermore, the value of $\beta = 3.209$ and $\kappa = 2.309$ imply a steady state labor force participation $\bar{s} + \bar{n} = 0.68$ and an unemployment rate $\bar{s}/(\bar{s} + \bar{n}) = 0.06$.

Our calibration implies a steady state quarterly inflation rate $\pi = 0.95\%$ and a real quarterly interest rate $r = 1.35\%$. The rate at which searching agents find a job and firms fill a vacancy amount to $q = 0.783$ and $\vartheta = 1.177$, respectively. The endogenous value of the consumption-capital ratio is equal to $c/k = 0.0743$.

2.7 Computation

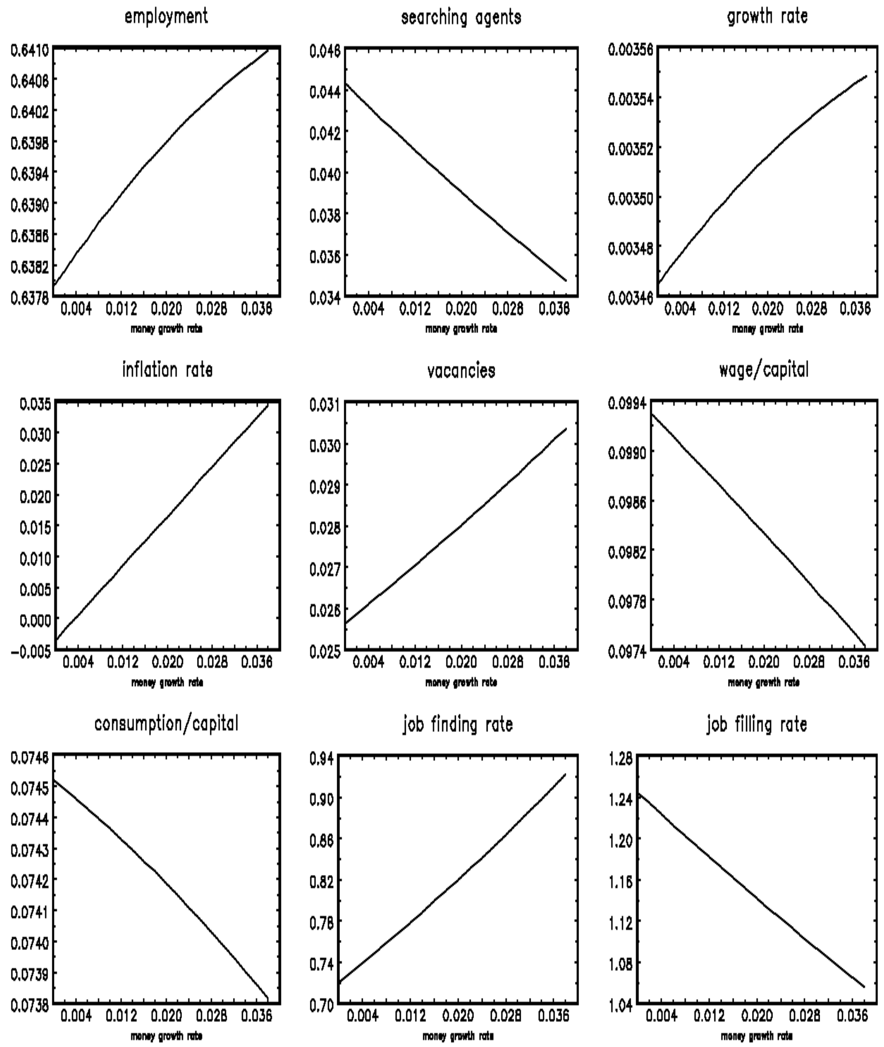
In section 4, the transition dynamics of employment n, the consumption/capital ratio c/k, search intensity s, and vacancies v following a change in the money growth rate μ are presented. In order to solve for the transition dynamics, we have to solve a two-point boundary value problem. For the one sluggish variable, n, the inital condition n_0 is given. In the long run, the endogenous variables approach their new steady state values. Accordingly, the endpoints of all variables are known. Further, the new steady state is found to be locally stable for all cases considered in this paper. The Jacobian matrix of the differential equation system (21)-(25) has one negative and four positive eigenvalues. The numerical two-point boundary value problem is solved with the method of reverse shooting.[3] For this reason, we perturbated the new steady state $(\tilde{c}_\infty, \pi, \tilde{\lambda}_{F\infty}, n_\infty, \tilde{\lambda}_{n\infty})$ by the magnitude of 10^{-8} and moved backwards in time (simply by solving the system $\dot{x} = -f(x)$ rather than $\dot{x} = f(x)$). The differential equation system was solved with the standard Runge-Kutta method of order 4.

3 Balanced growth analysis

In this section, we study the properties of the balanced growth path. Along the balanced growth path, the extensive variables c, k, y, w, and m grow at constant rates, while the intensive variables such as search effort s, employment n, vacancies v, the rate of inflation π, and the interest rate r are constant. The effects of a change in the money growth rate on equilibrium values of the endogenous variables are presented in figure 1.

[3] A description of this technique is provided by Judd (1998).

Figure 1: Balanced growth effects of the money growth rate μ



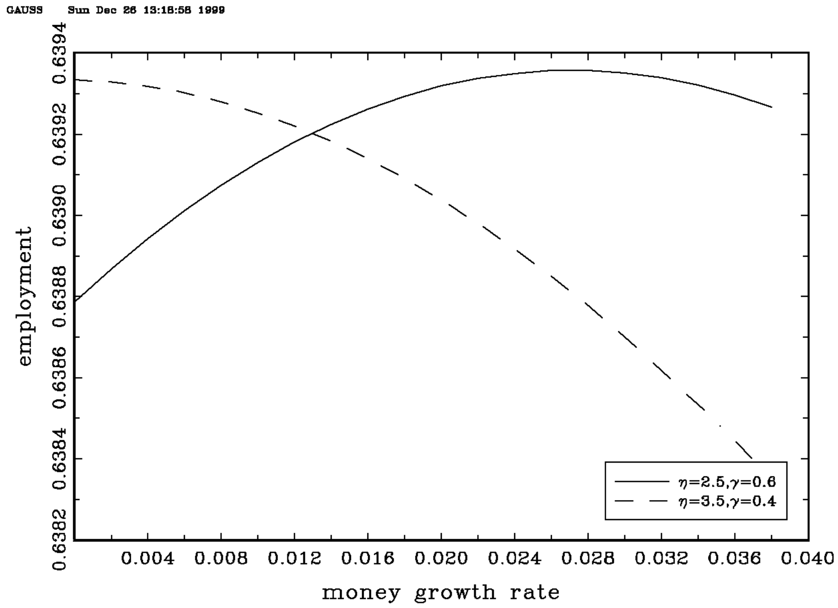
Following a rise in the money growth rate μ, the rate of inflation π goes up. As a consequence, agents increase leisure by reducing their search effort s. For given job finding probability q, this effect reduces equilibrium employment $n = qs/\theta$. However, the job finding probability q increases as firms increase their vacancies v. For the benchmark calibration, the latter effect dominates and equilibrium employment increases. There are two opposing effects of a higher inflation rate on the optimal number of posted vacancies. On the one hand, there are less agents searching for a job reducing the rate ϑ at which firms can fill a vacancy. On the other hand, high inflation reduces the optimal level of consumption. Households substitute capital k for real money balances m. As a consequence, both the marginal disutility from working and the reservation wage decrease. According to (19), wages fall. Even though the rate at which agents fill their vacancies decreases, the fall in wages induces firms to increase their hiring activities.

The quantitative effects are modest. A 4% increase of the quarterly money growth rate which corresponds to almost a 4% increase in the quarterly inflation rate, raises employment by less than 1%. In equilibrium, the growth rate is a simple positive function of employment, $g = \alpha A n^{1-\alpha} - \delta - \rho$. Accordingly, the growth rate increases as well. Following an increase of the money growth rate from $\mu = 1.3\%$ to $\mu = 2.3\%$, for example, the growth rate rises from $g = 0.350\%$ to $g = 0.352\%$.

Our qualitative results as presented in figure 1 are robust with regard to the choice of the parameters except for the two parameters η and γ.[4] If the labor supply elasticity ϵ increases (ie η decreases), the decline of the search effort s following a rise in inflation is more pronounced. Similarly, a decline in the elasticity of vacancy in job matches γ results in a smaller response of firms' vacancy posting to lower wages w. As a consequence, for lower values of η and γ, the employment effect as resulting from the decline in search effort may overcompensate the employment effect stemming from the increase in vacancies. Figure 2 illustrates the effects of money growth on equilibrium employment (and, hence, the growth rate of the economy) for $(\eta, \gamma) = (2.5, 0.6)$ and $(\eta, \gamma) = (3.5, 0.4)$. For these two parameter combinations, equilibrium employment n falls for higher rates of inflation.

[4] More specifically, following a change of either $\lambda \in \{0.25, 0.6\}$, $\kappa \in \{1, 5\}$, or $\psi \in \{0.25, 1\}$, employment and growth are found to be a positive function of moderate money growth rates $\mu \in [0, 0.04]$.

Figure 2: Sensitivity analysis for η=2.5 and γ=0.4



As our main result from the balanced-growth analysis, we concede that the growth-maximizing monetary policy depends crucially on the elasticity of labor supply and the elasticity of vacancies in job matches and may well imply strictly positive inflation rates.[5] For both critical parameters, empirical estimates vary considerably. Our benchmark case for the labor supply elasticity $\epsilon = 0.4$ is taken from Killingsworth (1983) and has been applied by Shi/Wen (1999) in computable general equilibrium (CGE) analysis. Other CGE studies even apply higher values of ϵ. For example, Lucas (1990) uses a value of $\epsilon \in [0.5, 5]$, while Jones et al. (1993) even take a value as high as 7.09 for the upper limit of ϵ. Similarly, the elasticity of vacancies in job matches γ has been estimated with British data and with US data by Pissarides (1986) and Blanchard/Diamond (1989) finding values of $\gamma = 0.4$ and $\gamma = 0.6$, respectively. The latter value has been applied by Shi/Wen (1999) which our benchmark calibration is based upon. Therefore, we carefully conclude that

[5] The qualitative effect of the inflation rate on economic growth further depends on the specification of the CIA constraint. If also investment purchases must be made with currency, $P(\psi_0 c + \psi_1 \dot{k}) \leq M$, inflation has a more adverse effect on economic growth.

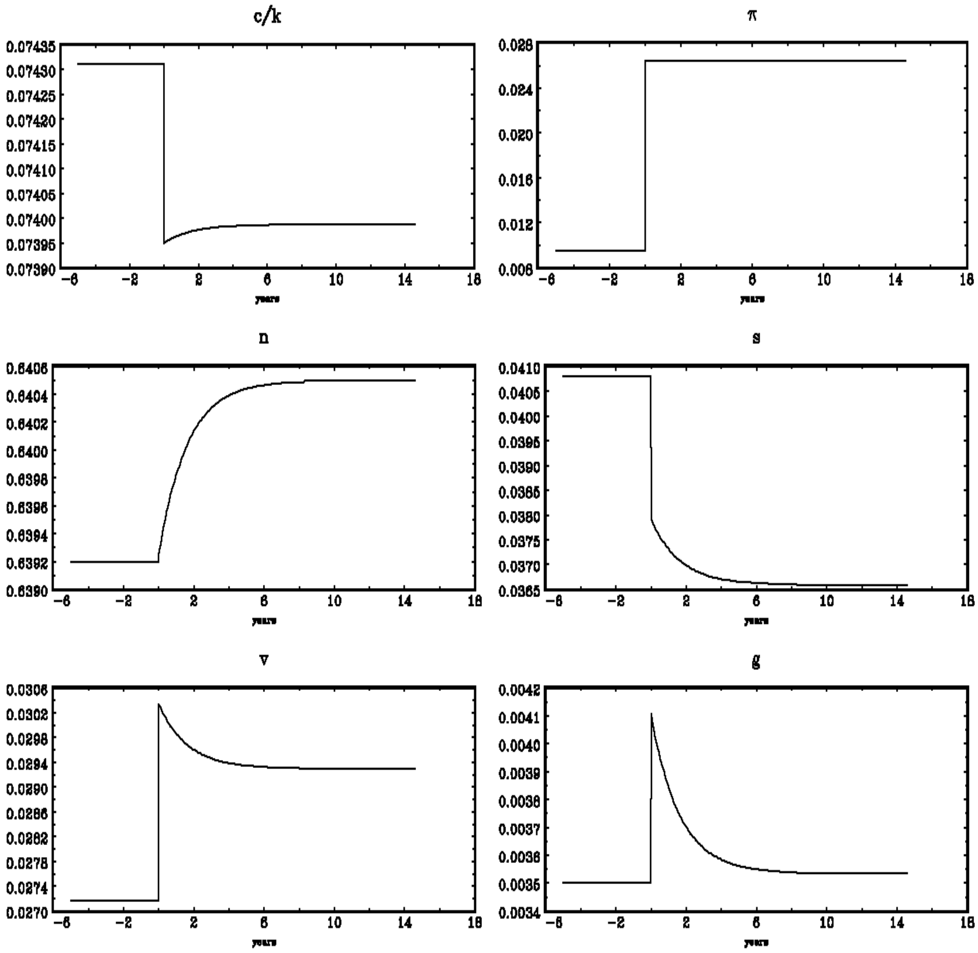
empirical observations do not allow for the firm conclusion that moderate rates of inflation and money growth well in excess of zero harm economic growth in the presence of labor market frictions.

4 Dynamic welfare analysis of inflation

In this section, the welfare costs of moderate rates of inflation are examined. As argued in the introduction, a sensible welfare analysis cannot ignore the transition dynamics following a change in monetary policy. For this reason, the dynamics are computed with the help of equations (21)-(25). The effects of a permanent and unexpected increase of the quarterly money growth rate from 1.3% to 3% at time 0 are graphed in figure 3 for the variables c/k, π, n, s, v, and g. The transition takes approximately 10 years. After this period, the deviation of the endogenous variables employment n, the consumption-capital ratio c/k, search effort s, and vacancies v are all less than 0.1% from the new steady-state values (compare figure 3).

In the new steady state, the values of c/k and s are lower than in the old steady state due to increased inflation tax, while the values of the new inflation rate π, the growth rate g, employment n, and vacancies v are higher. Immediately following the increase of money growth at time 0, the search intensity s and consumption c fall by 10% and 7.5%, respectively. As employment n gradually builds up, agents reduce their search effort s further. As a consequence, firms can fill their vacancies at a lower rate ϑ and reduce their posting of vacancies. For this reason, vacancies overshoot their new steady state value following a rise in inflation π. During transition, the growth rate also overshoots its new steady state value as i) agents increase their savings and ii) employment builds up (even though vacancies fall).

Figure 3: Transitional dynamics following an increase of μ to 3%

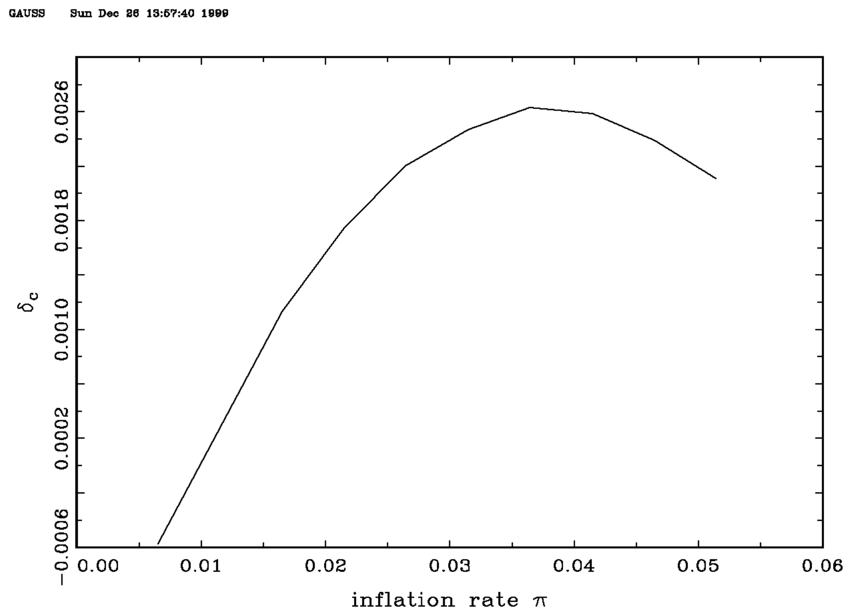


Welfare effects of a change in monetary policy are calculated by the change in utility (1) of the representative agent. In particular, the welfare gain of moving from allocation $\{c, s, n\}_0^\infty$ to allocation $\{c', s', n'\}_0^\infty$ will be measured by the consumption equivalent increase δ_c as suggested by McGrattan (1994):

$$\int_0^\infty U\left((1+\delta_c)c, s, n\right) e^{-\rho t} dt = \int_0^\infty U\left(c', s', n'\right) e^{-\rho t} dt. \tag{31}$$

There are three effects of a rise in the money growth rate on welfare. First, the level of consumption decreases and consumption will be lower for many years following the change in monetary policy. Second, agents increase leisure instantaneously. Third, in the long run, consumption grows at a higher rate (in our benchmark case). The net effect of change in the money growth rate are graphed in figure 4. For the benchmark calibration, the optimal money growth rate is well above zero and equals 3.5%. The welfare gain from a permanent change to the optimal rate of inflation implies welfare gains equal to $\delta_c = 0.26\%$. Notice that the sign of the effect is opposite to the one found in previous studies on monetary growth models such as Gomme (1993) and Wu/Zhang (1998).

Figure 4: Welfare effects of moderate inflation



Unsurprisingly, the optimal money growth rate depends crucially on the elasticity of labor supply and the elasticity of vacancies in job matches. Table 2 summarizes the effects of moderate inflation for different values of (η, γ). For lower values of η and γ, it may well be optimal to reduce the rate of inflation to zero (we only considered nonnegative inflation rates, $\pi \geq 0$). The reduction of the quarterly inflation rate from 0.95% to 0% for a lower, but still

Table 2

Sensitivity analysis of optimal monetary policy

	$\eta = 3.5$ $\gamma = 0.6$	$\eta = 3.5$ $\gamma = 0.4$	$\eta = 2.5$ $\gamma = 0.6$	$\eta = 2.5$ $\gamma = 0.4$
optimal nonnegative inflation rate	3.5%	0%	1.6%	0%
δ_c	0.26%	0.30%	0.02%	0.41%

empirically reasonable values of $\eta = 2.5$ and $\gamma = 0.4$ results in a welfare gain of 0.41% of total consumption. Again, the analysis in this section suggests that firm conclusions about the qualitative effects of a zero-inflation policy on welfare are difficult to obtain given the variety of empirical evidence on the values of the labor supply elasticity and the elasticity of vacancies in job matches.

5 Conclusion

This paper has examined the welfare costs of inflation in a search equilibrium model. Our results challenge conventional wisdom that higher rates of inflation reduce growth and welfare unanimously. In fact, for our benchmark calibration of the US economy, economic growth increases with moderate rates of inflation and the optimal quarterly inflation rate amounts to 3.5%. We show that two parameters from the labor market are crucial to the understanding of the qualitative effects of monetary policy. The lower the elasticity of labor supply and the elasticity of vacancies in job matches, the more likely is inflation to have adverse effects on both economic growth and welfare.

References

Blanchard, O.J., and P. Diamond, 1989, The Beveridge Curve, *Brookings Papers and Proceedings*, vol. 69, 1-78.

Cooley, T.F., and G.D. Hansen, 1995, Money and the business cycle, in: Cooley, T.F., (ed.), *Frontiers of Business Cycle Research*, Princeton University Press: Princeton, 175-216.

Gomme, P., 1993, Money and growth revisited: measuring the costs of inflation in an endogenous growth model, *Journal of Monetary Economics*, vol. 32, 51-77.

Grüner, H.P., and B. Heer, 2000, Optimal flat-rate taxes on capital — a reexamination of Lucas' supply-side model, *Oxford Economic Papers*, vol. 52, 289-305.

Jones, L.E., R.E. Manuelli, and P.E. Rossi, 1993, Optimal Taxation in Models of Endogenous Growth, *Journal of Political Economy*, vol. 101, 485-517.

Judd, K.L., 1998, *Numerical Methods in Economics*, MIT Press: Cambridge, MA.

Killingsworth, M.R., 1983, *Labor Supply*, Cambridge University Press: Cambridge.

Lucas, R.E., 1988, On the mechanics of economic growth, *Journal of Monetary Economics*, vol. 22, 4-42.

Lucas, R.E., 1990, Supply-Side Economics: An Analytical Review, *Oxford Economic Papers*, vol. 42, 293-316.

McGrattan, E., 1994, The macroeconomic effects of distortionary taxation, *Journal of Monetary Economics*, vol. 55, 491-514.

Pissarides, C.A., 1986, Unemployment and Vacancies in Britain, *Economic Policy*, vol. 3, 499-559.

Pissarides, C.A., 1990, *Equilibrium Unemployment Theory*, Basil Blackwell: Cambridge, MA.

Romer, P.M., 1986, Increasing returns and long run growth, *Journal of Political Economy*, vol. 94, 1002-38.

Shi, S., and Q. Wen, 1997, Labor market search and capital accumulation: Some analytical results, *Journal of Economic Dynamics and Control*, vol. 21, 1747-76.

Shi, S., and Q. Wen, 1999, Labor market search and the dynamic effects of taxes and subsidies, *Journal of Monetary Economics*, vol. 43, 457-95.

Tobin, J., 1965, Money and economic growth, *Econometrica*, vol. 33, 671-84.

Wu, Y., and J. Zhang, 1998, Endogenous growth and the welfare costs of inflation: a reconsideration, *Journal of Economic Dynamics and Control*, vol. 22, 465-82.