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# Bargaining and Conflict with Up-front Investments: How Power Asymmetries Matter

## Abstract

We examine settings - such as litigation, labor relations, or arming and war - in which players first make non-contractible up-front investments to improve their bargaining position and gain advantage for possible future conflict. Bargaining is efficient ex post, but we show that a player may prefer Conflict ex ante if there are sufficient asymmetries in strength. There are two sources of this finding. First, up-front investments are more dissimilar between players under Conflict, and they are lower than under Bargaining when one player is much stronger than the other. Second, the probability of the stronger player winning in Conflict is higher than the share received under Nash bargaining. We thus provide a rationale for conflict to occur under complete information that does not depend on long-term commitment problems. Greater balance in institutional support for different sides is more likely to maintain peace and settlements.

JEL-Codes: C700, D740, J530, K410.

Keywords: power asymmetries, war, litigation, contests.

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# 1 Introduction

Why does conflict occur? At its worst it is catastrophic; at its best, harmful and wasteful. Economists and other social scientists trying to understand the phenomena first posited asymmetric information as a cause of conflict. More recently, a second set of causes that have been examined involve incomplete contracting and the inability of adversaries to commit.[1] Within the broad category of incomplete contracting—the inability of adversaries to write long-term binding contracts—we show how large asymmetries in power could induce conflict.

Power asymmetries have been invoked as a cause of conflict in many settings. Wrangham and Glowacki (2012) present evidence that groups of chimpanzees and groups of human hunter-gatherers both follow the same strategy: attack only when you have overwhelming superiority over your adversary. Considerably more research—and controversy—surrounds the issue of power asymmetry in political science and international relations. Depending on the context, great power asymmetry can induce war or facilitate peace (see Wagner, 1994, for a synthetic view). The experience of U.S. foreign wars over the past forty years is consistent with wars taking place when there is overwhelming power asymmetry. All the wars in which the U.S. has been involved, at least since the invasion of Grenada in 1983, were against vastly inferior adversaries, militarily speaking. Such power asymmetry has been manifest in practice and even codified in semi-official policy such as the "Powell doctrine," named after former U.S. Secretary of State Colin Powell (see O'Sullivan, 2009). Of course, we neither claim that power asymmetries always lead to war nor that the particular mechanism we examine is the only one that may lead to war in the presence of asymmetries. Before we discuss the mechanism that we analyze, we first present the settings that we examine.

We consider economic and political environments in which the set of bargaining alternatives—the utility possibilities set—and the disagreement

[1] An early form of the asymmetric information argument can be found in Wittman (1979) in the context of wars. It was subsequently and extensively developed through game-theoretic models during the 1980s. Cramton and Tracy (2003) present the argument in the context of industrial conflict. Sanchez-Pages (2009) shows how information revelation can be part of the bargaining process itself.

Different forms of the inability to commit argument have been advanced by Garfinkel and Skaperdas (2000), Robson and Skaperdas (2008), Bevia and Corchon (2010), McBride and Skaperdas (2014), Kimbrough et al. (2015), and Garfinkel and Syropoulos (2018). Smith et al. (2014) show how conflict is less likely when the costs of conflict are endogenous to arming. Fearon (1995) and Skaperdas (2006) present overviews of how conflict could come about.

point are endogenous in the following sense:[2] Players first make up-front investments that determine both the range of alternatives available and each player's disagreement utility through probabilities of winning and losing a contest or conflict. Then, in the case of conflict, variable resources have to be expended to determine the probabilities of winning. We are thinking of the up-front investments as "capital" and the variable resources as "labor," combining through a production function that determines each player's total effort and chance of winning a contest. Both of the inputs are non-contractible. Conditional on having the opportunity to bargain, the players would have no incentive to choose their disagreement utility and enter into conflict. Yet, entering into conflict might be ex ante preferable by at least one of the players, causing them to commit to conflict. Our functional assumptions have been axiomatized by Rai and Sarin (2009) and Arbatskaya and Mialon (2010). Münster (2007) had earlier provided an analysis of such contests for all-pay auctions, Fu and Lu (2009) allow for investments that lower the marginal cost of effort, whereas Arbatskaya and Mialon (2012) analyzed a version of our Conflict game but without Bargaining.

Examples of the settings that our framework fits include the following:

- Military expenditures and wars. States and other parties to conflict invest in hardware, military personnel, and organizational infrastructure regardless of whether war is coming or not. If it does come, additional resources are deployed, therefore making war costly (beyond destruction and additional costs).

- Litigation and going to court. In such settings, the up-front investments cover the hiring of lawyers and expenditures on exploration and discovery; the variable inputs would include the extra expenses of going to court. We examine the conditions under which going to court might be preferable to settling out of court.

- Interactions of unions and firms. Unions expend resources on organizational infrastructure, building solidarity among their members, publicizing their perspective to the press and the wider public, and building

[2]Nash (1950) defined the bargaining problem in terms of two objects: The set of alternatives or utility possibilities set for the two players and the disagreement or threat point. These two objects can be derived in any economic environment that involves production, trade, even conflict, in either deterministic or stochastic environments. The beauty of Nash's approach (initially perhaps not sufficiently appreciated) was distilling such a variety of economic contexts in these two objects, as well as defining his Nash bargaining solution within the same paper. Much research on bargaining has concentrated on defining other bargaining solutions to that of Nash as well as developing non-cooperative bargaining games.

contingency funds in case of a strike. Firms hire lawyers and other experts to help handle relations with unions and the press and, in anticipation of work stoppage, they may build inventories above normal levels. These are representative of up-front investments meant to improve the respective side's bargaining position even if conflict is not expected.

- Lobbying and policy formation. Lobbying firms and think tanks invest in office space, researchers, lawyers, secretaries, public relations specialists, and, of course, lobbyists. This infrastructure behind the lobbyists themselves can be considered up-front investment that is used to promote different policies and bills. Such up-front investment is usually deployed on a range of policies, but the issue of whether to go all out and try to win or compromise with other interest groups is a choice they face.

With such settings in mind, we examine and compare the equilibria of two games, a Conflict game and a Bargaining game. In the Conflict game, each player first makes up-front investments that are mutually observed before conflict ensues. The two sides then devote additional variable resources to conflict. Under the Bargaining game, the two players make up-front investments and then negotiate to divide the prize. They do so, however, under the threat of conflict whereby the disagreement payoffs are determined by the variable resource choices under the subgame perfect equilibrium of the Conflict game. No conflict actually takes place in the Bargaining game, hence there are no additional resources expended beyond the up-front investments.

We find that up-front investments can differ significantly between Bargaining and Conflict, and the difference in investment levels between players is itself quite different between the two games. In particular, investments are similar between the players under Bargaining – only reflecting the ratio of player's marginal costs – but tend to differ substantially under Conflict when the players have significant differences in their underlying strength. Strength is measured conveniently by a summary index that reflects the following: differences in marginal costs of investments and variable resources, the effectiveness of one's efforts compared to that of the other player, and the relative importance of the two inputs. As will become clear in the model, the greater the difference in strength, the higher is the difference in the two player's investments but also the lower are total investments.

Partly as a result of the greater asymmetry of investments under Conflict, the probability of winning for the stronger player under Conflict is greater than the share the same player receives under Bargaining. Given then that the stronger player can have lower investments under Conflict and receives in expectation a bigger share of the pie, he or she would prefer Conflict to Bargaining. That occurs when there is sufficient asymmetry in strength, and up-front investments are sufficiently important to the production of effort. However, for high levels of this latter factor the weaker player receives a negative payoff under Conflict and in such a case would prefer not to participate in the game in the first place.[3] There are even cases in which total equilibrium payoffs under Conflict are higher than total payoffs under Bargaining, although this is driven by the payoff of the stronger player.

For cases without high differences in strength, both players prefer Bargaining. Interestingly, it is possible to have strong participants *and* peaceful settlement. The key, to the extent that the context allows it, is to create balance between agents and eliminate sources of bias in the overall environment. This result is applicable to the union-management context where economists have tried for a long time to explain the occurrence of strikes. Supreme Court Justice Louis Brandeis took this view even at the height of labor unrest. He argued, "Strong, responsible unions are essential to industrial fair play. Without them the labor bargain is wholly one-sided. The parties to the labor contract must be nearly equal in strength if justice is to be worked out, and this means that the workers must be organized and that their organizations must be recognized by employers as a condition precedent to industrial peace."[4] Hence it is the threat of strike and the balance of power that ironically establish conditions for harmony. To our knowledge, this is the first paper to present a conflict equilibrium as applied to labor relations under complete information without relying on complicated punishment strategies (e.g. Fernandez and Glazer, 1991). We show conditions under which work stoppage may be rational and preferred under complete information.

The possibility of power asymmetries inducing conflict has been analyzed experimentally by a number of papers, even though none of the experiments test the particular mechanism of up-front investments that we examine here. Sieberg et al. (2013) examined an alternating-offers bargaining game in which disagreement implies that the two players have different exogenous probabilities of winning. Although conflict occurred in the experiments more than predicted by theory, greater asymmetry did not induce more conflict. Kimbrough et al. (2014) and Herbst et al. (2017) allowed possibly asymmetric probabilities of winning in conflict to be endogenously determined. Kimbrough et al. (2014) employed an ex ante random device instead of a bar-

[3]Contrary to typical "Tullock" contests in which equilibrium payoffs are positive regardless of the number of players (see, e,g., Konrad, 2009).

[4]Brandeis (1934)

gaining game to resolve conflict, and found that greater asymmetries induced some additional conflict but not as much as expected theoretically. Herbst et al. (2017) allowed for exogenous divisions of the surplus (that reflect asymmetries) as well as endogenous bargaining (Nash demand game). Overall, they found power asymmetries induced conflict only in the case of endogenous bargaining, a result they attribute to the strategic uncertainty inherent in endogenous bargaining.

After specifying the two games—Conflict and Bargaining—in the next section, we completely characterize the equilibria of each and then make comparisons between equilibrium payoffs. As a robustness check, in the subsequent section we allow for a different bargaining protocol in which one player has all the bargaining power so that they play an ultimatum game. It turns out that when the stronger player is not the proposer (the one with the bargaining power), then that player almost always prefers Conflict. Thus, any strength imbalance in Conflict, even slight, that is not reflected in the bargaining rule leads to Conflict. In a Supplementary Appendix we examine a more general production function of effort and show that our qualitative results carry through.

# 2 The Conflict and Bargaining Games

Two sides, 1 and 2, have a surplus $S$ that they can either fight over or divide under the threat of a fight. The two sides cannot write a costless contract not to fight. Or, another way to put it is that fighting efforts (e.g., military expenditures in the case of warfare, or litigation expenditures in the case of litigation) are non-contractible. However, contracts to divide the surplus under the threat of conflict, in which the two sides have prepared for fighting, are possible.

For positive efforts $R_1$ and $R_2$, the probability of player 1 winning the whole surplus $S$ in conflict is

$$P(R_1, R_2) = \frac{\varepsilon R_1}{\varepsilon R_1 + R_2} \tag{1}$$

whereas player $2's$ winning probability is $1 - P(R_1, R_2) = \frac{R_2}{\varepsilon R_1 + R_2}$ and where $\varepsilon > 0$ is a source of asymmetry in conflict; when $\varepsilon > 1$ player 1 has the advantage and when $\varepsilon < 1$ player 2 has the advantage. The sources of asymmetries can vary depending on the context, of course. For the four cases of contests we have discussed, there are many examples of sources of asymmetry. In warfare, a defensive position or technological superiority are typical sources of advantage (Grossman, 2001). In litigation, having the truth with you (Hirshleifer and Osborne, 2001) or the degree of protection of property rights are sources of advantage, with a higher $\varepsilon$ implying a higher level of property rights protection (Robson and Skaperdas, 2008). In union-firm interactions, police intervention whether lawful or unlawful, judiciary bias either from an individual judge or the legal system as a whole, public opinion concerning unions, and, for modern times in the U.S., the composition of the NLRB are sources of advantage and disadvantage for the two sides (Gourevitch, 2015; Cooke et al., 1995) In the case of lobbying, access to and disposition from government officials on the part of different lobbies are sources of advantage and disadvantage.

Following Arbatskaya and Mialon (2010), the fighting efforts are functions of two variables. To allow for analytical solutions, we consider the functional form $R_i = K_i^\alpha L_i$ ($\alpha \in (0,1); i = 1,2$), where $K_i$ represents the up-front investment of player $i$, the coefficient $\alpha$ increases the marginal productivity of $K_i$, and $L_i$ represents their variable effort in the event of conflict. (In a Supplementary Appendix we examine the more general case of $R_i = K_i^\alpha L_i^\gamma$, $\gamma \in (0,1]$, with qualitatively similar results.) The players first make the up-front investments $K_1$ and $K_2$ and only if they were to engage in conflict would they choose variable levels of effort $L_1$ and $L_2$.

For simplicity, we compare two games, one in which Conflict ensues and one in which there is a Bargaining agreement (under the threat of Conflict). [5] The timing of the game under Conflict is the following:

1. Each player chooses whether to enter the game and make up-front investments or not. If a player does not enter the game, he or she receives a payoff of 0. If only one player chooses to enter, then that player receives the surplus. If both players choose to enter the game, they go to the next stage 2.

2. The two players simultaneously choose up-front investments $K_1$ and $K_2$.

3. The players enter into Conflict and choose variable fighting efforts $L_1$ and $L_2$. The total effort of each player $i$ is determined by $R_i = K_i^\alpha L_i$ and (1) provides the probability of winning for player 1.

The players are risk neutral and have constant marginal costs of up-front investments $r_1$ and $r_2$ and constant marginal costs of variable fighting efforts

[5] We could modify the timing and make the choice between Conflict and Bargaining endogenous to a larger game, with essentially the same results but with some added complication. The equilibrium choices of the two main variables will be the same but there might be some parameter values under which the Bargaining game would not be a subgame perfect equilibrium of the larger game.

$w_1$ and $w_2$. Then, given (1) and the way efforts are determined, the expected payoffs under Conflict are as follows:

$$
\begin{aligned}
V_1^C(K_1, L_1, K_2, L_2) &= \frac{\varepsilon K_1^\alpha L_1}{\varepsilon K_1^\alpha L_1 + K_2^\alpha L_2} S - r_1 K_1 - w_1 L_1 \\
V_2^C(K_1, L_1, K_2, L_2) &= \frac{K_2^\alpha L_2}{\varepsilon K_1^\alpha L_1 + K_2^\alpha L_2} S - r_2 K_2 - w_2 L_2
\end{aligned} \tag{2}
$$

While the surplus is in principle divisible under Conflict the nature of the game is such that winner takes all given the probabilistic function in (1). It could be that one or both players have engaged in a "burn-the-bridges" act (Schelling, 1960) or there is another commitment mechanism that prevents bargaining and a division of the surplus. These expressions apply in the event of Conflict, but for a given choice of up front investments they also form the threat point of a possible bargaining agreement under the Bargaining game which has the following timing:

1. Each player chooses whether to enter the game and make up-front investments or not. If a player does not enter the game, he or she receives a payoff of 0. If only one player chooses to enter, then that player receives the surplus. If both players choose to enter the game, they go to the next stage 2.

2. The two players simultaneously choose up-front investments $K_1$ and $K_2$.

3. The players arrive at a division of the surplus $S$ to be described below.

When the two sides reach a bargaining agreement, they do not pay the variable costs of conflict, $L_1$ and $L_2$, although they will already have paid their up-front investments, $K_1$ and $K_2$. In Bargaining, the disagreement payoffs are the Conflict payoffs that would be the induced subgame payoffs in (2) for the given combination of $K_1$ and $K_2$ that the two players have already chosen. Given the disagreement payoffs, here we suppose that the shares of $S$ are determined by the split-the-difference rule. Because of risk neutrality, this rule coincides with the Nash bargaining solution or of any other symmetric bargaining solution, as well as any noncooperative bargaining games (such as alternating-offers games) that might approximate such a rule.[6] (In section

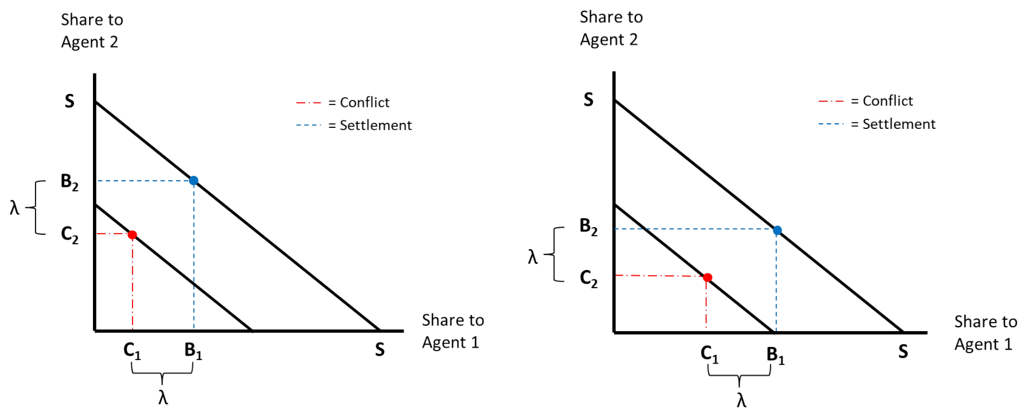
[6] Anbarci et al. (2002) show how different bargaining solutions can induce different outcomes when the utility possibilities frontier is strictly concave (which is not so in our case). Allison (2018) shows how alternating-offers games do not necessarily approximate a bargaining solution and might actually be more efficient than those employing axiomatic solutions as we do here.

five, we examine the case in which one side has all the bargaining power in an ultimatum game).

Define $\beta(K_1, K_2)$ as player 1's share under bargaining and $(1-\beta(K_1, K_2))$ as player 2's share. Given the split-the-difference rule, the share $\beta(K_1, K_2)$ is defined by

$$\begin{aligned} &\beta(K_1, K_2)S - P(K_1^\alpha L_1^*(K_1, K_2), K_2^\alpha L_2^*(K_1, K_2))S + w_1 L_1^*(K_1, K_2) \\ &= [1-\beta(K_1, K_2)]S - [1 - P(K_1^\alpha L_1^*(K_1, K_2), K_2^\alpha L_2^*(K_1, K_2))]S \\ &\qquad + w_2 L_2^*(K_1, K_2) \end{aligned} \tag{3}$$

where $L_i^*(K_1, K_2)$ for $i = 1, 2$ represent the Conflict subgame perfect equilibrium choices of variable efforts for any combination $(K_1, K_2)$. The probability of winning for player 1 for any combination $(K_1, K_2)$ is thus $P(K_1^\alpha L_1^*(K_1, K_2), K_2^\alpha L_2^*(K_1, K_2))$. Figure 1 provides a graphical representation of the setting.




(a) Stronger player 2

(b) Stronger player 1

Figure 1: The Threat Point and Split-the-Difference Rule

The outside line represents all the possible splits that the two players could achieve under Bargaining. It is Pareto superior to the inside line which is all the possible splits under Conflict. Each player will try to move the inside dot closer to their axis in order to secure a favorable result. Doing so will also drag the outside dot in their direction. Strategically, they accomplish this by committing to preparations (the up-front investments) that will boost their strength under Conflict and thus shift both the conflict split and the bargaining split to their side.

This structure emphasizes the role of threats in the bargaining process. Yet this raises questions for strategy: how do the two sides prepare differently if their aim is to affect the threat point rather than the Conflict outcome? It is far from guaranteed that investments will be the same when settlement is the expected outcome. We will return to this problem momentarily. Given this bargaining rule, it can be shown that the share of player 1 under Bargaining is

$$\beta(K_1,K_2) = P(K_1^\alpha L_1^*(K_1,K_2), K_2^\alpha L_2^*(K_1,K_2)) + \frac{w_2 L_2^*(K_1,K_2)}{2S} - \frac{w_1 L_1^*(K_1,K_2)}{2S} \tag{4}$$

This share equals the player's own probability of winning in the event of conflict, suitably adjusted by the variable costs of conflict of the two players ($w_i L_i^*(K_1,K_2)$ for player $i = 1,2$). In particular, a higher variable cost of conflict disadvantages a player and advantages his opponent. Both the probabilities of winning and the variable costs of conflict depend on the up-front investments $(K_1,K_2)$ in ways that we cannot a priori specify but which we plan to explore. What is clear, however, is that the probabilities of winning under Conflict can be expected to have different properties (in terms of the of their sensitivity to $(K_1,K_2)$) from those of the sharing function under Bargaining.

The payoff functions for the game under Bargaining are as follows:

$$\begin{aligned} V_1^B(K_1,K_2) &= \beta(K_1,K_2)S - r_1K_1 \\ V_2^B(K_1,K_2) &= [1-\beta(K_1,K_2)]S - r_2K_2 \end{aligned} \tag{5}$$

Note how the "sharing" function $\beta(K_1,K_2)$ depends both on the bargaining solution as well as the contest success function in (1) whereas Conflict payoffs in 2 depend solely on the (probabilistic) contest success function.

We now turn to analyzing each of the two games and then to comparing them.

# 3 Solving the Conflict Game

We use backwards induction to solve for a subgame perfect equilibrium. Begin by assuming a $(K_1,K_2)$ pair and let the players maximize their expected payoffs in (2) by the choice of their respective variable efforts $(L_1,L_2)$:[7]

$$\begin{aligned} V_1^C(L_1,L_2 \mid K_1,K_2) &= \frac{\varepsilon K_1^\alpha L_1}{\varepsilon K_1^\alpha L_1 + K_2^\alpha L_2} S - r_1K_1 - w_1L_1 \\ V_2^C(L_1,L_2 \mid K_1,K_2) &= \frac{K_2^\alpha L_2}{\varepsilon K_1^\alpha L_1 + K_2^\alpha L_2} S - r_2K_2 - w_2L_2 \end{aligned}$$

[7]Please note that, at this stage, the problem of choosing variable efforts is equivalent to choosing total efforts $(R_1,R_2)$ (that, by (1), equal $(K_1^\alpha L_1, K_2^\alpha L_2)$) but with $K_1$ and $K_2$

The first-order conditions imply

$$K_1^\alpha K_2^\alpha L_2^* = \frac{w_1(\varepsilon K_1^\alpha L_1^* + K_2^\alpha L_2^*)^2}{S\varepsilon} \quad and \quad K_1^\alpha K_2^\alpha L_1^* = \frac{w_2(\varepsilon K_1^\alpha L_1^* + K_2^\alpha L_2^*)^2}{S\varepsilon}$$

Solving simultaneously, we have $L_1^* = \omega L_2^*$ (where $\omega \equiv \frac{w_2}{w_1}$) which makes for the following subgame perfect equilibrium choices:

$$\begin{aligned} L_1^*(K_1, K_2) &= \frac{\varepsilon\omega K_1^\alpha K_2^\alpha}{w_1(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2} S \\ L_2^*(K_1, K_2) &= \frac{\varepsilon K_1^\alpha K_2^\alpha}{w_1(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2} S \end{aligned} \tag{6}$$

Just as with $\varepsilon$, the higher is the ratio of costs $\omega$ the better it is for player 1. This makes the winning probability of player 1 a simple function of $K_1$ and $K_2$:

$$P^*(K_1, K_2) = \frac{\varepsilon\omega K_1^\alpha}{\varepsilon\omega K_1^\alpha + K_2^\alpha} \tag{7}$$

Continuing backwards, the players choose optimal up-front investments, $K_1$ and $K_2$ for the case of open conflict given the implied variable conflict costs in (6). That is, the Conflict payoff functions as a function of the up-front investments become:

$$\begin{aligned} V_1^C(K_1, L_1^*(K_1, K_2), K_2, L_2^*(K_1, K_2)) &= \frac{\varepsilon\omega K_1^\alpha}{\varepsilon\omega K_1^\alpha + K_2^\alpha} S - r_1 K_1 - \frac{\varepsilon\omega K_1^\alpha K_2^\alpha}{(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2} S \\ &= \frac{(\varepsilon\omega K_1^\alpha)^2}{(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2} S - r_1 K_1 \\ V_2^C(K_1, L_1^*(K_1, K_2), K_2, L_2^*(K_1, K_2)) &= \frac{K_2^\alpha}{\varepsilon\omega K_1^\alpha + K_2^\alpha} S - r_2 K_2 - \frac{\varepsilon\omega K_1^\alpha K_2^\alpha}{(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2} S \\ &= \frac{(K_2^\alpha)^2}{(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2} S - r_2 K_2 \end{aligned}$$

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fixed. The constrained payoffs are then

$$\begin{aligned} V_1^C(R_1, R_2 \mid (K_1, K_2)) &= \frac{\varepsilon R_1}{\varepsilon R_1 + R_2} S - r_1 K_1 - \frac{w_1}{K_1^\alpha} R_1 \\ V_2^C(R_1, R_2 \mid (K_1, K_2)) &= \frac{R_2}{\varepsilon R_1 + R_2} S - r_2 K_2 - \frac{w_2}{K_2^\alpha} R_2 \end{aligned}$$

What is notable from this re-writing of the problem at this stage of the game is that the up-front investments are interpretable as reducing the marginal cost of total effort ($\frac{wi}{K_i^\alpha}$ for player $i$). That is, in the conflict and contest literature, marginal costs could be thought of as being partly the result of previous investments and endowments that the players have inherited from the past. See Fu and Lu (2009) that formulate "pre-contest" investments as lowering marginal costs.

First order conditions for an equilibrium imply:

$$\varepsilon\omega K_1^{*2\alpha-1}K_2^{*\alpha} = \frac{r_1(\varepsilon\omega K_1^{*\alpha}+K_2^{*\alpha})^3}{2\alpha\varepsilon\omega S} \quad and \quad K_1^{*\alpha}K_2^{*2\alpha-1} = \frac{r_2(\varepsilon\omega K_1^{*\alpha}+K_2^{*\alpha})^3}{2\alpha\varepsilon\omega S}$$

resulting in the relationship $K_1^* = (\varepsilon\omega\rho)^{\frac{1}{1-\alpha}}K_2^* = \theta\rho K_2^*$ where $\rho \equiv r_2/r_1$ and $\theta \equiv (\varepsilon\omega)^{\frac{1}{1-\alpha}}\rho^{\frac{\alpha}{1-\alpha}}$. The parameter $\theta$ represents a summary indicator of the asymmetry across the two players which, as we shall see, enters in all the key equilibrium variables of the model. In terms of the components of $\theta$, the asymmetry in the contest success function ($\varepsilon$) plays a similar role to the ratio of marginal costs in variable efforts ($\omega$), whereas the ratio in marginal costs in up-front investments ($\rho$) has a smaller exponent (given that $\alpha < 1$). Overall, as with its component variables, $\theta > 1$ implies player 1 has the advantage whereas $\theta < 1$ implies that player 2 has the advantage.

We can show that equilibrium up-front investments equal:

$$\begin{aligned} K_1^* &= \frac{2\alpha\theta^2}{r_1(\theta+1)^3}S \\ K_2^* &= \frac{2\alpha\theta}{r_2(\theta+1)^3}S \end{aligned} \tag{8}$$

Then, the (ex-ante) equilibrium probability of player 1 winning reduces to a function of the three sources of asymmetry:

$$P(K_1^*, K_2^*) = \frac{\theta}{\theta+1} \tag{9}$$

By substitution, we can obtain the subgame equilibrium variable fighting efforts:

$$\begin{aligned} L_1^* \equiv L_1^*(K_1^*, K_2^*) &= \frac{\theta}{w_1(\theta+1)^2}S \\ L_2^* \equiv L_2^*(K_1^*, K_2^*) &= \frac{\theta}{w_2(\theta+1)^2}S \end{aligned} \tag{10}$$

As already noted, variable fighting efforts differ across the two players only in terms of the ratio of marginal costs of these efforts (i.e., $L_1^* = \omega L_2^*$). By contrast, the difference in up-front investments does not just depend on the ratio of marginal costs of these efforts ($\rho$) but depends on the overall asymmetry parameter $\theta$ as well so that, as previously noted, $K_1^* = \theta\rho K_2^*$. As we shall see later, this asymmetry in up-front investments does not exist in the case of Bargaining.

Using the equilibrium values for the efforts, the expected Conflict equilibrium payoffs can be shown to be:

$$\begin{aligned} V_1^{C*} &= \frac{\theta^2(\theta+1-2\alpha)}{(\theta+1)^3}S \\ V_2^{C*} &= \frac{\theta(1-2\alpha)+1}{(\theta+1)^3}S \end{aligned} \tag{11}$$

Note for $2\alpha \leq 1$, both payoffs are guaranteed to be positive. However, for $2\alpha > 1$ they are not guaranteed to be so and therefore there might be an incentive for one player to not enter the Conflict game at all. In particular, when player 1 has the advantage ($\theta > 1$), it can be seen from (11) that player 1's equilibrium payoff is always positive but for player 2 it is only so if $\theta < \frac{1}{2\alpha-1}$. Similarly, when player 2 has the advantage $V_2^*$ is always positive but $V_1^*$ is positive only if $\theta > 2\alpha - 1$. Thus, for $\alpha > \frac{1}{2}$ a Conflict equilibrium exists only if one player does not have too high an advantage over the other; otherwise the weaker player will choose not to enter the game.

We summarize the main properties of the Conflict game equilibrium as a Proposition.

**Proposition 1:** *(i) A unique equilibrium of the Conflict game exists in which both players participate when* $\alpha \leq 1/2$ *and when* $\theta \in (2\alpha - 1, \frac{1}{2\alpha-1})$ *with* $\alpha > 1/2$. When $\alpha > 1/2$ and $\theta \in (0, 2\alpha - 1)$, *player 1 has negative payoff in the Conflict game. When* $\alpha > 1/2$ *and* $\theta \in (\frac{1}{2\alpha-1}, \infty)$, *player 2 has negative payoff in the Conflict game. The equilibrium payoffs are described in (11).*

*(ii) The equilibrium winning probabilities favor the stronger player so that* $P(K_1^*, K_2^*) = \frac{\theta}{\theta+1}$ (which is greater than 1/2 when $\theta > 1$ and less than 1/2 when $\theta < 1$)

*(iii) The effects of the asymmetry parameter* $\theta$ *on the equilibrium up-front investments are as follows:* $\frac{\partial K_1^*}{\partial \theta} \gtreqless 0$ *as* $\theta \lesseqgtr 2$ *and* $\frac{\partial K_2^*}{\partial \theta} \gtreqless 0$ *as* $\theta \lesseqgtr \frac{1}{2}$;

*(iv) The effects of the asymmetry parameter* $\theta$ *on the equilibrium variable conflict efforts are as follows: Both* $\frac{\partial L_1^*}{\partial \theta} \gtreqless 0$ *and* $\frac{\partial L_2^*}{\partial \theta} \gtreqless 0$ *as* $\theta \lesseqgtr 1$.
(Proof is in the Appendix)

Consistent with the greater asymmetry for the up-front investments, their levels are maximized at an asymmetry parameter $\theta$ that favors the player with the advantage (2 for player 1 and 1/2 for player 2, part *(iii)* of Proposition), whereas the variable fighting efforts are maximized at the symmetric level $\theta = 1$.

# 4 Solving the Bargaining Game

In the Bargaining game, the payoff functions are as in (5) with the bargaining share of player 1, $\beta(K_1, K_2)$ as defined in (4) and the continuation variable fighting efforts $L_i^*(K_1, K_2)s$ as in (6). It can then be shown that $\beta(K_1, K_2) = \frac{\varepsilon\omega K_1^\alpha}{\varepsilon\omega K_1^\alpha + K_2^\alpha}$ which is the same winning probability of player 1 in the Conflict game conditional on the up-front investments (i.e., $P^*(K_1, K_2)$ in (7)). The payoff functions under Bargaining then reduce to:

$$\begin{aligned} V_1^B(K_1, K_2) &= \frac{\varepsilon\omega K_1^\alpha}{\varepsilon\omega K_1^\alpha + K_2^\alpha} S - r_1 K_1 \\ V_2^B(K_1, K_2) &= \frac{K_2^\alpha}{\varepsilon\omega K_1^\alpha + K_2^\alpha} S - r_2 K_2 \end{aligned} \tag{12}$$

The Nash equilibrium conditions imply:

$$\begin{aligned} K_1^{\alpha-1} K_2^\alpha &= \frac{r_1(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2}{\alpha\varepsilon\omega S} \qquad \textit{and} \\ K_1^\alpha K_2^{\alpha-1} &= \frac{r_2(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2}{\alpha\varepsilon\omega S} \end{aligned}$$

which produce the relationship $\bar{K_1} = \rho\bar{K_2}$ and the following equilibrium expressions:

$$\begin{aligned} \bar{K_1} &= \frac{\alpha\theta^{1-\alpha}}{r_1(\theta^{1-\alpha} + 1)^2} S \\ \bar{K_2} &= \frac{\alpha\theta^{1-\alpha}}{r_2(\theta^{1-\alpha} + 1)^2} S \end{aligned} \tag{13}$$

The equilibrium share of player 1 then equals:

$$\beta(\bar{K_1}, \bar{K_2}) = \frac{\theta^{1-\alpha}}{\theta^{1-\alpha} + 1} \tag{14}$$

Note that this is less than $P(K_1^*, K_2^*) = \frac{\theta}{\theta+1}$ when $\theta > 1$ and greater than $P(K_1^*, K_2^*)$when $\theta < 1$. That is, the player with the advantage always receives a lower share of the surplus in the Bargaining game than she or he has equilibrium probability of winning in the Conflict game.

The equilibrium payoffs under Bargaining are then as follows:

$$\begin{aligned} \bar{V_1^B} &= \frac{\theta^{1-\alpha}(\theta^{1-\alpha} + 1 - \alpha)}{(\theta^{1-\alpha} + 1)^2} S \\ \bar{V_2^B} &= \frac{\theta^{1-\alpha}(1 - \alpha) + 1}{(\theta^{1-\alpha} + 1)^2} S \end{aligned} \tag{15}$$

Contrary to the case of the Conflict payoffs, the Bargaining payoffs are always positive and therefore both players would have an incentive to participate in the Bargaining game.

We summarize the main results of the Bargaining game in the following Proposition.

**Proposition 2:** *(i) A unique equilibrium of the Bargaining game exists in which both players participate for all parameter values. The equilibrium payoffs are described in (15).*

*(ii) The equilibrium shares under bargaining favor the stronger player so that* $\beta(\bar{K}_1, \bar{K}_2) = \frac{\theta^{1-\alpha}}{\theta^{1-\alpha}+1}$ (which is greater than 1/2 when $\theta > 1$ and less than 1/2 when $\theta < 1$).

*(iii) The effects of the asymmetry parameter* $\theta$ *on the equilibrium up-front investments are identical for the two players so that:* $\frac{\partial \bar{K}_1}{\partial \theta} \gtreqless 0$ *and* $\frac{\partial \bar{K}_2}{\partial \theta} \gtreqless 0$ *as* $\theta \lesseqgtr 1$.

(Proof is in the Appendix)

Contrary to the equilibrium up-front investments under Conflict, equilibrium up-front investments under Bargaining, as shown under *(iii)* move together as a function of the asymmetry parameter $\theta$. They are both maximal when strength is equal ($\theta = 1$). This result is similar to what occurs in simple contests where the efforts are greatest under symmetry but become lower as the asymmetry increases (see, for example, Konrad, 2009). As we shall shortly see, this is a key attribute in comparing payoffs under Conflict and Bargaining, to which we now turn.

# 5 Comparing Conflict to Bargaining

In Comparing Conflict to Bargaining, there are at least two issues of interest. One is distributional. Are the probabilities of winning under Conflict and the shares received under Bargaining similar? How do the up-front investments differ in the two games and how do the variable fighting efforts under Conflict influence outcomes? The second issue that is ultimately most important, and partly depends on the first one, is whether one side would ever prefer Conflict to Bargaining. Given that Conflict involves the extra variable effort costs, for Conflict to be ex ante preferable by at least one player a combination of low enough up-front investments under Conflict and a high enough probability of winning (relative to the share under Bargaining) would be necessary.

We summarize the main comparisons between Conflict and Bargaining in Proposition 3.

**Proposition 3:** *(i)* $K_1^*/K_2^* = \theta\rho \gtreqless \rho = \bar{K}_1/\bar{K}_2$ as $\theta \gtreqless 1$

*(ii) The stronger player has a higher probability of winning under Conflict than she has as a share of the surplus under Bargaining (i.e., $P(K_1^*, K_2^*) = \frac{\theta}{\theta+1} \gtreqless \frac{\theta^{1-\alpha}}{\theta^{1-\alpha}+1} = \beta(\bar{K}_1, \bar{K}_2)$ as $\theta \gtreqless 1$).*

*(iii) The strongest player prefers Conflict to Bargaining for high enough $\alpha$ and sufficiently favorable $\theta$.*

*(iv) Also for high enough $\alpha$ and sufficiently low or sufficiently high $\theta$ total equilibrium payoffs under Bargaining can be lower than under Conflict.*
(Proof is in the Appendix)

By part (i) of the Proposition, up-front investments under Conflict vary across the two players the more ex ante different the players are (i.e., the further $\theta$ is away from 1), whereas under Bargaining up-front investments differ only to the extent that the marginal costs of effort differ ($\bar{K}_1 = \rho\bar{K}_2$). Figure 2 shows how for $\rho = 1$ and $\alpha = 0.75$, the up-front investments under the two games compare.

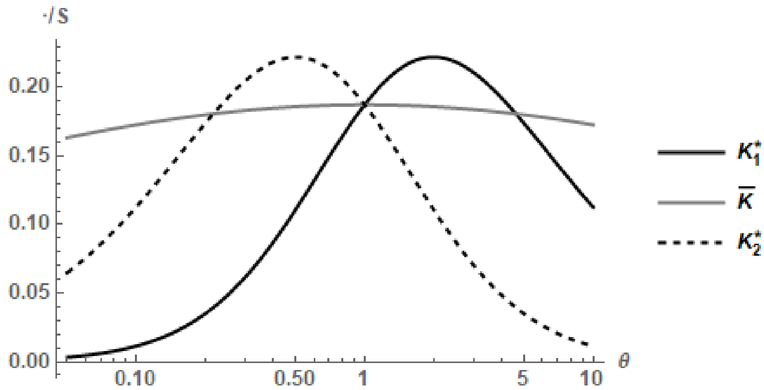


Figure 2: K comparison, $\alpha = 0.75$, $\rho = 1$

Given that $\rho = 1$, $\bar{K}_1 = \bar{K}_2$ for all values of $\theta$; but the difference between the two up-front investment under Conflict ($K_1^*$ and $K_2^*$) becomes larger the further $\theta$ is away from 1 (and the greater is the asymmetry between the players). Moreover, the total level of up-front investments under Conflict becomes smaller and lower than the up-front investments under Bargaining.

The greater asymmetry under Conflict for up-front investments along with the relative symmetry of variable fighting efforts ($L_1^* = \omega L_2^*$) implies (part (ii) of Proposition) that the stronger player has a higher probability of winning under Conflict than he or she has as a share of the surplus under Bargaining.

Therefore, the lower cost of up-front investments for a sufficiently strong player under Conflict (compared to Bargaining) but a higher probability of winning under Conflict (compared to the share under Bargaining) induces a payoff under Conflict that is higher than that under Bargaining (part (iii)).

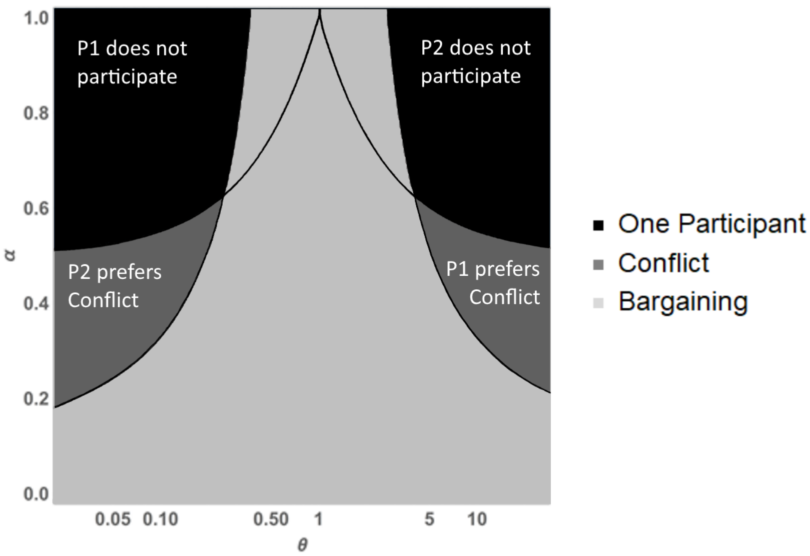


Figure 3: Outcome Regions

Figure 3 partitions $(\theta, \alpha)$ into regions that we expect to be induced given what we know about the payoffs under the different games. In the darker regions, the weaker player has a negative payoff under Conflict (Proposition 1, part (i)) and can be expected not to participate given that the stronger player prefers Conflict to Bargaining. Since the stronger player is the sole participant in this case, he or she receives the whole surplus.[8] In the less dark regions in which the stronger player prefers Conflict, the weaker player participates. Finally, in the remaining areas without strong asymmetries or with low returns on up-front investments (low $\alpha$), Bargaining is preferred by both sides.

Thus Conflict is possible when asymmetry is high enough and one side may even prefer it. Interestingly, this also implies that it is possible to have stout participants *and* peaceful settlement. The key is to create balance between agents and eliminate sources of bias in the overall environment.

[8]It is possible that staying out of the contest could yield a negative payoff, not 0. It could also be the case that No Participation could yield a positive payoff (by, for example, employing resources in alternative endeavors). However, as long as the No Participation payoff is constant the qualitative results of the model would not be affected. For example, in Figure 3 a negative No Participation payoff would shrink (up) the No Participation zone while a positive such payoff would expand (down) the No Participation area.

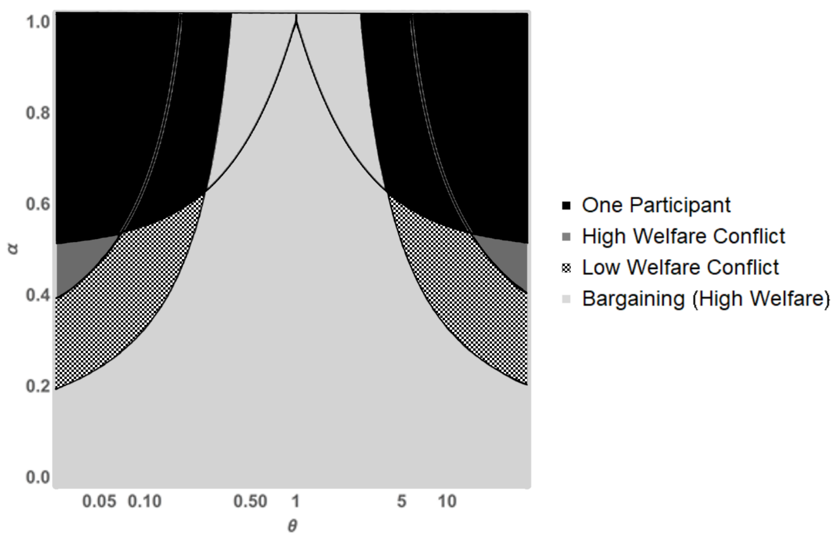


Figure 4: Welfare Regions

In Figure 4 we include regions that actually involve higher total payoffs under Conflict than under Bargaining. As can be expected these regions are strictly within the region that the stronger player prefers Conflict

# 6 Ultimatum Bargaining

We now alter the game slightly to allow for positional dominance in the bargaining structure itself; that is, we consider the case of ultimatum bargaining. We will see that the essential results of this model are robust to such a change, and that relaxing the symmetry of the process that is characteristic of Nash bargaining places greater power in the hand of the proposer. Furthermore, conflict becomes less likely when the proposer is strong, but more likely when the proposer is weak. This is because there is an additional first mover advantage in bargaining that makes a strong player very likely to prefer settlement if they gain the benefit of this additional share, and highly likely to prefer conflict if they must give it up.

Suppose player 1 is the proposer. To accomplish equilibrium settlement, she must propose share $x$ to herself and $1 - x$ to player 2 such that player 2 is indifferent between settlement and conflict. Naturally, this allows the proposer to extract all the gains from avoiding conflict. The worked out solution and equilibrium expressions are in the Appendix. We just highlight here the results and intuition of the extension.

Figure 5 shows that when player 1 is the proposer, investment is higher than the baseline solution for low $\theta$ values, and lower for high $\theta$ values. The pattern is flipped for when player 2 is the proposer. This means that proposers use their first mover advantage to shore up their bargaining position by investing more than in symmetric bargaining when they are otherwise disadvantaged. They do not invest as much when they already have the advantage of being a proposer.

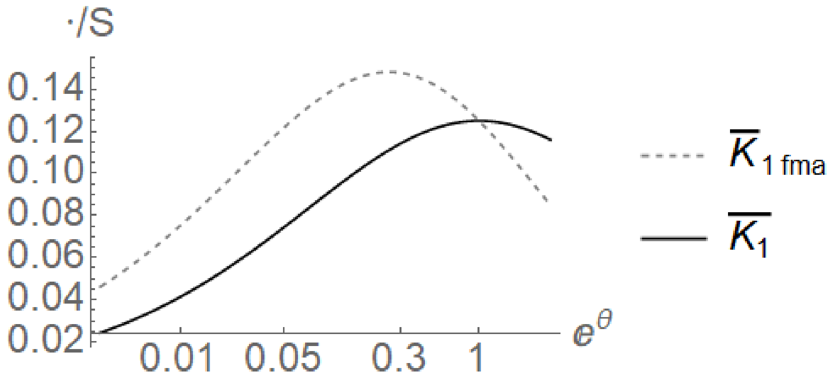


Figure 5: $\bar{K}_1$ Nash vs. Ultimatum Bargaining

The first mover advantage under ultimatum bargaining is shown in Figure 6 along with the difference between beta from the baseline model and $x$, the equilibrium share of the prize going to player 1 under this alternative structure. One can see that being the first mover confers a rather large advantage in this version of the game. The effect of the additional asymmetry introduced in this extension compared to the base model is smaller but still substantial.

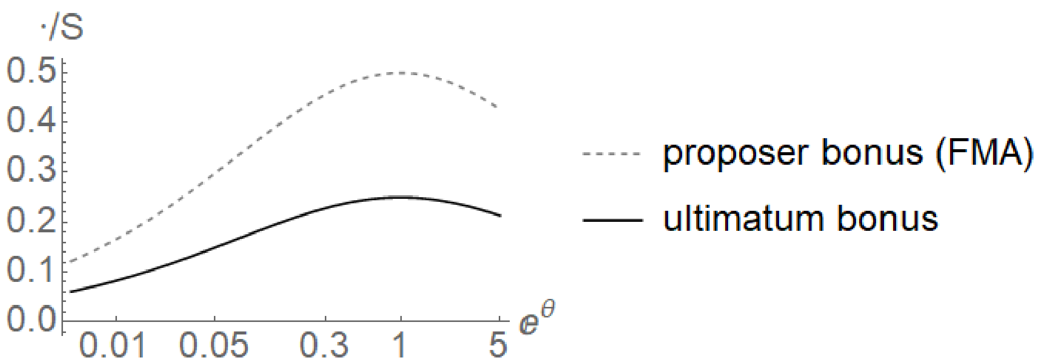


Figure 6

Moving on to the question of which outcome to expect, our essential result

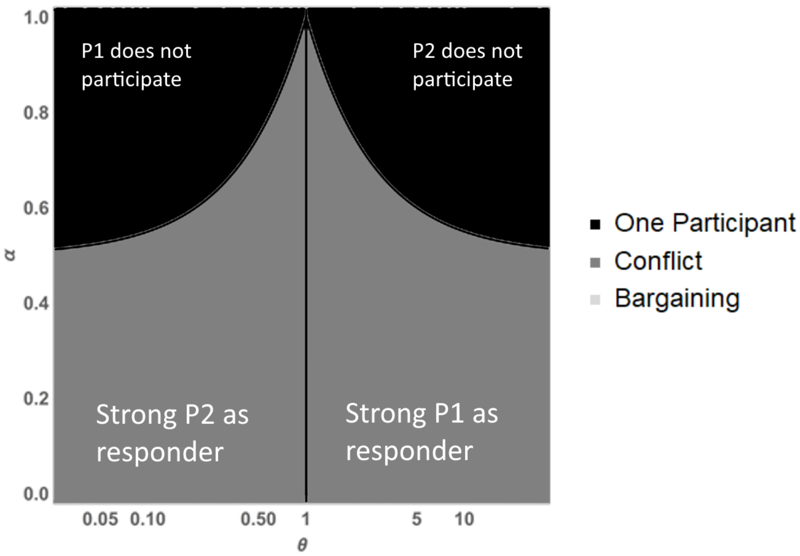


Figure 7: Outcome Regions, Ultimatum Bargaining

that one player may prefer conflict to settlement still holds with ultimatum bargaining. The parameter values necessary to see this outcome become more extreme, however, when the proposer already enjoys an advantage in terms of $\theta$. This is because their first mover advantage is only realized in settlement and the gains from conflict drop to nothing for all but extremely high values of $\theta$ where winning is almost a sure thing with little effort.

The flip side of this coin is interesting. Suppose that the proposer is instead disadvantaged in $\theta$. This player most surely wants settlement, yet their opponent, the responder, will prefer to commit to Conflict. Such a responder has to invest so much to overcome the first mover advantage and get a decent bargaining outcome that they are better off committing to much lower optimal conflict investments and bearing the additional costs of contest efforts. Figure 7 shows that Conflict is now the predominant outcome whenever the proposer is weak. The gray region to the right of $\theta = 1$ represents Conflict preferred by player 1 if they are the responder, the gray region to the left represents Conflict preferred by player 2 if they are the responder. As before, the black regions represent parameter values for which one player prefer Conflict, but the other chooses not to participate because Conflict would yield a negative payoff.

## 7 Concluding Remarks

We have seen how asymmetries in power can induce conflict. In some settings power asymmetries are not due to technology but are, at least, partly determined by policy. In labor relations, for example, how the courts and other state institutions treat labor unions relative to management is largely a result of government policies. In lobbying and litigation, governments can also influence the relative power of contestants. Therefore, to the extent that one wishes to avoid conflict, one possible policy implication is to maintain balanced institutions. This paper highlights that the threat of conflict can be high in highly biased environments and low in more balanced environments. It is possible to have two strong and well prepared agents interacting peacefully when the underlying rules of the game are balanced. Moreover, the strength of both is required to prevent the stronger from taking advantage of the weaker in outright conflict.

# Appendix

**Proof of Proposition 1:**

Part (i): The equilibrium and the conditions under which the players have positive or negative equilibrium payoffs have been derived in the main text. Corner solutions are thus handled and there is only one interior optimum for each choice variable. Second derivatives confirm that these optima are maxima:

$$\frac{\partial^2}{\partial L_1^2}[V_1^C(K_1, K_2, L_1, L_2)] = \frac{-2S\varepsilon^2 K_1^{2\alpha} K_2^{\alpha} L_2}{(\varepsilon K_1^{\alpha} L_1 + K_2^{\alpha} L_2)^3}$$

$$\frac{\partial^2}{\partial K_1^2}[V_1^C(K_1, K_2, L_1^*(K_1,K_2), L_2^*(K_1, K_2))] = \frac{2\alpha S\varepsilon^2\omega^2 K_1^{2\alpha-2} K_2^{\alpha}}{(\varepsilon\omega K_1^{\alpha} + K_2^{\alpha})^4}((2\alpha - 1)K_2^{\alpha} - (\alpha + 1)\varepsilon\omega K_1^{\alpha})$$

which is negative as long as:

$$(2\alpha - 1)K_2^{\alpha} < (\alpha + 1)\varepsilon\omega K_1^{\alpha}$$

looking at the optimum point, the condition becomes:

$$\begin{aligned}(2\alpha - 1)K_2^{\alpha} &< (\alpha + 1)\varepsilon\omega(\varepsilon\omega\rho)^{\frac{\alpha}{1-\alpha}} K_2^{\alpha} \\ (2\alpha - 1) &< (\alpha + 1)(\varepsilon\omega)^{\frac{1}{1-\alpha}} \rho^{\frac{\alpha}{1-\alpha}} \\ (2\alpha - 1) &< (\alpha + 1)\theta\end{aligned}$$

which holds when $\alpha \leq 1/2$ and when $\alpha > 1/2$ with $\theta > 2\alpha - 1$ as assumed. Similar expressions for player two yield the same outcomes.

Part (ii): as shown in (9)

Part (iii): Suppose $K_1^* = \frac{2\alpha\theta^2}{r_1(\theta+1)^3}S$ and $K_2^* = \frac{2\alpha\theta}{r_2(\theta+1)^3}S$ as derived in the text.

$$\begin{aligned}\frac{\partial K_1^*}{\partial\theta} = 2\alpha S/r_1\left(\frac{2\theta}{(\theta + 1)^3} - \frac{3\theta^2}{(\theta + 1)^4}\right) &> 0 \\ \frac{2\theta - \theta^2}{(\theta + 1)^4} &> 0 \\ 2 &> \theta\end{aligned}$$

$$\frac{\partial K_2^*}{\partial \theta} = 2\alpha S/r_2 \left( \frac{1}{(\theta+1)^3} - \frac{3\theta}{(\theta+1)^4} \right) > 0$$

$$\frac{1-2\theta}{(\theta+1)^4} > 0$$

$$1/2 > \theta$$

Therefore, $\frac{\partial K_1^*}{\partial \theta} \gtreqless 0$ as $\theta \lesseqgtr 2$ and $\frac{\partial K_2^*}{\partial \theta} \gtreqless 0$ as $\theta \lesseqgtr \frac{1}{2}$.

Part (iv): Suppose $L_i^* = \frac{\theta}{w_i(\theta+1)^2} S$ for $i \in \{1, 2\}$ as derived in the text.

$$\frac{\partial L_i^*}{\partial \theta} = S/w_i \left( \frac{1}{(\theta+1)^2} - \frac{2\theta}{(\theta+1)^3} \right) > 0$$

$$\frac{1-\theta}{(\theta+1)^3} > 0$$

$$1 > \theta$$

Therefore, both $\frac{\partial L_1^*}{\partial \theta} \gtreqless 0$ and $\frac{\partial L_2^*}{\partial \theta} \gtreqless 0$ as $\theta \lesseqgtr 1$.

■

**Proof of Proposition 2:**

Part (i): Equilibrium payoffs in (15) are nonnegative for all parameter values, hence there is only the unique interior solution derived in the main text. Second derivatives confirm that these optima are maxima:

$$\frac{\partial^2}{\partial K_1^2}[V_1^B(K_1, K_2)] = \frac{\alpha S \varepsilon \omega K_2^\alpha}{(\varepsilon \omega K_1^\alpha + K_2^\alpha)^3}((\alpha - 1)K_1^{\alpha-2}K_2^\alpha - \varepsilon \omega K_1^{2\alpha-2})$$

which is always negative. A similar exercise for player 2 yields the same outcome.

Part (ii): as shown in (14).

Part (iii): Suppose that $\bar{K}_i = \frac{\alpha \theta^{1-\alpha}}{r_i(\theta^{1-\alpha}+1)^2} S$ for $i \in \{1, 2\}$ as derived in the text.

$$\frac{\partial \bar{K}_i}{\partial \theta} = \frac{\alpha(1-\alpha)S}{r_i \theta^\alpha} \left( \frac{1}{(\theta^{1-\alpha}+1)^2} - \frac{2\theta^{1-\alpha}}{(\theta^{1-\alpha}+1)^3} \right) > 0$$

$$\frac{1-\theta^{1-\alpha}}{(\theta^{1-\alpha}+1)^3} > 0$$

$$1 > \theta$$

Therefore, both $\frac{\partial \bar{K_1}}{\partial \theta} \gtreqless 0$ and $\frac{\partial \bar{K_2}}{\partial \theta} \gtreqless 0$ as $\theta \lesseqgtr 1$.

■

**Proof of Proposition 3:**

Parts (i) and (ii) follow directly from equilibrium expressions given that $\alpha < 1$ and $\theta$ may be greater than 1.

Part (iii): Suppose $\alpha \in (0,1)$, and $\theta > 0$. Consider the difference in equilibrium payoffs for player 1.

$$\frac{\theta^2(\theta+1-2\alpha)}{(\theta+1)^3}S - \frac{\theta^{1-\alpha}(\theta^{1-\alpha}+1-\alpha)}{(\theta^{1-\alpha}+1)^2}S$$

after some algebra we have:

$$\frac{S\theta}{(\theta+1)^3(\theta+\theta^\alpha)^2}\big((\theta^2+(1-2\alpha)\theta)(\theta+\theta^\alpha)^2+((\alpha-1)\theta^\alpha-\theta)(\theta+1)^3\big)$$

$$\frac{S\theta}{(\theta+1)^3(\theta+\theta^\alpha)^2}\big((\alpha+1)\theta^{\alpha+3}+\theta^{2\alpha+2}-(2+2\alpha)\theta^3-(1+\alpha)\theta^{\alpha+2} \\ -(2\alpha-1)\theta^{2\alpha+1}-(3-3\alpha)\theta^{\alpha+1}-(1-\alpha)\theta^\alpha-3\theta^2-\theta\big)$$

which is always positive for sufficiently high $\theta$ values since the first two terms will dominate the remaining negative terms. The exact threshold at which this difference in payoffs becomes positive does not have a closed form solution; however, it can be easily characterized using numerical methods and graphs as shown in the main text.

A similar exercise can be shown for the difference in payoffs for player 2, only there we must have $\theta$ sufficiently small to guarantee positive sign. It is a trivial demonstration and omitted for brevity.

Therefore, player 1 (2) may prefer Conflict to Bargaining for a given $\alpha$ and sufficiently high (low) $\theta$.

Part (iv): Suppose $\alpha \in (0,1)$, and $\theta > 0$. Consider the difference in total equilibrium payoffs, that is $(V_1^{C*}+V_2^{C*})-(\bar{V_1^B}+\bar{V_2^B})$.

$$\frac{\theta^2(\theta+1-2\alpha)+\theta(1-2\alpha)+1}{(\theta+1)^3}S - \frac{\theta^{2-2\alpha}+2\theta^{1-\alpha}(1-\alpha)+1}{(\theta^{1-\alpha}+1)^2}S$$

after some algebra we have:

$$\frac{S\theta}{(\theta+1)^3(\theta+\theta^\alpha)^2}\big(2\alpha\theta^{\alpha+3}+2\alpha\theta^\alpha-(2+2\alpha)\theta^3-(4-2\alpha)\theta^{\alpha+2}- \\ (2+2\alpha)\theta^{2\alpha+1}-(2+2\alpha)\theta^2-(4-2\alpha)\theta^{\alpha+1}-(2+2\alpha)\theta^{2\alpha}\big)$$

which is always positive for sufficiently high $\theta$ values since the first term will dominate the remaining negative terms. Similarly, it is always positive for sufficiently low $\theta$ values since the fractional exponent in the second term will dominate. The exact thresholds at which this difference in total payoffs becomes positive do not have closed form solutions; however, it can be easily characterized using numerical methods and graphs as shown in the main text.

Therefore, total equilibrium payoffs under Bargaining can be lower than under Conflict.

■

**Solution to the ultimatum bargaining extension.**

Suppose player 1 is the proposer. To accomplish equilibrium settlement, she must propose share $x$ to herself and $1 - x$ to player 2 such that player 2 is indifferent between settlement and conflict. That is, $V_2^*(K_1, K_2) = V_2^B(K_1, K_2)$. Hence,

$$\frac{K_2^\alpha}{\varepsilon\omega K_1^\alpha + K_2^\alpha}S - r_2K_2 - w_2\frac{\varepsilon K_1^\alpha K_2^\alpha}{w_1(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2}S = (1-x)S - r_2K_2$$

$$1 - x(K_1, K_2) = \frac{K_2^{2\alpha}}{(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2}$$

Any offer less than this is rejected and the outcome is conflict. Optimal upfront investments in this case are the same as before. Settlement investments, however, must be reanalyzed since bargaining is no longer symmetric and simultaneous.

Player 2 thus chooses $K_2$ to maximize her payoff given the above expression, and Player 1 chooses $K_1$ to maximize

$$x(K_1, K_2) = \frac{\varepsilon^2\omega^2K_1^{2\alpha} + 2\varepsilon\omega K_1^\alpha K_2^\alpha}{(\varepsilon\omega K_1^\alpha + K_2^\alpha)^2}$$

giving us first order conditions

$$K_1^{\alpha-1}K_2^{2\alpha} = \frac{r_1(\varepsilon\omega K_1^\alpha + K_2^\alpha)^3}{2\alpha\varepsilon\omega S} \qquad \textit{and} \qquad K_1^\alpha K_2^{2\alpha-1} = \frac{r_2(\varepsilon\omega K_1^\alpha + K_2^\alpha)^3}{2\alpha\varepsilon\omega S}$$

which yield the same relationship as before, $\bar{K_1} = \rho\bar{K_2}$, but with somewhat different equilibrium expressions

$$\bar{K_1} = \frac{2\alpha\theta^{1-\alpha}}{r_1(\theta^{1-\alpha}+1)^3}S \qquad \textit{and} \qquad \bar{K_2} = \frac{2\alpha\theta^{1-\alpha}}{r_2(\theta^{1-\alpha}+1)^3}S$$

leading us to

$$\bar{x_1} = \frac{\theta^{1-\alpha}(\theta^{1-\alpha}+2)}{(\theta^{1-\alpha}+1)^2} \tag{16}$$

$$\bar{V_1^B} = \frac{\theta^{1-\alpha}(\theta^{2-2\alpha}+3\theta^{1-\alpha}+(2-2\alpha))}{(\theta^{1-\alpha}+1)^3}S \tag{17}$$

$$\bar{V_2^B} = \frac{\theta^{1-\alpha}(1-2\alpha)+1}{(\theta^{1-\alpha}+1)^3}S \tag{18}$$

Suppose instead that player 2 is the proposer. A similar exercise yields the following equilibrium expressions

$$\bar{K_1} = \frac{2\alpha\theta^{2-2\alpha}}{r_1(\theta^{1-\alpha}+1)^3}S \qquad \textit{and} \qquad \bar{K_2} = \frac{2\alpha\theta^{2-2\alpha}}{r_2(\theta^{1-\alpha}+1)^3}S$$

$$\bar{x_1} = \frac{\theta^{2-2\alpha}}{(\theta^{1-\alpha}+1)^2} \tag{19}$$

$$\bar{V_1^B} = \frac{\theta^{3-3\alpha}+\theta^{2-2\alpha}(1-2\alpha)}{(\theta^{1-\alpha}+1)^3}S \tag{20}$$

$$\bar{V_2^B} = \frac{\theta^{2-2\alpha}(2-2\alpha)+2\theta^{1-\alpha}+1}{(\theta^{1-\alpha}+1)^3}S \tag{21}$$

Therefore the proposer enjoys an additional $\frac{2\theta^{1-\alpha}}{(\theta^{1-\alpha}+1)^2}$ fraction of the prize.

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# SUPPLEMENTARY APPENDIX

"Bargaining and Conflict with Up-front Investments: How Power Asymmetries Matter"

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December 18, 2019

In this Appendix we consider a more general case than the one we analyzed in the main body of the paper, whereby $R_i = K_i^{\alpha} L_i^{\gamma}$ $(\alpha, \gamma \in (0,1]; i = 1,2)$, where $K_i$ represents the up-front investment of player $i$ and $L_i$ represents their variable effort in the event of conflict. Note that in the main body of the paper we have $\gamma = 1$, which is the only case for which we have found analytical solutions for both the Conflict and Bargaining equilibria.

When $\gamma < 1$, we presently show that, though the Bargaining game has analytical solutions similar to those in the main body of the paper, the Conflict game does not afford analytical solutions. Therefore, to make comparisons between the equilibrium payoffs under the two games we have made in the paper, we employ numerical methods. In particular, we focus on the cases with (i) $\alpha = 1$ and $\gamma$ allowed to vary over $(0,1]$ (ii) $\alpha = \gamma$ and (iii) the constant-returns case of $\gamma = 1 - \alpha$. We find that the qualitative results about the effects of asymmetries on preferences for Bargaining, Conflict, or No Participation that we derived in the paper carry through in these more general cases. If anything, for case (i) the results are stronger, in the sense that smaller asymmetries would lead to at least one player preferring Conflict over Bargaining.

We start by specifying the two games. The players first make up-front investments $K_1$ and $K_2$, and only if they were to engage in conflict would they choose variable levels of effort $L_1$ and $L_2$.

The expected payoffs under Conflict are as follows:

$$V_1^C(K_1, L_1, K_2, L_2) = \frac{\varepsilon K_1^{\alpha} L_1^{\gamma}}{\varepsilon K_1^{\alpha} L_1^{\gamma} + K_2^{\alpha} L_2^{\gamma}} S - r_1 K_1 - w_1 L_1$$

$$V_2^C(K_1, L_1, K_2, L_2) = \frac{K_2^{\alpha} L_2^{\gamma}}{\varepsilon K_1^{\alpha} L_1^{\gamma} + K_2^{\alpha} L_2^{\gamma}} S - r_2 K_2 - w_2 L_2$$

The share of player 1 under Bargaining is

$$\beta(K_1, K_2) = P(K_1 L_1^{*\gamma}(K_1, K_2), K_2 L_2^{*\gamma}(K_1, K_2)) + \frac{w_2 L_2^*(K_1, K_2)}{2S} - \frac{w_1 L_1^*(K_1, K_2)}{2S}$$

This share equals the player's own probability of winning in the event of conflict, suitably adjusted by the variable costs of conflict of the two players ($w_i L_i^*(K_1, K_2)$ for player $i = 1, 2$). In particular, a higher variable cost of conflict disadvantages that player and advantages his opponent. Both the probabilities of winning and the variable costs of conflict depend on the up-front investments $(K_1, K_2)$ in ways that we cannot a priori specify but which we momentarily explore. What is clear, however, is that the probabilities of winning under Conflict can be expected to have different properties (in terms of the of their sensitivity to $(K_1, K_2)$) from those of the sharing function under Bargaining.

The payoff functions for the game under Bargaining are as follows:

$$\begin{aligned} V_1^B(K_1, K_2) &= \beta(K_1, K_2)S - r_1 K_1 \\ V_2^B(K_1, K_2) &= [1 - \beta(K_1, K_2)]S - r_2 K_2 \end{aligned}$$

## Solving the Conflict Game

We use backwards induction to solve for a subgame perfect equilibrium.

Begin by assuming a $(K_1, K_2)$ pair and let the players maximize their expected payoffs in (1) and (2) by the choice of their respective variable efforts $(L_1, L_2)$:

$$V_1^C(L_1, L_2 \quad | \quad K_1, K_2) = \frac{\varepsilon K_1^\alpha L_1^\gamma}{\varepsilon K_1^\alpha L_1^\gamma + K_2^\alpha L_2^\gamma} S - r_1 K_1 - w_1 L_1 \tag{1}$$

$$V_2^C(L_1, L_2 \quad | \quad K_1, K_2) = \frac{K_2^\alpha L_2^\gamma}{\varepsilon K_1^\alpha L_1^\gamma + K_2^\alpha L_2^\gamma} S - r_2 K_2 - w_2 L_2 \tag{2}$$

We can show that $L_1^* = \omega L_2^*$ (where $\omega \equiv \frac{w_2}{w_1}$), and the subgame perfect equilibrium choices are as follows, where $\phi \equiv \varepsilon \omega^\gamma$:

$$L_1^*(K_1, K_2) = \frac{\gamma \phi K_1^\alpha K_2^\alpha}{w_1(\phi K_1^\alpha + K_2^\alpha)^2} S \tag{3}$$

$$L_2^*(K_1, K_2) = \frac{\gamma \phi K_1^\alpha K_2^\alpha}{w_2(\phi K_1^\alpha + K_2^\alpha)^2} S \tag{4}$$

Just as with $\varepsilon$, the higher is the ratio of costs $\omega$ the better it is for player 1. This makes the winning probability of player 1 a simple function of $K_1$ and $K_2$:

$$P^*(K_1, K_2) = \frac{\phi K_1^\alpha}{\phi K_1^\alpha + K_2^\alpha}$$

Conflict payoff functions in the first stage (as functions of $(K_1, K_2)$, conditional on subgame-perfect induced $L_1^*(K_1, K_2)$ and $L_2^*(K_1, K_2)$ :

$$
\begin{aligned}
V_1^C(K_1, L_1^*(K_1,K_2), K_2, L_2^*(K_1,K_2)) &= \frac{\phi K_1^\alpha}{\phi K_1^\alpha + K_2^\alpha} S - r_1 K_1 - \frac{\gamma\phi K_1^\alpha K_2^\alpha}{(\phi K_1^\alpha + K_2^\alpha)^2} S \\
&= \frac{(\phi K_1^\alpha)^2 + (1-\gamma)\phi K_1^\alpha K_2^\alpha}{(\phi K_1^\alpha + K_2^\alpha)^2} S - r_1 K_1 \\
V_2^C(K_1, L_1^*(K_1,K_2), K_2, L_2^*(K_1,K_2)) &= \frac{K_2^\alpha}{\phi K_1^\alpha + K_2^\alpha} S - r_2 K_2 - \frac{\gamma\phi K_1^\alpha K_2^\alpha}{(\phi K_1^\alpha + K_2^\alpha)^2} S \\
&= \frac{(K_2^\alpha)^2 + (1-\gamma)\phi K_1^\alpha K_2^\alpha}{(\phi K_1^\alpha + K_2^\alpha)^2} S - r_2 K_2
\end{aligned}
$$

First-order conditions for an interior equilibrium (all $K_i s$ evaluated at equilibrium):

$$
\frac{\partial V_1^{C*}}{\partial K_1} = \frac{\alpha\phi K_1^{\alpha-1} K_2^\alpha[(1+\gamma)\phi K_1^\alpha + (1-\gamma)K_2^\alpha]}{(\phi K_1^\alpha + K_2^\alpha)^3} S - r_1 = 0 \tag{5}
$$

and

$$
\frac{\partial V_2^{C*}}{\partial K_2} = \frac{\alpha\phi K_1^\alpha K_2^{\alpha-1}[(1+\gamma)K_2^\alpha + (1-\gamma)\phi K_1^\alpha]}{(\phi K_1^\alpha + K_2^\alpha)^3} S - r_2 = 0 \tag{6}
$$

Which imply $K_1[(1+\gamma)K_2^\alpha + (1-\gamma)\phi K_1^\alpha] = \rho K_2[(1+\gamma)\phi K_1^\alpha + (1-\gamma)K_2^\alpha]$. We can't find analytical solutions for this equation, but we use the first-order conditions to numerically derive equilibrium strategies and payoffs. Comparisons similar to those of the main text follow the next subsection.

## Solving the Bargaining Game

The payoff functions under Bargaining reduce to:

$$
\begin{aligned}
V_1^B(K_1, K_2) &= \frac{\phi K_1^\alpha}{\phi K_1^\alpha + K_2^\alpha} S - r_1 K_1 \\
V_2^B(K_1, K_2) &= \frac{K_2^\alpha}{\phi K_1^\alpha + K_2^\alpha} S - r_2 K_2
\end{aligned}
$$

which produce the relationship $\bar{K_1} = \rho\bar{K_2}$ and the following equilibrium expressions (where $\mu \equiv \varepsilon\omega^\gamma\rho^\alpha = \phi\rho^\alpha$)[1]:

$$
\bar{K_1} = \frac{\alpha\mu}{r_1(\mu+1)^2} S \tag{7}
$$

[1]Note that $\mu = \theta^{1-\alpha}$ where $\theta$ is used in the main body of the paper.

$$\bar{K_2} = \frac{\alpha\mu}{r_2(\mu+1)^2} S \tag{8}$$

The equilibrium share of player 1 then equals:

$$\beta(\bar{K_1}, \bar{K_2}) = \frac{\mu}{\mu+1} \tag{9}$$

The equilibrium payoffs under Bargaining are then as follows:

$$\bar{V_1^B} = \frac{\mu(1-\alpha+\mu)}{(\mu+1)^2} S \tag{10}$$

$$\bar{V_2^B} = \frac{1+\mu(1-\alpha)}{(\mu+1)^2} S \tag{11}$$

## Comparing Payoffs under Conflict and Bargaining

We next compare the equilibrium payoffs under Conflict and Bargaining based on numerical results, given that we cannot analytically solve for the equilibrium under Conflict (but use the first-order-conditions in (5) and (6)).

Figure A1 shows how payoffs under Conflict and Bargaining compare as $\gamma$ varies from 0 to 1 (with $\alpha = 1$) in the vertical axis and the log of the asymmetry parameter $\phi$ varies in the horizontal axis.

Conflict is preferred to Bargaining by a wider range of parameter than in the case of the main text as can be seen in Figure A2 which includes an overlay over Figure 3. For sufficient asymmetries in $\phi$, Conflict is preferred to Bargaining for at least one player for all values of $\gamma$.

Figure A3 considers the case of $\alpha = \gamma$. Note that for sufficiently low values of $\alpha$ and $\gamma$ (but less than 0.5) Bargaining is preferred by both players regardless of the asymmetry. For higher values of $\alpha$ and $\gamma$, however, there are wide areas for which Conflict is preferred to Bargaining by at least one player. The dark area of No Participation by one player is non-monotonic in $\alpha$ and $\gamma$ (as a function of the asymmetry parameter $\phi$). The last figure also indicates that, for a given level of asymmetry ($\phi$) between the players, one factor that appears to matter for Conflict to be preferable to Bargaining is the total value of the coefficients $\alpha$ and $\gamma$, something that emerges in the main body of the paper when $\alpha$ increases (but $\gamma$ is fixed there at 1).

Finally, Figure A4 shows what occurs when $\gamma = 1 - \alpha$ and the production function of effort has constant returns to scale. As $\alpha$ increases (and $\gamma$ decreases) the region for which Conflict becomes preferable increases substantially. In this case, however, No Participation zones do not arise. The reason for this appears to be the fact that as $\gamma$ goes to 0 (and $\alpha$ goes to 1), the variable cost of conflict becomes very small and Conflict becomes not that costly for the weaker player. This result is similar to what was shown before since the regions of No Participation in the other figures also appear only for $\alpha$ close enough to 1, but with values of $\gamma$ that are substantial.

Overall, then, these figures show that the results we have derived in the main text carry through qualitatively.

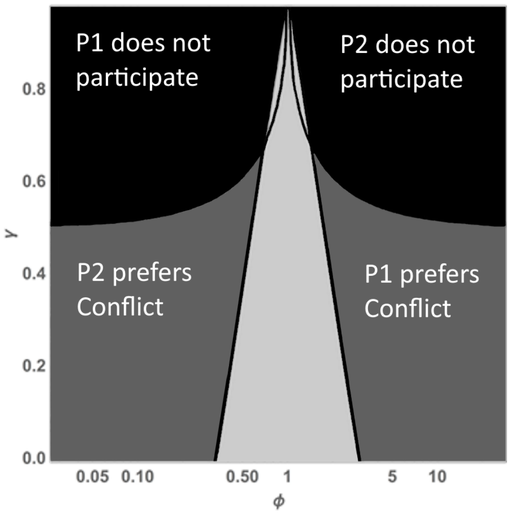


Figure A1: $\alpha = 1$, $\gamma \in (0, 1]$

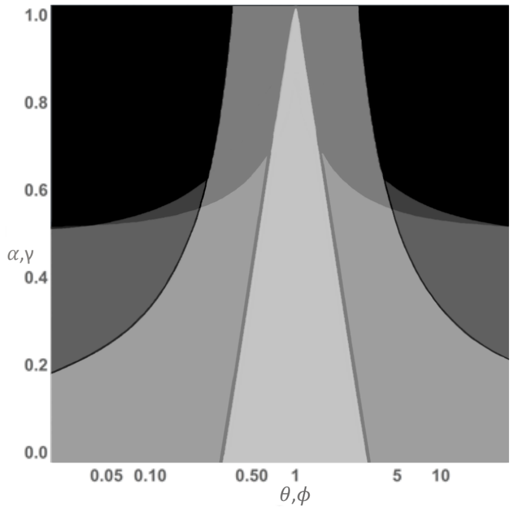


Figure A2: $K^{\alpha}L$ and $KL^{\gamma}$

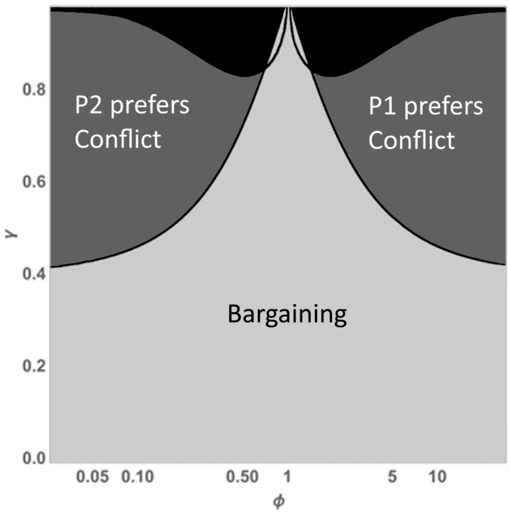


Figure A3: $K^{\alpha}L^{\gamma}$ with $\alpha = \gamma$

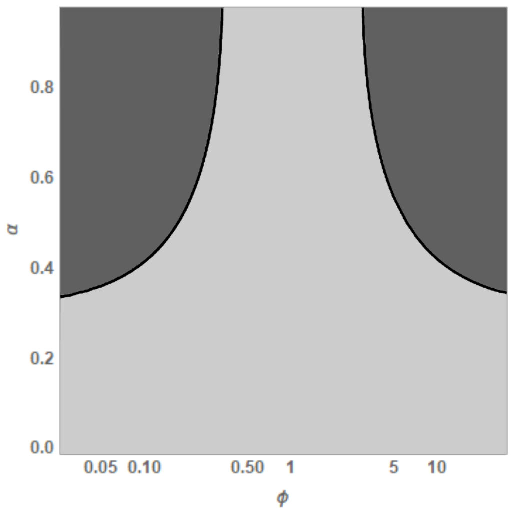


Figure A4: $K^{\alpha}L^{1-\alpha}$