CESifo WORKING PAPERS

7956
2019

November 2019

Real and Nominal Effects of Monetary Shocks under Time-Varying Disagreement

Vania Esady

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
www.cesifo-group.org/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: www.CESifo-group.org/wp

CESifo *Working Paper No. 7956*

Real and Nominal Effects of Monetary Shocks under Time-Varying Disagreement

Abstract

This paper investigates the heterogeneity of monetary policy transmission under time-varying disagreement regimes using a threshold VAR. Empirically, I establish that during times of high disagreement, prices respond more sluggishly in response to monetary shocks. These stickier prices cause a flatter Phillips curve, leading to the empirical result that monetary policy has stronger real (output) effects in high disagreement periods. I develop a tractable theoretical model that show rationally inattentive price-setters produce this result. The model also links disagreement and uncertainty – two fundamentally different concepts, and bridges the results of this paper to the literature on state-dependent monetary transmission. The main result highlights a role for improved central bank communications that reduce disagreement among economic agents, which lessens output falls when implementing disinflationary monetary policies.

JEL-Codes: E320, E520, E580, D830.

Keywords: time-varying disagreement, monetary policy, threshold VAR, rational inattention.

Vania Esady
Department of Economics
City University of London
Northampton Square
United Kingdom – EC1V 0HB London
vania.esady@city.ac.uk

October 2019
A previous version of this paper was circulated under the title “Real and Nominal Effects of Monetary Shocks under Uncertainty”. I am grateful for the advice and constant support of Joseph Pearlman and Michael Ben-Gad. I appreciate the financial support from City, University of London. I would also like to thank Ana Galvão, Marco Del Negro, Kris Nimark and Tiziano Ropele and seminar participants at the Norges BankWorkshop, RES 2018 and CESifo.

"... the literature has convincingly shown that disagreement is not uncertainty. They are conceptually different."

— Ricardo Reis, *ECB Forum in Central Banking*, Sintra (June 2018)

1 Introduction

Uncertainty plays an important role in many economic decisions, ranging from firm pricing and investment decisions to household saving and consumption choices. This has spurred a large collective effort to measuring economic uncertainty, with an established literature that proxies it with the disagreement of individual forecasts in surveys. However, the contemporary literature now considers uncertainty and disagreement as fundamentally different concepts (Rich and Tracy, 2018), and empirically, various measures of macroeconomic uncertainty and disagreement have positive, but weak, correlations (Kozeniauskas et al., 2018).[1]

This paper dissects the relationship between uncertainty and disagreement, and focuses on how they distinctly affect an important policy-relevant question: how state-dependent are the effectiveness of monetary policy? The contribution of this paper is two-fold. Firstly, I design a tractable rational-inattention model that unifies uncertainty and disagreement together, to highlight when there is a positive link between them, and when they break down. I utilise the model to examine how the two concepts affect price-setting behaviour of firms, and thus, the effect of monetary policy on central banks' goal variables. Secondly, using a threshold VAR and a measure of disagreement of professional forecasters, I empirically document how in periods of heightened disagreement, monetary policy has *less* control over inflation, but *more* influence over output.

How does disagreement amongst price-setters affect their response to monetary shocks? As with many other imperfect information models, the rational inattention model suggests that when firms are only able to imperfectly observe factors that affect their optimal prices, they attach a positive (but less than unity) weight to the signals they receive (the 'Kalman gain') on these factors. This implies that their prices respond sluggishly to aggregate monetary shocks. The slower prices respond, the more 'sticky' prices

[1] 'Disagreement' in this paper is close to the Kozeniauskas et al. (2018) definition of higher-order uncertainty.

appear, leading to a flatter Phillips curve and thus output would correspondingly react by more to the monetary shock.

A novel insight from the rational inattention model is that plausible causes of the variation in disagreement has different effects on how price-setters respond to monetary shocks, than uncertainty. For example, a reduction in firms' information processing capacity worsens the quality of information available, leading firms to attach less weight to signals they receive.[2] Prices would then be more sluggish and disagreement increases. Note that this is the case even when the fundamental uncertainty has not changed, illustrating the one of cases where uncertainty and disagreement do not co-move together. Another insight from the rational inattention model is that endogenous optimal attention allocation could cause disagreement to change non-monotonically in response to fluctuations in aggregate uncertainty. In particular, an *increase* in demand uncertainty raises the benefits to monitoring demand conditions, the firms could optimally re-allocate much more attention to demand, and actually *decrease* disagreement of firms' assessment of demand.

These results also shed light on how increased communication by monetary policymakers can affect their ability to deliver on their stabilisation objectives. There is a recent trend of vastly increased central bank transparency — from releasing detailed minutes of monetary policy deliberations, increased frequency of speeches, to developing material more easily accessible to the general public (for example, the Bank of England's *Inflation Report* infographics). However, much of the literature focuses on how inflation expectations helps anchoring inflation. The mechanism that explains the empirical results in this paper suggests that, in addition, communicating aggregate *real* conditions can also help central banks achieve their objectives. As improved communication helps economic agents form expectations about current and future conditions, this reduces the disagreement of agents and potentially lowers the sacrifice ratio. During a disinflation, inflation can be reduced by more, with smaller output losses.

The general idea of the empirical threshold VAR methodology (Tsay, 1998) is to pick an endogenous 'threshold variable' that contains information

[2] For example, the reduction of information available to a firm from the bankruptcy of a supplier or customer.

about the different regimes — in this case, high and low disagreement. The threshold variable is endogenous, and thus allows for endogenous regime switching. As will be discussed in greater detail, the particular threshold variable that is chosen is the dispersion of the cross-sectional GDP forecasts from the U.S. Survey of Professional Forecasters, which I term as disagreement.

There is more work on differentiating the effects of recessions and expansions on the strength of monetary transmission. However, there appears little agreement across the literature. Tenreyro and Thwaites (2016) and Caggiano et al. (2014) find that monetary shocks have less impact on output and prices in *recessions*, while others such as Peersman and Smets (2001) and Lo and Piger (2005) find the opposite. Methodologically, this paper is most related to the literature examining how monetary policy transmission is state-dependent on uncertainty (in contrast to disagreement). For example, Aastveit et al. (2017) uses an interacted VAR, treating uncertainty as an exogenous interaction variable, while Castelnuovo and Pellegrino (2018) also works with with two-regime threshold VAR model but also focuses on uncertainty, rather than disagreement. Both papers find monetary policy to be less effective in affecting both output and prices in *high uncertainty*. A key difference with this paper is that the threshold variable or interaction term in their papers are treated exogenously, instead of as an endogenous variable. This means that uncertainty cannot react to monetary policy shocks. In practice, as shown by Pellegrino (2017), uncertainty can indeed respond to monetary policy shocks which indicates the importance of allowing threshold variable to be endogenous. In this paper, I compute the impulse responses using Generalised IRFs (GIRFs) which accounts for the endogenous threshold variable that creates nonlinearities in the threshold model.[3] It is important to note, as highlighted previously, the relationship between uncertainty and disagreement is not always monotonic and thus the results from the uncertainty literature do not necessarily conflict with the disagreement results in this paper.

The disagreement (cross-sectional forecast dispersion) measure is related to some recent empirical works that measure aggregate volatility. This

[3] GIRFs allow the regime to change after the shock such that uncertainty reacts to monetary policy shocks.

approach builds on a long literature, for example Baker et al. (2016) and D'Amico et al. (2008) to study the *direct* effect of uncertainty shocks (rather than the *indirect* impact on monetary transmission). There are many other uncertainty proxies, such as the Volatility Index (VIX), newspaper-based Economic Policy Uncertainty (Baker et al., 2016) and the Jurado et al. (2015) factor-based estimates. Kozeniauskas et al. (2018) emphasise the importance in distinguishing the different uncertainty measures. In the recent uncertainty literature in understanding monetary policy transmission, the most often use uncertainty is 'macro uncertainty' – uncertainty about aggregate variable such as Jurado et al. (2015) or VIX. Whereas the disagreement measure in this paper is related closer to the concept of higher-order uncertainty that Kozeniauskas et al. (2018) define as the uncertainty (shocks) about others' beliefs that arises when forecasts differ. Although they are positively correlated, as shown later in the paper, their relationship could break down.

The theoretical model is also closely related to the the rational inattention literature studying the impact of monetary policy shocks under different states. Menkulasi (2009) considers a dynamic general equilibrium model in which firms optimally allocate their limited attention across aggregate and idiosyncratic states. An increase in the volatility of aggregate shocks causes an optimal re-allocation of attention to the aggregate environment.

The analysis of this paper is closest to Zhang (2017) who investigates the endogenous information processing capacity as a channel through which uncertainty affects price dynamics, and empirically tests it with a Markov-switching FAVAR. The key is that with higher uncertainty, the more effort firms would exert into monitoring the economic state. I expand on the Zhang (2017) model to examine the implications on disagreement, and its links with aggregate uncertainty. Empirically, the main difference of this paper's threshold VAR methodology and the more common Markov-switching approach is that Markov-switching models examine the whole empirical model for regime breaks, while the threshold variable more precisely pins down the regimes, enabling the threshold VAR to differentiate across disagreement regimes. As a result, Markov-switching approaches tend to only pick up the large regime change from the Great Inflation to the Great Moderation period, but I show that there is significant variation in disagreement even *within* the Great Moderation period.

This paper is structured as follows. Section 2 presents the theoretical model that eludicates the fine distinction between uncertainty and disagreement, and their implications for pricing behaviour. Section 3 describes the data, measure of disagreement and econometric methodology. Section 3.4 highlights the main empirical results from the model. Section 4 concludes.

2 Stylised Rational Inattention Model

To illustrate the mechanisms that generate the empirical results, I present a stylised price-setting model with rational inattention, with closed-form solutions that allow us to compute comparative statics. I analyse how disagreement endogenously evolve to changes in information processing of firms and various uncertainties relevant for pricing decisions, and how that relates to how monetary shocks affect optimal prices.[4]

In this model, the price-setters in the firms face an unobserved aggregate demand y_t, composed of a normally-distributed demand shock b_t, and a 'monetary policy' component $c \cdot r_t$. The demand shock has a variance σ_b^2, which I refer to as fundamental demand uncertainty.[5] For tractability, without the loss of generality, the demand shock is assumed to be mean-zero. The monetary policy component is fully known: price-setters observe the policy rate r_t and the interest-elasticity of demand $c > 0$.

$$y_t = b_t - c \cdot r_t, \qquad \text{where} \quad b_t \sim N(0, \sigma_b^2) \tag{1}$$

In this simple model, I assume demand is insensitive to prices. The full-information optimal price p_{it}^* purely depends on the marginal costs, which is increasing with respect to demand y_t, and decreasing to an unobserved,

[4] The model is partially based on the simple model in Zhang (2017), but I add more economic structure and differ informational structure to aid interpretation. I also further expand on the analytic to focus on the behaviour of disagreement to changes in uncertainty and how rationally inattentive price-setters respond to monetary shocks.

[5] The simplifying assumption that the shock is white noise enables us to get analytical solutions, as the optimal information decision is independent across time periods. We abstract away from dynamics, as we are interested in the cross-sectional attention allocation.

stochastic firm-specific productivity term a_{it}.

$$p_{it}^{*}=\varphi y_t-a_{it},\qquad \text{where}\quad a_{it}\sim N(0,\sigma_a^2) \tag{2}$$

This simple structure can be micro-founded by a profit-maximising firm with decreasing returns to scale (thus marginal costs are increasing in output) that is common with rational inattention models, or a firm that faces labour market rigidities (thus needs to pay higher wages to produce more output). The second interpretation lends to the interpretation of the constant $\varphi>0$ as the inverse of the Frisch elasticity of labour supply.

To help set optimal prices, firms receive the signals $s_{it}=\{s_{it}^{y},s_{it}^{a}\}$ on key variables:

$$s_{it}^{y}=y_t+\varepsilon_{it}^{y},\qquad \varepsilon_{it}^{y}\sim N(0,\sigma_{\varepsilon_{y,t}}^{2}) \tag{3}$$

$$s_{it}^{a}=a_{it}+\varepsilon_{it}^{a},\qquad \varepsilon_{it}^{a}\sim N(0,\sigma_{\varepsilon_{a,t}}^{2}) \tag{4}$$

The firms choose the variance of the noise on the two signals, but this decision is subject to an information constraint:

$$I\left(p_{it}^{*};s_{it}\right)=H\left(p_{it}^{*}\right)-H\left(p_{it}^{*}\mid s_{it}\right)\leq K \tag{5}$$

where the firms are limited to how much entropy $H(\cdot)$ they could reduce the uncertainty on the two state variables b_t and a_{it} after observing the signal s_{it}. Given that the signals are uncorrelated and Gaussian, this can be shown to simplified to (see Appendix A):

$$\underbrace{H\left(y_t\right)-H\left(y_t\mid s_{it}^{y}\right)}_{K_{it}^{y}}+\underbrace{H\left(a_{it}\right)-H\left(a_{it}\mid s_{it}^{a}\right)}_{K_{it}^{a}}\leq K \tag{6}$$

$$\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_y^2}{\sigma_{\varepsilon_{y,t}}^2}+1\right)}_{K_{it}^{y}}+\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_{a_i}^2}{\sigma_{\varepsilon_{a,t}}^2}+1\right)}_{K_{it}^{a}}\leq K \tag{7}$$

where K_{it}^{y} and K_{it}^{a} are the entropy reduction to the uncertainty on the two unobserved state variables . Hereafter, I will refer to K_{it}^{y} and K_{it}^{a} as the 'attention' firm i allocates to monitoring y_t and a_{it}, which will be chosen optimally.

Based on the previous equation, an attention allocation implies the following perceived volatility of the tracking noises:

$$\sigma^2_{\varepsilon_{y,t}} = \frac{1}{2^{2K^y_{it}} - 1}\sigma^2_y \tag{8}$$

$$\sigma^2_{\varepsilon_{a,t}} = \frac{1}{2^{2K^a_{it}} - 1}\sigma^2_{ai} \tag{9}$$

In other words, the more attention paid to each variable, the associated variance of the noise on the signals would be lower. As the signals are e and the only source of information on y_t is s^y_{it}, any dispersion in the expectations of y_t across firms i is captured by $\sigma^2_{\varepsilon_{y,t}}$. Thus, $\sigma^2_{\varepsilon_{y,t}}$ is a sufficient summary statistic of demand nowcast disagreement.

In the Zhang (2017) model, K is pinned down by ensuring the marginal benefit of information equates to a fixed marginal cost of information, as the firms 'purchase' information with a linear cost in K. This model has a small, but important, departure by assuming maximum information gain constraint K is exogenous to the firm. This makes it more tractable to see the impact of changes in uncertainty of different variables, as well as changes in the information capacity, on attention allocation and price-setting.

2.1 Optimal Pricing and Attention Allocation

Each firm i minimises the expected profit losses due to mispricing by setting prices given its information choice, subject to the maximum information gain constraint (equation (6)):

$$\min_{\{K^y_{it}, K^a_{it}\} \in \mathcal{R}^+} E\left[(p_{it} - p^*_{it})^2 | s_{it}\right] \qquad \text{subject to } K^y_{it} + K^a_{it} \leq K \tag{10}$$

As Maćkowiak and Wiederholt (2009) show, minimising the quadratic loss around the full-information optimal price subject to information constraints is equivalent to profit-maximisation. The quadratic loss function is symmetric, so it is trivial to show that the optimal price is the firms' best guess of what the true optimal price is given the signal it receives:

$$p_{it} = E\left[p^*_{it} \mid s_{it}\right] = \varphi E\left[y_t \mid s^y_{it}\right] - E\left[a_{it} \mid s^a_{it}\right] \tag{11}$$

As in Zhang (2017), the model is solved by a backward two-step procedure. Firstly, the optimal price is solved for a given attention allocation $\{K_{it}^y, K_{it}^a\}$. Secondly, I use the result from the first step to substitute for the profit loss (from the optimal profit) in the firm's objective as a function of the information choice. The attention allocation decision can then be solved by optimising the objective.

Similarly, the optimal price setting decision for a given attention allocation then can be inferred from standard Bayesian updating, the pricing rule (equation (11)) and noise volatilities (equations (8) and (9)):

$$p_{it} = \varphi \frac{\sigma_y^2}{\sigma_y^2 + \sigma_{\varepsilon_{y,t}}^2} s_{it}^y - \frac{\sigma_a^2}{\sigma_a^2 + \sigma_{\varepsilon_{a,t}}^2} s_{it}^a \tag{12}$$

$$= \varphi \left(1 - 2^{-2K_{it}^y}\right) s_{it}^y - \left(1 - 2^{-2K_{it}^a}\right) s_{it}^a \tag{13}$$

This optimal pricing behaviour substituted into the expected profit loss due to mispricing, noting the independence of fundamental and noise shocks, results in:

$$E\left[(p_{it} - p_{it}^*)^2 \mid s_{it}\right] = \varphi^2 2^{-2K_{it}^y} \sigma_y^2 + 2^{-2K_{it}^a} \sigma_a^2 \tag{14}$$

$$= \varphi^2 2^{-2K_{it}^y} \sigma_b^2 + 2^{-2K_{it}^a} \sigma_a^2 \tag{15}$$

where the last equality results from the prior variances $\sigma_y^2 = \sigma_b^2$, as the monetary policy component of demand $c \cdot r_t$ is observable. Substituting the maximum information gain constraint, it is trivial to show the expected profit loss is strictly convex for any finite and strictly positive combination of $\{\sigma_b^2, \sigma_a^2\}$. Thus, there exists a unique interior solution for the optimal attention allocation:[6]

$$K_{it}^{y*} = \frac{1}{2} \log_2 \left(\frac{\varphi \sigma_b}{\sigma_a}\right) + \frac{1}{2} K \tag{16}$$

$$K_{it}^{a*} = \frac{1}{2} \log_2 \left(\frac{\sigma_a}{\varphi \sigma_b}\right) + \frac{1}{2} K \tag{17}$$

The optimal attention allocation results are very intuitive: the attention paid to demand is increasing with the total attention available K and the uncertainty surrounding demand σ_b (as higher demand uncertainty in-

[6]See Appendix A

creases the benefits to monitoring demand conditions y_t), while decreasing in productivity uncertainty σ_a. The last result suggests that an increase in productivity uncertainty would make firms reallocate attention away from monitoring demand conditions. This contrasts to Zhang (2017), where the attention paid to a variable only depends on the prior variance of the variable itself and the marginal cost of attention.[7]

2.2 Comparative Statics: Disagreement

As we have now solved for the optimal attention allocation, in this subsection, we examine how disagreement of demand conditions $\sigma^2_{\varepsilon_{y,t}}$ responds to changes in: (1) total attention available K, (2) productivity uncertainty σ^2_a, and (3) demand uncertainty σ^2_b. In the next subsection, we examine the prices' reaction to monetary policy shock in response to changes in the mentioned variables.

Firstly, for demand disagreement, we revisit equation (8). From this equation, it is clear that disagreement is a function of (exogenous) fundamental uncertainty, but also related to the endogenous decision of attention allocation:

$$\sigma^2_{\varepsilon_{y,t}} = \frac{1}{2^{2K^y_{it}} - 1}\sigma^2_y$$

Differentiating it with respect to K, σ^2_a and σ^2_b results in:

$$\frac{d\sigma^2_{\varepsilon_{y,t}}}{dK} = -\sigma^2_b \ln(2) 2^{2K^y_{it}} \left(\frac{1}{2^{2K^y_{it}} - 1}\right)^2 < 0 \tag{18}$$

$$\frac{d\sigma^2_{\varepsilon_{y,t}}}{d\sigma^2_a} = \frac{1}{2}\frac{\sigma^2_b}{\sigma^2_a} 2^{2K^y_{it}} \left(\frac{1}{2^{2K^y_{it}} - 1}\right)^2 > 0 \tag{19}$$

$$\frac{d\sigma^2_{\varepsilon_{y,t}}}{d\sigma^2_b} = \frac{-2 + 2^{2K^y_{it}}}{2(2^{2K^y_{it}} - 1)^2} \gtrless 0 \tag{20}$$

The first two derivatives are simple and fairly intuitive: changes in total information processing available to firms K and productivity uncertainty σ^2_a only affect demand disagreement only through the endogenous response

[7] In her model, an increase in (the equivalent of) demand uncertainty would mean firms increase K, to ensure that the marginal benefit of attention equates the exogenous marginal cost.

of attention K^y_{it}. A lowering of the total information processing capacity of firms lead firms to pay less attention to aggregate demand (and productivity), leading to a poorer quality of information and thus increased disagreement across firms. Similarly, an increase of fundamental idiosyncratic productivity uncertainty lead firms to reallocate attention away from monitoring aggregate demand conditions, also resulting to increase in demand disagreement.

The more interesting case is what happens when fundamental demand uncertainty σ_b^2 rises. The sign of the derivative is ambiguous: it is positive when $K^y_{it} > \frac{1}{2}$ and negative when $K^y_{it} < \frac{1}{2}$.[8] In other words, when attention on aggregate demand is relatively high, fundamental demand uncertainty *positively* co-moves with demand disagreement, but when attention is relatively low, uncertainty and disagreement *negatively* co-move. This is because there are two opposing forces: a direct effect of increase in fundamental uncertainty, and an indirect effect from the endogenous reallocation of attention towards monitoring demand. When attention is relatively low, the re-allocation of attention towards aggregate demand conditions could be strong enough that it overturns the direct effect (as the marginal benefits of re-allocating attention towards demand is high).

Maćkowiak and Wiederholt (2009) argue that to explain the sluggish response of prices to aggregate monetary shocks, it must be that idiosyncratic productivity matters a lot more for firm profits than demand uncertainty ($\sigma_a^2 \gg \sigma_b^2$), implying that firms pay little attention to aggregate conditions. While my model is clearly not quantitative, the Maćkowiak and Wiederholt (2009) result at least points to the plausibility of negative co-movement between uncertainty and disagreement. Empirically, Kozeniauskas et al. (2018) document that the correlation between various uncertainty and disagreement measures are quite low.

2.3 Comparative Statics: Price Setting

This subsection returns to the key research question: how do prices respond to monetary shocks under different conditions? By combining equa-

[8] See Appendix A.2.

tion (11) and $s_{it}^y = y_t + \varepsilon_{it}^y = b_t - cr_t + \varepsilon_{it}^y$, we arrive at:

$$\frac{dp_{it}}{dr_t} = \frac{dp_{it}}{ds_{it}^y} \cdot \frac{ds_{it}^y}{dr_t} = \left(1 - 2^{-2K_{it}^y}\right) \cdot (-c)\varphi < 0 \tag{21}$$

$$= -\varphi c \left(1 - \frac{\sigma_a}{\sigma_b \varphi} 2^{-K}\right) \tag{22}$$

where we derive the second line by substituting in K_{it}^y (equation (16)). Intuitively, firms set lower prices as demand falls (as full-information optimal prices also fall). However, the extent that this occurs depends on the level of attention on aggregate demand conditions.

Taking the second-order comparative statics of equation (22) with respect to the same variables in the previous subsection:

$$\frac{d^2 p_{it}}{dr_t dK} = -\ln(2)\frac{\sigma_a}{\varphi \sigma_b} 2^{-K} \varphi c < 0 \tag{23}$$

$$\frac{d^2 p_{it}}{dr_t \sigma_a} = \frac{1}{\varphi \sigma_b} 2^{-K} \varphi c > 0 \tag{24}$$

$$\frac{d^2 p_{it}}{dr_t d\sigma_b} = -\frac{\sigma_a}{\varphi}\frac{1}{\sigma_b^2} 2^{-K} \varphi c < 0 \tag{25}$$

These results are also fairly intuitive: prices are less responsive to monetary shocks when firms pay less attention. This could be generated by: (1) a reduction in total information processing capacity, (2) an *increase* in productivity uncertainty, or (3) a *decrease* in aggregate demand uncertainty.

The key takeaway from this simple model is that the mechanisms of increased disagreement and uncertainty to the monetary transmission mechanism can be very different, and thus explain why the results with disagreement regimes contrast with those in the literature on uncertainty. For example, a reduction of information processing capability of agents raises disagreement and weakens monetary policy, but this change has no effect on fundamental uncertainty. Meanwhile, an increase in productivity uncertainty also increase demand disagreement, and the same time, reduce the effectiveness of monetary policy. But a *decrease* in demand uncertainty could cause an endogenous attention response, that is an *increase* in disagreement, but also weakens monetary transmission.

3 Empirical Analysis

3.1 Data

I obtained quarterly data of real GDP, GDP deflator, commodity price index, and effective Federal Funds Rates from Federal Reserves Economic Data (FRED) for the sample period from 1970Q1 to 2015Q3. Real GDP and GDP deflator are measures of economic activity and prices, sourced from the Bureau of Economic Analysis, and are seasonally adjusted. I include commodity price index is to control for oil price shocks and captures supply side factors that may influence output and prices. This data is from the Bureau of Labour Statistics, and is originally not seasonally adjusted.[9] The choice of these variables is standard in the empirical literature studying monetary policy transmission as noted by Christiano et al. (1994), Sims (1992), and Bernanke and Gertler (1995). I transform real GDP, GDP deflator and commodity price index with log first-differences.

I replaced the effective Federal Funds Rates (FFR) between 2009Q1 and 2015Q3 with the Wu and Xia (2016) shadow rate to account for the zero lower bound (ZLB) and quantitative easing.[10] During these periods, the effective Federal funds rate was in the 0 to 0.25 percent range, so the Wu-Xia shadow rate captures the overall monetary policy stance better than FFR on its own.

3.2 Measuring Disagreement

I draw the data to calculate disagreement among forecasters from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF). This quarterly survey covers a wide range of macroeconomic variables.

[9] I have seasonally adjusted commodity price index using the Census Bureaus X-13 ARIMA-SEATS, with near identical results.

[10] After the 2007-09 financial crisis, the Fed took drastic measures that took the FFR in to the zero lower bound from December 2008 to 2015. Additionally, the Fed took unconventional measures such as quantitative easing, to further ease credit conditions and lower long-term interest rates. Thus, after December 2008, the FFR is less likely to describe the monetary policy stance well. To overcome this issue, Wu and Xia (2016) propose a nonlinear term structure model to construct a 'shadow interest rate' that captures the effect of QE on the overall stance of monetary policy.

Each quarter, every forecaster receives a form in which to fill out values corresponding to forecasts for a variety of variables in each of the next five quarters (including the current quarter), as well as annualised values for the following 2 years.

The SPF's *cross sectional forecast dispersion* is a measure defined as the difference between the 75th percentile and the 25th percentile of the projections in levels or growth. SPF dispersion measures how close the individual forecasters' projections in the SPF with each other.

Following this, I calculate the benchmark disagreement among forecasters measure using the interquartile range of *real* GDP for the *current* quarter divided by the median of the current quarter as a normalisation. Interquartile range is widely used in the literature to ensure that any outliers do not unfairly influence the variable of interest – the measure of disagreement.[11]

Furthermore, as the aim of this paper is to study the responses of output and prices to a monetary shock, I focus on the variable that is representative of the business cycle, such as real GDP. As a robustness check, I also include the 1-year ahead cross sectional forecast disagreement of the real GDP as well as the current quarter and 1-year ahead forecast disagreement of the nominal GDP.[12][13]

[11] This is similar to using standard deviation as a measure of disagreement. However, as Sill (2014) shows, the standard deviation in cross-sectional forecasts is clearly more volatile, though tracks the interquartile range measure fairly closely. This volatility may partly reflect reporting errors by forecasters. Thus, in line with the literature, I measure disagreement using the interquartile range.

[12] The SPF provides individual forecasts for the quarterly and annual level of chain-weighted real GDP. The dataset is seasonally adjusted. Prior to 1992, these are forecasts for real GNP. Annual forecasts are for the annual average of the quarterly levels.

[13] The SPF provides individual forecasts for the quarterly and annual level of chain-weigh forecasts for the quarterly and annual level of nominal GDP. The data is seasonally adjusted. Prior to 1992, these are forecasts for nominal GNP. Annual forecasts are for the annual average of the quarterly levels.

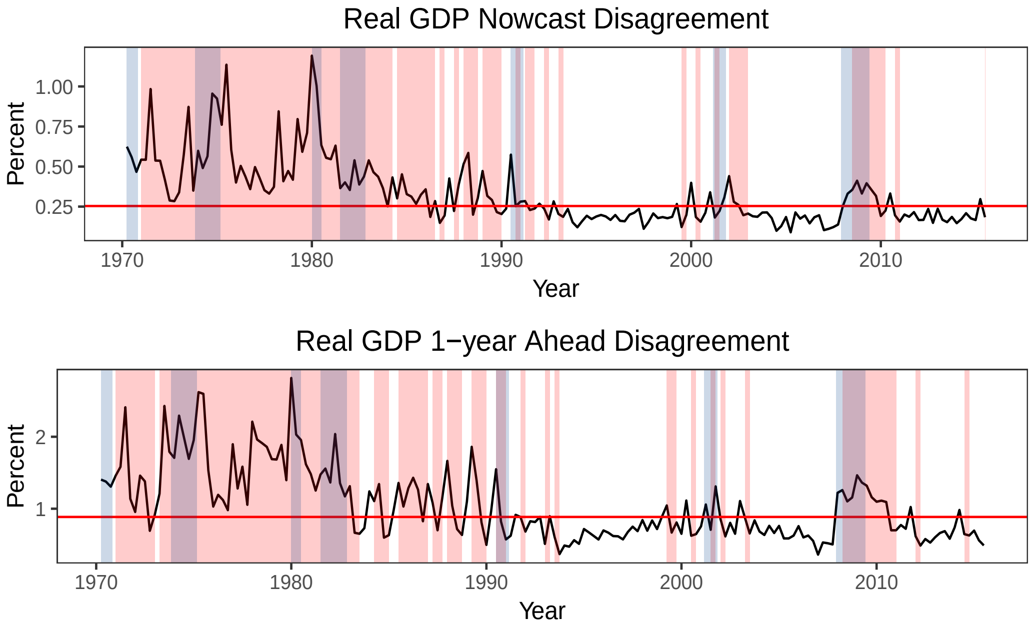

Figure 1: Time-varying disagreement – Time series of the real GDP disagreement index based on the dispersion (interquartile range) of SPF nowcasts and 1-year (4 quarters) ahead forecasts. The grey shaded areas indicate NBER-dated recessions. The red shaded areas indicate high disagreement periods. The red line indicate the estimated threshold VAR endogenous threshold.

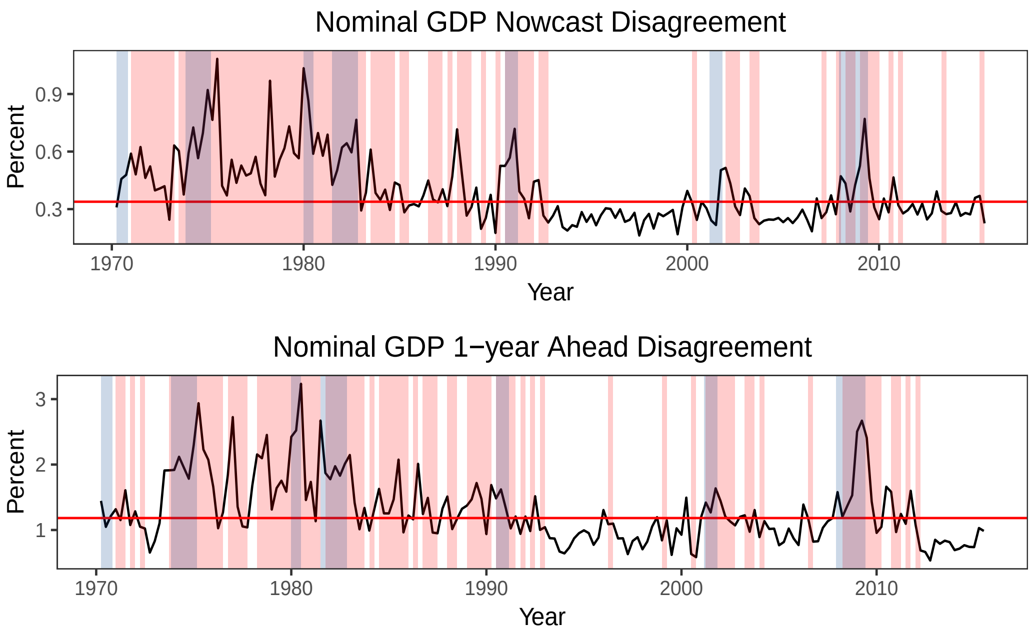

Figure 2: Time-varying disagreement – Time series of the nominal GDP disagreement index based on the dispersion (interquartile range) of SPF nowcasts and 1-year (4 quarters) ahead forecasts. The grey shaded areas indicate NBER-dated recessions. The red shaded areas indicate high disagreement periods. The red line indicate the estimated threshold VAR endogenous threshold.

Figure 1 and Figure 2 plots the Survey of Professional Forecaster cross-section disagreement, interquartile range of individual responses divided by the median, for the current quarter (nowcasts) and 1-year ahead forecasts of real and nominal GDP, respectively. The estimated value of the threshold parameter, as explained in the following subsection, is the solid red line in Figure 1 and Figure 2. High disagreement periods are defined as the periods where the disagreement is above the threshold (as estimated by the model) – depicted in the red shaded area. Grey shaded areas indicate the NBER business cycle contraction dates. Each regime contains half of the disagreement data, ensuring consistency across different disagreement measures. The delay parameter is set to 1, hence the regimes change with a lag of one period, after crossing the threshold. The charts show that disagreement generally tend to be higher in the survey early years compared with the latter half of the sample. Broadly speaking, this pattern of declining disagreement tracks the period known as the Great Moderation from 1984 to 2008, when the overall volatility of the economic data was lower than in the pre-1984 period. Although, we still observe high disagreement regimes, especially around business cycle contraction dates. While high disagreement is correlated with recessions, high disagreement episodes are more prolonged than recessions, and disagreement regime changes typically occur at a higher frequency than business cycles.

3.3 Methodology

3.3.1 Estimation of the Threshold Variable

The estimation of the threshold uses conditional maximum likelihood, following Galvão (2006). If the threshold is known, it is possible to simply split the sample (above and below the threshold variable) and estimate the parameters with OLS, as well as the variance-covariance matrix Σ of the residuals U_t in each of the two regimes. Thus, we can iterate across the threshold values, to find the optimal threshold θ^*:

$$\theta^* = \min_{\theta} \left[\frac{T_1}{2} \log |\widehat{\Sigma}^{(1)}(\theta)| + \frac{T_2}{2} \log |\widehat{\Sigma}^{(2)}(\theta)| \right]$$

where $|\widehat{\Sigma}^{(i)}(\theta)|$ is the determinant of the covariance matrix of the residuals

U_t in regimes $i = 1, 2$ (low and high disagreement regimes). The set of threshold values that are searched over is based on restrictions so that half of observations are in each regime.

3.3.2 Threshold Vector Autoregression Model

The baseline methodology of this paper is a threshold VAR that allows us to capture potentially different effect of monetary policy shocks to differ high and low disagreement regimes. The VAR model parameters are allowed to differ across (disagreement) regimes, and the transition between the regimes being governed by the evolution of a single *endogenous* variable of the VAR crossing a threshold (the 'threshold variable'). Therefore, this makes it possible that regime switches may occur after the shock to each variable. Because of this, the magnitude (and even the sign) of the impulse response may be affected by: (1) the state of the system at the time of the shock, (2) the sign of the shock, and (3) the magnitude of the shock.

The difference between a threshold VAR and the more common Markov-switching approach, is that Markov-switching models examine the whole empirical model for regime breaks (which may be affected by various shocks and structural changes unrelated to the underlying states). As a result, Markov-switching approaches tend to pick up the large regime change from the Great Inflation to the Great Moderation period, and very small number of regime changes within the Great Moderation era. Instead, by specifying a 'threshold variable' to the threshold VAR – which would then determine the threshold that govern which regime a particular point in time is in – I show that there suggests a significant variation in states even *within* the Great Moderation period.

The threshold VAR model is described below. The first term in on the right hand side of the equation is analogous to a linear VAR. The non-linearity of the model comes from introducing different regimes on the second term of the right hand side.

$$Y_t = \left[c_1 + \sum_{j=1}^{p} \gamma^1(L) Y_{t-j} \right] + \left[c_2 + \sum_{j=1}^{p} \gamma^2(L) Y_{t-j} \right] I(y^*_{t-d} > \theta) + U_t$$

where Y_t is a vector of endogenous (stationary) variables as mentioned in the previous section. I is an indicator function that takes the value of 1 when the threshold variable is higher than the *estimated* threshold parameter θ, and 0 otherwise, with time lag d set to 1. U_t are reduced-form disturbances.

$\gamma^1(L)$ and $\gamma^2(L)$ are lag polynomial matrices with order p. The lag order selection by Akaike information criteria marginally chose 2 lags in the threshold VAR and 4 lags in the linear VAR. This is as expected, as the threshold VAR has more parameters to estimate. As the middle ground, I chose 3 lags for threshold VAR which is more consistent with the findings in the literature that monetary policy's effect is long and variable. In terms of the Akaike information criteria, the AIC for two and three lags are almost identical, which suggests that the third lag is capturing some additional information.

The specific identification – real GDP, GDP deflator, the commodity price index, the Federal Funds Rates and the SPF disagreement – reflects some assumptions about the links in the economy. The ordering of the first four variables associated with the Cholesky decomposition of the covariance matrix of U_t is widely used, such as in Bernanke and Gertler (1995). Ordering SPF dispersion last implies that it reacts contemporaneously to all other variables. The results are robust to other orderings.

As this is a non-linear model, I use the generalised impulse response (GIRF) approach of Tsay (1998). The full algorithm, including the computation of bootstrap confidence intervals, is described in Appendix C of Caggiano et al. (2015).

3.4 Baseline Results

Figure 3, Figure 4, Figure 6 and Figure 5 show the impulse responses to a 1 standard deviation positive shock to FFR, while the shaded area corresponds to a 68% bootstrapped confidence interval. Figure 3 show the IRFs of linear vector autoregressive without differentiating the level of disagreement in economy. Figure 4, Figure 6 and Figure 5 show the GIRFs of the baseline threshold VAR, allowing for a shock that occurs initially in a low disagreement regime (blue line) and high (red-dash line) disagreement regime.

In the linear VAR, the peak effect on real GDP is 0.5% after around 8 quarters or 2 years, which is a typical horizon in the literature for output to respond to a contractionary monetary shock. The commodity price index drops more quickly than GDP deflator as expected by Bernanke and Gertler (1995). The sluggish responses in real GDP and price level, as well as the persistent decline in GDP deflator is fairly consistent with the literature e.g. Galí (2015) and Christiano et al. (1999). The GDP deflator depiction of a weak 'price-puzzle' – prices increase after an increase in FFR – is a common finding for monetary shocks identified with a recursively identified VAR.

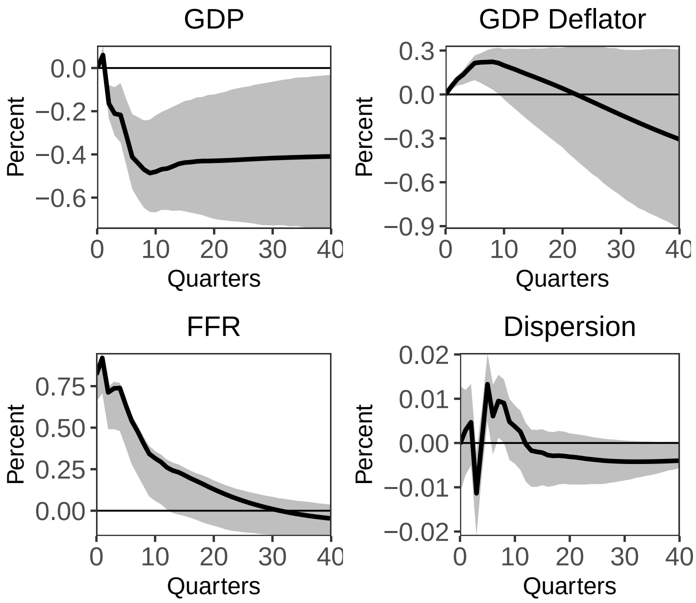

Figure 3: The shock corresponds to a positive one standard deviation change in the FFR. The IRFs are generated with 68% bootstrapped confidence intervals using Cholesky-identified structural VAR. Sample: 1970Q1 - 2015Q3.

The main result shown in Figure 4 is the heterogeneity in the effect of monetary policy shock across high (red) and low (blue) disagreement regimes. In high disagreement periods, monetary policy shocks have a strong impact on real activity yet a weak impact on nominal variable. And in low disagreement periods, monetary policy is more powerful in controlling prices.

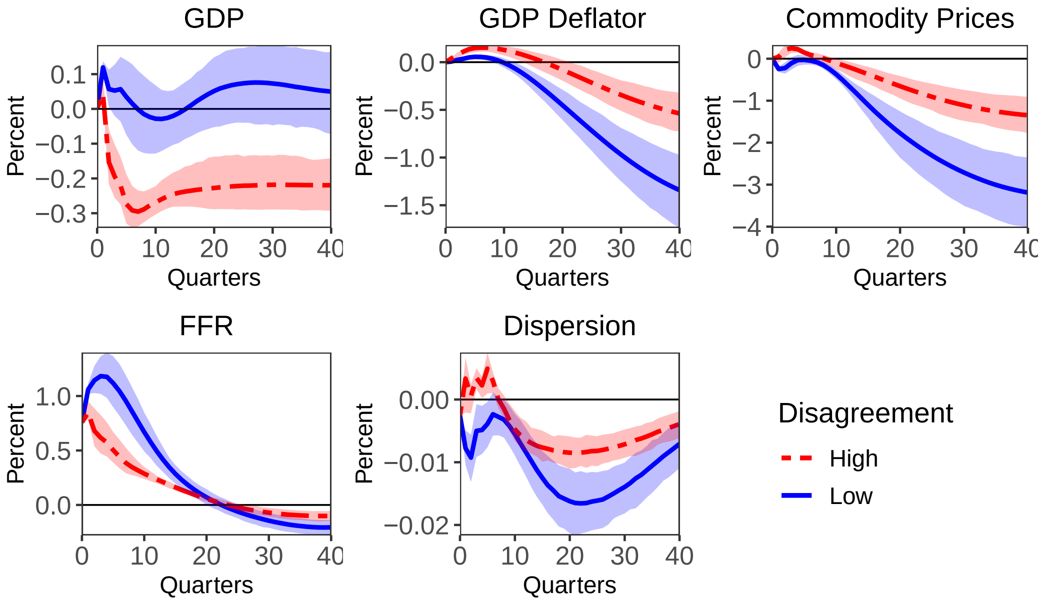

Figure 4: The shock corresponds to a positive one standard deviation change in the FFR. The GIRFs are generated with 68% bootstrapped confidence intervals using threshold VAR. The threshold is estimated using SPF disagreement of the nowcasts of **real** GDP. Red lines indicate high disagreement period and blue lines indicate low disagreement period. Sample: 1970Q1 - 2015Q3.

There is a long debate in the literature on the predictions of monetary policy transmission in different economic regimes. When looking at recessions or higher uncertainty, the typical intuition would be that agents becomes more cautious and therefore responds more slowly. Tenreyro and Thwaites (2016) find strong evidence that the effects of monetary policy on real and nominal variables are less powerful in recessions. Castelnuovo and Pellegrino (2018) and Aastveit et al. (2017) also point to a weak impact of monetary policy shocks on real activity under high uncertainty – the period they relate with recessions.

In contrary, I show that in high disagreement periods, a positive shock to FFR is more powerful in controlling output, yet less powerful in controlling prices. The GDP deflator under low disagreement becomes statistically significant from zero at a horizon less than half of the GIRF under high disagreement. Specifically, under low (high) disagreement, the GDP deflator is statistically significant from zero at 68% confidence interval by quarter 10 (18) – a difference of 8 quarters. Furthermore, in contrast to findings in the uncertainty literature, a contractionary monetary policy is more powerful in controlling output under high, than low, disagreement. Real GDP is broadly statistically insignificant different from zero when disagreement of real GDP among forecasters is low. In the presence of heightened disagreement, the trade-off between output and inflation worsens, as output falls faster after a positive monetary policy shock. This means inflation-output trade-off is even trickier to deal with when disagreement is high, which I discuss in more detail later in the discussion of policy implications.

The rational inattention model offers three explanations for the empirical findings. All explanations have a common theme that to produce the more sluggish response of prices to a monetary shock, attention paid by price-setters to aggregate conditions must be lower. Thus, firms react less to monetary shocks, making prices more 'sticky'. A standard New Keynesian model with stickier prices would predict that output would respond more to a monetary shock.

Firstly, the information processing capacity of firms could be lower, leading firms to reduce attention to aggregate conditions (and others). This could be caused by a variety of causes — for example, the exit of firms over the business cycle break down existing supplier-customer relationships that facilitate information flows across the supply chain. This would also reduce the quality the information that the firm processes, leading to higher disagreement, consistent with the empirical finding.

Secondly, higher uncertainties in state variables other than aggregate conditions (in the model, idiosyncratic productivity was one example), lead firms to re-allocate attention away from aggregate conditions. This has the same effect in increasing disagreement and stickier prices. This result also holds in larger general equilibrium models. Maćkowiak and Wiederholt (2009) show that by increasing the variance of idiosyncratic productivity shocks, rationally inattentive firms pay very little attention to monetary shocks, resulting in prices reacting slowly and by a small amount to the monetary shock.

Lastly, a *decrease* in aggregate demand uncertainty have potential impact to make prices more sticky. A rationally inattentive firm would respond to this by reducing attention allocated to monitoring aggregate conditions. As the model shows, in some parameter regions, the endogenous response of attention allocation has the potential to increase disagreement by reducing the information quality used to monitor on aggregate conditions. These regions typically occur when the overall variance of aggregate conditions is low compared, the marginal benefits are high. This is exactly the parameter space that Maćkowiak and Wiederholt (2009) suggest is plausible to create the effect that prices respond sluggishly to monetary shocks.

These theoretical results bridge the disagreement results with the broader literature on the effect of uncertainty on monetary transmission, which typically find that monetary policy has a weaker effect on prices and output during heightened uncertainty. The effect of rising uncertainty on the responsiveness of prices is potentially non-monotonic, and the three different posited mechanisms could be more important at different times. As discussed earlier, Kozeniauskas et al. (2018) measure of macro uncertainty is positively but not strongly correlated with higher-order uncertainty measured in dispersions in forecasts.[14]

3.5 Robustness

The main finding of the heterogeneity in the effect of monetary policy shock across high and low disagreement regimes holds subject to a variety of robustness checks. Figure 5 uses SPF disagreement of 1-year ahead forecast of real GDP, while Figure 6 estimates the threshold using *nominal* GDP nowcasts of SPF disagreement.

Qualitatively, the main result that there exist a heterogeneity in the response of output and prices to a monetary shock still holds given the 1-year ahead forecast disagreement of real GDP. The only notable difference is in the response of output during a low disagreement period. The response of output is now also statistically significant – implying inflation-output trade-off exists in both, low and high disagreement periods. However, the effect of monetary shock to output is still relatively weaker in lower disagreement, while monetary policy is still relatively stronger in affecting prices. This is inline with the baseline result, that the trade-off during high disagreement period is higher.

[14]The cross-section disagreement among forecasters used here is closer to the higher-order uncertainty measure rather than macro uncertainty measures such as Jurado et al. (2015) or VIX used in many uncertainty papers.

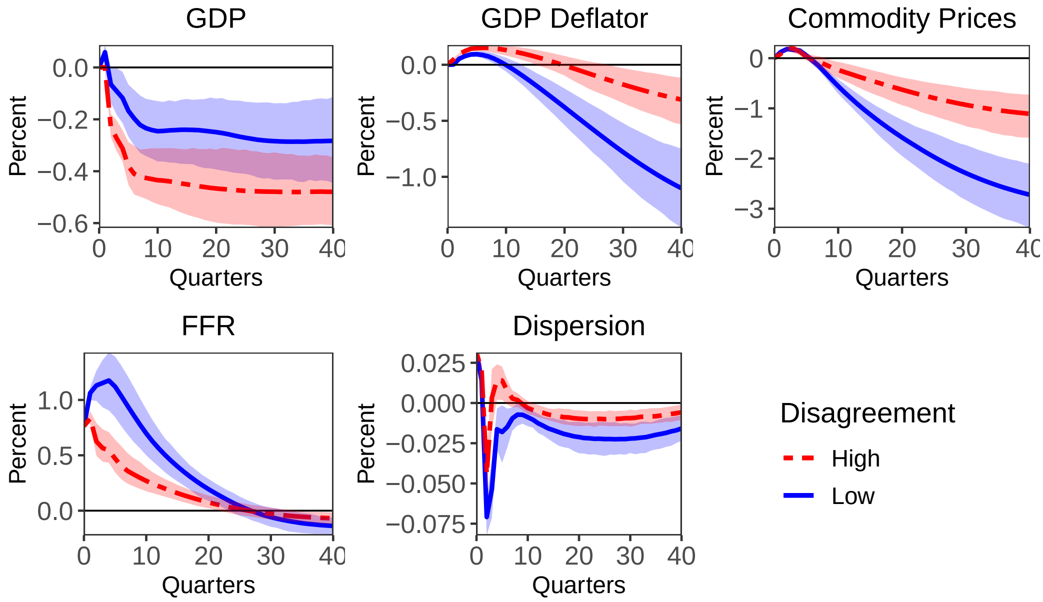

Figure 5: The shock corresponds to a positive one standard deviation change in the FFR. The GIRFs are generated with 68% bootstrapped confidence intervals using threshold VAR. The threshold is estimated using SPF disagreement of 1-year ahead forecast of real GDP. Red lines indicate high disagreement period and blue lines indicate low disagreement period. Sample: 1970Q1 - 2015Q3.

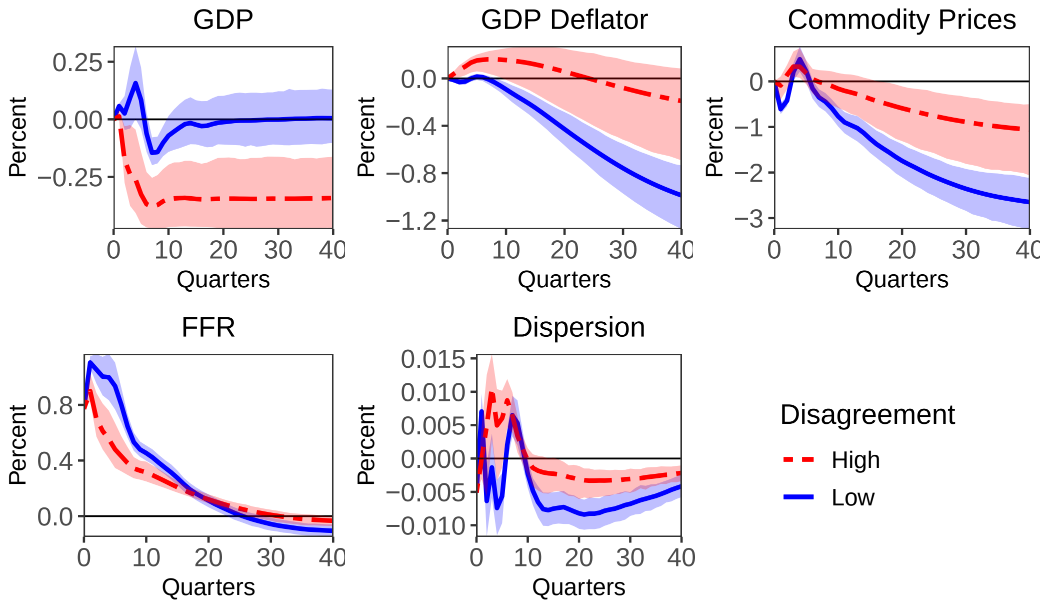

Figure 6: The shock corresponds to a positive one standard deviation change in the FFR. The GIRFs are generated with 68% bootstrapped confidence intervals using threshold VAR. The threshold is estimated using SPF disagreement of the nowcasts of nominal GDP. Red lines indicate high disagreement period and blue lines indicate low disagreement period. Sample: 1970Q1 - 2015Q3.

Figure 6 plots the responses (GIRFs) of the same variables as in the benchmark threshold VAR to a one-standard deviation shock to FFR. Since the aim of this paper is to study the responses of output and prices to a monetary shock, I focus on the variable that is representative of the business cycle, such as GDP. A dual-mandate central bank such as the Federal Reserves cares about stabilising both inflation and output growth. Thus, it would be relevant to also look at nominal GDP disagreement. Also, nominal GDP is directly influenced by monetary policy actions and subject to less data revisions than real GDP.

In comparison to Figure 4, the response of output is similar when the threshold is estimated using SPF disagreement of nominal GDP. In high disagreement periods, the peak response of output in both figures is around 0.3% at the 8^{th} quarter. While in low disagreement periods, the response of real GDP is broadly insignificant. However, Figure 6 shows that monetary policy is powerful in controlling prices *only* during low disagreement periods. Nonetheless, even when using nominal, instead of real GDP disagreement, monetary policy is still more powerful in controlling output in high disagreement periods and more powerful in controlling prices in low disagreement periods.

Moreover, using a longer time forecast horizon does not change the main findings. The main difference in the GIRFs when we use SPF disagreement of real GDP 1-year ahead forecast shows real GDP is that under low disagreement periods is not statistically insignificant from zero any more, although it is still weaker than the response of output under high disagreement periods. The peak response of output under high (low) disagreement is around 0.5% (0.2%). In comparison to the response of GDP deflator in Figure 4, here prices responds a little slower – by 2 quarters – during both disagreement periods.

Overall, while the impulse response of output has relatively small quantitative differences with the different cross-section forecast disagreement measures, the response of prices is very consistent — the key variable and prediction in the theoretical model — across the various specifications.

Lastly, across all the different estimated threshold for disagreement regimes, the response of the Fed Funds Rate is higher for longer in the low disagreement regime. One explanation for this is, in the high disagreement regime, output falls significantly by more and thus the endogenous monetary policy component is forced to relax monetary policy. On the other hand, as this does not occur under the low disagreement regime, this enables the central bank to keep monetary policy tight for longer to lower inflation. This suggests that, at least empirically, the inflation expectations channel does not operate by as much as the fall in inflation created by the drag on output gap.

4 Conclusion

This paper dissects the relationship between uncertainty and disagreement, and focuses on how they distinctly affect an important policy-relevant question: how state-dependent are the effects of monetary policy?

The contribution of this paper is two-fold. Firstly, I design a tractable rational-inattention model that unifies uncertainty and disagreement together, to highlight when there is a positive link, and when they break down. I utilise the model to examine how the two concepts affect price-setting behaviour of firms, and thus, the effect of monetary policy has on prices and output. This model suggests that in periods of higher disagreements the response of prices is unambiguously more sluggish in response to a monetary shock. This contrasts to how prices respond when fundamental uncertainty is higher. Another insight from the rational inattention model is that endogenous optimal attention allocation could cause disagreement to change non-monotonically in response to fluctuations in aggregate uncertainty.

Secondly, using a threshold VAR and a measure of disagreement of professional forecasters, I empirically document how in periods of heightened disagreement, monetary policy has less control over inflation, but more influence over output. This is a novel finding over the conventional results in the uncertainty literature.

One policy takeaway from these results is the role of central bank communications in disinflations and the sacrifice ratio. As noted, in low disagreement regimes, contractionary monetary policy is able to reduce inflation significantly with relatively little output losses. This raises the potentially important role of central bank in communicating aggregate conditions to economic agents, enabling them to internalise the disinflationary policy (effectively, making prices more flexible). Thus, the sacrifice ratio is lower and enables an inflation-targeting central bank to better achieve its objective. This mechanism complements the literature results in having a credible central bank moving inflation expectations down during a disinflationary policy episode, which further reduces the sacrifice ratio.

References

Aastveit, K. A., Natvik, G. J., and Sola, S. (2017). Economic uncertainty and the influence of monetary policy. *Journal of International Money and Finance*, 76:50–67.

Baker, S. R., Bloom, N., and Davis, S. J. (2016). Measuring economic policy uncertainty. *The Quarterly Journal of Economics*, 131(4):1593–1636.

Bernanke, B. S. and Gertler, M. (1995). Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives*, 9(4):27–48.

Caggiano, G., Castelnuovo, E., Colombo, V., and Nodari, G. (2015). Estimating fiscal multipliers: News from a non-linear world. *The Economic Journal*, 125(584):746–776.

Caggiano, G., Castelnuovo, E., and Groshenny, N. (2014). Uncertainty shocks and unemployment dynamics in us recessions. *Journal of Monetary Economics*, 67:78–92.

Castelnuovo, E. and Pellegrino, G. (2018). Uncertainty-dependent effects of monetary policy shocks: A new-keynesian interpretation. *Journal of Economic Dynamics and Control*.

Christiano, L. J., Eichenbaum, M., and Evans, C. (1994). The effects of monetary policy shocks: some evidence from the flow of funds. Technical report, National Bureau of Economic Research.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end? *Handbook of macroeconomics*, 1:65–148.

D'Amico, S., Orphanides, A., et al. (2008). *Uncertainty and disagreement in economic forecasting*. Divisions of Research, Statistics and Monetary Affairs, Federal Reserve Board.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.

Galvão, A. B. C. (2006). Structural break threshold vars for predicting us recessions using the spread. *Journal of Applied Econometrics*, 21(4):463–487.

Jurado, K., Ludvigson, S. C., and Ng, S. (2015). Measuring uncertainty. *The American Economic Review*, 105(3):1177–1216.

Koop, G., Pesaran, M. H., and Potter, S. M. (1996). Impulse response analysis in nonlinear multivariate models. *Journal of econometrics*, 74(1):119–147.

Kozeniauskas, N., Orlik, A., and Veldkamp, L. (2018). What are uncertainty shocks? *Journal of Monetary Economics*, 100:1–15.

Lo, M. C. and Piger, J. M. (2005). Is the response of output to monetary policy asymmetric? evidence from a regime-switching coefficients model. *Journal of Money, Credit, and Banking*, 37(5):865–886.

Maćkowiak, B. and Wiederholt, M. (2009). Optimal sticky prices under rational inattention. *The American Economic Review*, 99(3):769–803.

Menkulasi, J. (2009). Rational inattention and changes in macroeconomic volatility, university of maryland. Technical report, Working Paper.

Peersman, G. and Smets, F. (2001). The monetary transmission mechanism in the euro area. *ECB Working Paper 91*.

Pellegrino, G. (2017). Uncertainty and monetary policy in the us: A journey into non-linear territory.

Rich, R. W. and Tracy, J. (2018). A closer look at the behavior of uncertainty and disagreement: Micro evidence from the euro area. *FRB of Cleveland Working Paper*.

Sill, K. (2014). Forecast disagreement in the survey of professional forecasters. *Business Review*, (Q2):15–24.

Sims, C. A. (1992). Interpreting the macroeconomic time series facts: The effects of monetary policy. *European Economic Review*, 36(5):975–1000.

Tenreyro, S. and Thwaites, G. (2016). Pushing on a string: Us monetary policy is less powerful in recessions. *American Economic Journal: Macroeconomics*, 8(4):43–74.

Tsay, R. S. (1998). Testing and modeling multivariate threshold models. *journal of the american statistical association*, 93(443):1188–1202.

Wu, J. C. and Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking*, 48(2-3):253–291.

Zhang, F. (2017). Rational inattention in uncertain business cycles. *Journal of Money, Credit and Banking*, 49(1):215–253.

A Rational Inattention Model Details

Optimal price setting decision:

$$p_{it} = E\left[p_{it}^* \mid s_{it}, I_{i,t-1}\right] = \varphi E\left[y_t \mid s_{it}^y, I_{i,t-1}\right] - E\left[a_{it} \mid s_{it}^a, I_{i,t-1}\right] \tag{26}$$

Information constraint:

$$I\left(p_{it}^*; s_{it} \mid I_{i,t-1}\right) = H\left(p_{it}^* \mid I_{i,t-1}\right) - H\left(p_{it}^* \mid s_{it}, I_{i,t-1}\right) \leq K \tag{27}$$

Note that for Gaussian distributed random variable X, the unconditional and conditional entropy is:

$$H(X) = \frac{1}{2}\log_2\left[2\pi e Var(X)\right] \tag{28}$$

$$H(X \mid I) = \frac{1}{2}\log_2\left[2\pi e Var(X \mid I)\right] \tag{29}$$

So:

$$\underbrace{H\left(y_t \mid I_{i,t-1}\right) - H\left(y_t \mid s_{it}^y, I_{i,t-1}\right)}_{K_{it}^y} + \underbrace{H\left(a_{it} \mid I_{i,t-1}\right) - H\left(a_{it} \mid s_{it}^a, I_{i,t-1}\right)}_{K_{it}^a} \leq K \tag{30}$$

Taking the profit maximising price and signals (where the noises of the signals follow unit-variance Gaussian processes and independent of one another), the information constraint becomes:

$$\begin{aligned}&\underbrace{\frac{1}{2}\log_2\left[2\pi e Var\left(y_t \mid I_{i,t-1}\right)\right] - \frac{1}{2}\log_2\left[2\pi e Var\left(y_t \mid s_{it}^y, I_{i,t-1}\right)\right]}_{K_{it}^y} \\ &+ \underbrace{\frac{1}{2}\log_2\left[2\pi e Var\left(a_{it} \mid I_{i,t-1}\right)\right] - \frac{1}{2}\log_2\left[2\pi e Var\left(a_{it} \mid s_{it}^a, I_{i,t-1}\right)\right]}_{K_{it}^a} \leq K\end{aligned}$$

$$\underbrace{\frac{1}{2}\log_2\left[2\pi e\sigma_y^2\right]-\frac{1}{2}\log_2\left[2\pi e\frac{\sigma_{\varepsilon y}^2}{\sigma_{\varepsilon y}^2+\sigma_y^2}\sigma_y^2\right]}_{K_{it}^y}$$
$$+\underbrace{\frac{1}{2}\log_2\left[2\pi e\sigma_{ai}^2\right]-\frac{1}{2}\log_2\left[2\pi e\frac{\sigma_{\varepsilon ai}^2}{\sigma_{\varepsilon ai}^2+\sigma_{ai}^2}\sigma_{ai}^2\right]}_{K_{it}^a}\leq K$$

$$\underbrace{-\frac{1}{2}\log_2\left[\frac{\sigma_{\varepsilon y}^2}{\sigma_{\varepsilon y}^2+\sigma_y^2}\right]}_{K_{it}^y}\underbrace{-\frac{1}{2}\log_2\left[\frac{\sigma_{\varepsilon ai}^2}{\sigma_{\varepsilon ai}^2+\sigma_{ai}^2}\right]}_{K_{it}^a}\leq K \tag{31}$$

$$\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_y^2}{\sigma_{\varepsilon y}^2}+1\right)}_{K_{it}^y}+\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_{ai}^2}{\sigma_{\varepsilon ai}^2}+1\right)}_{K_{it}^a}\leq K \tag{32}$$

Based on the previous equation, an attention allocation implies the following perceived volatility of the tracking noises

$$\sigma_{\varepsilon y}^2=\frac{1}{2^{2K_{it}^y}-1}\sigma_y^2 \tag{33}$$

$$\sigma_{\varepsilon ai}^2=\frac{1}{2^{2K_{it}^a}-1}\sigma_{ai}^2 \tag{34}$$

A.1 Optimal Pricing Rule and Attention allocation

For a given attention choice, Kalman filtering equation, pricing rule, and the noise volatility above, the optimal price setting decision is

$$\begin{aligned}p_{it}&=E\left[p_{it}^*\mid s_{it},I_{i,t-1}\right]\\&=\varphi E\left[y_t\mid s_{yit},I_{i,t-1}\right]-E\left[a_{it}\mid s_{ait},I_{i,t-1}\right]\\p_{it}&=\varphi\left(1-2^{-2K_{it}^y}\right)s_{it}^y-\left(1-2^{-2K_{it}^a}\right)s_{it}^a\end{aligned} \tag{35}$$

Conditional profit loss due to mispricing becomes:

$$E\left[(p_{it}-p^*_{it})^2 \mid I_{i,t-1}\right] \tag{36}$$

$$= E\left[\varphi\left(1-2^{-2K^y_{it}}\right)s^y_{it} - \left(1-2^{-2K^a_{it}}\right)s^a_{it} - (\varphi y_t - a_{it})\right]^2 \tag{37}$$

$$= E\left[\varphi\left(-2^{-2K^y_{it}}y_t + \left(1-2^{-2K^y_{it}}\right)\varepsilon^y_{it}\right) - \left(-2^{-2K^a_{it}}a_{its} + \left(1-2^{-2K^a_{it}}\right)\varepsilon^a_{it}\right)\right]^2 \tag{38}$$

$$= E\left[\varphi^2\left(2^{-4K^y_{it}}y^2_t + \left(1-2^{-2K^y_{it}}\right)^2\varepsilon^{y2}_{it}\right) + \left(2^{-4K^a_{it}}a^2_{it} + \left(1-2^{-2K^a_{it}}\right)^2\varepsilon^{a2}_{it}\right)\right] \tag{39}$$

taking expectations and substituing $\sigma^2_{\varepsilon y}$ and $\sigma^2_{\varepsilon a}$

$$E\left[(p_{it}-p^*_{it})^2 \mid I_{i,t-1}\right] \tag{40}$$

$$= \left[\varphi^2\left(2^{-4K^y_{it}}\sigma^2_y + \left(1-2^{-2K^y_{it}}\right)^2\sigma^2_{\varepsilon y}\right) + \left(2^{-4K^a_{it}}\sigma^2_{ai} + \left(1-2^{-2K^a_{it}}\right)^2\sigma^2_{\varepsilon ai}\right)\right] \tag{41}$$

$$= \left[\varphi^2\left(2^{-4K^y_{it}}\sigma^2_y + \frac{\left(1-2^{-2K^y_{it}}\right)^2}{2^{2K^y_{it}}-1}\sigma^2_y\right) + \left(2^{-4K^a_{it}}\sigma^2_{ai} + \frac{\left(1-2^{-2K^a_{it}}\right)^2}{2^{2K^a_{it}}-1}\sigma^2_{ai}\right)\right] \tag{42}$$

$$= \left[\varphi^2\left(\frac{1-2^{-2K^y_{it}}}{2^{2K^y_{it}}-1}\right)\sigma^2_y + \left(\frac{1-2^{-2K^a_{it}}}{2^{2K^a_{it}}-1}\right)\sigma^2_a\right] \tag{43}$$

$$= \varphi^2 2^{-2K^y_{it}}\sigma^2_y + 2^{-2K^a_{it}}\sigma^2_a \tag{44}$$

$$= \varphi^2 2^{-2K^y_{it}}\sigma^2_b + 2^{-2K^a_{it}}\sigma^2_a \tag{45}$$

The objective function becomes

$$\min_{K^y_{it}} \varphi^2 2^{-2K^y_{it}}\sigma^2_b + 2^{-2(K-K^y_{it})}\sigma^2_a \tag{46}$$

first-order conditions:

$$\varphi^2(-2)\ln(2)2^{-2K^{y*}_{it}}\sigma^2_b + 2\ln(2)2^{-2(K-K^{y*}_{it})}\sigma^2_a = 0 \tag{47}$$

$$\varphi^2 2^{-2K^{y*}_{it}}\sigma^2_b = 2^{-2(K-K^{y*}_{it})}\sigma^2_a \tag{48}$$

taking $\log_{(2)}$ of everything:

$$-2K_{it}^{y*} + \log_2(\varphi^2\sigma_b^2) = -2K_{it} + 2K_{it}^{y*} + \log_2 \sigma_a^2 \tag{49}$$

$$K_{it}^{y*} = \frac{1}{4}\log_2\left(\varphi^2 \frac{\sigma_b^2}{\sigma_a^2}\right) + \frac{1}{2}K \tag{50}$$

$$K_{it}^{y*} = \frac{1}{2}\log_2\left(\varphi \frac{\sigma_b}{\sigma_a}\right) + \frac{1}{2}K \tag{51}$$

A.2 Comparative Statics: Disagreement

Using the perceived volatility of the tracking noises and optimal attention allocation

$$\sigma_{\varepsilon y}^2 = \frac{1}{2^{2K_{it}^y} - 1}\sigma_y^2, \quad K_{it}^{y*} = \frac{1}{4}\log_2\left(\varphi^2 \frac{\sigma_b^2}{\sigma_a^2}\right) + \frac{1}{2}K$$

Differentiating it with respect to σ_b^2:

$$\frac{d\sigma_{\varepsilon y}^2}{d\sigma_b^2} = \frac{1}{2^{2K_{it}^y} - 1} + \sigma_b^2 \frac{d}{dK_{it}^y}\left(\frac{1}{2^{2K_{it}^y} - 1}\right)\frac{dK_{it}^y}{d\sigma_b^2}$$

where

$$\frac{d}{dK_{it}^y}\left(\frac{1}{2^{2K_{it}^y} - 1}\right) = \frac{(-2)\ln(2)2^{2K_{it}^y}}{(2^{2K_{it}^y} - 1)^2}$$

$$\frac{d}{d\sigma_b^2}\left(\frac{1}{4}\log_2\left(\varphi^2 \frac{\sigma_b^2}{\sigma_a^2}\right) + \frac{1}{2}K\right) = \frac{1}{4}\frac{1}{\sigma_b^2 \ln(2)}$$

therefore,

$$\begin{aligned}
\frac{d\sigma^2_{\varepsilon y}}{d\sigma^2_b} &= \frac{1}{2^{2K^y_{it}}-1} + \sigma^2_b \frac{(-2)\ln(2)2^{2K^y_{it}}}{(2^{2K^y_{it}}-1)^2} \frac{1}{4} \frac{1}{\sigma^2_b \ln(2)} \\
&= \frac{1}{2^{2K^y_{it}}-1} - \frac{1}{2} \frac{2^{2K^y_{it}}}{(2^{2K^y_{it}}-1)^2} \\
&= \frac{2(2^{2K^y_{it}}-1) - 2^{2K^y_{it}}}{2(2^{2K^y_{it}}-1)^2} \\
&= \frac{-2 + 2^{2K^y_{it}}}{2(2^{2K^y_{it}}-1)^2} \gtrless 0
\end{aligned}$$

Differentiating it with respect to K and σ_a^2 results in:

$$\begin{aligned}
\frac{d\sigma_{\varepsilon y}^2}{dK} &= \frac{d\sigma_{\varepsilon y}^2}{dK_{it}^y}\frac{dK_{it}^y}{dK} \\
&= \sigma_b^2(-1)\frac{d2^{2K_{it}^y}}{dK_{it}^y}\left(\frac{1}{2^{2K_{it}^y}-1}\right)^2\frac{dK_{it}^y}{dK} \\
&= \sigma_b^2(-1)2\ln(2)2^{2K_{it}^y}\left(\frac{1}{2^{2K_{it}^y}-1}\right)^2\frac{dK_{it}^y}{dK} \\
&= -\sigma_b^2 2\ln(2)2^{2K_{it}^y}\left(\frac{1}{2^{2K_{it}^y}-1}\right)^2\frac{1}{2} \\
&= -\sigma_b^2\ln(2)2^{2K_{it}^y}\left(\frac{1}{2^{2K_{it}^y}-1}\right)^2 < 0
\end{aligned}$$

$$\begin{aligned}
\frac{d\sigma_{\varepsilon y}^2}{d\sigma_a^2} &= \frac{d\sigma_{\varepsilon y}^2}{dK_{it}^y}\frac{dK_{it}^y}{d\sigma_a^2} \\
&= \sigma_b^2(-1)2\ln(2)2^{2K_{it}^y}\left(\frac{1}{2^{2K_{it}^y}-1}\right)^2\frac{dK_{it}^y}{d\sigma_a^2} \\
&= \sigma_b^2(-1)2\ln(2)2^{2K_{it}^y}\left(\frac{1}{2^{2K_{it}^y}-1}\right)^2\left(-\frac{1}{4\ln(2)\sigma_a^2}\right) \\
&= \frac{1}{2}\frac{\sigma_b^2}{\sigma_a^2}2^{2K_{it}^y}\left(\frac{1}{2^{2K_{it}^y}-1}\right)^2 > 0
\end{aligned}$$

A.3 Comparative Statics: Price Setting

$$p_{it} = \varphi\left(1-2^{-2K_{it}^y}\right)s_{it}^y - \left(1-2^{-2K_{it}^a}\right)s_{it}^a \tag{52}$$

where

$$s_{it}^y = y_t + \varepsilon_{it}^y = b_t - cr_t + \varepsilon_{it}^y$$

$$\frac{ds_{it}^y}{dr_t} = -c \tag{53}$$

$$\frac{dp_{it}}{dr_t} = \frac{dp_{it}}{ds_{it}^y}\frac{ds_{it}^y}{dr_t} = \varphi\left(1-2^{-2K_{it}^y}\right)(-c) < 0 \tag{54}$$

Which means that as r $\uparrow$, $p_{it} \downarrow$

Then, we can replace $2^{-2K_{it}^y}$ using

$$K_{it}^{y*} = \frac{1}{2}\log_2\left(\varphi\frac{\sigma_b}{\sigma_a}\right) + \frac{1}{2}K$$

such that

$$2^{-2K_{it}^y} = \frac{\sigma_a}{\sigma_b\varphi}2^{-K}$$

and thus

$$\frac{dp_{it}}{dr_t} = \left(1 - 2^{-2K_{it}^y}\right)(-c) = -\varphi c\left(1 - \frac{\sigma_a}{\sigma_b\varphi}2^{-K} < 0\right)$$

again, an expansionary monetary policy shock, price

$$\frac{d}{dK_{it}}\left(\frac{dp_{it}}{dr_t}\right) = -\ln(2)\frac{\sigma_a}{\sigma_b\varphi}2^{-K}\varphi c < 0 \tag{55}$$

$$\frac{d}{d\sigma_a}\left(\frac{dp_{it}}{dr_t}\right) = \frac{1}{\sigma_b\varphi}2^{-K}\varphi c > 0 \tag{56}$$

$$\frac{d}{d\sigma_b}\left(\frac{dp_{it}}{dr_t}\right) = \frac{\sigma_a}{\varphi}(-1)\frac{1}{\sigma_b^2}2^{-K}\varphi c < 0 \tag{57}$$

B GIRF Algorithm

GIRF Bootstrap Algorithm

I follow the algorithm in Koop et al. (1996):

1. Pick a history and Ω_{t-1} contains the sequence of lagged data up to time $t-1$, which defines the history of the model at date t. Also, pick a structural shock of size δ.

2. Use Monte-Carlo integration to compute the *conditional* response for: variable y, shock size δ, history Ω_{t-1} and horizon $h = 0, 1, \ldots, H$

3. Then average out over each regime's set of random histories Ω^r, to get the *unconditional* responses for each regime

4. Subtract the **second** from **first** time path. The difference is the estimate of GIRF.

5. However, Step 4 is a noisy estimate. To eliminate the random variation in the GIRF, repeat steps **2 - 4** many times and take the mean of the resulting impulse responses as the central tendency. I also take the empirical quantiles from these draws to compute the confidence intervals.

C Asymmetric Responses to Positive and Negative Monetary Shocks

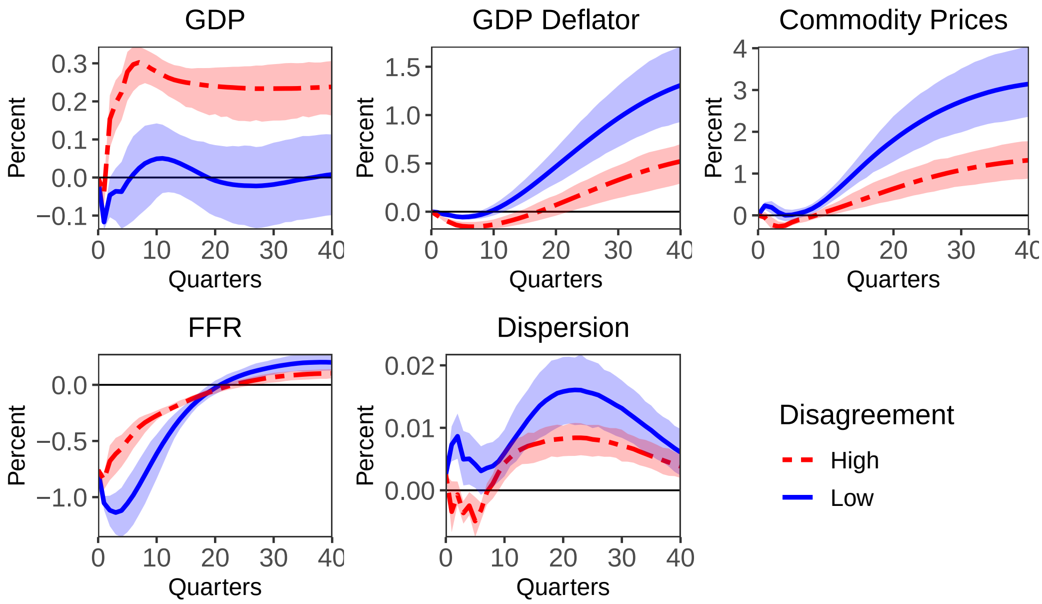

Figure 7: **Standard shock (-1) SD shock to FFR**. Red (blue) lines indicates high (low) disagreement, with 68% bootstrapped confidence intervals. The threshold is estimated using SPF disagreement of nowcast of real GDP.

One of the reasons for studying non-linear effects is to see whether asymmetric response to positive and negative shock exists. Previous studies on monetary policy transmission in different economic regimes do not often discuss any potentially of asymmetries between the response of economic variables to positive and negative shocks. We observe asymmetry in the response to real GDP and Fed fund rates only to a certain extent. It is most apparent on the median and confidence interval of real GDP shows a slightly different response under low disagreement, although still insignificantly different from zero. While the response of prices appears only in the *magnitudes* of the responses, rather than the shape. In a high disagreement regime, positive shock creates price-puzzle for about 15 quarters, while a negative shock only for 12 quarters, a difference of about one year. If the downward wage rigidity argument pass on to prices, via the virtue that labour is an input to production, then this is fairly consistent with that hypothesis. In times of low disagreement, this is similar except that there is a weaker price-puzzle. Also, by quarter 40, the response of a positive shock is only up to -1% yet for a negative shock the price drops to 1.2%. This asymmetry becomes more noticeable with large shocks.

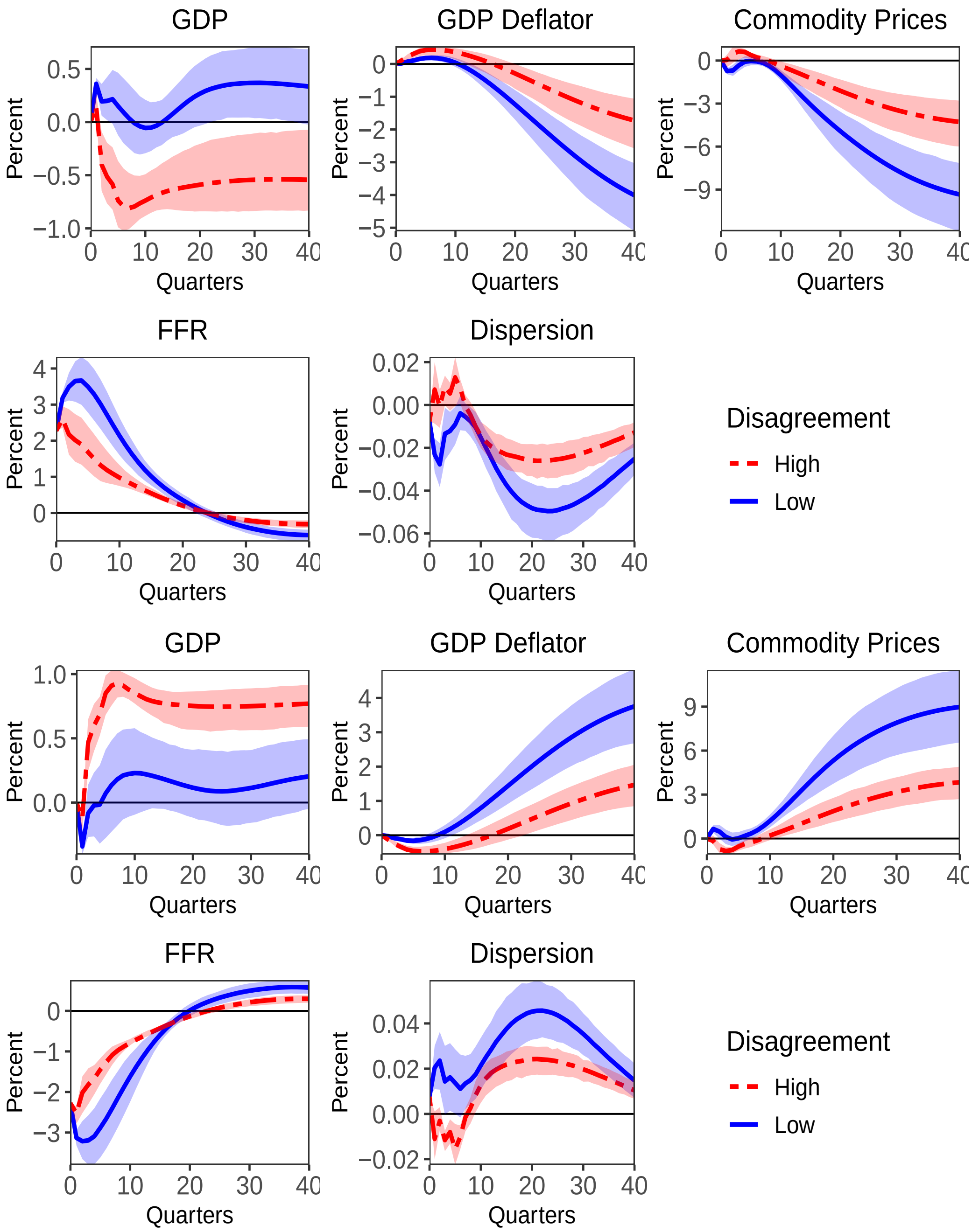

Figure 8: **Large Shocks (±3) SD shock to FFR** Top (bottom) figure is the responses to positive (negative) shock. Red (blue) lines indicates high (low) disagreement, with 68% bootstrapped confidence intervals.

In Figure 8, I compare the response to positive and negative large shock – 3 standard deviation shocks to FFR – on real GDP and prices. I investigate whether a larger shock result in more asymmetry than with a 1 standard deviation shocks to FFR. In low disagreement, the peak response of GDP deflator to a positive shock is to drop up to 4% and to negative shock it rises up to slightly under 4% in the 40^{th} quarter. Even if it is a slightly larger difference than with the standard shock, the economic significance cannot lead us to expect prices to be more sticky with larger shocks. The explanation of the relative differences between low and high disagreement in this case of a large shock is analogous to the response to the case of 1 standard deviation shocks.

In low and high disagreement periods, a 1 standard deviation shocks to FFR generate the corresponding response in output that is less responsive to an expansionary monetary shock than relative to a contractionary shock. The weak display of asymmetry in the response in output is more visible when the shocks to FFR is larger. Here, in response to a contractionary shock, the real GDP is significantly below zero for a couple of quarters when the economy is in a low disagreement period. However, real GDP is not significant at any quarter in response of an expansionary shock.

Nevertheless, either contractionary/expansionary monetary policy shocks or standard/large shocks, the key results do not change. In a high disagreement state, monetary policy has stronger real effects, while in low disagreement, monetary policy has stronger price effects. The GIRFs show more noticeable, although weak, asymmetry in larger shocks, in terms of the magnitude of the responses.