



Earnings Inequality and the Minimum Wage: Evidence from Brazil

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Abstract

We quantify the effect of a minimum wage on compression throughout the earnings distribution. Using the case of Brazil, which experienced a large decrease in earnings inequality while its real minimum wage increased from 1996-2012, we document that the inequality decrease was bottom-driven yet widespread, with compression up to the 75th earnings percentile. We develop an equilibrium search model with heterogeneous firms and workers and find that effects of the minimum wage are consistent with the above facts, explaining 70 percent of the observed inequality decrease, with half of the decrease due to spillovers further up the earnings distribution.

JEL-Codes: E240, E610, J310.

Keywords: worker and firm heterogeneity, minimum wage, matched employer-employee data, equilibrium search model.

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1 Introduction

To what extent does economic policy shape earnings inequality? Given that inequality has increased significantly in many economies over the past decades, a quantitative answer to this question has become more urgent. Aiming to boost earnings at the bottom of the distribution, many countries have advocated a minimum wage. While this may come at a cost, including increased unemployment, proponents of the minimum wage defend the policy as an effective way to reduce labor income inequality. Despite its importance, given an ongoing debate in the empirical literature and limited theoretical guidance on how to reconcile different findings, the quantitative effect of a minimum wage on earnings inequality is far from clear.¹

Skeptics of the benefits associated with a minimum wage point to the small fraction of workers bound by the wage floor as evidence that its impact is likely to be limited.² Furthermore, *spillover effects* of the minimum wage (i.e. effects higher up in the earnings distribution), while potentially promising, have been hard to identify given data limitations and methodological disagreements. Previous work in this area primarily builds on reduced-form evidence from household survey data as in [Lee \(1999\)](#) and [Autor et al. \(2016\)](#), with the latter concluding that spillover effects are indistinguishable from measurement error. Complementing this literature, we use large administrative data combined with a structural and testable model to quantify the effects of a minimum wage throughout the earnings distribution. The size and nature of the administrative data allow us to exploit more detailed variation with higher estimation precision than has previously been possible. The model lets us quantify the causal equilibrium effects of the minimum wage on inequality and unemployment in a counterfactual policy experiment, enabling us to discuss welfare implications.

To address this problem, we study the case of Brazil between 1988 and 2012, which has two key advantages for our purpose. First, Brazil has exceptional data availability, with administrative matched employer-employee data that we merge with administrative firm financial data covering a long time horizon. Second, there was a large policy change implemented in Brazil, with the real minimum wage increasing by 119 percent in real terms, starting out at 30 percent of median earnings and reaching 60 percent by the end of the period. The combination of a large policy

¹See [Flinn \(2010\)](#) for a comprehensive survey of the literature.

²In 2015, the share of hourly paid workers earning at or below the minimum wage was 3.3 percent in the US ([U.S. Bureau of Labor Statistics, 2016](#)).

change and detailed microdata provide us with an ideal testing ground for quantifying the effects of the minimum wage on earnings inequality.

To this end, we carry out the following three steps. In the first step, we use matched employer-employee data on workers and firms in Brazil to document a 26 log points drop in the variance of earnings between 1996 and 2012.³ We show that this decrease is characterized by three key facts: (i) the decrease was bottom-driven yet pervasive throughout large parts of the earnings distribution; (ii) reductions in the firm productivity-pay premium and in the worker skill premium were the key drivers behind the decrease; and (iii) the bindingness of the minimum wage is correlated across Brazilian states and over time with compression up to the 75th percentile of the earnings distribution.⁴

In the second step, we build an equilibrium model of frictional wage dispersion based on the canonical framework by [Burdett and Mortensen \(1998\)](#). Motivated by our empirical findings, we extend this framework to tractably feature heterogeneity in worker ability in addition to firm productivity differentials described in the original paper. We close the model by introducing a vacancy margin, allowing job creation to respond to the minimum wage increase. Theoretically and in line with our empirical facts, we show that minimum wage effects ripple through the earnings distribution and cause a decrease in the pass-through from firm productivity and worker skill to pay. Spillover effects arise because firms compete for workers by setting wages strategically relative to one another and in reference to the minimum wage. Therefore, the effects of the minimum wage reach above the wage floor but slowly fade toward the top of the earnings distribution by reducing the productivity-pay gradient across firms and the skill premium across workers.⁵

In the third step, we use our model to quantify the causal effect of the minimum wage on earnings inequality. We estimate the model on Brazilian matched employer-employee data from 1996-2000 and use it to conduct a counterfactual policy experiment, simulating the equilibrium

³To put Brazil's 26 log points decrease in the variance of log earnings into perspective, the same measure for adult male workers in the US increased by approximately eight log points over this period ([Heathcote et al., 2010](#); [Kopczuk et al., 2010](#)). Other countries experiencing rising inequality in recent decades include the UK ([Blundell and Etheridge, 2010](#)), Germany ([Fuchs-Schündeln et al., 2010](#)), Canada ([Brzozowski et al., 2010](#)), Italy ([Jappelli and Pistaferri, 2010](#)), and Sweden ([Domeij and Flodén, 2010](#)). Yet the Brazilian case is no anomaly: large inequality decreases have been documented for Mexico ([Binelli and Attanasio, 2010](#)), Spain ([Pijoan-Mas and Sánchez-Marcos, 2010](#)), Russia ([Gorodnichenko et al., 2010](#)), and large parts of Latin America ([Tsounta and Osueke, 2014](#)).

⁴By exploiting cross-sectional variation in the data, our methodology can identify effects of the minimum wage even in the presence of aggregate trends, so the fact that overall inequality declined in Brazil is not crucial to our analysis.

⁵While we estimate the strength of spillovers due to strategic complementarity in firms' wage setting, similar effects arise in other environments through comparative advantage in skills ([Teulings, 2000](#)), preferences ([Lopes de Melo, 2012](#)), fairness considerations ([Card et al., 2012](#)), substitutability across tasks/goods ([Stokey, 2016](#)), or educational investment ([Bárány, 2016](#)).

effects of the observed minimum wage increase. In line with our three stylized facts characterizing Brazil's inequality decrease, the estimated model predicts that the rise in the minimum wage caused: (i) a 21 log points decrease in the P50-P10 earnings ratio, or 68 percent of its empirical counterpart, but only a six log points decrease in the P90-P50 earnings ratio, or 46 percent of the empirical change; (ii) essentially all of the explained decline as a consequence of a lower firm productivity-pay gradient and lower worker skill premium; and (iii) significant spillover effects reaching up to the 75th percentile of the earnings distribution. Due to large effects of the minimum wage higher up the earnings distribution, the model attributes 70 percent of the total decrease in the variance of log earnings observed in the data over this period to the rise in the minimum wage. Half of the total inequality decrease in the data and in the model are due spillover effects of the minimum wage reaching up in the earnings distribution.

In contrast to a competitive theory of labor markets, our model also predicts modest disemployment effects of the minimum wage, consistent with the data. Thus, a general insight from our analysis is that frictional labor markets can propagate effects of policies like the minimum wage on the inequality while also buffering negative employment effects.

Related literature. Our work relates to three strands of the literature that aim to understand inequality in labor markets. The first strand is concerned with reduced-form earnings decompositions into a worker component, a firm component, and their covariance. The seminal work in this area is [Abowd, Kramarz, and Margolis \(1999](#), henceforth AKM) who prove identification of a two-way fixed effects model controlling for unobserved worker and firm heterogeneity, and apply the framework to French matched employer-employee data. Building on their methodology, a large number of papers have used this fixed effects methodology to study the sources of cross-sectional earnings dispersion, including [Andrews et al. \(2008\)](#) for Germany; [Iranzo et al. \(2008\)](#) for Italy; [Bagger and Lentz \(2016\)](#) for Denmark; [Card et al. \(2016\)](#) for Portugal; as well as [Abowd et al. \(1999b\)](#), [Woodcock \(2015\)](#), and [Sorkin \(2016\)](#) for the US. Using versions of the AKM framework or similar decompositions to study the dynamics of earnings inequality over time, ([Card et al., 2013](#); [Alvarez et al., 2016](#); [Barth et al., 2016](#); [Song et al., 2015](#); [Abowd et al., 2016](#)), highlight that changes in firm-level pay are important for understanding observed inequality over the last decades in Germany, Brazil, and the US. We complement this literature by rationalizing the AKM

two-way fixed effects framework within a structural equilibrium model.⁶ Such microfoundations are important to interpret moments of the aforementioned empirical literature, particularly the pattern of sorting between workers and firms (Lentz and Mortensen, 2010; Eeckhout and Kircher, 2011; Lopes de Melo, 2017; Bonhomme et al., 2016). We thus provide a theory for the contraction in between-firm pay differences, arguing that a rise in the minimum wage can lead to sizable compression throughout the earnings distribution.

Second, our theoretical framework is closely related to the literature on wage dispersion arising from labor market frictions. While work in this area goes back at least to Stigler (1961) and McCall (1970), a more recent class of equilibrium search models pioneered by Burdett (1978) and Burdett and Mortensen (1998) lays the foundation for our analysis. A rich body of follow-up work has used versions of this model to study wage dispersion and labor dynamics (van den Berg and Ridder, 1998; Bontemps et al., 1999, 2000; Postel-Vinay and Robin, 2002; Jolivet et al., 2006; Moscarini and Postel-Vinay, 2013; Lise et al., 2016; Lise and Robin, 2017). We contribute to this literature a tractable model of the minimum wage with two-sided heterogeneity in firm productivity and worker ability, an environment that previous research highlighted as important but difficult to study (Cahuc et al., 2006).

Third, our focus on determinants of the earnings distribution complements a long-standing debate on how a minimum wage affects labor market outcomes. Much of the literature has focused on employment effects of the minimum wage (Card and Krueger, 1994; Neumark and Wascher, 1994; Manning, 2005; Clemens and Wither, 2014; Dube et al., 2016), with mixed findings pointing in the direction of small, negative employment effects. In related work, a revisionist set of papers argue that weakening labor market institutions have contributed to rising earnings inequality in the US (DiNardo et al., 1996; Card and DiNardo, 2002). Using cross-state variation in the bindingness of the federal minimum wage, Lee (1999) concludes that much of the increased dispersion throughout the earnings distribution in the 1980s is explained by spillover effects of a declining wage floor. In a recent extension to this work, Autor et al. (2016) propose an alternative specification and estimation strategy that leads them to find spillover effects that are small and indistinguishable from noise in the US Current Population Survey. We complement this reduced-form literature by applying their empirical methodology to large administrative data and providing a

⁶That is, the AKM empirical wage decomposition is a special case of the equilibrium wage equation in our model. Recent alternative approaches include the bargaining framework in Bagger et al. (2014), the partial equilibrium model in Card et al. (2016), and the piece rate model in Burdett et al. (2016).

micro-founded equilibrium model that reconciles some of the conflicting findings of the previous literature.

Outline. The rest of the paper is structured as follows. Section 2 provides the required background on the minimum wage in Brazil. Section 3 documents three empirical facts characterizing Brazil's decrease in earnings inequality. To interpret these facts, Section 4 develops an equilibrium search model and theoretically characterizes the effects of a rise in the minimum wage on worker and firm pay differences. Section 5 estimates the model, which we then use in Section 6 to quantitatively quantify the effects of the minimum wage. Section 7 discusses implications of the minimum wage for employment and welfare as well as its relation to Brazil's informal economy. Finally, Section 8 concludes.

2 The minimum wage in Brazil

The minimum wage in Brazil is primarily a federal institution with only minor adjustments for regional price level differences. It was institutionalized as Decree-Law No. 2162 in 1940 and consolidated in 1943 under new labor laws (*Consolidação das Leis do Trabalho*, or *CLT*).⁷ While the minimum wage was initially region-specific and not automatically adjusted to inflation or even legally enforced, it underwent several reforms under different political regimes between the 1940s and 1984, when it was unified across regions. Yet it was not until when president Fernando Henrique Cardoso of the centrist Brazilian Social Democracy Party took office in 1995, following Brazil's monetary stabilization, that the minimum wage became a renewed policy focus.

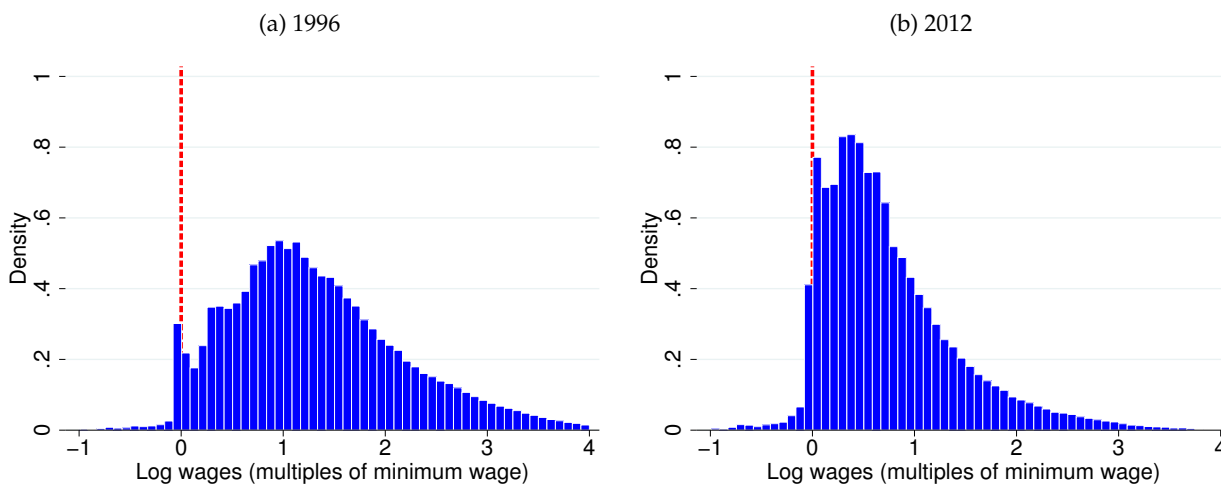
Between 1996 and 2012, the federal minimum wage grew by a total of 119 percent in real terms, fueled by a sequence of discretionary increases and reaching 622 BRL (410 PPP-adjusted USD) per month by the end of the period. To put these numbers into context, the minimum wage as a fraction of median earnings increased from around 34 percent in 1996 to 60 percent in 2012.⁸ Nowadays, the minimum wage is enforced by the the Brazilian Ministry of Labor (*Ministério do Trabalho e Emprego*, or *MTE*), ensuring high compliance rates through business audits in the form of on-site visits and surveys of local employees.

⁷The original law was based in parts on Mussolini's Carta del Lavoro in Italy.

⁸Over the same period, average labor productivity in manufacturing and mining increased by 16.6 percent; hence the ratio of the minimum wage to average labor productivity increased by 56.3 percent over this period.

Visual inspection of the earnings distribution over this period, presented in Figure 1, shows pronounced compression in the left tail, suggesting that the minimum wage was an important contributor towards falling inequality.⁹

Figure 1. Histogram of the Earnings Distribution



Notes: Density plots based on 60 equi-spaced histogram bins for population of male workers aged 18–49 in RAIS data.

Yet a key challenge for the minimum wage hypothesis is the fact that few people are directly affected by the minimum wage.¹⁰ The small share of workers directly affected by the minimum wage may cast doubt on the minimum wage as a potential explanation behind Brazil’s inequality decline.

The minimum wage still has some hope to explain Brazil’s wide-spread inequality decline according to a literature in the general equilibrium tradition, which has suggested that spillover effects of the minimum wage may reach higher up in the earnings distribution. Theories of such indirect effects of the minimum wage go back to at least [Burdett and Mortensen \(1998\)](#). At the core of this framework lies the idea that the minimum wage disrupts the equilibrium wage order in a labor market populated by strategic wage setters, leading to indirect effects of the minimum wage higher up in the distribution. How large such equilibrium effects of the minimum wage can be remains an open question, which we will turn to next.

⁹ Annual histograms over the full period 1996-2012 are presented in Appendix B.2.

¹⁰ Appendix B.1 shows that by three empirical measures of “bindingness”—the share of workers earning exactly the legal minimum wage, the share at or below the minimum wage, and the share within a five percent band around the minimum wage—throughout the period 1996-2012 at most seven percent of workers are “binding” at the minimum wage in the data.

3 Empirical facts

Motivated by the large fall in inequality in Brazil over the past 20 years, the following section establishes three facts on the evolution of earnings inequality and the importance of the minimum wage in Brazil. Our analysis combines data from two administrative data sources: a linked employer-employee dataset called *Relação Anual de Informações Sociais (RAIS)*, containing annual information from 1988-2012 on earnings and demographic characteristics of formal sector workers as reported by employers, and the *Pesquisa Industrial Anual Empresa (PIA)*, which contains information on the revenue and cost structure of large firms in Brazil's mining and manufacturing sectors from 1996-2012.

3.1 Data description

The RAIS data are based on a mandatory survey filled in annually by all formally registered firms in Brazil. The data are confidential and administered by the Brazilian Ministry of Labor and Employment. Data collection was initiated in 1986 within a nationally representative set of regions, reaching complete coverage of all employees at tax-registered establishments across all sectors of the economy in 1994. The data contain unique, anonymized, and time-invariant person identifiers as well as firm identifiers, allowing us to follow workers and their employers over time.

The PIA dataset details firm characteristics from 1996 to 2012. It is constructed from annual surveys filled by firms in the manufacturing and mining sector and collected by the Brazilian Statistics and Geography Institute (*Instituto Brasileiro de Geografia e Estatística*, or IBGE). This survey is mandatory for all firms with either more than 30 employees or more than \$300,000 in revenues. As with RAIS, completion of the survey is mandatory and non-compliance is subject to a fine by national authorities. Each firm has a unique, anonymized identifier, which we use to link firm characteristics data from PIA data to worker-level outcomes in the RAIS data. Each firm has a unique, completely anonymized identifier which we use to link the PIA dataset with employee data from RAIS.

Variable definitions. Throughout this paper, earnings or the wage from employer j in year t refers to total payments, including regular salary payments, holiday bonuses, performance-based and commission bonuses, tips, and profit-sharing agreements, divided by total months worked

during the year for that employer. Unless otherwise noted, we restrict attention to a unique observation per worker-year by choosing the highest-paying among all longest employment spells in any given year. In addition, we observe the age, gender, educational level, and occupation¹¹ of each worker. On the firm side, we also use sub-sector categories from IBGE, the national statistical institute.¹² Our firm size measure is the number of full-time equivalent workers during the reference year.

The PIA dataset includes a breakdown of operational and non-operational revenues, costs, investment and capital sales, number of employees and payroll. All nominal values are converted to real values using the CPI index provided by the IBGE. Instead of the measure of firm size in the PIA, we prefer our measure of full-time-equivalent employees constructed from the RAIS as it accounts for workers only employed during part of the year. We define operational costs as the cost of raw materials, intermediate inputs, electricity and other utilities, and net revenues as the gross sales value due to operational and non-operational firm activities net of any returns, cancellations, and corrected for changes in inventory.¹³ We finally construct value added as the difference between net revenues and intermediate inputs, and value added per worker as value added divided by full-time equivalent workers. This is our main measure of firm productivity.¹⁴

Sample selection. We exclude individual observations that have either firm IDs or worker IDs reported as invalid as well as data points with missing earnings or dates of employment. This leads us to drop a very small share of the original population, indicative of the high quality of the administrative dataset. Furthermore, we restrict attention to male workers aged 18-49. We follow the literature in focusing on prime age males in order to obtain a group of workers with a relatively strong attachment to the labor force, for which the model we develop in the next section is a good approximation. Appendix A contains additional details on the data sources and summary statistics for our final sample.

¹¹We use occupations from the pre-2003 Classificação Brasileira de Ocupações (CBO) at the two-digit level.

¹²Both the industry and occupation classification systems changed during the period we study. We use conversion tables provided IBGE to standardize classification between different years and choose categories for both occupations and sectors coarse enough in order to avoid potential biases arising from mechanical changes in the classification system over time.

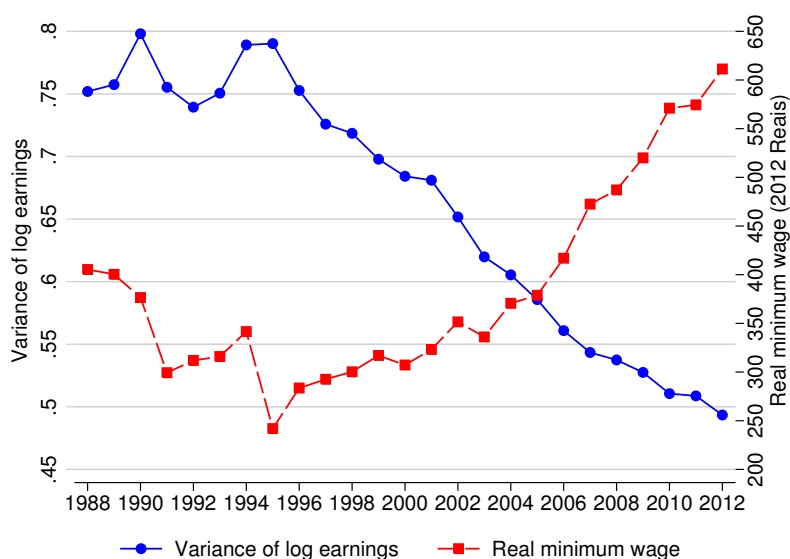
¹³We have explored alternative revenue definitions such as only restricting attention to operational revenues or excluding certain types of non-operational revenues. Such robustness checks yield very similar results to what we report below.

¹⁴We have also constructed alternative measures of firm productivity by cleaning value added per worker off industry-year effects and some measures of worker skill.

3.2 Three facts about Brazil's inequality decline

Between 1996 and 2012, the variance of earnings in Brazil fell by 26 log points at the same time as the minimum increased by 119 percent in real terms. The comovement of the minimum wage and earnings inequality, summarized in Figure 2, may suggest that the minimum wage was an important factor behind inequality dynamics in Brazil over this period. The following section presents three facts characterizing the large inequality decrease over this period.

Figure 2. Variance of Log Earnings and Real Minimum Wage in Brazil, 1988-2012



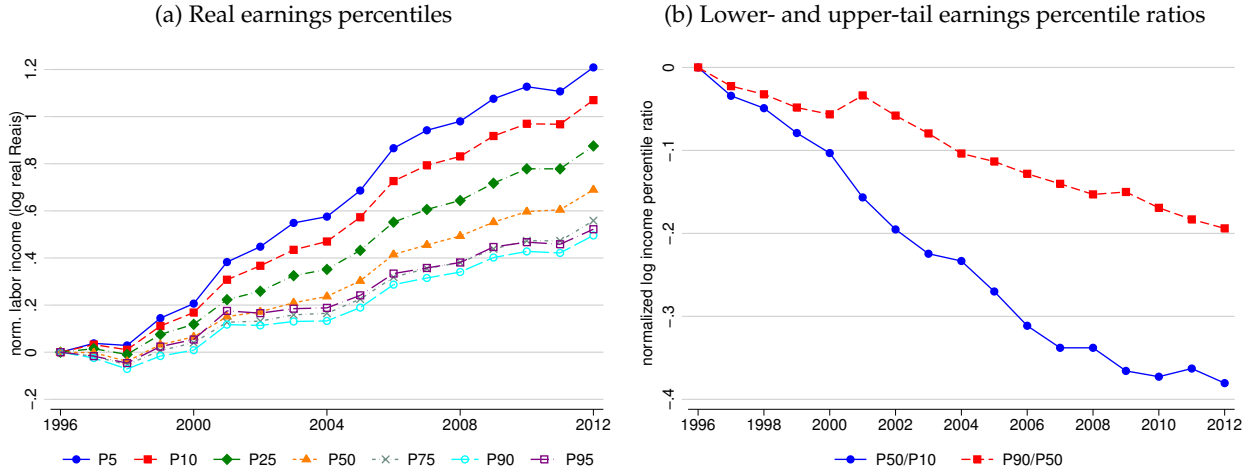
Notes: The variance of log earnings is computed for the population of male formal sector workers aged 18-49 in the Relação Anual de Informações (RAIS) data. Real minimum wage is the annual average of data provided by Brazil's Institute of Applied Economic Research (IPEA).

Fact 1. *The inequality decline is due to bottom-driven but wide-spread real wage growth.*

While overall inequality fell rapidly, some parts of the earnings distribution compressed more rapidly than others. Replicating the analysis of Alvarez et al. (2016), panel (a) of Figure 3 illustrates the bottom driven nature of the fall by plotting the real earnings evolution of various percentiles of the distribution from 1996-2012, normalized to zero in the initial year.

All earnings percentiles increased in real levels, with the 90th percentile (long dashed teal line with hollow circles) growing by 50 log points. But lower earnings percentiles experienced relatively higher earnings growth, with the tenth percentile (dashed red line with filled squares) growing by 120 log points over this period. It is this 70 log points relative real earnings growth at the bottom of the distribution that we seek to explain.

Figure 3. Evolution of the Earnings Distribution, 1996-2012



Notes: Earnings percentiles computed for male workers aged 18–49 in RAIS data.

Panel (b) of Figure 3 summarizes the implied dynamics of top- and bottom-inequality by plotting the log 90–50 percentile ratio (solid blue line with filled circles) and the log 50–10 percentile ratio (dashed red line with filled squares) of the earnings distribution. Both measures decline significantly but the log 50–10 percentile ratio markedly more so than the log 90–50 percentile ratio. Specifically, the log 50–10 percentile ratio declined by 38 log points while the log 90–50 percentile ratio declined by 19 log points at the same time. Indeed, earnings compressed little above the 75th percentile, with the very top of the distribution actually diverging slightly over the period.

Fact 2. *A lower firm productivity-pay premium and lower worker skill premium account for essentially all of the explained inequality decline.*

Noting that seemingly identical workers experience large pay differences across employers in Brazil, Alvarez et al. (2016) estimate the following two-way fixed effect regression of projecting log monthly earnings on a set of worker fixed effects, firm fixed effects and year dummies in five-year sub-periods:

$$\log y_{it} = \alpha_i^p + \alpha_{J(i,t)}^p + \gamma_t + \varepsilon_{it} \quad (1)$$

for $t \in p = \{t_1, \dots, t_5\}$ and where α_i^p denotes the individual fixed effect of worker i in period p , $\alpha_{J(i,t)}^p$ denotes the firm effect representing the employer of worker i at year t , γ_t is a year dummy, and ε_{it} is an error term that satisfies the strict exogeneity condition $\mathbb{E}[\varepsilon_{it} | i, J(i, t), t] = 0$.¹⁵ Imple-

¹⁵Note that the estimating equation (1) is identified off workers switching employers across years for the largest set of workers connected by between-employer worker flows. Alvarez et al. (2016) present a range of specification tests and robustness checks, concluding that the model fits well the Brazilian data during this period.

menting equation (1) via ordinary least squares, the overall fall in inequality over this period can be decomposed into a worker component, a firm component, and a covariance term.

Table 1 reports results such a decomposition of the observed 20 log points decline in the variance of earnings over time. The variance of firm effects falls from 17 log points in 1996-2000 to eight log points in 2008-2012, constituting 45 percent of the overall inequality decline over the period. At the same time, the variance of worker effects falls from 36 log points to 31 log points, explaining another 24 percent of the overall decline. There is also a proportionate fall in the covariance term, with the correlation between worker effects and firm effects staying approximately constant, as well as a small decline in residual variance. Thus, more equal pay across firms explains a disproportionate share of Brazil's inequality decline.

Table 1. AKM Variance Decomposition Between Periods

	(1)	(2)	(3)
	1996-2000	2008-2012	Change
Total variance of log earnings	0.72 (100%)	0.52 (100%)	-0.20 (100%)
Variance of worker fixed effects	0.36 (50%)	0.31 (60%)	-0.05 (24%)
Variance of firm fixed effects	0.17 (23%)	0.08 (15%)	-0.09 (45%)
2×Covariance b/w workers and firms	0.14 (20%)	0.10 (20%)	-0.04 (22%)
Residual variance	0.06 (8%)	0.04 (7%)	-0.02 (10%)
Worker-years	90.2	151.0	
R ²	0.92	0.93	

Notes: Cells contain variance (share) explained by each component. Year dummies are omitted but account for a negligible share of the overall variation. Number of worker-years is in millions.

In a second step, we relate the overall decline in the variance of firm effects and worker effects to observable firm and worker characteristics:

$$\hat{\alpha}_{J(i,t)}^p = \zeta_p \text{VAPW}_{J(i,t)} + \text{sector}_{J(i,t)} \zeta_s + \text{state}_{J(i,t)} \zeta_r + \eta_{J(i,t)}$$

$$\hat{\alpha}_i^p = \text{age}_i \zeta_a + \text{edu}_i \zeta_e + \eta_i$$

where $\text{VAPW}_{J(i,t)}$ denotes log value added per worker at the firm-level, ζ_s is a vector of sector effect, ζ_r a vector of state effects, ζ_a a vector of four age group effects, and ζ_e a vector of four education groups effects. All firm and worker characteristics are averaged across years within a period, hence have no time subscript. Based on these regressions, two potential explanations could be behind declining variance in each of these two components.

The first potential explanation is that the distribution of firm and worker characteristics compressed (holding fixed estimated loadings). In contrast to this hypothesis, we find that pay-relevant firm characteristics—including value added per worker, firm size, and export intensity—all have become more dispersed over this period. Thus, in spite of greater underlying inequality, Brazilian firms offer more equal pay over time. Similarly, on the worker side we find that changes in the distribution of pay-relevant worker characteristics—such as age and education—have contributed little to the declining dispersion of worker pay components (Alvarez et al., 2016).

The second potential explanation, which we confirm in the data, is that the relative returns in pay to firm and worker characteristics compressed (holding fixed initial distributions). On the firm side, Table 2 illustrates the declining productivity-pay gradient among manufacturing and mining firms covered in the PIA data. Between the two periods 1996-2000 and 2008-2012, the regression coefficient on value added per worker dropped from 0.210 to 0.112, implying a five log points reduction in the variance of log earnings while keeping fixed the initial productivity distribution (which widened over this period).

Table 2. Regression of Firm Pay Component on Productivity

	(1)	(2)	(3)
	1996-2000	2008-2012	Change
PANEL A. REGRESSION RESULTS			
Value added p.w.	0.210	0.112	-0.098
Worker-years	16.6	26.3	
R^2	0.711	0.657	
PANEL B. VARIANCE DECOMPOSITION			
Explained variance	0.10	0.05	-0.05
—due to returns			-0.05
—due to composition			0.01

Notes: Dependent variable is AKM estimate of firm effect on wages, $\hat{\alpha}_{j(i,t)}^p$, controlling for state and industry indicators. Explained variance holds R^2 fixed in 1996-2000. Number of worker-years in millions.

Another five log points decline in the variance of log earnings is due to compression in estimated worker effects in the AKM framework, which Table 3 shows is mostly due to compression in coefficients on age and education in our second-stage regression. As on the firm side, changes in the composition of workers did not contribute towards the inequality decline.¹⁶

¹⁶One may suspect that also the returns to unmeasured ability have declined over this period. In this case, our results should be interpreted as a lower bound on the true decline explained by a compression in returns.

Table 3. Regression of Estimated Worker Effects on Worker Characteristics

	(1) 1996-2000	(2) 2008-2012	(3) Change
PANEL A. REGRESSION RESULTS			
Age 25-29	0.20	0.16	-0.04
Age 30-39	0.39	0.30	-0.09
Age 40-49	0.52	0.42	-0.10
Middle school	0.21	0.11	-0.10
High school	0.61	0.27	-0.34
College or more	1.21	1.10	-0.11
Worker-years	90.2	151.0	
R^2	0.34	0.37	
PANEL B. VARIANCE DECOMPOSITION			
Explained variance	0.11	0.08	-0.03
—due to returns			-0.03
—due to composition			0.01

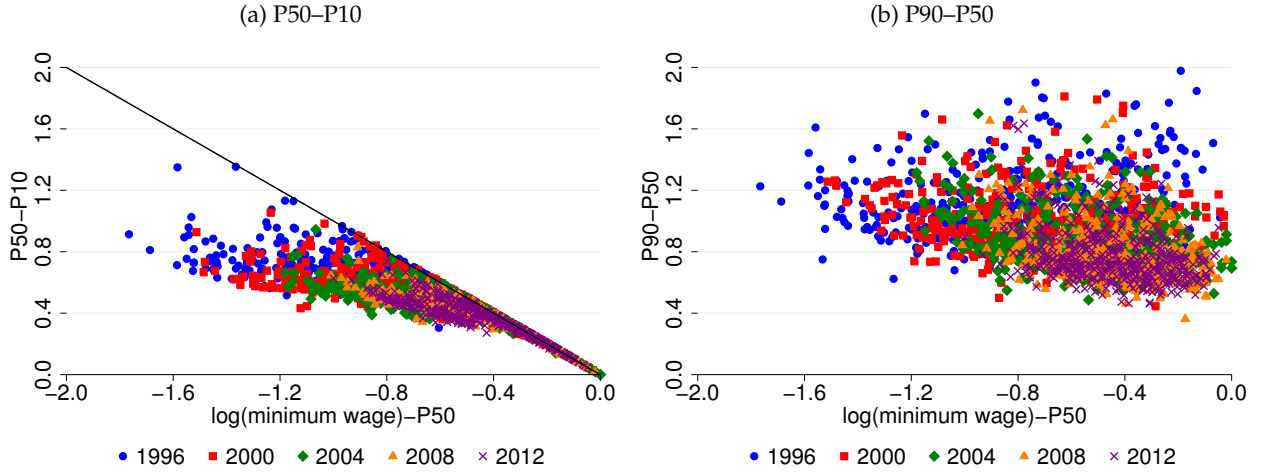
Notes: Dependent variable is the estimated worker effect a_i . Number of workers in millions. Age estimates are relative to “age 18–24” category. Education estimates are relative to “less than middle school (<7 years)” category. Number of worker-years is in millions.

Fact 3. *Greater bindingness of Brazil’s federal minimum wage across regions and over time is associated with compression up the 75th percentile of the earnings distribution.*

To what extent can the rise in the minimum wage account for Brazil’s concurrent decline in earnings inequality? As a first step towards answering this question, we relate the differential bindingness of the federal minimum wage across Brazilian states to state-levels of inequality following the seminal empirical framework developed by Lee (1999). Figure 4 plots two percentile ratios relative to median earnings—the log P50-P10 ratio and the log P90-P50 ratio—across states in Brazil over time. Panel (a) plots the relationship between bottom tail inequality and the bindingness of the minimum wage for five select years. There is a clear negative correlation between the bindingness of the minimum and bottom tail inequality across states, which becomes stronger as the minimum wage is raised over time. For comparison, panel (b) plots the relationship with upper tail inequality, where we see little covariation. This suggests that the minimum wage may be an important driver of bottom tail inequality in Brazil.¹⁷

¹⁷In Appendix B.3 we repeat the exercise for more earnings percentile ratios (Figure 16) and at the state-level (Figure 17). We have produced similar graphs by industry, education, and age groups, all confirming our insight in this section.

Figure 4. Inequality by Microregion And Initial Bindingness of the Minimum Wage, 1996-2012



Notes: Each marker represents one microregion-year combination, where microregions correspond to the 450 localities defined as the first four digits of the six-digit municipality (*municipio*) code.

We formalize these results by projecting various earnings inequality measures on the effective bindingness of the minimum wage, or the “effective minimum wage” (Lee, 1999; Autor et al., 2016).¹⁸ Specifically, we regress the log earnings of percentile p relative to the median in state s in year t on a polynomial in the effective minimum wage and year effects:

$$w_{st}(p) - w_{st}(50) = \sum_{n=1}^N \beta_n(p) \left[w_t^{min} - w_{st}(50) \right]^n + \gamma_t(p) + \varepsilon_{st}(p) \quad (2)$$

where N is the order of the polynomial in the effective minimum wage. After estimating equation (2), we compute the marginal effects of minimum wage on percentile p of the earnings distribution as $\rho_p \equiv \sum_{n=1}^N n\beta_n(p) \left[w_t^{min} - w_{st}(50) \right]^{n-1}$.

Table 4 shows the results of this exercise for regressions of polynomial order $N = 2$.¹⁹ We find evidence in support of significant spill-over effects of the minimum wage between the fifth percentile (significant point estimate of 0.664) and the 45th percentile (significant point estimate of -0.096) of the earnings distribution. But these spill-over effects die out towards higher percentiles, becoming statistically indistinguishable from zero at the 90th percentile and above. At

¹⁸Appendix B.4 discusses in detail the relation between our empirical approach and that taken in Lee (1999) versus the recent work by Autor et al. (2016), as well as how to interpret these estimates through the lens of our structural model presented in Section 4. Importantly, we find considerable support for the identification assumption of (Lee, 1999), namely that the latent earnings distribution is invariant across states.

¹⁹The distinction between polynomial orders $N = 1$ and $N = 2$ is qualitatively important as one would expect the minimum wage to have greater effects as it gradually becomes more binding. We also tried higher order polynomials without obtaining significantly different results to those presented below.

the same time, the explanatory power of the effective minimum wage declines from 0.930 at the fifth percentile to 0.322 at the 90th percentile, indicating that the minimum wage is relatively more important for wage setting at the bottom of the earnings distribution.

Table 4. Marginal Effects of Minimum Wage Throughout the Earnings Distribution (ρ_p)

	$p = 5$	$p = 10$	$p = 25$	$p = 40$	$p = 60$	$p = 75$	$p = 90$	$p = 95$
Marginal effect ρ_p	0.664*** (0.011)	0.467*** (0.014)	0.223*** (0.013)	0.072*** (0.007)	-0.048*** (0.008)	-0.096*** (0.020)	-0.022 (0.038)	0.072 (0.047)
Observations	9,334	9,334	9,334	9,334	9,334	9,334	9,334	9,334
R^2	0.930	0.827	0.602	0.420	0.322	0.322	0.310	0.318

Notes: * = significant at the 10% level, ** = 5%, *** = 1%. Underlying regression is equation (2) with polynomial degree $N = 2$ estimated at the micro-region level on the earnings distributions of male workers age 18–49 across 450 localities defined as the first four digits of the six-digit municipality (*municipio*) code in the RAIS data from 1992 (earliest available data) to 2012. All specifications include year effects. Table shows predicted marginal effects evaluated at the worker-weighted average across years.

4 Model

In this section, we develop an equilibrium search model to interpret our empirical findings in Section 3.2, particularly the effects of the minimum wage throughout the earnings distribution. The model extends the [Burdett and Mortensen \(1998\)](#) framework to allow for differences in worker ability, in addition to firm productivity differentials described in the original paper. Both dimensions of heterogeneity feature prominently in our empirical analysis and are crucial for our quantitative analysis.²⁰ A key property of the model is that identical workers are paid differently across employers in a frictional labor market populated by monopsonistic firms. In the model, as in the data, on-the-job mobility leads workers to climb a “job ladder” by gradually moving to better-paying employers. Consequently, firms compete for workers by setting wages strategically relative to one another and in reference to the minimum wage.

4.1 Environment

Time is continuous and we restrict attention to a stationary environment without aggregate shocks. A unit mass of heterogeneous workers and a positive mass of heterogeneous firms meet in a labor market subject to search frictions. In the spirit of [van den Berg and Ridder \(1998\)](#), search is

²⁰Models without worker heterogeneity struggle to produce realistic wage dispersion ([Hornstein et al., 2011](#)) and may produce misleading results with respect to minimum wage effects—two challenges that our paper overcomes.

segmented in the sense that different worker types search in separate markets while firms decide how many vacancies to create and what wage to offer in each market. Search is also random in the sense that within each market workers cannot direct their search toward specific firms.²¹

4.2 Workers

Workers differ in their permanent ability level θ , which is time-invariant and distributed continuously according to H over support $[\underline{\theta}, \bar{\theta}]$. They are infinitely-lived and value a stream of expected consumption discounted at rate ρ .

Search occurs both from non-employment and while employed in labor markets segmented by worker types. Let λ_θ^u denote the instantaneous rate at which a non-employed worker receives a job offer and let λ_θ^e be the arrival rate for an employed worker. A job offer is an opportunity to work for a wage w drawn from distribution $F_\theta(w)$ with support $[\underline{w}_\theta, \bar{w}_\theta]$. Although a worker treats job finding rates and the distribution of job offers as given, they will be determined endogenously in equilibrium through firms' optimal job creation and wage posting decisions.²² A job is terminated either endogenously when workers move towards a preferred job opportunity, or exogenously with probability δ_θ , in which case workers flow back to the non-employed pool.

Denoting by W_θ the value function of a non-employed worker of ability θ and by $S_\theta(w)$ the value of such a worker employed at wage w , the following Bellman equations are satisfied:

$$\begin{aligned}\rho W_\theta &= b_\theta + \lambda_\theta^u \int_{\underline{w}_\theta}^{\bar{w}_\theta} \max \{S_\theta(w) - W_\theta, 0\} dF_\theta(w) \\ \rho S_\theta(w) &= w + \lambda_\theta^e \int_w^{\bar{w}_\theta} [S_\theta(w') - S_\theta(w)] dF_\theta(w') + \delta_\theta [W_\theta - S_\theta(w)]\end{aligned}$$

Strict monotonicity of the value function S_θ in w implies that the optimal strategy of a non-employed worker will be characterized by a reservation wage ϕ_θ . A non-employed worker accepts wage offers above ϕ_θ and rejects offers below that threshold. Following [Burdett and Mortensen \(1998\)](#), the reservation wage ϕ_θ is implicitly defined as the flow value of unemployment plus the

²¹The assumption of random search in segmented markets makes the model analytically tractable while capturing the notion that filling a vacancy is costly and firms can condition job offers on certain worker attributes (e.g. education).

²²The distinction between endogenous versus exogenous contact rates is important, as pointed out by [Flinn \(2006\)](#). Alternatively or in addition to our firm-side approach, one could endogenize workers' search effort as in [Lentz \(2010\)](#) but we abstract from this as it would complicate our analysis substantially.

option value forgone when leaving unemployment:

$$\phi_\theta = b_\theta + (\lambda_\theta^u - \lambda_\theta^e) \int_{\phi_\theta}^{\bar{w}_\theta} \frac{1 - F_\theta(w)}{\rho + \delta_\theta + \lambda_\theta^e (1 - F_\theta(w))} d\tau$$

The lowest wage at which a worker of type θ can be employed is thus $\max\{\phi_\theta, w^{min}\}$, and we refer to $w^{min} > \phi_\theta$ as a binding minimum wage in market θ .

Denote by u_θ the unemployment rate in market θ . In the stationary equilibrium, a standard flow balance equation solved for the stationary solution implies that

$$u_\theta = \frac{\delta_\theta}{\delta_\theta + \lambda_\theta^u} \quad (3)$$

Let G_θ denote the wage distribution in market θ . Because employed workers gradually move to better jobs, G_θ in general differs from the offer distribution F_θ . Given the law of motion for G_θ and solving for the stationary solution we get:

$$G_\theta(w) = \frac{F_\theta(w)}{1 + \kappa_\theta (1 - F_\theta(w))} \quad (4)$$

where $\kappa_\theta \equiv \lambda_\theta^e / \delta_\theta$ governs the relative speed of climbing up the job ladder.

4.3 Firms

Firms are characterized by a constant productivity level p , drawn from a continuous distribution Γ_0 with support $P = [p_0, \bar{p}]$. Firms produce output by combining workers of different ability levels using a linear production technology. Together with the assumption of perfect segmentation of labor markets by ability types, the assumption of a linear production technology improves tractability because it abstracts from interactions across θ markets.²³ Letting l_θ denote the number of employees from market θ , flow output of a firm with productivity p is

$$y(p, \{l_\theta\}_{\theta \in \Theta}) = p \int_{\theta \in \Theta} \theta l_\theta d\theta$$

Extending the endogenous vacancy framework of [Mortensen \(2000\)](#) to the case of heterogeneous firms and workers, a firm attracts workers of type θ by posting job openings in that market,

²³Consequently, as will become clear soon, our model produces spillover effects of the minimum wage within θ markets but not across.

v_θ , subject to an increasing and strictly convex flow cost $c_\theta(v_\theta)$. A job opening is a promise to pay a wage, w_θ , for the remainder of the match. Firms are assumed to commit to the posted wage.

In equilibrium, the number of jobs and the wage a firm posts jointly determine the amount of workers it employs, $l_\theta = l_\theta(w_\theta, v_\theta)$. In choosing what wage to post, a firm trades off two forces. On the one hand, a higher wage relative to the pool of competing offers increase total output by poaching more workers per posted vacancy and shielding its own workforce from a larger share of competitor firms. On the other hand, a higher wage reduces profits per employed worker. In addition to the wage posting margin, conditional on its position in the pay ranks, a firm can increase the mass of workers it attracts at some cost post per additional vacancy.

Because workers of different ability are perfect substitutes, firms maximize profits in each labor market separately. A firm with productivity p in market θ chooses a mass of jobs to create and a wage to associate with those jobs in order to maximize steady-state flow output:

$$\max_{w_\theta \geq w^{min}, v_\theta} \{(p\theta - w_\theta) l_\theta(w_\theta, v_\theta) - c_\theta(v_\theta)\}$$

A firm makes strictly positive profits in market θ if and only if they post a wage strictly between workers' outside option, ϕ_θ , and their own productivity, p . As a result, only firms with productivity above $\underline{p}_\theta \equiv \max\{w^{min}, \phi_\theta\} / \theta$ are active in that market. The distribution of active firms in market θ is thus given by $\Gamma_\theta(p) = \Gamma_0(p | p > \underline{p}_\theta) = [\Gamma_0(p) - \Gamma_0(\underline{p}_\theta)] / [1 - \Gamma_0(\underline{p}_\theta)]$.

Denote by $v_\theta(p)$ the optimal vacancy posting rule that solves the firm's problem in market θ , and by $w_\theta(p)$ the optimal wage posting rule. The total mass of jobs in market θ is

$$V_\theta = \int_{p' > \underline{p}_\theta}^{\bar{p}} v_\theta(p') d\Gamma_\theta(p') \quad (5)$$

Postulating that equilibrium wages are monotonic in productivity, the wage offer distribution is

$$F_\theta(w_\theta(p)) = \int_{p' > \underline{p}_\theta}^p \frac{v_\theta(p')}{V_\theta} d\Gamma_\theta(p') \quad (6)$$

4.4 Matching

We assume that employed workers search with efficiency s_θ relative to unemployed workers and that an aggregate matching function brings together searching workers and firms. Following the

literature, we assume that the matching function is on the Cobb-Douglas form, $M(u + s(1 - u), V) = \chi [u + s(1 - u)]^{1-\alpha} V^\alpha$, where α governs the elasticity of matches with respect to vacancies and χ is matching efficiency. Letting q_θ denote the rate of an open job in market θ being filled, we can then express the finding rates for unemployed workers, employed workers, and firms as

$$\lambda_\theta^u = \chi \left(\frac{V_\theta}{u_\theta + s_\theta(1 - u_\theta)} \right)^\alpha, \quad \lambda_\theta^e = s_\theta \lambda_\theta^u, \quad \text{and} \quad q_\theta = \chi \left(\frac{u_\theta + s_\theta(1 - u_\theta)}{V_\theta} \right)^{1-\alpha} \quad (7)$$

4.5 Equilibrium

Before we are ready to define an equilibrium in our economy, we need to characterize the number of workers that a firm obtains if it posts v_θ jobs paying wage w_θ in market θ . The following law of motion characterizes the evolution of firm size,

$$\dot{l}_\theta(w, v) = -\delta_\theta l_\theta(w, v) - s_\theta \lambda_\theta^u (1 - F_\theta(w)) l_\theta(w, v) + v q_\theta \left[\frac{u_\theta}{u_\theta + (1 - u_\theta) s_\theta} + \frac{(1 - u_\theta) s_\theta}{u_\theta + (1 - u_\theta) s_\theta} G_\theta(w) \right]$$

where $\dot{l}_\theta(w, v)$ denotes the instantaneous rate of change of firm size for given wage and vacancy posting policies. A fraction δ_θ of a firm's employees exit to unemployment and a fraction $s_\theta \lambda_\theta^u (1 - F_\theta(w))$ move on to better employers. A vacancy meets with a worker with probability q_θ , who is unemployed with probability $\frac{u_\theta}{u_\theta + (1 - u_\theta) s_\theta}$ and employed with complementary probability. All unemployed workers accept the offer, while a fraction $G_\theta(w)$ of employed workers accept the offer. Solving for the stationary solution,

$$l_\theta(w, v) = \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w))} \right)^2 \frac{v_\theta}{V_\theta} u_\theta \lambda_\theta^u (\delta_\theta + s_\theta \lambda_\theta^u) \quad (8)$$

Definition 1. A *stationary search equilibrium* is a set of reservation policies $\{\phi_\theta\}_{\theta \in \Theta}$; wage policies and job creation policies $\{w_\theta(p), v_\theta(p)\}_{\theta \in \Theta}$; wage offer distributions $\{F_\theta(w)\}_{\theta \in \Theta}$; firm sizes $\{l_\theta(w, v)\}_{\theta \in \Theta}$; unemployment rates $\{u_\theta\}_{\theta \in \Theta}$; total jobs created, $\{V_\theta\}_{\theta \in \Theta}$; and worker transition rates $\{\lambda_\theta^u, \lambda_\theta^e\}_{\theta \in \Theta}$ such that:

1. *Worker optimality:* Given the labor market transition rates and the offer distribution, the reservation policies solve each worker type's problem;
2. *Firm optimality:* Taking as given equation (8), wage policies and job creation policies solve firms' problem in each market;

3. *Labor market consistency*: The unemployment rates are consistent with equation (3), total vacancies are the sum of individual firms' job creation decisions as in equation (5), and the transition rates are determined by the aggregate matching function and relative search intensity in equation (7); and
4. *Aggregation*: Wage policies and job creation policies map into firm sizes according to equation (8), and the wage offer distributions are given by equation (6).

4.6 Equilibrium characterization

We define the piece rate, \tilde{w}_θ , such that $w_\theta = \theta\tilde{w}_\theta$. Using the stationary mapping (8) from wages and job offers to firm size, we can define $T_\theta = \theta [u_\theta\lambda_\theta^u(\delta_\theta + s_\theta\lambda_\theta^u)] / V_\theta$ and re-state the problem of firm p in market θ as

$$\max_{v, \tilde{w}} \left\{ T_\theta v (p - \tilde{w}) \left(\frac{1}{\delta_\theta + s_\theta\lambda_\theta^u(1 - F_\theta(\tilde{w}))} \right)^2 - c_\theta(v) \right\} \quad \text{s.t.} \quad \tilde{w} \geq \frac{\max\{w^{min}, \phi_\theta\}}{\theta}$$

The associated first-order conditions with respect to vacancies and piece rates are

$$\begin{aligned} c'(v_\theta(p)) &= T_\theta (p - \tilde{w}) \left(\frac{1}{\delta_\theta + s_\theta\lambda_\theta^u(1 - F_\theta(\tilde{w}))} \right)^2 \\ 1 &= (p - \tilde{w}_\theta(p)) \frac{2s_\theta\lambda_\theta^u f_\theta(\tilde{w}_\theta(p))}{\delta_\theta + s_\theta\lambda_\theta^u(1 - F_\theta(\tilde{w}_\theta(p)))} \end{aligned}$$

Since profits are increasing in productivity and c_θ is strictly convex, it follows that $v'_\theta(p) > 0$. That is, more productive employers create more jobs. Using an argument akin to [Burdett and Mortensen \(1998\)](#), we can show that as a consequence of the single-crossing property of the profit function with respect to productivity and wages for a given vacancy decision, $\tilde{w}_\theta(p)$ is strictly increasing in productivity. Similarly, the equilibrium wage offer distribution has no mass points.²⁴

Appendix D details the algorithm we use to numerically solve the problem based on these first-order conditions. Before numerically solving the model, however, it is instructive to illustrate its mechanics in a partial equilibrium version. Abstracting for this purpose from vacancy creation,

²⁴As predicted by the model, in Appendix B.1 we find consistently small shares of workers in the vicinity of the minimum wage, even as the minimum rapidly increases over time.

it is straight-forward to show that the unique equilibrium wage offered by firm p in market θ is:

$$w(p, \theta) = \theta p - \theta \int_{\underline{p}_\theta}^p \left[\frac{1 - \Gamma_0(\underline{p}_\theta) + \kappa_\theta (1 - \Gamma_0(p))}{1 - \Gamma_0(\underline{p}_\theta) + \kappa_\theta (1 - \Gamma_0(x))} \right]^2 dx \quad (9)$$

Our model nests as a special case the environment without a binding minimum wage and $(\underline{p}_\theta, \kappa_\theta)$ shared across θ markets. In this case, wages in our model satisfy exactly the log additive wage specification of AKM. That is, firms pay different workers a constant multiple of their worker ability, so that log wages are additively separable into worker and firm components:

$$\log w(p, \theta) = \underbrace{\log \theta}_{\text{"worker effect"}} + \underbrace{\log \tilde{w}(p)}_{\text{"firm effect"}} \quad (10)$$

where the “firm effect” term on the right-hand side is independent of worker attributes:

$$\tilde{w}(p) = p - \int_{\underline{p}}^p \left[\frac{1 - \Gamma_0(\underline{p}) + \kappa (1 - \Gamma_0(p))}{1 - \Gamma_0(\underline{p}) + \kappa (1 - \Gamma_0(x))} \right]^2 dx$$

With a binding minimum wage, the second term in equation (10) depends on θ , perturbing the exact decomposition in equation (10).²⁵ We now turn to characterizing the effects of the minimum wage on the determinants of pay in relation to our previously documented facts characterizing Brazil’s inequality decline.

The rising minimum wage naturally speaks to Fact 1—the bottom-driven decline—and Fact 3—lower bottom tail inequality for higher effective minimum wage levels. Fact 2 of our empirical part establishes that the main driver behind Brazil’s inequality decline was a fall in the pass-through from firm productivity to pay as well as lower returns to measures of worker ability. The following proposition shows that our model also rationalizes this fact:

Proposition 1. *Suppose that worker types share the same mobility parameter $\kappa_\theta = \bar{\kappa}$ and that the minimum wage is sufficiently low to begin with, $w^{\min} / \underline{\theta} < p_0$. Then a marginal increase in the minimum wage:*

²⁵In Appendix E.2, we show that more generally there is a tight link between model worker ability and firm productivity on one hand, and empirical AKM worker and firm effects on the other hand. Variance decompositions confirm that the empirical AKM model explains more than 99.9 percent of earnings variation in our model-generated data.

1. increases worker pay in markets for which the minimum wage binds relative to higher worker types:

$$\partial \left[\frac{w(p, \theta; w^{\min})}{w(p, \theta'; w^{\min})} \right] / \partial w^{\min} > 0, \quad \forall \theta, \theta' : \phi_{\theta} < w^{\min} < \phi_{\theta'}$$

2. reduces the productivity-pay gradient across firms in affected markets:

$$\partial \left[\frac{\partial w(p, \theta; w^{\min})}{\partial p} \right] / \partial w^{\min} < 0, \quad \forall \theta : \phi_{\theta} < w^{\min}$$

3. reduces the returns to ability across workers in affected markets:

$$\partial \left[\frac{\partial w(p, \theta; w^{\min})}{\partial \theta} \right] / \partial w^{\min} < 0, \quad \forall \theta : \phi_{\theta} < w^{\min}$$

Proof. See Appendix C. □

The intuition for the first part of Proposition 1 is straight-forward: An increase in the minimum wage reduces the monopsony power of firms in markets where it binds, which raises average pay in those markets relative to markets where the minimum wage does not bind. Since the minimum wage is more likely to bind in low ability markets, this reduces the pay gap between high and low ability workers.

To understand the second part, note that in binding markets a rise in the minimum wage forces the least productive firms to increase pay one-for-one. Higher productivity firms in equilibrium want to retain a positive wage premium by adjusting wages upwards, but by a smaller amount than firms below them. This is because the elasticity of employment with respect to wages, which depends on how many competing wage offers can be dominated by an incremental wage raise, tends to zero towards the top of the wage distribution. As a consequence, the minimum wage affects firms initially paying above the wage floor but its effects fade towards higher productivity levels. As a result, the pay-productivity gradient across firms falls.

Similarly, the third result shows that a rise in the minimum wage increases wages by the greatest amount among the lowest worker types, for which the minimum wage is originally the most binding, thereby reducing the ability-pay gradient across workers.

5 Estimating the model

The previous section showed that the minimum wage can qualitatively rationalize our three facts on Brazil’s earnings inequality decline from 1996-2012. To quantitatively evaluate the effects of the rise in the minimum wage, we estimate our model to key moments in the microdata on Brazilian labor markets for the “pre-period” 1996–2000. To this end, we target cross-sectional moments of the joint distribution of worker and firm heterogeneity on the one hand, and information on worker type-specific transitions between employment states on the other hand.

5.1 Estimation strategy

Parameters guiding the job ladder structure of our model are identified off ordinal information on worker flows across firm pay ranks, while worker and firm heterogeneity are tightly linked to the observed pay structure associated with worker and firm ranks. Following [Cahuc et al. \(2006\)](#), we thus adopt a two-stage estimation procedure. In the first stage, we rank workers by an ordinal metric and non-parametrically identify worker type-specific parameters guiding labor market transitions. In a second stage we then employ the method of simulated moments via indirect inference ([Smith, 1993](#)) in order to estimate parameters guiding the distributions of worker ability and firm productivity.

First stage. The first stage of our estimation routine uses panel information on worker flows between employment states together with non-parametric estimates of the worker type-specific distribution of firm pay ranks in order to estimate the four labor market parameters of our model for each worker type θ : the separation rate, δ_θ , the job offer arrival rate from non-employment, λ_θ^u , the job offer arrival rate from employment, λ_θ^e , and the reservation wage, ϕ_θ .

We first classify workers into deciles²⁶ according to their rank in the estimated AKM worker fixed effects distribution.²⁷ We then construct a monthly worker panel from the RAIS data in order to calculate by worker type θ the fraction of formal sector entrants, exiters, and job-to-job switchers by worker type in every year between 1996 and 2012.²⁸ We use this panel to estimate

²⁶We later interpolate estimated labor market parameters across worker ability deciles using a linear function for our model simulations using an arbitrarily high number of worker types θ on the computer.

²⁷Appendix E.2 justifies this approach and shows that there is a tight relationship between worker ability and estimated AKM worker effects on the one hand, and firm productivity and estimated AKM firm effects on the other.

²⁸Since we are unable to distinguish flows between formal sector employment on the one hand and unemployment, informal employment, or out of the labor force status on the other hand, we use the term “non-employment” to stand

the four labor market parameters of interest by worker deciles for the “pre-period” 1996–2000.

The worker ability-specific monthly separation rate is straight-forward to compute as the average rate of leaving the formal labor force, $\hat{\delta}_\theta = \mathbb{E}(\text{exit} | \text{ability } \theta)$.

To estimate the monthly rate of finding a job from non-employment λ_θ^u , we use a proportional hazards model in order to predict re-entry of workers that left formal sector employment within the previous 24 months. We invert the model in order to recover the employment finding rate using the relationship $\log \mathbb{P}(\# \text{ months until re-entry} \geq t | \text{ability } \theta) = t \times \log \pi_\theta^u$, where $\pi_\theta^u = 1 - \lambda_\theta^u$ is the monthly probability of remaining unemployed. We then solve for $\hat{\lambda}_\theta^u = 1 - \exp\left(\widehat{\log \pi_\theta^u}\right)$, where $\widehat{\log \pi_\theta^u}$ denotes the ordinary least squares (OLS) estimate of the coefficient on t from above.

Third, the job offer arrival rate on the job, λ_θ^e , cannot be directly inferred from observed job-to-job flows, since an employed worker only accepts offers paying more than their current employer. Our model, however, suggests that this parameter guides the speed of upwards employer mobility (Jolivet et al., 2006), where we interpret rungs of the job ladder as estimated AKM firm effects ranks. Specifically, λ_θ^e is tightly linked to the distance between the cross-sectional wage distribution G and the wage offer distribution F . Thus, we recover κ_θ^e from equation (4) using a non-parametric density estimate²⁹ of the cross-sectional firm effects distribution $\hat{G}(fe_\theta)$ and of the firm effects offer distribution $\hat{F}(fe_\theta)$ for worker types θ , approximated in a model-consistent manner as the worker-weighted distribution of firm effect ranks for new formal sector entrants. This nonparametric estimate of κ^e is given by³⁰

$$\hat{\kappa}_\theta^e = \frac{\hat{F}(fe_\theta) - \hat{G}(fe_\theta)}{(1 - \hat{F}(fe_\theta)) \hat{G}(fe_\theta)} \quad (11)$$

Using our earlier estimate of δ_θ , we then use equation (11) to back out the implied value for λ_θ^e and hence the relative on-the-job search intensity $s_\theta = \lambda_\theta^e / \lambda_\theta^u$.

Next, we infer workers’ reservation wage as $\phi_\theta = \max\{\min_i \{w_\theta^i\}, w^{min}\}$. Note that we need not identify the reservation wage for workers with $\phi_\theta \leq w^{min}$ as for them the minimum wage will be the relevant binding constraint. To limit the role of measurement error, we implement this relationship in the data by identifying the first percentile of the earnings distribution conditional

in for any of the latter terms.

²⁹We use a 1000 quantile bin approximation to the empirical cumulative distribution functions, although our results are unchanged for any reasonable bin number, and alternative estimation methods such as kernel density estimates.

³⁰In a previous version of this paper we estimated κ_θ^e in three different model-consistent ways, yielding similar results.

on being at or above the minimum wage and setting the reservation wage equal to that value.

Second stage. In the second stage of our estimation routine, we assume that worker ability is log normally distributed, $\log \theta \sim \mathcal{N}(\mu, \sigma^2)$ with mean μ and standard deviation σ , and that firm productivity is Pareto distributed, $p \sim \text{Pareto}(\zeta)$ with tail parameter ζ . (we normalize the scale parameter to one). These distributional assumptions allow us to roughly capture the empirical shape of the wage distribution in Brazil.

Next, we assume that the cost of creating jobs is of the form $c_\theta(v) = c_\theta v^{c_1+1} / (c_1 + 1)$ for $c_\theta, c_1 > 0$. Without vacancy data, we cannot separately identify match efficiency, χ , from the cost of creating jobs, c_θ . We hence normalize $\chi \equiv 1$. Furthermore, we follow [Petrongolo and Pissarides \(2001\)](#) in setting the Cobb-Douglas parameter of the matching function to $\alpha = 0.5$. Finally, absent data on vacancies we cannot estimate the curvature of the vacancy cost function and hence proceed to assume the quadratic cost case with $c_1 = 1$ ([Shephard, 2017](#)).

We estimate the remaining parameters of the model by indirect inference: average worker ability, μ , the dispersion in worker ability, σ , the shape of the firm productivity distribution, ζ , and the cost of creating jobs, c_θ . An appealing aspect of our model is its sparse parameterization. Heuristically, the following moments identify the following parameters: The bindingness of the minimum wage as measured by the minimum to mean log wage identifies average worker ability. The dispersion in AKM worker fixed effects identifies the dispersion in underlying worker ability, and the dispersion in AKM firm effects identifies the shape of the productivity distribution. Finally, the cost of creating jobs is identified by λ_θ^u that we estimated in the first step. Although each of these moments is particularly informative about one particular parameter, all parameters are jointly determined.

Our choice of target moments is motivated by our discussion in the previous section. We argued there that absent a minimum wage and with identical labor market transition hazards across markets, wages in our model are log additively separable into log worker ability and a firm effect that is independent of worker ability. We showed that the latter is a strictly increasing transformation of firm productivity. Although an exact decomposition no longer prevails in our more general environment, it seems plausible that the dispersion in AKM firm and worker effects provides valuable information for identifying the underlying dispersion in worker ability and firm productivity. We show in [Appendix E](#) that these two moments indeed do appear to well identify

these two parameters, in the sense that the distance between the relevant data and model moment quickly increases as we change the parameter of interest. The same is true with respect to average worker ability: the minimum-to-mean log wage in the model quickly departs from the data target as we change average worker ability.³¹

5.2 Parameter estimates

Table 5 contains the resulting parameterization of worker type-specific labor market parameters that we use in order to simulate our model with many more worker types: the monthly separation rate δ_θ , the job finding rate from nonemployment λ_θ^u , the relative on-the-job search intensity s_θ , and the reservation wage ϕ_θ .³²

Table 5. Monthly Labor Market Parameters

Description		Parameter	Value	Target
EN rate	Intercept	δ_0	0.039	} EN hazard by worker type
	Slope	δ_1	-0.030	
NE hazard	Intercept	λ_0^u	0.050	} NE hazard by worker type
	Slope	λ_1^u	0.023	
Relative search intensity	Intercept	s_0	0.192	} Speed of moving up the job ladder
	Slope	s_1	0.213	
Reservation wage	Intercept	ϕ_0	-0.90	} Lowest wage by worker type
	Slope	ϕ_1	1.60	

Notes: Targets are computed by AKM worker fixed effect decile for period 1996–2000. Model parameters over the normalized worker type interval $\theta \in [0, 1]$ are then filled in as a linear approximation to the estimated relationship.

We notice that while our estimate of the employment-to-nonemployment (EN) hazard from formal sector employment is similar to what is commonly found in the U.S., the estimated nonemployment-to-employment (NE) hazard is more in line with continental European countries. We find substantial heterogeneity in labor market transition rates by worker ability, particularly for the exit rate from formal sector employment.³³ The EN hazard of the lowest decile of worker effects is over four times as high as for the highest decile of workers, while the NE hazard is 32 percent lower and relative search intensity while employed 53 percent lower. As we show below, this large heterogeneity in transition rates implies substantial sorting of more able workers to higher paying

³¹We discuss our estimation routine in greater detail in Appendix E.

³²Figure 21 in the Appendix shows estimated labor market parameters by worker ability decile for the period 1996–2000.

³³In related work, Pessoa Araujo (2017) finds a negative relation between separation rates and wages offered across jobs in Brazil’s RAIS data.

firms. Finally, the estimated reservation wage is increasing in worker ability.

Table 6 shows the estimated structural parameters of our model guiding worker and firm heterogeneity as well as the initial relative bindingness of the minimum wage. Furthermore, we find that the estimated cost of hiring is increasing in worker ability in absolute terms, but decreasing relative to the productivity of the worker, detailed results of which are presented in Appendix E.

Table 6. Structural Estimates of Worker Ability and Firm Productivity Distributions

Description	Parameter	Value	Target
Mean worker ability	μ	1.81	Min-to-mean earnings ratio
Variance of worker ability	σ	0.50	Variance of AKM worker effect
Tail index of firm productivity	ζ	6.50	Variance of AKM firm effect

Notes: Mean and variance of worker ability refer to log-normal distribution parameters. Tail index of firm productivity refers to shape parameter of the Pareto distribution, with mean firm productivity normalized to one. See text for details.

5.3 Model fit

Our estimated model successfully replicates both cross-sectional and longitudinal facts relating to the distribution of earnings and labor market dynamics in Brazil during the “pre-period” 1996–2000. Table 7 compares the model fit in terms of estimated AKM components.

Table 7. Data vs. Model: AKM Variance Decomposition, 1996–2000

Component of AKM earnings decomposition	(1)	(2)
	Data	Model
Total variance of log earnings	0.72	0.65
Variance of worker fixed effects	0.36	0.36
Variance of firm fixed effects	0.17	0.16
2×Covariance b/w workers and firms	0.14	0.12
Residual variance	0.06	0.00
Worker years	90.2	0.43
R^2	0.92	100.0

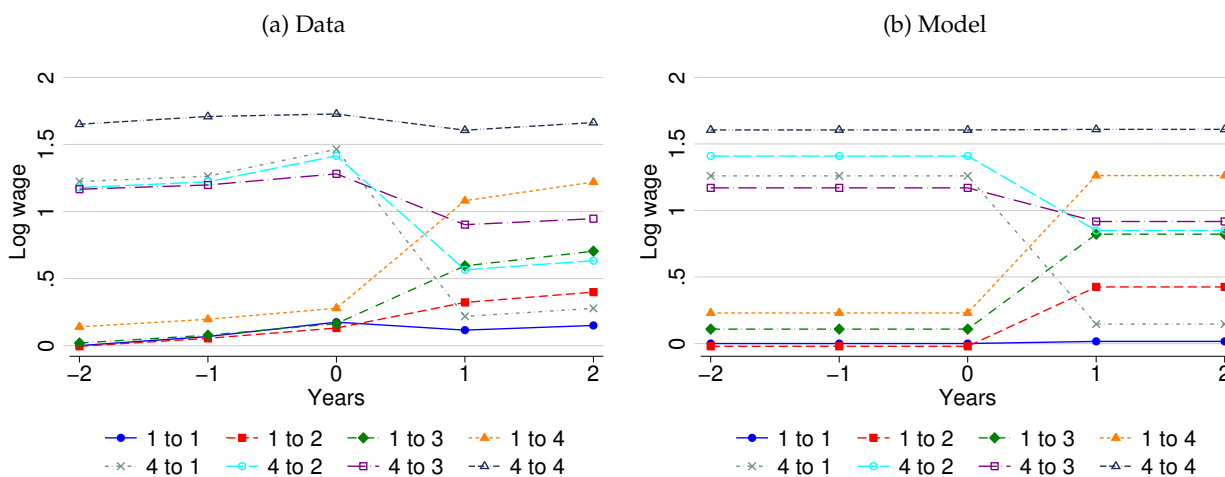
Notes: Variance decomposition is based on AKM regression $\log y_{it} = \alpha_i^p + \alpha_{f(i,t)}^p + \gamma_t + \varepsilon_{it}$. Variance of time effects and covariance terms involving time effects are small and omitted for brevity. See text for details.

By ways of our indirect inference step, the model replicates closely the variances of worker effects and firm effects. The model also replicates a substantial share of the covariance between the worker effect and the firm effect. The latter is not mechanical, but a result of the independently estimated (not targeted) labor market parameter heterogeneity across worker types, with higher ability workers traveling up the job ladder more quickly. Given that the covariance is not directly

targeted in our indirect inference estimation, this is a success of the model. As noted above, the model does not capture the full raw dispersion in earnings in the data since we do not target the residual in the AKM regression.³⁴ Overall, the model replicates 98.5 percent of the empirical variance of log earnings, net of the residual term.

The model also replicates two key exercises that the empirical literature tends to conduct to verify the AKM methodology, which we view as a validation of the model given that they are not targeted. First, as documented by Alvarez et al. (2016), wage gains and losses of workers switching up and down the firm ladder are roughly symmetric in Brazil. The left panel of Figure 5 reproduces their results for the 1996–2000 sub-period by plotting an event study graph of average wages for workers switching out of the first and fourth quartile of firm effects up to three years prior to the switch and two years after.³⁵

Figure 5. Data vs. Model: Average Wage Gains from Switching Employers, 1996–2000



Notes: Event study graph of changes in mean log wage upon switching employers between year 0 and year 1. Different colored lines show transitions from first and fourth quartile of AKM firm fixed effects distribution in data and model for period 1996–2000.

The right panel of Figure 5 plots the model-generated data. The model captures both the qualitative and quantitative aspects of the data well. Wages increase for workers that switch up the ladder and decline for those that move down. Furthermore, the model matches well the fact that average wages of those who move up to quartile j are lower than average wages of those who

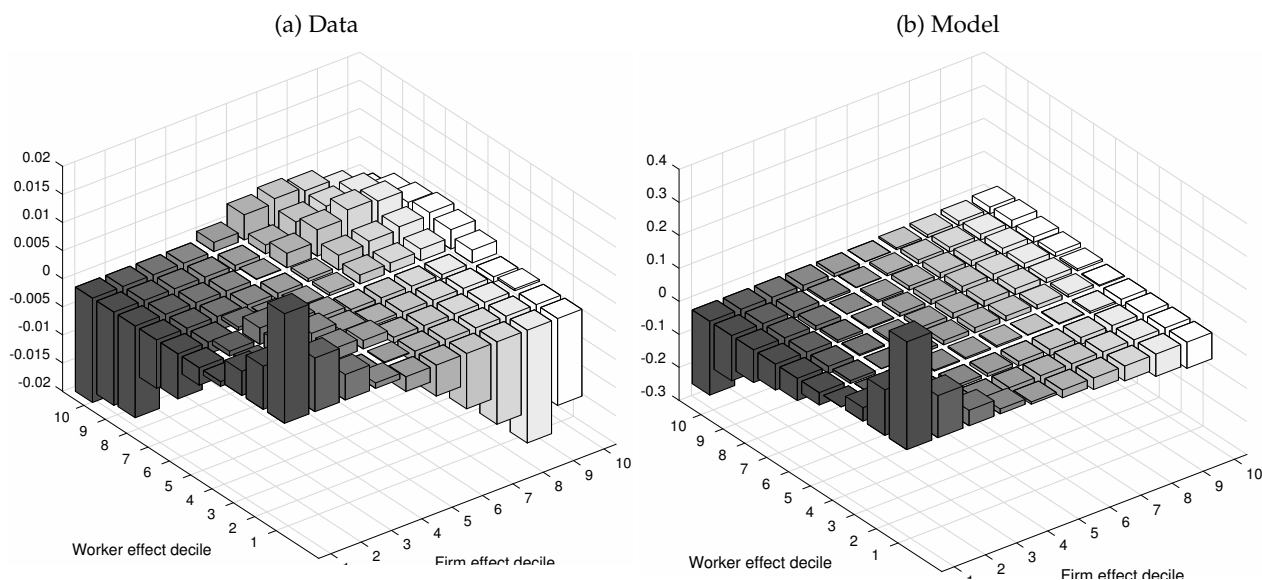
³⁴It is easy to add white noise to earnings in order to fit also the residual, but since this has no major impact on our results we refrain from doing so.

³⁵Following the methodology of Card et al. (2013), we produce this graph both in the model and data conditioning on the worker having been employed by the same employer for the past three years prior to the switch and the staying with the same employer for the two years after the switch. To avoid clogging up the graph, we only include switchers out of the first and fourth quartiles, but the other quartiles follow a similar pattern.

move down to quartile j . This is surprising in light of the fact that a worker can only move down the ladder by starting over from the bottom. As a result, we might for instance have expected workers who move from the fourth to the third quartile to have the same wage on average as workers who move into the third quartile from the first quartile. The reason this is not true in the model is sorting. The average worker who makes a switch from the fourth to the third quartile is of higher ability than the average worker who switches from the first to the third quartile, and as a result the former earns a higher wage than the latter conditional on moving into the same quartile.

Second, the empirical literature tends to investigate the behavior of the average residual from the AKM regression over worker and firm effects. The left panel of Figure 6 reproduces results from Alvarez et al. (2016) by plotting the average AKM residual by deciles of worker and firm effects in the 1996–2000 subperiod.

Figure 6. Data vs. Model: AKM Residual Plot, 1996–2000



Notes: The figure shows residual plots of $\varepsilon_{it} = \log y_{it} - \alpha_i^p - \alpha_{f(i,t)}^p - \gamma_t$ by AKM worker fixed effect decile and AKM firm fixed effect decile in the data and model for period 1996–2000.

The right panel shows the same figure on model-generated data. We note three things. First, the model captures well the key feature of the data that over most of the support, the average residual is close to zero. Second, the model qualitatively matches the fact that the lowest worker effect deciles have a positive residual while employed at the lowest paying firms. In the model, this is driven by the minimum wage pushing up wages of the lowest paid workers when they are employed at the lowest paying firms. Finally, the model substantially overstates this pattern at

the bottom of the distribution. This pattern becomes stronger in both the model and the data as the minimum wage is raised.

5.4 Policy experiment in the model

To evaluate the impact of a rise in the minimum wage, we consider the following experiment: we hold all parameters fixed at their initial estimated values, and change only the minimum wage in line with the data.³⁶ We calibrate the increase in the minimum wage to match the growth in the productivity adjusted real minimum wage between 1996–2000 and 2008–2012, which we compute in the following way. The average real minimum wage (in 2012 values) is 384 BRL in 1996–2000 and 701 BRL in 2008–2012, implying an 60.2 log point growth in the real minimum wage. Average log value added per worker grew by 15.4 log points between 1996–2000 and 2008–2012. Thus, we estimate that the real, productivity-adjusted minimum wage grew by 44.7 log points between 1996–2000 and 2008–2012.³⁷ This implies a hike in the minimum wage from 0.189 to 0.315 or approximately 67 percent. We evaluate the implications for income inequality of imposing this higher minimum wage through the lens of our model.

6 Quantitative results

To isolate the effect of the rise in the minimum wage on the earnings distribution, we consider the counterfactual experiment of holding all parameters fixed at their initial estimates for the 1996–2000 period and change only the minimum wage in the model to match the growth in the productivity adjusted minimum wage until the 2008–2012 period.

We first present results on the aggregate impact of a change in the minimum wage on earnings inequality, addressing our first empirical fact that the decline in inequality was most pronounced at the bottom of the distribution, yet widespread. Second, we decompose the decline in inequality

³⁶The assumption that the reservation wage remains constant in response to a change in the minimum wage deserves special mentioning, given that it is an equilibrium outcome rather than a parameter. This assumption is innocuous for the following reasons. Suppose first that the minimum wage when raised becomes binding in market θ . In this case, the reservation wage is no longer relevant, and it is without loss of generality to disregard any changes to it. Second, suppose that the minimum wage remains non-binding in market θ . In this case, nothing has changed in market θ , leaving the reservation wage unchanged.

³⁷For robustness, we also explored alternative targets for the increase in the minimum wage, including the growth rate of the minimum wage relative to productivity growth in Brazil's services, commerce, and construction sectors (for which we have firm-level productivity data); or relative to growth in aggregate output per capita from national accounts. These alternative targets imply similar increases in the minimum wage and therefore lead to comparable results.

in the model in the same way as in the data into a worker and a firm component, and we study changes in the pass-through from productivity to the firm component of pay and from worker ability to the worker component. We find that our model captures well our second empirical fact that the decline was driven by a fall in the pass-through from fundamentals to pay. Third, we show that the model correctly captures our third empirical fact about the co-variation between bottom tail inequality and the bindingness of the minimum wage across Brazilian states over time.

6.1 Explaining Fact 1: Effects throughout the earnings distribution

Table 8 compares log percentile ratios in the model and in the data over time.

Table 8. Data vs. Model: Evolution of Log Earnings Percentile Ratios

	1996-2000		2008-2012		Change		
	(1) Data	(2) Model	(3) Data	(4) Model	(5) Data	(6) Model	(7) % Explained
P50-P10	0.86	0.95	0.55	0.74	-0.31	-0.21	68%
P50-P25	0.48	0.59	0.33	0.48	-0.15	-0.11	73%
P75-P50	0.60	0.67	0.50	0.62	-0.10	-0.05	50%
P90-P50	1.30	1.21	1.17	1.15	-0.13	-0.06	46%

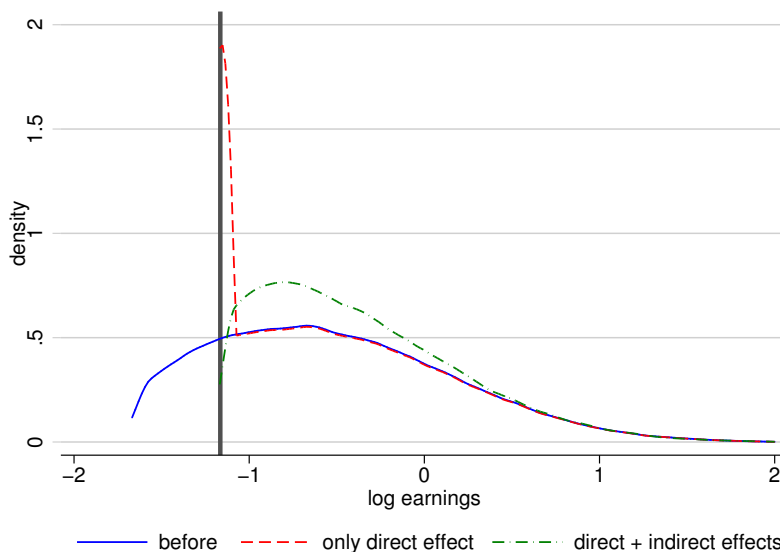
Notes: Earnings percentile ratios for male workers aged 18-49 in RAIS data and for model-simulated data.

The model does a very good job at matching the empirical compression in earnings at different percentiles in response to an increase in the minimum wage. The model predicts roughly 70 percent of the compression in inequality at the bottom of the distribution, and about 50 percent at the top of the distribution. For instance, the 50-10 log ratio compresses by 31 log points in the data versus 21 log points in the model (or 68 percent) whereas the log 90-50 ratio compresses by six log points in the model versus 13 log points in the data (or 46 percent). It is worth noting that, given the share of workers binding at the minimum wage is below seven percent throughout this period, the compression in percentile ratios shown in Table 8 is amplified by indirect effects of the minimum wage further up in the earnings distribution.

Another way to quantify the relative importance of spillover effects of the minimum wage is to consider a decomposition of the overall inequality decline into two components. We define the first component as the change in inequality implied by comparing the initial distribution of workers to a hypothetical distribution that moves workers (conditional on remaining profitably employed) up to the new minimum wage. The second component is defined as the change in

inequality implied by comparing the previous hypothetical distribution to the new equilibrium distribution resulting after feeding the minimum wage change into the model. Intuitively, the first component captures the direct or partial equilibrium effect, while the second component captures the indirect or general equilibrium effect of the minimum wage. Figure 7 illustrates this two-step decomposition graphically.

Figure 7. Illustration: Direct and Indirect Minimum Wage Effects on the Earnings Distribution



Notes: “Before” illustrates earnings distribution before minimum wage increase. “Only direct effect” shows distribution after shifting workers below the new minimum wage up to the new wage floor (conditional on remaining employed). “Direct + indirect effects” shows resulting equilibrium earnings distribution after allowing firm wage offers to adjust and workers to travel up the job ladder. See text for details.

Implementing this decomposition in the data and in the estimated model using the variance of log earnings as our inequality measure, we find that roughly half of the total inequality decrease is due to the direct effect (54.2% in the data and 42% in the model) and half of it is due to the indirect effect of the minimum wage (45.8% in the data and 57.6% in the model).

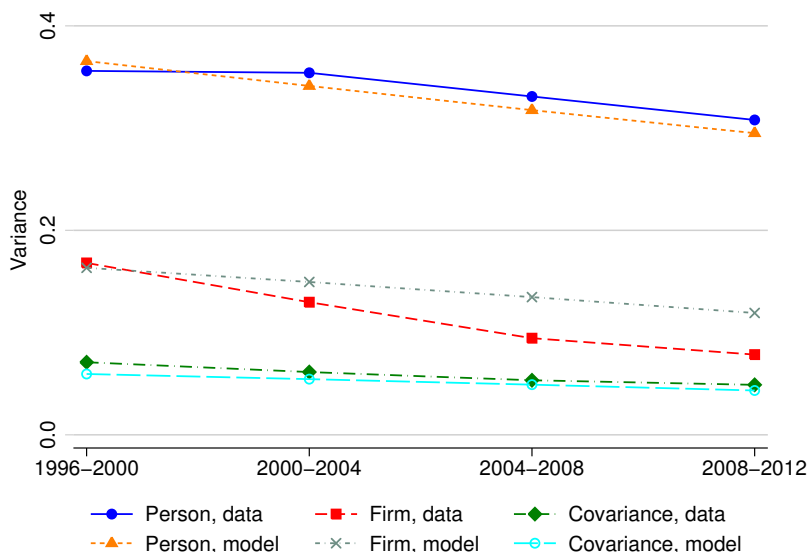
Thus, in line with Fact 1 from our empirical section, the model predicts significant compression above the wage floor that declines in magnitude toward the top of the earnings distribution, resulting in spillover effects that are as important as the direct effects of the minimum wage.

6.2 Explaining Fact 2: Changes in returns as the main driver of declining inequality

Figure 8 plots the variance of worker effects, firm effects and their covariance in the model and the data in each of the four subperiods. The model somewhat overstates the fall in the variance of the

worker effect, explains three quarters of the fall in the covariance, and matches close to half of the fall in the variance of the firm component. As in the data, the correlation between worker and firm effects remains stable. That the model somewhat over-predicts the fall in the worker component may be the result of technological change over the last two decades pushing against the force of the minimum wage, as we may expect from the literature on skill-biased technical change.

Figure 8. Data vs. Model: AKM Decomposition



Notes: Variance decomposition is based on AKM regression $\log y_{it} = \alpha_i^p + \alpha_{j(i,t)}^p + \gamma_t + \varepsilon_{it}$. “Person” denotes variance of worker fixed effects, “Firm” denotes variance of firm fixed effects, and “Covariance” denotes two times the covariance terms. See text for details.

To summarize the effects of the minimum wage on different components of the earnings distribution, Table 9 presents numbers on the 1996-2000 and 2008-2012 subperiods in the data versus the model.³⁸ Overall, our estimated model attributes a 14 log points decrease in the variance of earnings, or 70 percent of the 20 log points empirical decrease, to the concurrent rise in the minimum wage over the period.³⁹

In the model, the exogenous distribution of worker ability and firm productivity is held constant by construction. However, the model endogenously generates changes in the distribution of worker and firm characteristics among employed workers over time. The former may change in response to some workers experiencing relative falls in their employment rates (such as in the form of permanent exit of the lowest ability workers from the formal sector labor market). The

³⁸Note that by construction the model does not track the empirical drop in residual variance over time.

³⁹Using a completely different identification strategy, [Komatsu and Menezes Filho \(2016\)](#) also reach the conclusion that the minimum wage explains the majority of the inequality decline in Brazil.

Table 9. Data vs. Model: AKM Decomposition

	1996-2000		2008-2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
Total variance of log earnings	0.72	0.65	0.52	0.51	-0.20	-0.14	70%
Variance of worker fixed effects	0.36	0.37	0.31	0.30	-0.05	-0.07	140%
Variance of firm fixed effects	0.17	0.16	0.08	0.12	-0.09	-0.04	44%
2×Cov. b/w workers and firms	0.14	0.12	0.10	0.09	-0.04	-0.03	75%
Residual variance	0.06	0.00	0.04	0.00	-0.02	0.00	0%

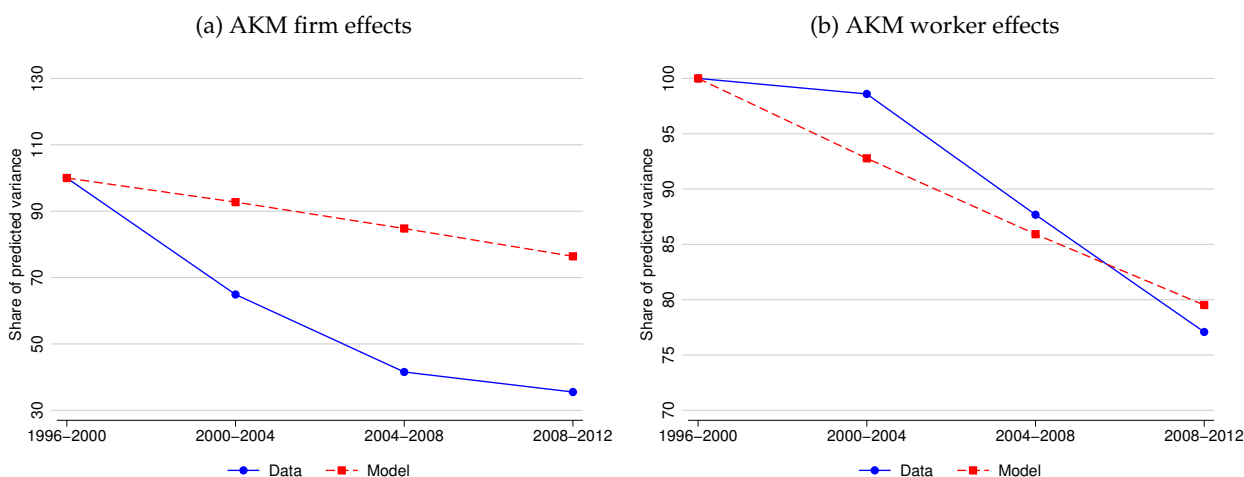
Notes: Variance decomposition is based on AKM regression $\log y_{it} = \alpha_i^p + \alpha_{j(i,t)}^p + \gamma_t + \varepsilon_{it}$. Variance of time effects and covariance terms involving time effects are small and omitted for brevity. See text for details.

latter changes in response to shifts in the relative amounts of vacancies posted by different firm types (such as in the form of permanent exit of the lowest productivity firms from the lowest ability markets). To investigate to what extent the fall in inequality is driven by changes in the distributions versus changes in the returns to worker ability and firm productivity, we consider two counterfactual scenarios. In the first scenario, we hold constant the distribution of worker and firm productivity and allow only the degree of pass-through to change. In the second scenario, we hold pass-through constant and allow only the distribution of ability and productivity to change.

As a result of this exercise, the left panel of Figure 9 plots the percentage fall in the explainable component of AKM firm effects driven by a changing pass-through from productivity to pay in the data (solid blue line with circles) and in the model (dashed red line with squares). In the data, the component of pay that is accounted for by value added falls by over 60 percent over this period, while in the model the decline is a more moderate 25 percent. In absolute numbers, the variance of the explainable part falls by 0.05 log points in the data and 0.03 log points in the model. In neither the model nor the data is there much compression due to less dispersion in underlying fundamentals.

The right panel of Figure 9 plots the percentage fall in the explainable component of AKM worker effects driven by a changing pass-through from worker ability to pay in the data (solid blue) and in the model (dashed red). Since the model does not contain as easily interpretable measures of ability as education and age, we construct this by regressing the worker component of pay on a linear in log ability. The explainable component falls by 25 percent in the data and 20 percent in the model. Also on the worker side is there no meaningful compression due to changes in the underlying distribution in worker ability in the model or the data.

Figure 9. Data vs. Model: Fall in Inequality Due to Declining Pass-Through



Notes: Graphs show predicted variance of AKM firm effects in panel (a) and variance of AKM worker effects in panel (b) while holding fixed underlying distribution of firm and worker characteristics in initial period, thereby isolating effects of changes in pass-through from firm and worker characteristics into pay in the data vs. in the model.

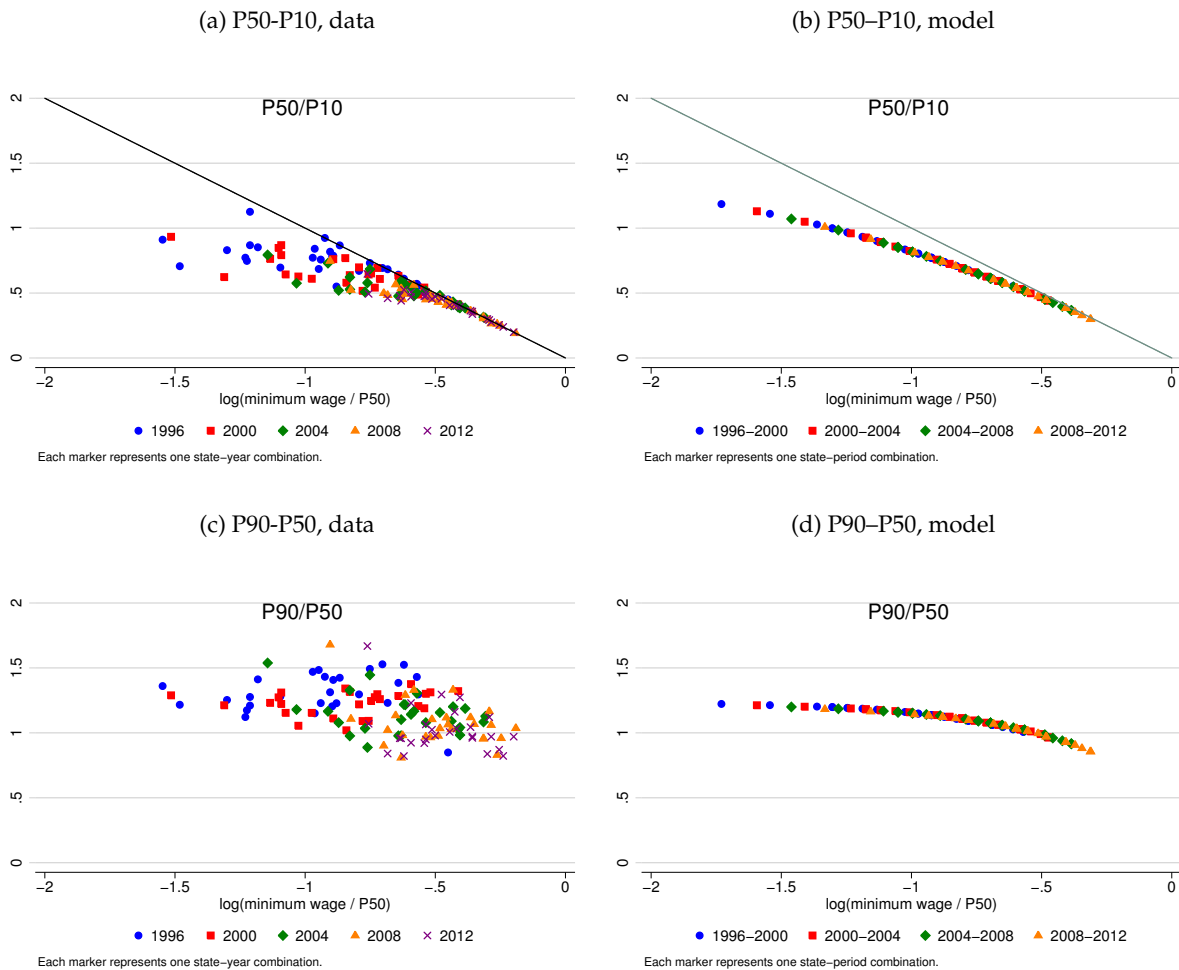
We conclude from these exercises that the model is able to successfully reproduce roughly half of the fall in the dispersion of the firm component of pay, and that in line with the data this is driven by a lower pass-through from firm productivity to pay. It somewhat over-predicts the compression in the worker effects, but again correctly captures the fact that this is driven by a lower return to worker ability, not changes in the underlying distribution of worker ability.

6.3 Explaining Fact 3: Spill-over effects identified off cross-regional variation

In order to investigate the model's ability to reproduce our second empirical fact that bottom tail inequality is lower in states with a higher bindingness of the minimum wage, we implement the following minor change to our above methodology. Instead of estimating the model to fit the aggregate economy in 1996-2000, we estimate it to fit the least binding state in that subperiod. The model again fits the data well initially. We subsequently re-estimate average worker ability in each state to hit the initial level of the bindingness of the minimum in that state, while holding all other parameters fixed at their estimated values for the least binding state. Finally, we change the minimum wage as in the data over time, while holding all parameters fixed at their initial values. This approach aligns closely with the spirit of Lee (1999) that there is an underlying latent distribution of wages that is shared across all states, but that this is differentially masked by a federal minimum wage due to differences in wage levels across states.

Figure 10 plots the results of this exercise. The top left panel plots the relationship between bottom tail inequality, measured by the log 50-10 ratio, and the bindingness of the minimum wage, measured by the log median to minimum ratio, across Brazilian states over time. The right panel repeats this exercise on model generated data. The plots show a clear negative relationship between bottom tail inequality and the bindingness of the minimum wage, that grows more pronounced for more binding minimum wages over time and across states. The model captures this relationship well, both qualitatively and quantitatively. For comparison, the bottom two panels show that in both the model and data there is only a weak relationship between the bindingness of the minimum wage and top inequality, measured by the log 90-50 ratio.

Figure 10. Data vs. model: Tail Inequality Across Brazilian States Over Time

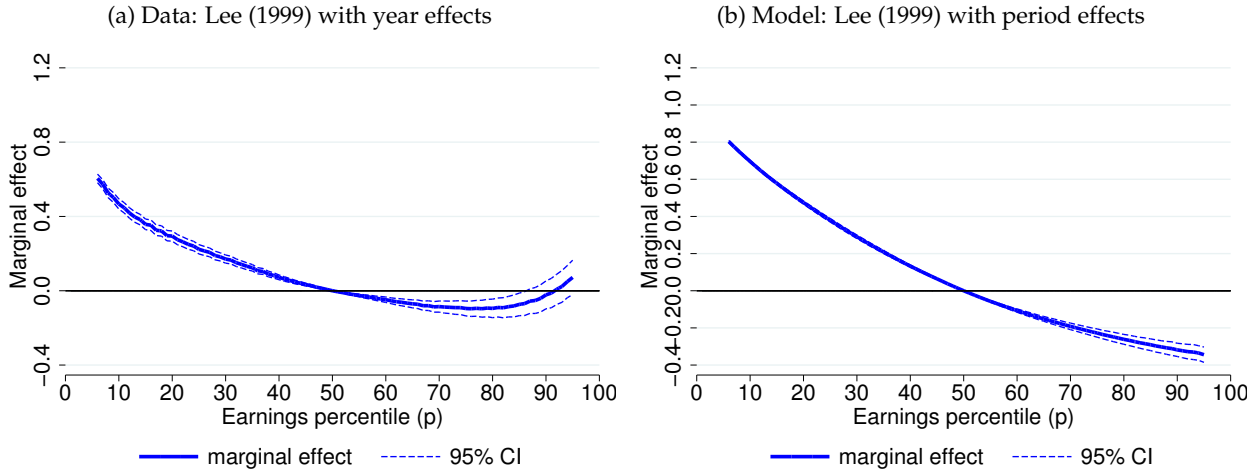


Notes: Each marker represents one state-year combination, where states correspond to the 27 Federative Units (*Unidades Federativas*, or *UF*) of Brazil in the data, and 27 separate model simulations with estimated levels of initial bindingness of the minimum wage in the model.

How high do the spill-over effects of the minimum wage reach in the model versus the data?

Figure 11 compares the empirical vs. model-predicted marginal effects of the minimum wage for various earnings percentiles. In the spirit of Lee (1999), we estimate equation (2) of our empirical section with polynomial degree $N = 2$ and only time fixed effects (i.e. year effects in the data, and period effects in the model). The figure then plots the estimated marginal effect of the effective minimum wage on various earnings percentiles p relative to the median, denoted ρ_p .

Figure 11. Data vs. Model: Marginal Effect of w^{min} -P50 on P p -P50



Notes: Underlying regression is equation (2) with polynomial degree $N = 2$ estimated on the middle 90 percentiles of the earnings distributions of male workers age 18–49 across 450 localities defined as the first four digits of the six-digit municipality (*municipio*) code in the RAIS data from 1992 (earliest available data) to 2012, and on model-simulated data for four periods 1996–2000, 2000–2004, 2004–2008, and 2008–2012. All specifications include time effects. Graphs show predicted marginal effects evaluated at the worker-weighted average across years.

Both the data and the model produce significant spill-over effects of the minimum wage below the median. While the model slightly over-predicts the magnitude of the marginal effects in the lower half of the distribution, the general shape of the marginal effects is matched well. Above the median and up to the 90th percentile, the data suggests negative point estimates (i.e. compression in the earnings distribution) in line with the model predictions, although the empirical estimates are become insignificant past the 85th percentile. Past the 90th percentile, the data suggest that the bindingness of the minimum is positively correlated with inequality, in the sense that the difference between the p th percentile and the 90th increases with the minimum wage. Our model, by construction, cannot rationalize this behavior at the top, but predicts declining yet positive marginal compression in this range of the earnings distribution.

As Autor et al. (2016) note, the positive point estimate toward the top of the empirical earnings distribution may result from measurement error combined with the fact that the median is on

both sides of our estimating equation, potentially giving rise to a spurious positive relationship. Another explanation for the observed pattern includes factors that impact the top of the earnings distribution over this period that are correlated with the initial bindingness of the minimum wage but not captured by our model. That our model provides less than perfect fit for the top ten percent of the earnings distribution is not a prime concern, given our focus on the bottom end of the distribution and common wisdom that more complex theories are needed to explain the top of the earnings distribution. Yet over most of the support, the model does a good job at capturing the heterogeneity of the response of inequality to the minimum wage in the data.

7 Discussion

While the focus of our paper lies on the distributional consequences of a rise in the minimum wage in the formal sector of the Brazilian economy, we discuss briefly in this section the effects on employment, welfare, and the informal economy.

7.1 Employment effects and welfare

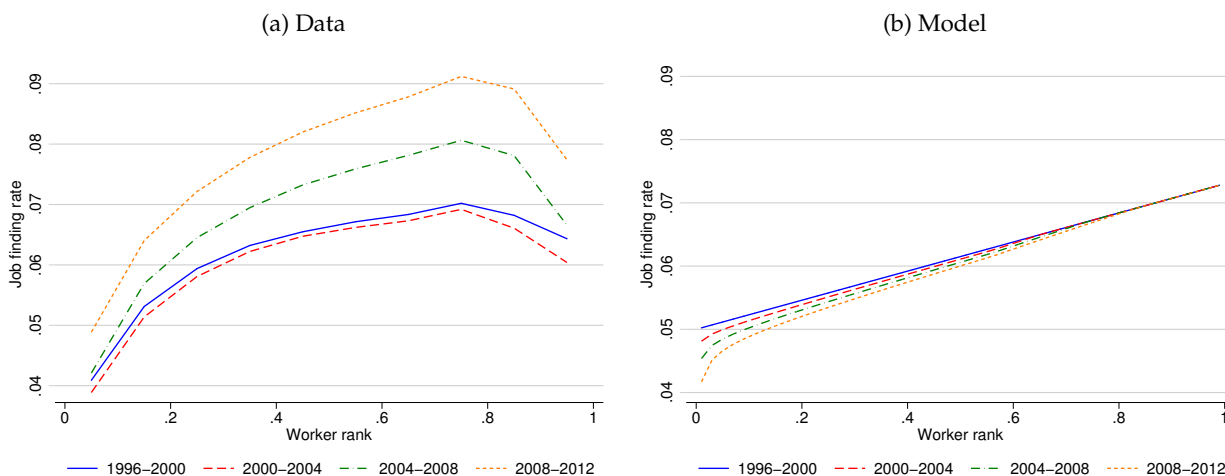
The minimum wage transfers rents from firms to workers, thereby potentially reducing firms' incentives to create jobs. The reduced demand for particularly low-skill workers leads to greater unemployment as well as slower job-to-job transitions for those skill groups. As we showed in the previous section, however, the model predicts only small effects on the composition of the workforce, suggesting that such general equilibrium effects are relatively minor. This is verified by Figure 12, which plots the job finding rates, λ_{θ}^u , in each period as a function of worker rank in the worker effect distribution (normalized to be on the unit interval). The left panel plots the predicted finding rates in the model, while the right pane plots those in the data.

Job finding rates in the model fall in response to the hike in the minimum wage, with the most pronounced effect at the bottom of the worker ability distribution. The negative employment effects are significant towards the bottom of the distribution, with the lowest worker ability decile experiencing a 10 percent decline in their job finding rate. Further up in the skill distribution, the employment effects of the minimum wage are modest and eventually zero.

The data, on the other hand, display a general increase in the job finding rates over this period in Brazil, potentially as a result of positive economic shocks that are outside of our model. Im-

portantly though, in line with the predictions of the model the data show a steepening of the job finding rates in worker ability. Relative to the top, the empirical job finding rate of the bottom of the skill distribution fell by 11 percent, which is close to the prediction of our model.

Figure 12. Data vs. model: Job Finding Rates by Worker Ability Rank, by Subperiod



Notes: Job finding rates are computed from proportional hazard model of re-entry into formal sector at monthly frequency within 5-year periods in the RAIS data, and from initial estimation plus equilibrium vacancy creation response to the minimum wage in the model. See text for details.

Although our framework has rich implications for labor market outcomes in response to the minimum wage, we abstract from product market responses such as the pass-through to output prices, as considered by [MaCurdy and McIntyre \(2001\)](#) for the US and [Harasztosi and Lindner \(2015\)](#) for Hungary. It is possible that such a response would further moderate the impact of the minimum wage of job finding rates, while potentially resulting in consumers ending up paying higher prices for final goods. Given that we simplify the structure of the final goods market—by assuming linear preferences over homogeneous output, and abstracting from rebates of firm profits back to households—we leave an investigation of both price effects of the minimum wage in Brazil for future research.

7.2 The informal sector

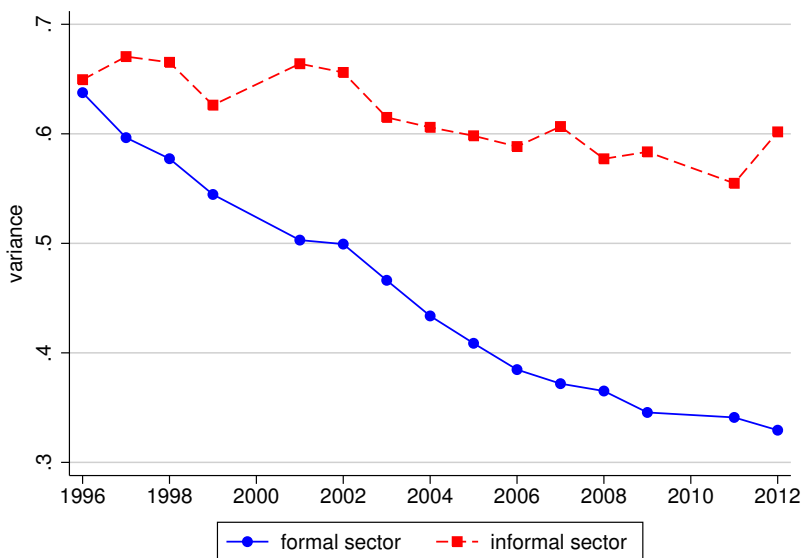
Our current framework abstracts from the informal sector, which is an important component of aggregate economic activity in Brazil. Although it would be interesting to introduce an informal sector into the model along the lines of [Albrecht et al. \(2009\)](#) or [Meghir et al. \(2015\)](#) and consider

spillover effects of the minimum wage between sectors, this exercise is beyond the scope of the current paper for two reasons.

First, adding an informal sector would significantly add to the complexity of the theoretical model and our estimation procedure. Absent matched employer-employee data for the informal sector, such an approach would also require several strong assumptions in order to quantify the degree of spillovers between the informal and formal sector.

Second, we find that most of the inequality decline occurred within Brazil’s formal sector, motivating our focus on this part of the economy. As we show in Figure 13 using the available data on the informal sector from the Brazilian *Pesquisa Nacional por Amostra de Domicílios* (PNAD) household data,⁴⁰ inequality fell also in the informal sector over this period, although not by as much.

Figure 13. More Pronounced Decline of Earnings Inequality In Formal Sector, 1996-2012



Notes: Statistics are computed for male workers aged 18–49 in the PNAD household survey. To classify workers as “formal” or “informal,” the survey asks respondents about their current employment status and whether they hold a valid employment license (*Carteira de Trabalho e Previdência Social*, or *CTPS*).

Since the minimum wage only applies to the formal sector, this is in line with the idea that the minimum wage was an important driver of reduced inequality. Over the same period, informal sector employment among prime-age male employees fell from 36 to 26 percent. Hence, although the counterfactual is admittedly hard to pin down, it appears that to a first order formal sector

⁴⁰The PNAD data are described in more detail in Appendix Section A.3.

jobs did not shift into the informal sector in response to the minimum wage hike.

To sum up, while we think that it would be very interesting to extend the model to include an informal sector that is unbound by the minimum wage, we view our approach as sufficiently rich to capture the dynamics of the majority of Brazilian employment over this period.

8 Conclusion

What is the impact of a minimum wage on income inequality? We evaluate the hypothesis that a minimum wage may reduce inequality through spill-over effects cascading up the earnings distribution. Using Brazil's inequality decrease from 1996-2012 as a case study, we find that the minimum wage in our simulated model replicates 70 percent of the observed inequality decrease with compression reaching up to the 75th percentile of the earnings distribution. Furthermore, in line with other empirical studies, an increase in the minimum wage in our model predicts only a modest negative effect on employment. We conclude that the equilibrium effects of a minimum wage on earnings inequality can be quantitatively large in an economy with frictional labor markets.

Our findings point to interesting future work. First, spillover effects we characterized between firms in the formal sector may also affect wage setting in the informal sector, which is not constrained by the minimum wage. Given that informality is common in many developing economies, the interactions between formal and informal sector earnings distributions are worth exploring. Second, in light of the large equilibrium effects we attribute to the minimum wage, a range of other labor market institutions may have quantitatively important effects on inequality, including unions, unemployment benefits, and other social safety net provisions.

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Appendix

A Data description and sample selection

A.1 Matched employer-employee data

Table 10 provides key summary statistics for the population of adult male workers aged 18–49 in the RAIS matched employer-employee data between 1996 and 2012. The group of adult males represents 55% of the total population in 2000.

Table 10. RAIS Summary Statistics

	(1)	(2)	Log earnings	
			(3)	(4)
	# Workers	# Firms	Mean	Std. dev.
1996	18.05	0.98	1.32	0.87
1997	18.31	1.06	1.32	0.85
1998	18.65	1.12	1.28	0.85
1999	18.54	1.18	1.25	0.84
2000	19.15	1.22	1.20	0.83
2001	20.45	1.30	1.12	0.83
2002	21.22	1.37	1.06	0.81
2003	21.70	1.42	0.99	0.79
2004	22.78	1.48	0.98	0.78
2005	23.96	1.54	0.94	0.77
2006	25.14	1.61	0.86	0.75
2007	26.58	1.68	0.83	0.74
2008	28.45	1.76	0.83	0.73
2009	29.17	1.84	0.80	0.73
2010	31.01	1.95	0.78	0.71
2011	32.38	2.05	0.81	0.71
2012	33.23	2.13	0.78	0.70

Notes: All statistics are for male workers age 18–49. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a subperiod.

A.2 Firm characteristics data

Table 11 shows key summary statistics for the PIA firm financial data for 1996–2012. All results are weighted by the number of full-time equivalent workers employed by the firm in a given year.

Table 11. PIA Summary Statistics

	(1) # Firm-years	Log revenues per worker		Log value added per worker	
		(2) Mean	(3) Std. dev.	(4) Mean	(5) Std. dev.
1996	21,840	11.83	1.00	11.15	1.08
1997	21,022	11.86	1.03	11.15	1.11
1998	22,096	11.88	1.09	11.17	1.19
1999	22,771	12.01	1.16	11.29	1.27
2000	22,751	12.00	1.19	11.22	1.30
2001	24,920	12.01	1.24	11.23	1.32
2002	26,418	12.02	1.30	11.26	1.39
2003	27,853	11.96	1.31	11.18	1.37
2004	28,708	12.00	1.32	11.21	1.35
2005	30,628	11.94	1.30	11.16	1.33
2006	31,962	11.94	1.28	11.18	1.32
2007	31,808	11.97	1.28	11.21	1.31
2008	33,349	12.01	1.27	11.26	1.30
2009	34,200	12.01	1.23	11.31	1.27
2010	34,650	12.03	1.22	11.32	1.25
2011	36,773	12.06	1.20	11.34	1.23
2012	37,858	12.07	1.18	11.36	1.20

Notes: Sample includes all firms covered by the PIA dataset in the mining and manufacturing sectors. The number of firm-years and number of unique firms are reported in thousands. All means and standard deviations are weighted by the number of employees. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a subperiod.

A.3 Household survey data

The *Pesquisa Nacional por Amostra de Domicílios* (PNAD) household survey data consist of a double-stratified sampling scheme by region and municipality, interviewing a representative of households in Brazil. The survey asks the household head to respond on behalf of all family members and report a rich set of demographic and employment-related questions. In particular, the survey asks a question about whether the respondent holds a legal work permit. We use the answer to this survey question to identify individuals as working in the formal or in the informal sector. Survey questions regarding income and demographics of the respondent household members are comparable to the US March Current Population Survey (CPS). We keep only observations that satisfy our selection criteria and have non-missing observations for labor income, whose variable definition we harmonize across years.⁴¹ Table 12 presents basic summary statistics on the PNAD data.

⁴¹Standardized cleaning procedures are adopted from the Data Zoom suite developed at PUC-Rio and available for replication online at <http://www.econ.puc-rio.br/datazoom/english/index.html>.

Table 12. PNAD Summary Statistics

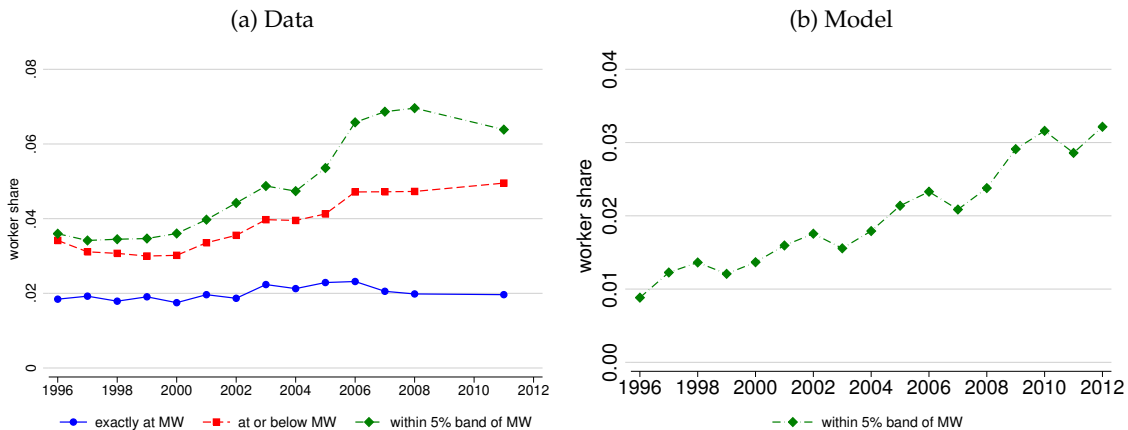
	Log earnings			
	(1)	(2)	(3)	(4)
	# Workers	Mean	Std. dev.	Formal share
1996	60,176	6.81	0.98	0.65
1997	64,204	6.79	1.00	0.64
1998	63,016	6.78	0.97	0.64
1999	64,328	6.72	0.95	0.63
2000	-	-	-	-
2001	70,558	6.68	0.95	0.63
2002	72,273	6.66	0.93	0.63
2003	71,959	6.59	0.93	0.64
2004	75,617	6.61	0.91	0.64
2005	78,064	6.64	0.90	0.65
2006	78,627	6.71	0.89	0.66
2007	76,616	6.76	0.87	0.68
2008	76,571	6.80	0.85	0.69
2009	77,037	6.83	0.84	0.70
2010	-	-	-	-
2011	67,884	6.93	0.80	0.73
2012	69,297	6.98	0.80	0.73

Notes: All statistics are for adult male workers of age 18–49. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers. Means are computed by period. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a subperiod. Surveys are not available in years 2000 and 2010.

B Additional material on the minimum wage in Brazil

B.1 Bindingness of the minimum wage

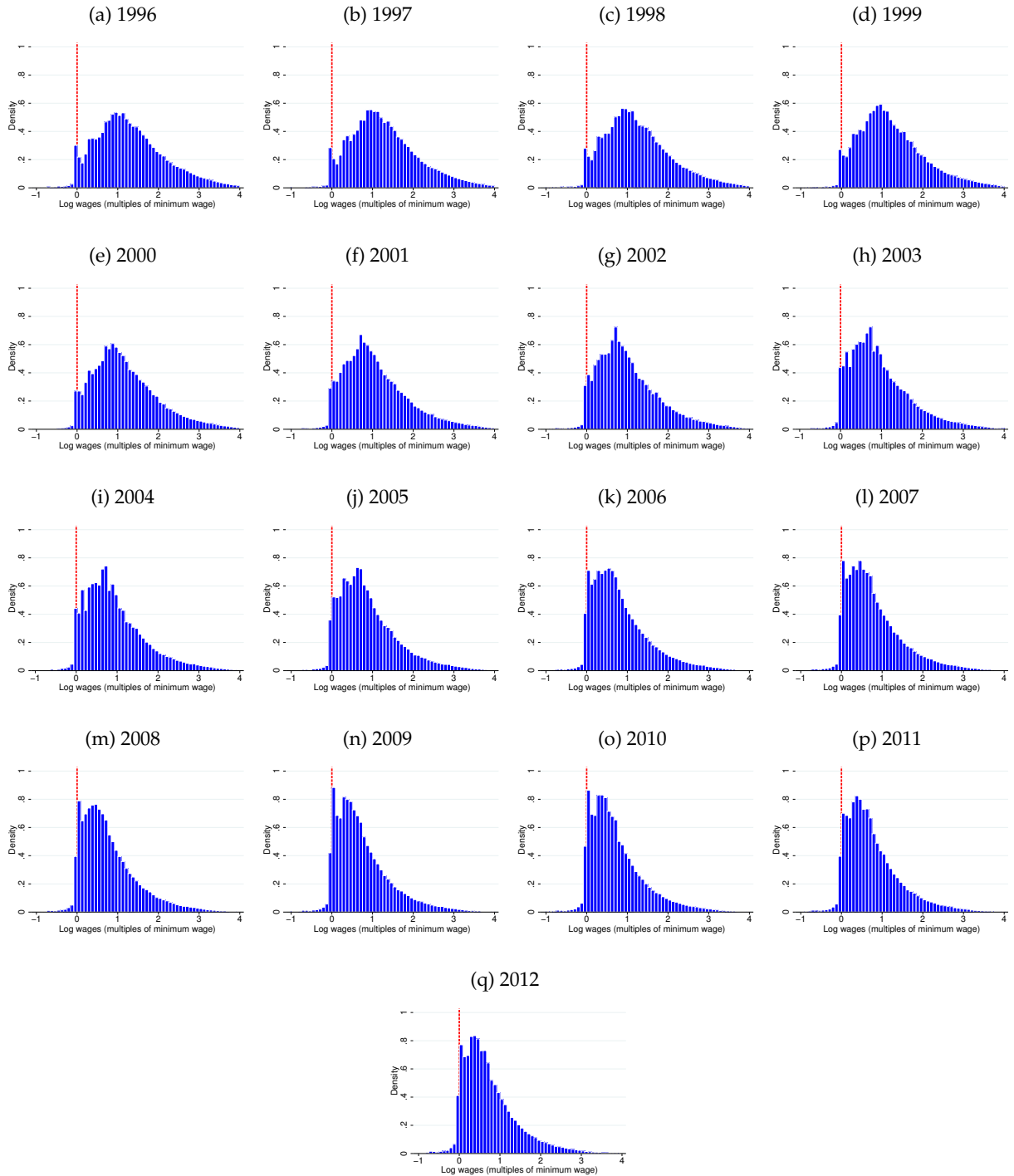
Figure 14. Data vs. Model: Bindingness of the Minimum Wage, 1996–2012



Notes: Blue line with circles represents share of workers earning exactly the minimum wage. Red dashed line with squares shows share at or below the minimum wage. Green dash-dotted line with diamonds plots share within five percent of the minimum wage.

B.2 Earnings distributions by year

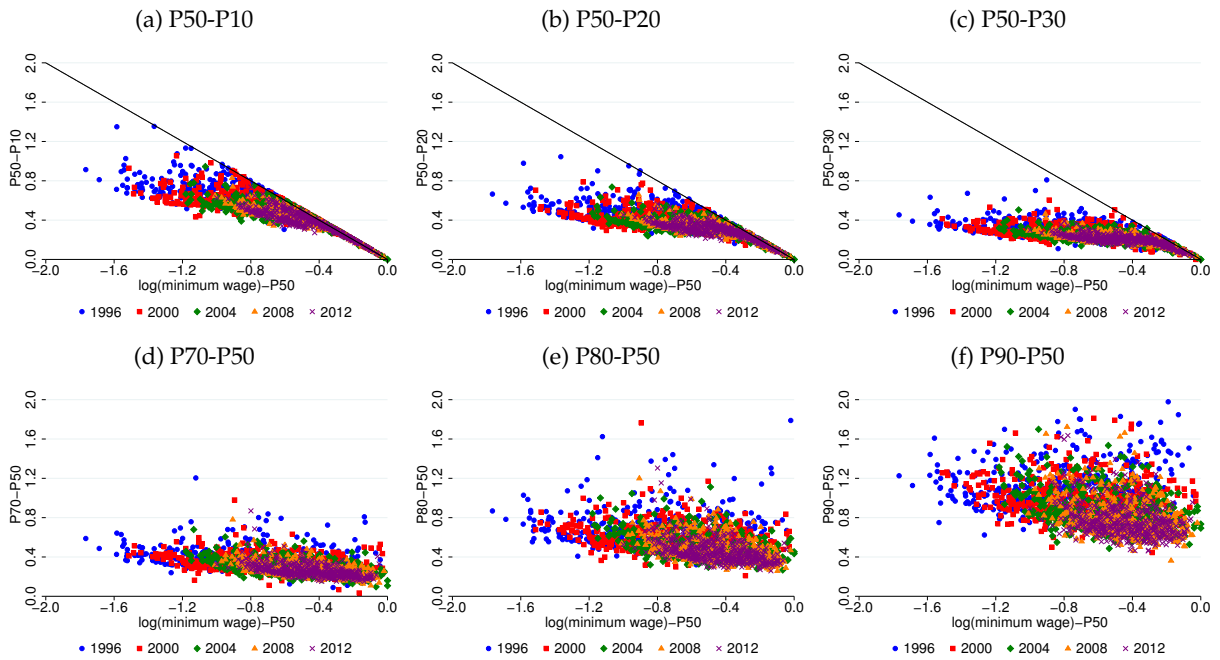
Figure 15. Data: Earnings Distributions by Year, 1996-2012



Notes: Density plots based on 60 equi-spaced histogram bins for population of male workers aged 18-49 in RAIS data.

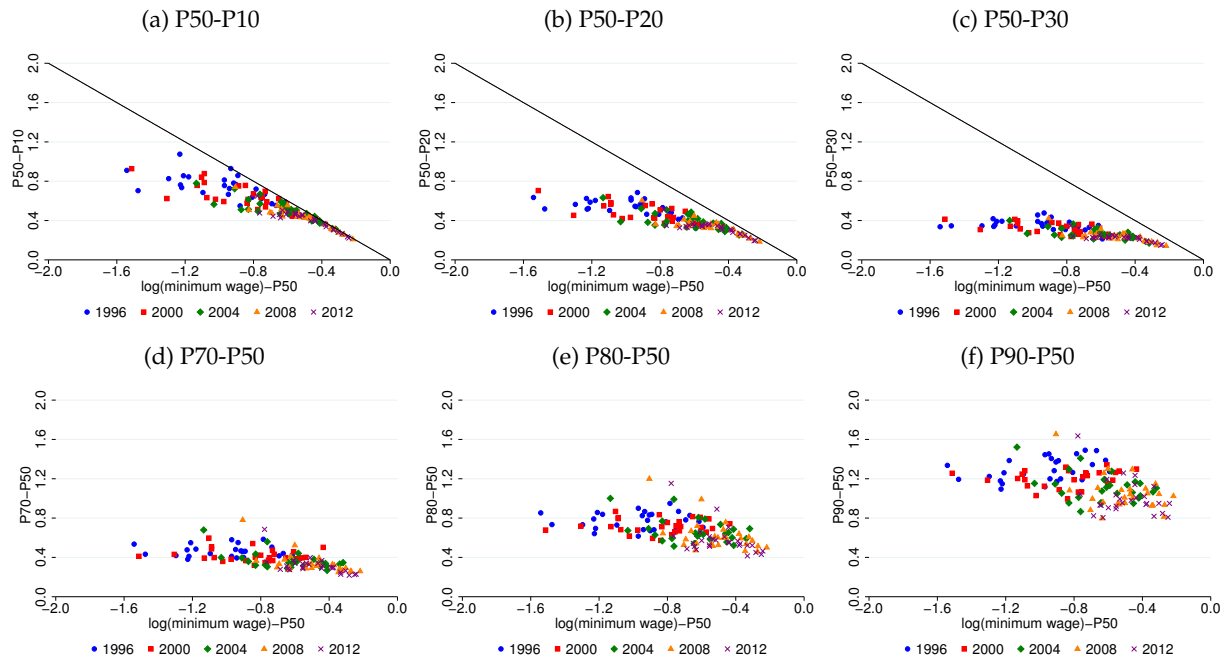
B.3 Additional exercises using cross-regional variation

Figure 16. Data: Inequality Evolution Across Microregions and Years, 1996–2012



Notes: Each marker represents one microregion-year combination, where microregions correspond to the 450 localities defined as the first four digits of the six-digit municipality code. A small number of outliers are dropped in order to facilitate presentation.

Figure 17. Data: Inequality Evolution Across States and Years, 1996–2012



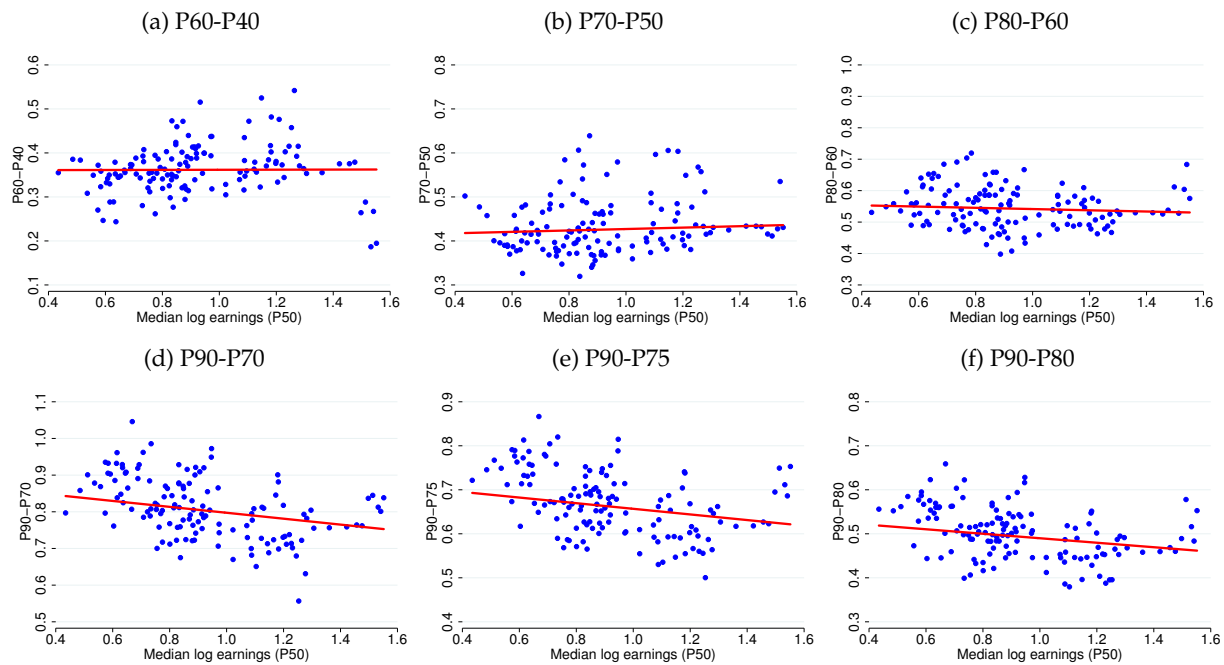
Notes: Each marker represents one state-year combination, where states correspond to the 27 Federative Units (*Unidades Federativas*, or *UF*) of Brazil.

B.4 Relation to empirical literature

This section provides support of the identifying assumptions underlying the Lee (1999)-type exercises in Section 3 and discusses why the framework with region fixed effects as in Autor et al. (2016) is not well suited to the Brazilian case.

Evidence supporting the identifying assumption. Figures 18–19 support the identifying assumption that states⁴² share the same underlying latent wage distribution that is masked by the minimum wage. Specifically, they plot the relationship between measures of upper tail inequality against the median in the region, showing consistently little systematic co-variation between the two. In the period 2008–2012 when the minimum wage is raised there is some evidence of positive covariation up to the 70th percentile, in line with our model predictions of substantial spillover effects of the minimum wage.

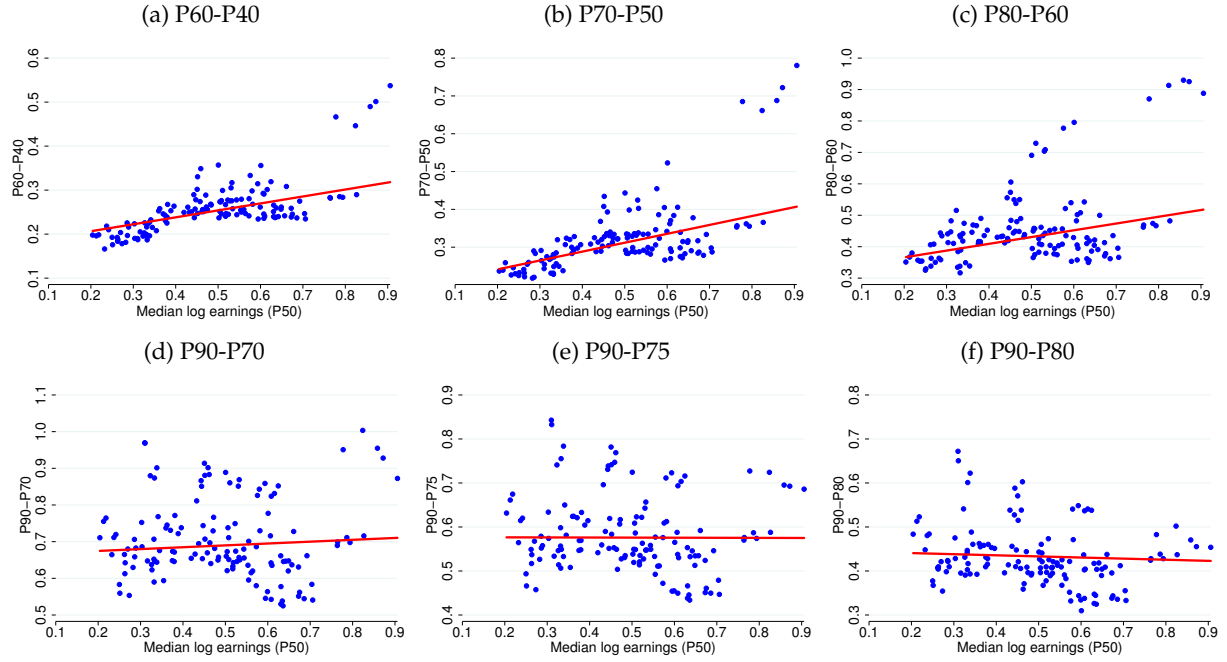
Figure 18. Data: Upper Tail Inequality versus Median Earnings Across States, 1996–2000



Notes: Blue dots represent state-year observation. Red line represents worker-weighted linear fit. Specification with no state dummies or state trends.

⁴²A similar analysis applies to microregions.

Figure 19. Data: Upper Tail Inequality versus Median Earnings Across States, 2008–2012



Notes: Blue dots represent state-year observation. Red line represents worker-weighted linear fit. Specification with no state dummies or state trends.

Why a region fixed effect specification may not be applicable. To see why the framework with region fixed effects is not well suited to our environment with only a federal minimum wage, consider a two-period case with only two regions, s and \bar{s} . With only two periods, differencing is the same as including region fixed effects. Denote by bottom_t^s bottom tail inequality in region s in year t , by $l50_t^s$ the log median in region s in year t , and by \min_t the federal minimum wage in year t . There always exist some values α and β such that

$$\text{bottom}_{t+1}^{\bar{s}} - \text{bottom}_t^{\bar{s}} = \alpha + \beta [\min_{t+1} - l50_{t+1}^{\bar{s}} - (\min_t - l50_t^{\bar{s}})]$$

$$\text{bottom}_{t+1}^s - \text{bottom}_t^s = \alpha + \beta [\min_{t+1} - l50_{t+1}^s - (\min_t - l50_t^s)]$$

Subtracting the first expression from the second,

$$\text{bottom}_{t+1}^{\bar{s}} - \text{bottom}_t^{\bar{s}} - (\text{bottom}_{t+1}^s - \text{bottom}_t^s) = \beta [l50_{t+1}^s - l50_{t+1}^{\bar{s}} - (l50_t^s - l50_t^{\bar{s}})] \quad (12)$$

Hence, β relates the change in the difference in medians to the differential change in bottom tail inequality.

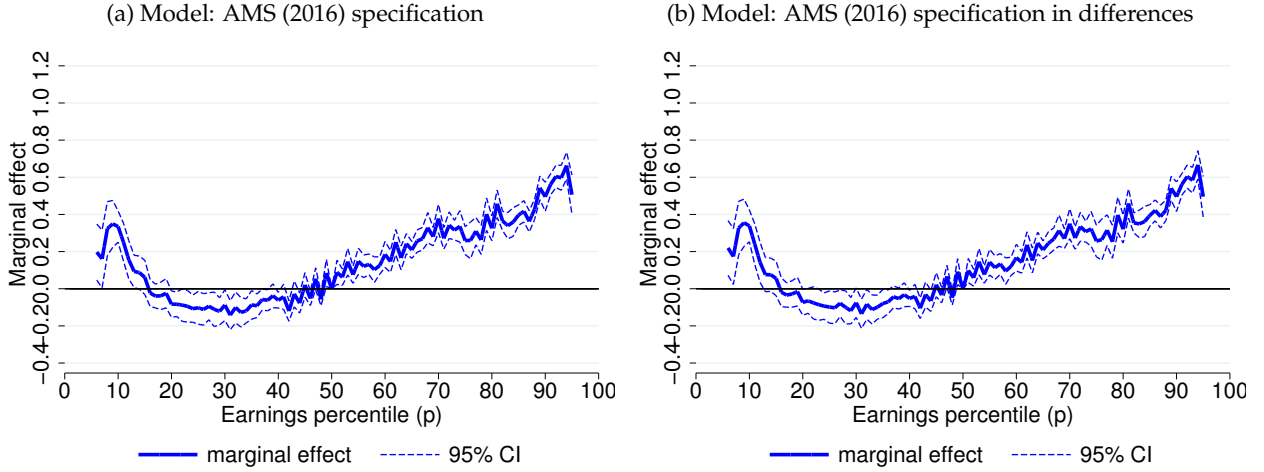
Suppose without loss of generality that region s has a lower median in the initial period than region \bar{s} , and suppose that the change in the minimum wage is the only force driving changes in the median over time. Suppose further that the two regions have identical latent wage distributions, and consider a hike in the federal minimum wage.

Since the minimum is initially more binding in region s , region s sees a larger relative increase in its median wage in response to the federal minimum wage hike. This is a robust prediction of the model—the minimum wage boosts wages more if it was initially more binding. Hence the right hand side of equation (12) is positive. If it were always the case that bottom tail inequality falls more in the initially more binding region in response to a hike in the minimum, we would have $\beta < 0$, as is the presumption in [Autor et al. \(2016\)](#).

This is, however, not generally true in our equilibrium model. To see why this fails, consider an extreme case in which the minimum wage is so binding in region s that bottom tail inequality is essentially nonexistent. Suppose further that the minimum wage is only somewhat binding in region \bar{s} , so that bottom tail inequality is relatively large. In this case, it may be that bottom tail inequality in region s changes only marginally in response to the minimum, because the p th percentile and the median move up by a similar amount in response to a hike in the minimum wage. It may fall by more in region \bar{s} , resulting in a positive left hand side of equation (12).

Essentially, the model predicts a non-monotone relationship between the initial bindingness of the minimum wage and the size of the subsequent fall in bottom tail inequality in response to a minimum wage hike. As a result of this non-monotonicity, even in the presence of strong and global effects of the minimum wage, resulting estimates of the marginal minimum wage effect β (or ρ_p in the full specification) may be either positive and negative in the model, depending on the initial bindingness of the minimum wage. We confirm this intuition in [Figure 20](#), which plots estimates of the marginal impact of the minimum wage on inequality at different percentiles, ρ_p , implied by the regression framework (2) with region fixed effects and region trends in panel (a), and estimated in differences in panel (b).

Figure 20. Model: Marginal Effect of w^{\min} -P50 on Pp -P50



Notes: Underlying regression is equation (2) with polynomial degree $N = 2$, including year fixed effects, state fixed effects, and state trends, estimated on the middle 90 percentiles of the earnings distributions of model-simulated data for four periods 1996–2000, 2000–2004, 2004–2008, and 2008–2012. Graphs show predicted marginal effects evaluated at the worker-weighted average across years.

In spite of the minimum wage having large spillover effects in our model, the within-region specification produces estimated marginal effects of the minimum wage that are positive at the bottom, quickly become statistically insignificant, and then start sloping upwards for higher earnings percentiles.

We conclude that through the lens of our model the graphical analysis and regression specification with only time effects as in Lee (1999) is a valid test of the degree of spillovers, while through the lens of our model interpreting specifications with region fixed effects or specifications in differences can be misleading vis-a-vis true spillover effects in the underlying data.

C Proof of Proposition 1

Under the assumption that labor market parameters are the same across types and that the minimum wage is low enough to never cut off firms, equation (9) reduces to

$$w(p, \theta) = \theta \left[p - \int_{p_\theta}^p \left[\frac{1 + \kappa_\theta^e (1 - \Gamma_0(p))}{1 + \kappa_\theta^e (1 - \Gamma_0(x))} \right]^2 dx \right] \quad (13)$$

Part 1. In markets where the minimum wage is not binding, a rise in the minimum has no effect on wages. Hence, it is sufficient to show that an increase in the minimum wage raises earnings in

markets where it is binding. Differentiating equation (13) with respect to the minimum wage

$$\frac{\partial w(p, \theta)}{\partial w^{\min}} = \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)} \right]^2 > 0$$

which establishes the first part of the proposition.

Part 2. Consider a market where the minimum wage is binding. Differentiating equation (13) with respect to productivity gives that the productivity-pay gradient is given by

$$\frac{\partial w(p, \theta)}{\partial p} = \theta 2\kappa_{\theta}^e \gamma_0(p) [1 + \kappa_{\theta}^e (1 - \Gamma_0(p))] \int_{p_{\theta}}^p \left(\frac{1}{1 + \kappa_{\theta}^e (1 - \Gamma_0(x))} \right)^2 dx$$

Differentiating this with respect to the minimum wage

$$\frac{\partial \left(\frac{\partial w(p, \theta)}{\partial p} \right)}{\partial w^{\min}} = \theta 2\kappa_{\theta}^e \gamma_0(p) [1 + \kappa_{\theta}^e (1 - \Gamma_0(p))] \left(-\frac{1}{\theta} \right) \left(\frac{1}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)} \right)^2 < 0$$

Hence the firm productivity-pay gradient falls with the minimum wage.

Part 3. Consider markets where the minimum wage is binding. Differentiating equation (13) with respect to ability gives that the ability-pay gradient is given by

$$\frac{\partial w(p, \theta)}{\partial \theta} = p - \int_{p_{\theta}}^p \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e (1 - \Gamma_0(x))} \right]^2 dx - \frac{w^{\min}}{\theta} \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)} \right]^2$$

Differentiating this with respect to the minimum wage

$$\begin{aligned} \frac{\partial \left(\frac{\partial w(p, \theta)}{\partial \theta} \right)}{\partial w^{\min}} &= \frac{1}{\theta} \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)} \right]^2 - \frac{1}{\theta} \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)} \right]^2 - \\ &\quad - \frac{w^{\min}}{\theta} [1 + \kappa_{\theta}^e (1 - \Gamma_0(p))]^2 (-2) \left(-\kappa_{\theta}^e \gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right) \frac{1}{\theta} \left[\frac{1}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)} \right]^3 \\ &= -\frac{2w^{\min}}{\theta^2} [1 + \kappa_{\theta}^e (1 - \Gamma_0(p))]^2 \kappa_{\theta}^e \gamma_0 \left(\frac{w^{\min}}{\theta} \right) \left[\frac{1}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)} \right]^3 < 0 \end{aligned}$$

Hence in markets where the minimum wage is binding, the worker ability-pay gradient falls with the minimum wage. \square

D Solution algorithm

This section discusses the solution algorithm we employ to solve and estimate the model. Under the assumption that the vacancy cost is quadratic, we can write the first-order conditions for the firm as

$$c_\theta v_\theta(p) = T_\theta (p - w(p)) \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w(p)))} \right)^2$$

and

$$1 = (p - w_\theta(p)) \frac{2s_\theta \lambda_\theta^u f_\theta(w_\theta(p))}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w_\theta(p)))}$$

where $T_\theta = \theta[u_\theta \lambda_\theta^u (\delta_\theta + s_\theta \lambda_\theta^u)]/V_\theta$. Define $h_\theta(p) = F_\theta(w_\theta(p))$ so that $f_\theta(w_\theta(p)) = h'(p)/w'_\theta(p)$ and $v_\theta(p) = \frac{V_\theta h'(p)}{\gamma(p)}$. Substituting this into the first-order equations, we have

$$h'_\theta(p) = \frac{T_\theta}{c_\theta V_\theta} (p - w(p)) \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - h_\theta(p))} \right)^2 \gamma(p) \quad (14)$$

and

$$w'_\theta(p) = \frac{2s_\theta \lambda_\theta^u T_\theta}{c_\theta V_\theta} (p - w_\theta(p))^2 \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - h_\theta(p))} \right)^3 \gamma(p) \quad (15)$$

Our empirical section estimates λ_θ^u , δ_θ and s_θ . Under these parameter estimates, the unemployment rate and total vacancies are given by the expressions

$$u_\theta = \frac{\delta_\theta}{\delta_\theta + \lambda_\theta^u}, \quad V_\theta = \lambda_\theta^{u \frac{1}{\alpha}} (u_\theta + s_\theta (1 - u_\theta))$$

Having obtained unemployment and total vacancies, we solve the system of differential equations (14)–(15) for a given guess for the cost of creation jobs, c_θ , subject to the boundary conditions

$$w_\theta(\underline{p}_\theta) = \frac{\max\{\phi_\theta, w^{min}\}}{\theta}, \quad h(\underline{p}) = 0$$

We update the guess for the cost c_θ until the vacancy policy integrated across firms replicates the total amount of vacancies necessary to match empirical transition hazards.

We adopt the above algorithm when using the model in order to evaluate effects of a rise in the

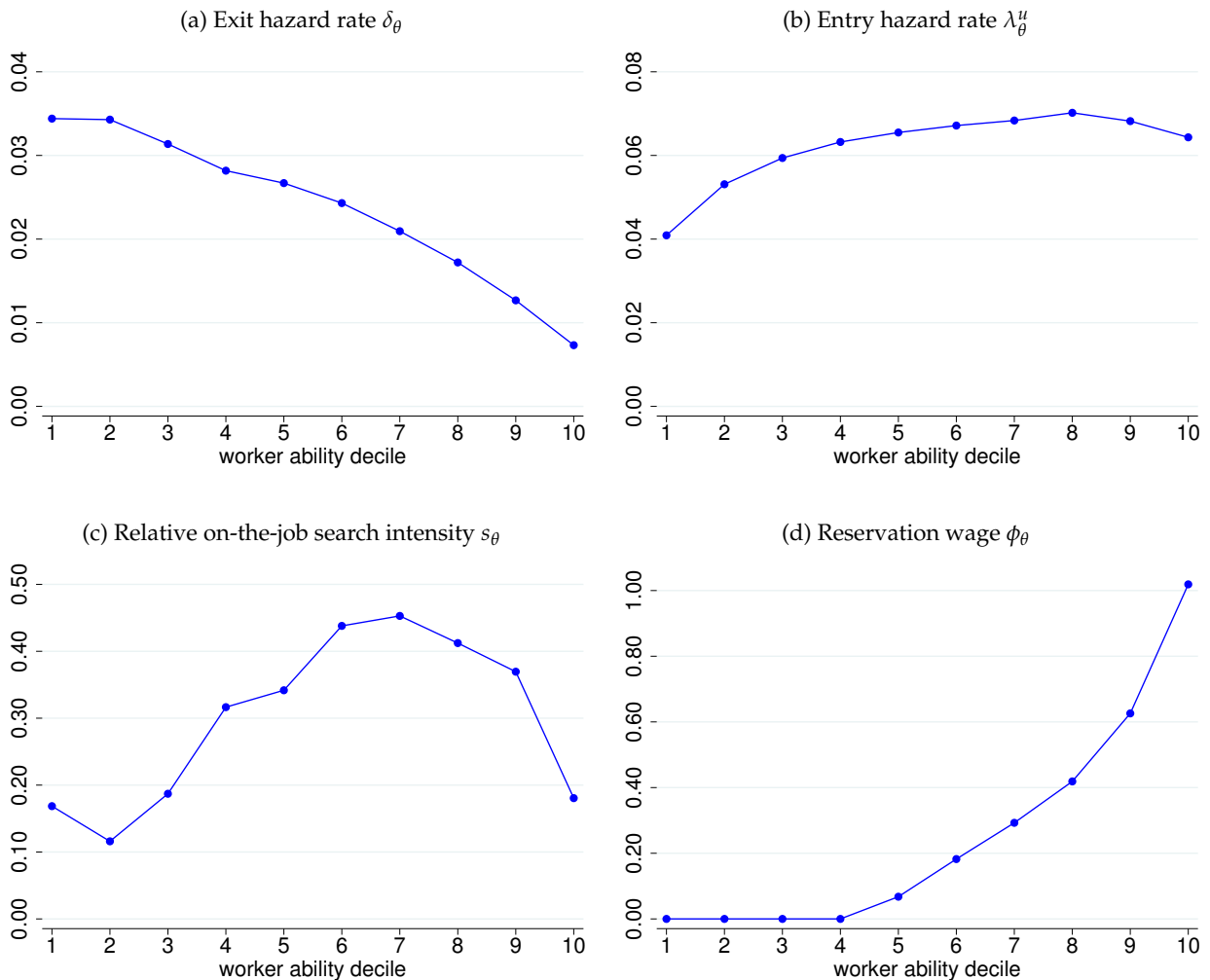
minimum wage. Instead of taking λ_{θ}^u from the data and iterating over c_{θ} to match that, we keep c_{θ} fixed at its “pre-period” estimate and then loop over the new job finding rate λ_{θ}^u until implied total vacancies are consistent with optimizing firm behavior in the post-period.

E Details of estimation routine

E.1 First stage: Labor market parameters

Figure 21 plots estimated labor market parameters by worker ability type.

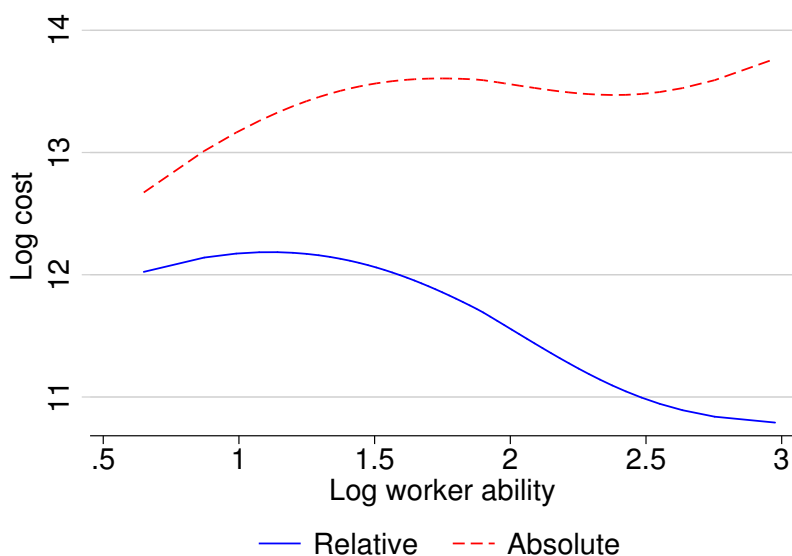
Figure 21. Estimated Labor Market Parameters by Worker Ability Decile, 1996–2000



Notes: Each worker ability decile contains around 9 million observations. Standard errors of point estimates are all of magnitude $< 10^{-3}$ and omitted for ease of presentation. Exit hazard rate in panel (a) and entry hazard rate in panel (b) are monthly rates. Relative on-the-job search intensity in panel (c) is a scalar. Estimated reservation wage in panel (d) is in log multiples of the current minimum wage.

The implied vacancy cost parameter c_θ of the model that matches the estimated job arrival rates from the data shown above is shown in Figure 22 as a function of worker ability. Note that this step takes as an input the estimated distributions of worker ability and firm productivity discussed in Section E.2.

Figure 22. Implied vacancy posting cost c_θ by worker ability

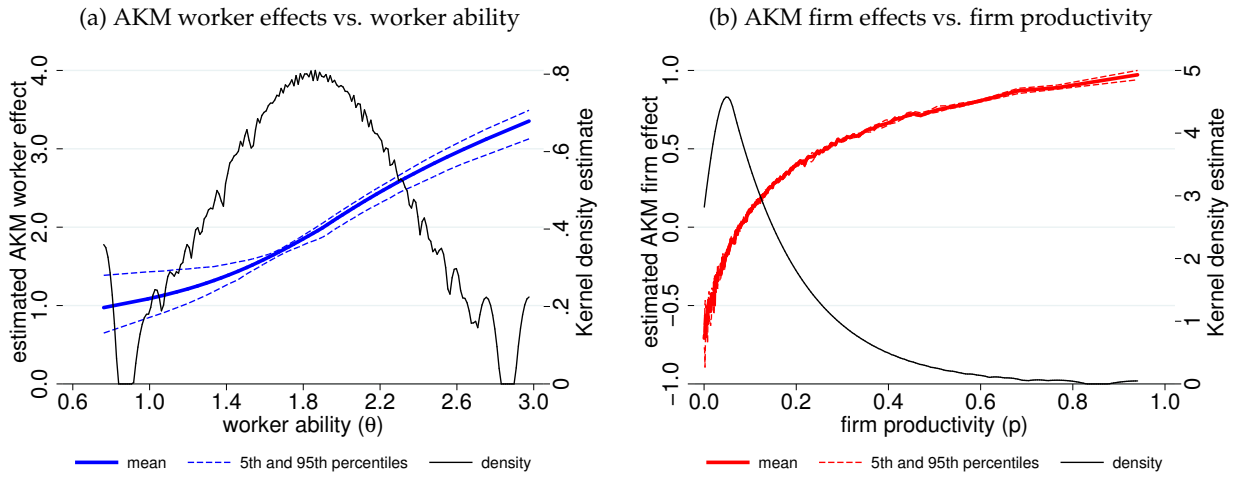


Notes: Figure plots the estimated vacancy posting cost c_θ that matches the differences in labor market hazard rates and value of leisure across worker types. The declining cost relative to worker ability (solid blue line) is implied by the higher job finding rate of high ability workers. The increasing absolute cost (dashed red line) is implied by the magnitude of the estimated increase in job finding rates with ability.

E.2 Second stage: Firm and worker heterogeneity

AKM as an auxiliary model. Figure 23 shows that applying the AKM estimation equation (1) to our model-simulated data is informative about the underlying structural parameters guiding worker ability and firm productivity.

Figure 23. Model: Estimated AKM Components Across Worker and Firm Types, 1996–2000



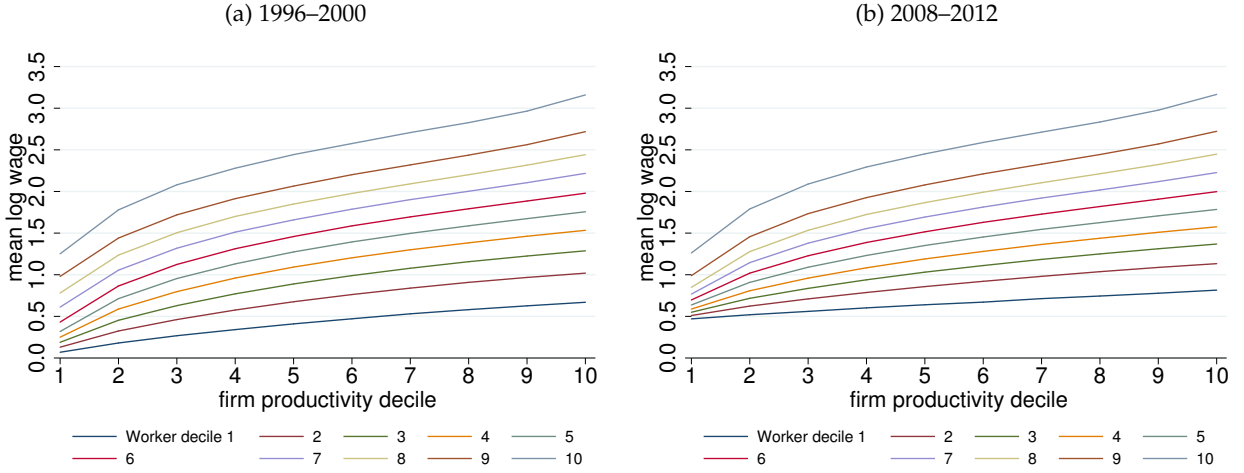
Notes: Worker ability density is approximated using an Epanechnikov kernel density estimate with 50 bins for 50 worker types. Firm productivity density is approximated using an Epanechnikov kernel density estimate with 200 bins for 200 firm types.

Estimated AKM worker effects in panel (a) are on average strictly increasing in worker ability θ and have tighter error bands around the center of the ability distribution, with a correlation between worker ability and estimated AKM worker effects above 0.97. Estimated AKM firm effects in panel (b) are also strictly increasing in firm productivity and have tight error bands throughout the distribution.

Figure 24 shows model-simulated mean log earnings of workers by ability deciles across firm productivity deciles. In line with the AKM estimation equation (1), panel (a) shows that for the period 1996–2000 log earnings are approximately evenly spaced across worker groups for individuals employed at or above the third firm productivity decile. Panel (b) shows that the same statement is true for the period 2008–2012, although the gap in log earnings has shrunk as a result of the rising minimum wage. For lower firm productivity deciles, relative pay premiums across worker ability ranks become more compressed, and the returns to finding a better employer differ by worker type, thereby producing an error term as viewed through the lens of the AKM regression.⁴³

⁴³Because the AKM framework controls flexibly for worker and firm identity, the average shape of wages for a given worker across firm productivity groups, or the average distance in wages across worker groups within a given firm is not relevant here—any average shape and distance are reconcilable with the AKM framework.

Figure 24. Model: Mean Log Earnings Across Firm Productivity Deciles by Worker Ability Decile



Notes: Figures plot mean log wages by firm productivity decile and worker ability decile from model-simulated data for periods 1996–2000 and 2008–2012.

Model estimation via Method of Simulated Moments. As outlined in Section 5, we estimate the average worker ability, the dispersion in worker ability, the shape of the productivity distribution, and the cost of creating jobs by indirect inference. We implement this by repeatedly solving the model over a pre-specified grid for the first three parameters and recording the model-predicted values for the targeted moments. Within each loop, we iteratively solve for the cost of creating jobs that allows the model to match the UE hazard rate estimated in the first step of our estimation.

We use a 25-by-25-by-25 point grid in the three parameters of interest defined over a sufficiently large domain. Solving and parallel-simulating the model 15,625 times is relatively efficient and runs for about 16 hours on a modern quad-core desktop computer. We search on a given parameter grid for the triplet (μ, σ^2, ζ) that minimizes the sum of squared log deviations between three target moments in the data versus the model:

$$\operatorname{argmin}_{\hat{\mu}, \hat{\sigma}^2, \hat{\zeta}} \left\{ \log \left[\frac{\operatorname{Var}(\alpha_i^M)}{\operatorname{Var}(\alpha_i^D)} \right]^2 + \log \left[\frac{\operatorname{Var}(\alpha_j^M)}{\operatorname{Var}(\alpha_j^D)} \right]^2 + \log \left[\frac{mM^M}{mM^D} \right]^2 \right\} \quad (16)$$

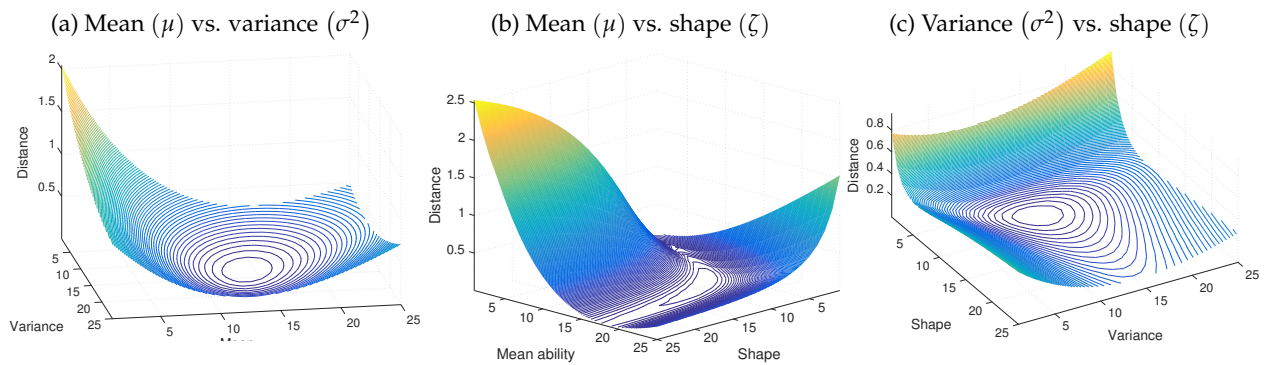
where $\operatorname{Var}(\alpha_i)$ denotes the variance of AKM worker fixed effects, $\operatorname{Var}(\alpha_j)$ denotes the variance of AKM firm fixed effects, and mM denotes the minimum-to-mean earnings ratio.

To verify that the grid is large enough as well as the uniqueness of the solution, we analyzed

the behavior of the objective function in two dimensions at a time, fixing the third parameter at its estimated value. This is plotted in Figure 25. The variance of worker ability appears well identified, while there is some ambiguity in the worker ability-firm productivity shape dimension. Specifically, a higher mean worker ability—implying a less binding minimum wage—can be compensated for by a higher shape of the firm productivity distribution—implying less dispersed firm productivity.

We have re-evaluated the impact of an increase in the minimum wage for different combinations of (μ, ζ) that produce only a slightly worse fit with the data, and find very similar quantitative results.

Figure 25. Distance Metric From Estimation Procedure for Period 1996–2000



Notes: Mean (μ) and variance (σ^2) denote mean and variance of log-Normal worker ability distribution, while shape (ζ) denotes Pareto tail parameter of firm productivity distribution. The plots show the distance minimization criterion in equation (16) used to estimate the model parameters (μ, σ^2, ζ) vis-a-vis the data.