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Central Bank Policy Rates: Are they Cointegrated?

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Abstract

This paper analyses the stochastic properties of and the bilateral linkages between the central bank policy rates of the US, the Eurozone, Australia, Canada, Japan and the UK using fractional integration and cointegration techniques respectively. The univariate analysis suggests a high degree of persistence in all cases: the fractional integration parameter d is estimated to be above 1, ranging from 1.26 (US) to 1.48 (UK), with the single exception of Japan, for which the unit root null cannot be rejected. Concerning the bivariate results, Australian interest rates are found to be cointegrated with the Eurozone and UK ones, Canadian rates with the UK and US ones, and Japanese rates with the UK ones. The increasing degree of integration of international financial markets and the coordinated monetary policy responses following the global financial crisis might both account for such linkages.

JEL-Codes: C220, C320, E470.

Keywords: interest rates, long memory, fractional integration and cointegration.

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1. Introduction

Interest rate linkages have been analysed in numerous empirical studies. There are two main ways to interpret them (see Barassi et al., 2005). If interest rates are viewed as similar to other asset prices, then financial flows should be their main determinant. In particular, the uncovered interest parity condition (or the open arbitrage condition) implies that interest rate differentials should equal the (expected) change in exchange rates. Therefore if exchange rates are at most I(1) series (a common finding in the literature for the G-7), and if the risk premium is stationary, one should find that interest rates are cointegrated on a bilateral basis. It is noteworthy that in recent years many countries have liberalized their capital accounts, and there has been a shift in capital flows towards portfolio and other short-term flows. Cross-border capital flows have risen in search of higher yields given the low interest rates resulting from a global liquidity surplus. It has been suggested that these recent developments in international financial markets and their increasing globalization might have led to interest rate convergence.

By contrast, if interest rates are seen as policy instruments, policy objectives should be their main driving factor, and therefore co-movement should result from policy convergence. A key question is the extent to which domestic monetary authorities can still conduct an independent interest rate policy despite the fact that international financial markets have become increasingly integrated. Another important issue is whether the creation of EMU and the role of the euro as an international currency has resulted in the Eurozone having a more global role.

The present study focuses on central bank policy rates in the US, Japan, the Eurozone, the UK, Canada and Australia, and makes a twofold contribution. First, it applies long-memory techniques to provide evidence on the stochastic properties (in particular, the degree of persistence) of the interest rates series. Second, it examines their long-run linkages on a bilateral basis using a cointegration approach. Unlike the majority of earlier studies, it adopts a *fractional* integration/cointegration framework that is much more general than the standard approach based

on the I(0)/I(1) dichotomy since it allows for fractional values of the integration/cointegration parameter and therefore does not impose restrictive assumptions on the dynamic behaviour of the individual series and their linkages.

The structure of the paper is as follows: Section 2 provides a brief review of the empirical literature on interest rates, focusing specifically on the stationarity/nonstationarity debate and its relation to fractional integration and cointegration; Section 3 outlines the empirical methodology; Section 4 describes the data and the main empirical findings; Section 5 offers some concluding remarks.

2. Are Interest Rates Stationary?

The statistical properties of interest rates have been extensively analysed in the literature. Earlier studies usually focused on testing whether interest rates can be described as stationary I(0) or nonstationary I(1) series. For instance, Cox et al. (1985) concluded that the short-term nominal interest rate is a stationary and mean-reverting I(0) process, whereas Campbell and Shiller (1987) found that they exhibit a unit root. A drawback of I(0) models is that they imply long-term rates that are not volatile enough (Shiller, 1979), whereas a problem with I(1) models is that they imply that the term premium increases with bond maturities (Campbell, Law and MacKinlay, 1997). Other studies have analysed whether or not real rates are stationary, since a unit root in ex-ante real rates is inconsistent not only with the Fisher hypothesis but also with the consumption-based capital asset pricing model (CCAPM) of Lucas (1978) (see Rose, 1988). Various papers in the earlier literature found a unit root in the real interest rate (see, e.g., Goodwin and Grennes, 1994; Phylaktis, 1999; Rapach and Wohar, 2004).

However, the low power and limitations of traditional unit root testing methods are now well known. More recent studies have tried to deal with these issues by using long-horizon data (see, e.g., Sekioua and Zakane, 2007), or applying long-memory or fractional integration approaches. These are much more flexible than the usual I(0)/I(1) framework, since they allow the

degree of integration to be between 0 and 1, as well as above 1. This is particularly useful for series which, although mean-reverting, might exhibit long memory and therefore be characterised by a high degree of persistence. For example, Shea (1991) investigated the consequences of long memory in interest rates for tests of the expectations hypothesis of the term structure; he found that allowing for long memory and fractional integration can significantly improve the performance of the model, even though the expectations hypothesis cannot be fully resurrected. In a related study, Backus and Zin (1993) reported that the volatility of bond yields does not decline exponentially when the maturity of the bond increases; in fact, they noticed that the decline is hyperbolic, which is consistent with a fractionally integrated specification. Lai (1997) provided evidence based on semi-parametric methods that ex-ante and ex-post US real interest rates are fractionally integrated. Tsay (2000) employed an Autoregressive Fractionally Integrated Moving Average (ARFIMA) model to show that US real interest rates can be described as an $I(d)$ process. Further evidence can be found in Barkoulas and Baum (1997), Tkacz (2001), Meade and Maier (2003), Sun and Phillips (2004), Gil-Alana (2004a, b), and Karanasos, Sekioua and Zeng (2006). Couchman, Gounder and Su (2006) estimated ARFIMA models for ex-post and ex-ante interest rates in sixteen countries. Their results suggest that, for the majority of countries, the fractional differencing parameter lies between 0 and 1, and is considerably smaller for the ex-post than for the ex-ante real rates.

Fractional cointegration tests have also been employed in recent studies. Lardic and Mignon (2003) tested for fractional cointegration between nominal interest rates and inflation under the assumption that both individual series were $I(1)$. They tested this hypothesis with standard unit root procedures (Dickey-Fuller, ADF, 1979; Phillips-Perron, PP, 1988; and the Kwiatkowski et al., KPSS, 1992 tests). However, these methods have extremely low power if the alternatives are of a fractional form (Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996). Barkoulas and Baum (1997) also used fractional integration to model

nominal interest rates and found evidence of long memory in the differenced series. Mean reversion in nominal rates was reported for Asian and emerging countries respectively in Gil-Alana (2004a) and Candelon and Gil-Alana (2006).

3. Fractional integration and Cointegration

As already mentioned, a fractional integration approach, rather than imposing integer degrees of differentiation, allows for real values including fractional ones. If $\{x_t, t = 1, 2, \dots\}$ is the series of interest, the model is specified as:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots \quad (1)$$

where L is the lag operator (i.e., $Lx_t = x_{t-1}$), d can be any real number, and u_t assumed to be integrated of order 0, denoted by $I(0)$, and defined as a covariance stationary process with a finite value for the infinite sum of the autocovariances. In this context, x_t is said to be $I(d)$, and the differencing parameter d plays a crucial role in determining the degree of dependence of the series. The higher the value of d is the higher the dependence between observations distant in time will be. Moreover, values of d below 1 indicate that shocks have transitory effects, whereas values of d equal to or higher than 1 imply permanent effects.

The methodology employed here to estimate the differencing parameter is based on the Whittle function in the frequency domain; we use both parametric (Dahlhaus, 1989; Robinson, 1994) and semi-parametric (Robinson, 1995a,b, Abadir et al., 2007) methods. Additionally, other standard approaches in the time domain (Sowell, 1992; Beran, 1995) are also followed.

In order to examine bivariate relationships homogeneity in the order of integration of the series should be tested in the first instance. For this purpose, we follow the Robinson and Yajima's (2002) approach, which has a standard normal limit distribution (see Gil-Alana and Hualde (2009) for evidence on the finite sample performance of this procedure). If homogeneity is found, then the Hausman test for no cointegration of Marinucci and Robinson (2001) is performed; this compares

the Whittle estimates of d for the individual series with the more efficient bivariate one of Robinson (1995b), which uses the information that $d_x = d_y = d^*$. Marinucci and Robinson (2001) show heuristically that this statistic has a standard limit distribution.

4. Data and Empirical Results

The series analysed are the central bank policy rates, monthly, from January 1999 to October 2011, for the US (the Federal funds target rate), Japan (the target call rate), the Eurozone (the Repo rate), the UK (the base rate), Canada (the target rate) and Australia (the target cash rate); the data sources are the corresponding Central Banks.

The first step is the estimation of the (fractional) differencing parameter using the Whittle method in the frequency domain, under the assumption of white noise and autocorrelated disturbances respectively. In the latter case we implement the non-parametric method of Bloomfield (1973) that produces autocorrelation values decaying exponentially as in the autoregressive case. The model specification is the following:

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (2)$$

where y_t is the observed time series for each country, α and β are the unknown coefficients on an intercept and a linear trend, and the errors, x_t , are assumed to be $I(d)$.

Table 1 reports the results for white noise errors while Table 2 focuses on the case of Bloomfield-type disturbances; for both we consider the three standard cases of i) no deterministic terms, ($\alpha = \beta = 0$ in (2)), an intercept (α unknown and $\beta = 0$) and an intercept with a linear time trend (α and β unknown). An intercept seems to be sufficient to describe the deterministic components of the series. In the white noise case, only for Japan the unit root hypothesis (i.e., $d = 1$) cannot be rejected, whereas for all other countries the value of d is significantly above 1, ranging from 1.26 (US) to 1.48 (UK)

[Insert Tables 1 and 2 about here]

The same happens with (weakly) autocorrelated errors: the unit root null hypothesis is rejected in all cases in favor of $d > 1$ except for Japan ($d = 1.07$); in the other cases, d ranges between 1.39 (US) and 1.66 (Australia).

[Insert Table 3 about here]

We also employ a semi-parametric approach based on a “local” Whittle estimate that degenerates to zero (Robinson, 1995a). We report in Table 3 the values of d for a selected number of bandwidth parameters from $m = 10$ to 20. Focusing on those where m is approximately $(T)^{0.5}$, i.e., 12 and 13 one can see that the unit root null cannot be rejected for Australia, Japan and the UK, whilst it is in the remaining cases in favor of higher degrees of integration. Similar results were obtained using the extension of this method as in Abadir et al. (2007). On the whole, the univariate results indicate a high degree of persistence, with orders of integration equal to or higher than 1 in all cases, which implies that shocks have permanent effects.

[Insert Tables 4 and 5 about here]

Next we carry out the bivariate analysis. Tables 4 and 5 report the statistics of Robinson and Yajima (2003) for the equality in the order of integration using respectively $m = 12$ and $m = 13$ as the bandwidth parameters: Australia displays the same degree of integration as Japan (with $m = 12$), with the Eurozone (with $m = 13$) and with the UK regardless of the bandwidth parameter selected; Canada displays the same order of integration as the Eurozone, and the UK ($m = 12$) and the USA ($m = 13$); there is also homogeneity between the Eurozone and Japan for both bandwidth parameters, and between Japan and the UK with $m = 13$.

[Insert Tables 6 and 7 about here]

The cointegration results (again for $m = 12$ and $m = 13$ respectively) are reported in Tables 6 and 7. Australia is found to be cointegrated with the Eurozone and the UK; Canada with the UK and the US, and Japan with the UK. In all these cases, the reduction in the degree of integration is

quite small (around 0.2-0.3 with respect to the parent series), indicating slow mean reversion in the dynamic adjustment towards the long-run equilibrium.

5. Conclusions

This paper analyses the stochastic properties of and the bilateral linkages between the central bank policy rates of the US, the Eurozone, Australia, Canada, Japan and the UK using fractional integration and cointegration techniques respectively; this approach allows for much richer dynamics than the classical models based on the $I(0)/I(1)$ dichotomy. The univariate analysis suggests a high degree of persistence in all cases: the fractional integration parameter d is estimated to be above 1, ranging from 1.26 (US) to 1.48 (UK), with the single exception of Japan, for which the unit root null cannot be rejected; therefore it appears that interest rates are not mean-reverting, with shocks having permanent effects, a finding clearly important for the design of appropriate monetary policies.

Concerning the bivariate results, Australian interest rates are found to be cointegrated with the Eurozone and UK ones, Canadian rates with the UK and US ones, and Japanese rates with the UK ones. As pointed out previously, such linkages can be interpreted alternatively as the result of capital flows or policy convergence. Both factors are likely to have played a role over the sample period examined, during which international financial markets became increasingly integrated, and the global financial crisis led to similar, coordinated monetary policy responses (low interest rates, quantitative easing) aimed at injecting liquidity into the system and preventing the collapse of the banking system and stimulating the economy. The ability of national monetary authorities to conduct independent policies (for instance, for stabilisation purposes) has possibly been curtailed in the circumstances. Not surprisingly, linkages are found between US and Canada, two tightly connected economies. There is limited evidence of the supposedly more global effects of

developments in the Eurozone, since linkages are found only with Australian rates; it is the UK instead that appears to have a more global role, perhaps because of the size of its financial sector.

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Table 1: Estimated values of d and 95% confidence bands with white noise errors

	No regressors	An intercept	A linear trend
AUSTRALIA	1.08 (0.98, 1.21)	1.36 (1.27, 1.48)	1.36 (1.27, 1.48)
CANADA	1.07 (0.98, 1.10)	1.35 (1.25, 1.47)	1.35 (1.25, 1.47)
EUROZONE	1.11 (1.03, 1.23)	1.29 (1.20, 1.39)	1.29 (1.20, 1.39)
JAPAN	0.99 (0.89, 1.13)	1.08 (0.98, 1.22)	1.08 (0.98, 1.22)
UNITED KINGDOM	1.03 (0.93, 1.15)	1.48 (1.36, 1.63)	1.49 (1.37, 1.64)
UNITED STATES	1.11 (1.03, 1.22)	1.26 (1.19, 1.35)	1.26 (1.19, 1.35)

The values in bold refer to the significant models according to the deterministic terms. The values in parenthesis refer to the 95% confidence intervals for the differencing parameters.

Table 2: Estimated values of d and 95% confidence bands with autocorrelated errors

	No regressors	An intercept	A linear trend
AUSTRALIA	1.10 (0.92, 1.36)	1.66 (1.34, 2.01)	1.66 (1.34, 2.01)
CANADA	1.06 (0.90, 1.30)	1.54 (1.29, 1.86)	1.56 (1.29, 1.86)
EUROZONE	1.17 (1.00, 1.38)	1.51 (1.29, 1.85)	1.55 (1.29, 1.85)
JAPAN	0.96 (0.78, 1.20)	1.07 (0.87, 1.35)	1.07 (0.87, 1.37)
UNITED KINGDOM	1.06 (0.89, 1.29)	1.39 (1.16, 1.69)	1.40 (1.17, 1.78)
UNITED STATES	1.20 (1.05, 1.41)	1.62 (1.43, 1.88)	1.62 (1.43, 1.88)

The values in bold refer to the significant models according to the deterministic terms. The values in parenthesis refer to the 95% confidence intervals for the differencing parameters.

Table 3: Estimates of d based on a “local” Whittle semiparametric method

M	10	11	12	13	14	15	16	17	18	19	20
AUSTRALIA	0.838	0.947	1.089*	1.184*	1.301	1.299	1.347	1.374	1.410	1.488	1.497
CANADA	1.378	1.341	1.309	1.362	1.290	1.357	1.344	1.352	1.402	1.453	1.447
EUROZONE	1.212	1.258	1.280	1.324	1.348	1.423	1.432	1.498	1.500	1.500	1.500
JAPAN	1.089	1.178	1.067*	0.959*	1.010	1.023	1.059	1.102	1.045	1.072	1.018
U. K.	1.181	1.239	1.183*	1.179*	1.202	1.218	1.198	1.242	1.257	1.291	1.305
U.S.A.	1.411	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.457	1.479	1.467
95% low	0.739	0.752	0.762	0.771	0.780	0.787	0.794	0.800	0.806	0.811	0.816
95% high	1.260	1.247	1.237	1.228	1.219	1.212	1.205	1.199	1.193	1.188	1.183

The values in the first row are the bandwidth numbers. Note that $m = (T)^{0.5} = 12.40$

Table 4: Homogeneity condition tests (Robinson and Yajima, 2002)

m = 12	CANADA	EUROZONE	JAPAN	U.K.	U.S.A.
AUSTRALIA	-2.640	-2.291	0.264	-1.127	-4.932
CANADA	xxx	0.348	2.903	1.512	-2.291
EUROZONE	xxx	xxx	2.556	1.164	-2.640
JAPAN	xxx	xxx	xxx	-1.391	-5.915
U.K.	Xxx	xxx	xxx	Xxx	-3.804

m is the bandwidth number and the values in bold indicate no rejection of the homogeneity condition at the 5% level.

Table 5: Homogeneity condition tests (Robinson and Yajima, 2002)

m = 13	CANADA	EUROZONE	JAPAN	U.K.	U.S.A.
AUSTRALIA	-2.314	-1.819	-2.925	0.065	-4.107
CANADA	xxx	0.493	5.238	2.378	-1.794
EUROZONE	xxx	xxx	4.744	1.884	-2.288
JAPAN	xxx	xxx	xxx	-2.860	-7.033
U.K.	xxx	xxx	xxx	xxx	-4.173

m is the bandwidth number and the values in bold indicate no rejection of the homogeneity condition at the 5% level.

Table 6: Testing the null hypothesis of no cointegration against fractional cointegration

m = 12	EUROZONE	JAPAN	U.K.	U.S.A.
AUSTRALIA	H₁₀: 5.529 H₂₀: 17.833 d₁ = 1.089 d₂ = 1.280 d* = 0.849	H ₁₀ : 2.645 H ₂₀ : 1.190 d ₁ = 1.089 d ₂ = 1.067 d* = 0.923	H₁₀: 8.697 H₂₀: 14.978 d₁ = 1.089 d₂ = 1.183 d* = 0.788	xxxxxx
CANADA	H ₁₀ : 2.973 H ₂₀ : 2.074 d ₁ = 1.309 d ₂ = 1.280 d* = 1.133	xxxxxx	H₁₀: 27.374 H₂₀: 15.980 d₁ = 1.309 d₂ = 1.183 d* = 0.775	H₁₀: 21.660 H₂₀: 42.581 d₁ = 1.309 d₂ = 1.500 d* = 0.834
EUROZONE	xxxxxx	xxxxxx	H ₁₀ : 1.291 H ₂₀ : 4.355 d ₁ = 1.280 d ₂ = 1.183 d* = 1.396	xxxxxx
JAPAN	xxxxxx	xxxxxx	H₁₀: 5.034 H₂₀: 11.426 d₁ = 1.067 d₂ = 1.183 d* = 0.838	xxxxxx
UNITED KINGDOM	xxxxxx	xxxxxx	xxxxxx	xxxxxx

In bold the cases with significant evidence of (fractional) cointegration at the 5% level.

Table 7: Testing the null hypothesis of no cointegration against fractional cointegration

m = 13	EUROZONE	JAPAN	U.K.	U.S.A.
AUSTRALIA	H₁₀: 5.483 H₂₀: 13.789 d₁ = 1.184 d₂ = 1.324 d* = 0.945	H ₁₀ : 4.274 H ₂₀ : 0.018 d ₁ = 1.184 d ₂ = 0.959 d* = 0.973	H₁₀: 8.989 H₂₀: 8.686 d₁ = 1.184 d₂ = 1.179 d* = 0.890	xxxxxx
CANADA	H ₁₀ : 1.951 H ₂₀ : 1.019 d ₁ = 1.362 d ₂ = 1.324 d* = 1.225	xxxxxx	H₁₀: 23.564 H₂₀: 8.928 d₁ = 1.362 d₂ = 1.179 d* = 0.886	H₁₀: 21.173 H₂₀: 36.079 d₁ = 1.362 d₂ = 1.500 d* = 0.911
EUROZONE	xxxxxx	xxxxxx	H ₁₀ : 2.695 H ₂₀ : 9.738 d ₁ = 1.324 d ₂ = 1.179 d* = 1.485	xxxxxx
JAPAN	xxxxxx	xxxxxx	H₁₀: 4.499 H₂₀: 19.051 d₁ = 0.959 d₂ = 1.179 d* = 0.751	xxxxxx
UNITED KINGDOM	xxxxxx	xxxxxx	xxxxxx	xxxxxx

In bold the cases with significant evidence of (fractional) cointegration at the 5% level.