Optimum Commodity Taxation with a Non-Renewable Resource

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CESifo Working Paper No. 5270
Category 1: Public Finance
March 2015

ISSN 2364-1428

CESifo *Working Paper No. 5270*

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Abstract

Under standard assumptions, optimum commodity taxation (OCT) should target non-renewable resources (NRRs) in priority. NRRs should be taxed at a higher rate than otherwise-identical conventional commodities. NRR substitutes and complements should receive a particular tax treatment. When reserves are endogenous, OCT for NRRs distorts both developed reserves, which are reduced, and their depletion, which is slowed down. Reserves are a form of capital and royalties tax its income: our results contradict Chamley's conclusion that capital should not be taxed in the long run. In a NRR-importing economy, Ramsey taxes are further increased because they allow the capture of foreign rents.

JEL-Code: Q310, Q380, H210.

Keywords: optimum commodity taxation, inverse elasticity rule, non-renewable resources, Hotelling resource, supply elasticity, demand elasticity, capital income taxation.

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March, 2015

We thank participants at various seminars and conferences: University Panthéon-Assas (Paris II); ETH Zurich; Montreal Natural Resources and Environmental Economics Workshop; Toulouse Business School; University Panthéon-Sorbonne (Paris I); SURED Conference; Bonn Max Planck Institute; Marseille Journées Louis-André Gérard-Varet 2010; WCERE Montréal; Helsinki School of Economics; University of Basel; AERE Seattle; Tilburg University; University of Montpellier; University of Oxford; Université du Québec à Montréal; CPEG Montréal; CESifo Munich. Particular thanks go to Michael Burda, Helmuth Crémer, Gérard Gaudet, John Hartwick, Martin Hellwig, Matti Liski, Ngo van Long, Rick van der Ploeg, Larry Samuelson, Eytan Sheshinski and Tony Venables. Financial support from the Social Science and Humanities Research Council of Canada, the Fonds Québècois de recherche pour les sciences et la culture, the CESifo, the CIREQ, and the Agence Nationale de la Recherche (ANR-09-BLAN-0350-01) is gratefully acknowledged.

1 Introduction

In this paper we reexamine the theory of optimal commodity taxation (OCT) in presence of natural non-renewable resources (NRRs). The theory of OCT addresses the following question: how should a government concerned with total welfare distribute the burden of commodity taxation across sectors in such a way as to collect a set amount of tax income while minimising the deadweight loss? The literature originated with Ramsey's "A Contribution to the Theory of Taxation" published in the *Economic Journal* in 1927. It was further developed by Pigou (1947), Baumol and Bradford (1970), Diamond and Mirrlees (1971), Auerbach (1985), and others. Surprisingly, it was never extended to economies involving NRRs despite the appearance of "The Economics of Exhaustible Resources" in 1931 by Hotelling. This extension was not attempted in the optimal taxation literature. It was not considered in the resource literature that has focused on neutrality and distortions without looking at optimal Ramsey distortions nor their dynamics. It was not examined in the literature on capital taxation either, despite the fact that NRRs are a form of capital.

The most famous OCT result is the "inverse elasticity rule" which says that, under simplifying conditions, the tax rate applied on each good should be proportional to the sum of the reciprocals of its elasticities of supply and of demand. In an economy that includes NRRs, the Ramsey-Pigou inverse elasticity rule of optimal taxation should be modified or reinterpreted in several ways, with implications on the form that it takes, on its interpretation, and on policy prescriptions. First, a NRR should be taxed in priority. Second, the non-renewability of a natural resource adds an intertemporal dimension to the optimum tax distortion, which in turn confers a special role to demand elasticity as in Stiglitz's intertemporal resource monopoly (Stiglitz, 1976; Lewis *et al.*, 1979). Nonetheless, and this is the third new aspect of our results, the supply side is what makes NRRs different from other commodities; supply elasticity is manifest in the flow of extraction and in the stock of reserves when the latter is endogenous; we show how this translates into an augmented, dynamic, version of Ramsey's rule and how this relates to the taxation of capital.

1.1 The relevance of Ramsey's approach for NRRs

Before elaborating on these and other results, let us point to the relevance and modernity of Ramsey's point of view. Ramsey's approach rules out the direct taxation of profits, rents, or incomes, leaving it open to the criticism that it ignores the possibility of neutral taxation. Nevertheless, as Sandmo pointed out in 1976 "... it seems definitely sensible to admit the unrealism of the assumption that the public sector can raise all its revenue from neutral

(...) taxes, and once we admit this we face the second-best problem of making the best of a necessarily distortionary tax system. This is the problem with which the optimal tax literature is mainly concerned." This view was shared by Stiglitz and Dasgupta (1971), who pointed out that "no government imposes 100 per cent taxes on profits and the income of fixed factors, in spite of the desirability of such non-distortionary taxes".[1] Their remark applies more definitely nowadays, where most countries are struggling to meet debt and budget constraints in institutional environments where profits or rents are only partially taxed and no increase is considered a feasible alternative.

This limitation to the ability to use direct non-distortionary taxes is even more apparent in NRR sectors. On the demand side, resource taxes are almost exclusively linear commodity taxes. On the supply side, non-linear taxes such as the resource rent tax have been advocated as non distortionary (see *e.g.* Boadway and Flatters, 1993; Boadway and Keen, 2010). However, they are contested on theoretical grounds (Gaudet and Lasserre, 1986; Garnaut, 2010) and meet strong opposition in practice for reasons ranging from institutions (property rights) to feasibility (information and agency issues, *e.g.* Boadway and Keen, 2014). In any case royalties and other linear commodity taxes are dominant forms of resource taxation (Daniel *et al.*, 2010). To sum up, the Ramsey-Pigou framework needs analysis in a NRR economy both because it has been overseen theoretically and because Ramsey's tax instruments are prevalent in NRR taxation systems.[2] To explain the special tax treatment received by most energy NRRs, it is also often noted that energy demand, oil demand in particular, is relatively price inelastic (Berndt and Wood, 1975; Pindyck, 1979; Hamilton, 2009): Ramsey's framework seems perfectly natural to examine the importance of this property.

While clearly relevant empirically, OCT was dealt a serious theoretical blow by Atkinson and Stiglitz (1976) who showed that an optimal non-linear labour income tax does not need to be supplemented with any commodity taxes if leisure choice is weakly separable from consumption choices and preferences for consumption are homogeneous. The Atkinson-Stiglitz result assumes that profits are fully taxed or that they are absent because returns to scale are constant. While constant returns to scale may be considered a useful simplification for economies limited to conventional goods, that assumption would deprive the economics of

[1] According to Stiglitz and Dasgupta (1971), "two possible explanations for this limitation suggest themselves. (1) It is difficult if not impossible (...) to separate out pure profits from, say, income to capital, and few if any governments – or national income accountants – have even attempted the task. (2) In at least some western economies, where the rights of private property are considered to be very important, a 100 per cent profits tax would be considered equivalent to nationalisation of the fixed factors."

[2] For a good practical example of a relatively advanced system, see Alberta Royalty Review (2007, pp. 54-60).

NRRs of its very substance.[3,4] There is also renewed interest in the profit-capturing dimension of commodity taxes. In the carbon taxation literature, Barrage (2014, p. 50) shows that absent 100 per cent profits taxes, the optimal tax on carbon resources acquires a Ramsey component. The recent literature on capital taxation (*e.g.* Piketty, 2015) sees commodity taxation as an alternative to the direct taxation of wealth. As Auerbach and Hassett (2015) put it, consumption taxation has "the ability (...) to hit existing sources of wealth". In our paper, a similar logic applies; Ramsey taxes have the ability to tax NRR rents.

1.2 Structure of the analysis and principal findings

The role of supply elasticity permeates the three sets of results mentioned above. It is conventional to establish the inverse elasticity rule of OCT under the simplifying assumption that supply elasticity is infinite, so that distortions are determined on the demand side only. Such long-run perspective fits nicely with the assumption that profits are not taxed since competitive-equilibrium profits are zero under constant returns. On the other hand, the supply of a NRR is not infinitely elastic even if marginal extraction costs are constant. This is because the short-run supply of a non-renewable resource consists in allocating the production from a finite stock of reserves over time. A resource supplier that increases production at any date reduces the stock of reserves remaining for production in subsequent periods, so that the instantaneous supply elasticity is finite. An extreme example of this link between the fixity of long-run reserves and the behaviour of short-run supply occurs when a constant-rate commodity tax is imposed on a costlessly extracted resource, as assumed by Bergstrom (1982). Short-run supply is then insensitive to the tax.

In this paper, the commodity tax rate is allowed to vary over time and extraction is not costless, so that short-run supply is not inelastic. In order to facilitate comparisons with the conventional analysis involving non-resource sectors, we proceed in several steps. In the first step, presented in Section 2, we follow the traditional optimal taxation literature in assuming

[3] This includes Hotelling models when reserves are endogenous.

[4] In the absence of any NRR, a substantial literature questions the relevance of Atkinson and Stiglitz's framework of analysis. Even with non-linear income taxation, commodity taxation is considered to remain an issue if (1) income taxation is a limited option (but see Kaplow, 2006); or (2) the tax jurisdictions or authorities for direct income taxation and for indirect commodity taxation are not identical, are not perfectly coordinated, or have different objectives (Belan *et al.*, 2008, Footnote 1, p. 1739, give the example of a federal state); or (3) any other condition for the result of Atkinson-Stiglitz (1976) on the uselessness of commodity taxation is violated: besides the non-separability of preferences and issues related to the heterogeneity of endowments and tastes (Cremer *et al.*, 2001; Saez, 2002; Diamond and Spinnewijn, 2011), see Blomquist and Christiansen (2008) on self-selection constraints, Aronsson and Sjögren (2003) on unemployment, Naito (1999) and Saez (2004) on the production technology, Boadway *et al.* (1994) on tax evasion, as well as Cremer and Gahvari (1995) on uncertainty.

constant marginal costs of production. This implies that supply is infinitely elastic in non-resource sectors as should be the case in a long-run analysis when no factors are fixed. In the NRR sector, the same assumption on the technology, constant marginal extraction cost, implies that there is no limit to short-run supply; however Hotelling's long-run exhaustibility of the resource retains its central role.

It is in that setup that we obtain the sharpest version of the first result mentioned above, that the resource should be taxed in priority over producible commodities. More precisely we find that, if government revenue needs are low, they may even be satisfied by using the ability of linear royalties to tax the NRR without distortions, thus dispensing from distortionary taxes in the rest of the economy. A similar situation arises in Sandmo's (1975) time-honoured paper, as well as in Bovenberg and van der Ploeg (1994) and Barrage (2014), where the economy suffers from externalities that may be corrected by Pigovian taxes: if government revenue needs are low, they may be covered by such Pigovian taxes.

When revenue needs are too high to be covered by neutral royalties, distortions are inevitable and should be spread optimally in the economy. In that case, we find that the resource should be taxed at a higher rate than conventional commodities having the same demand elasticity.

Other new results concern the way that the Ramsey tax is conferred a dynamic dimension. The optimal distortion to the dynamics of resource exploitation has not been explored in the literature. The distortion to the resource sector takes the form of slower extraction at given level of remaining reserves so that the path of reserves over time does not diminish as fast as if the tax was neutral.

In the rest of the paper, we examine the role of some basic assumptions affecting resource demand and supply.

There are two basic ways to alleviate resource supply limitations; one is to rely on resource substitutes; the other one is to produce reserves for subsequent extraction. In Section 3, we introduce non-zero cross-price elasticities between the resource and other commodities. As far as the resource is concerned, exhaustibility retains the central role identified in the original setup. Yet the existence of resource substitutes and complements raises the question of whether they should receive a particular tax treatment as such. Our results differ from Sandmo's (1975) OCT analysis with externalities. Sandmo finds that Ramsey taxes on substitutes for externality-generating commodities should not acquire any Pigovian dimension. In contrast, we find that resource substitutes should be taxed at a different rate than similar commodities that are substitutes for conventional goods. Perhaps surprisingly, that rate should be lower; it also exhibits a dynamic dimension.

In Section 4, we assume that reserves are endogenous; their production is determined by the net-of-tax rents derived during the extraction phase completed by subsidies (negative Ramsey taxes) that the owner receives towards the production of reserves if any. This means that resource supply is allowed to be elastic not only in the short run as in the first part of the paper, but also in the long run. A first implication is that resources should never in that case be singled out as sole targets for OCT. The reason is that it is now impossible to avoid distortions, whatever the government's revenue needs. We establish the proper Ramsey rule for that case. It shows how reserve-supply elasticity combines with demand elasticity to determine how the taxation burden should be spread across resource and non-resource sectors. As far as the resource sector is concerned, we show that there exists a continuum of mixed tax systems, combining subsidies towards reserve supply with taxes on resource extraction, that achieve government's objectives in terms of NRR exploitation and tax revenues.[5]

All such optimal combinations of extraction taxes with exploration and development subsidies imply a tax load at least as high on the resource as on conventional commodities having the same demand elasticity. However, the distortion induced by NRR taxes is split between a distortion on the extraction profile extending the inverse demand elasticity rule dynamically, and a distortion on the level of induced reserves, obeying a rule reminiscent of the inverse elasticity rule for commodities of finite supply elasticity.

Section 5 concludes the analysis and confronts two aspects of our findings with important existing results. One is related to the literature on capital taxation; the other one to the literature on the capture of foreign NRR rents. Ramsey's framework explicitly rules out the direct taxation of capital income, whether in the form considered by Chamley (1986), or in a form mimicking profit taxation as with Lucas' (1990) capital levies, or via some form of resource rent taxes as described by Boadway and Keen (2010). However, a NRR is a form of capital and applying a commodity tax to NRR extraction over time is not unlike taxing the income of that capital. We find that taxing NRR extraction is optimal, in apparent contradiction with Chamley who shows that no tax should be applied on the income of capital in the long run. We explain why these results differ despite their similar OCT logic. The elasticity of supply again plays the central role, although in a way completely distinct

[5]Indeed most commonly observed extractive resource tax systems combine incentives to exploration and development with the taxation of production; this includes the polar case of a nationalised extraction sector where the government, perhaps because it is unable to commit to less drastic a tax system, appropriates itself the totality of resource rents during the extraction phase but also finances the totality of reserve development.

from that in Picketty and Saez (2013).

Concerning the capture of foreign rents, a NRR importer cannot apply any form of direct resource rent taxation to foreign suppliers; in that sense Ramsey's assumption that direct taxation is not possible applies to the foreign suppliers of an importing country implacably. However that country can apply commodity taxes to home consumption as substitute for the taxation of foreign resource rents. Since the capture of foreign rents involves the exercise of market power, the OCT problem for a NRR importer connects with the famous result of Bergstrom (1982) on rent capture. Here again, the OCT tax rate in a NRR importing country is higher than the rate on conventional commodities having the same demand elasticity.

The model used throughout the paper adheres to the conventional Ramsey-Pigou framework of pure OCT; the tax structure is restricted in such a way that optimum tax differences solely reflect differences in commodity attributes.

In particular, commodity taxes are the sole available tax instrument. Lump-sum transfers are impossible[6]; direct taxation is not an option, whether it aims at income[7], or at pure profits like resource rents[8]; indirect linear taxes or subsidies can be applied on the final consumption or on the production of any commodity or service; taxes (or subsidies) may take the form of *ad valorem* taxes or of *unit* taxes, proportional to quantities. The government is not concerned with individual differences; in fact we assume a representative consumer.[9] The optimal supply of public goods is not addressed either; we assume that the government faces exogenous financial needs in order to fulfil its role as a supplier of public goods so that the government's problem is to raise that amount of revenues in the least costly way, given the available tax instruments.

Proofs that are economically enlightening are provided in the main text; proofs involving algebraic manipulations are in Appendices.

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[6]In particular, this means that neither labour endowment nor leisure consumption are taxable (*e.g.* Auerbach, 1985, p. 89); it is standard to interpret the untaxed numeraire as being leisure.

[7]When income taxation is linear, there is no loss of generality in letting labour income be untaxed (*e.g.* Atkinson and Stiglitz, 1976). With non-linear income taxation, see Footnote 4.

[8]As long as profits and rents are not fully taxed, assuming that they are not taxed at all amounts to rescaling the production cost function and does not involve any loss of generality.

[9]Using a representative agent is a simplification that should not be interpreted to mean that a poll tax is feasible. With heterogeneous consumers and concerns about equity, the standard Ramsey tax formula for one commodity takes into account the social contribution of consumers' incomes (Diamond, 1975; see Belan *et al.*, 2008, for a partial-equilibrium exposition closer to ours).

2 OCT with a non-renewable Hotelling resource

There are n produced commodities (goods and services) indexed by $i = 1, ..., n$, one non-renewable resource indexed by s and extracted from a finite reserve stock S_0, and a numeraire which is not taxed and should be interpreted as the negative of labour. We adopt the standard partial-equilibrium restrictions under which Baumol and Bradford (1970) obtain the inverse elasticity rule; that is: all commodities $i = 1, ..., n$ and s are final-consumption, non-leisure goods. Assuming a single non-renewable resource simplifies the exposition without affecting the generality of the results. At each date $t \geq 0$, quantity flows are denoted by $x_t \equiv (x_{1t}, ..., x_{nt}, x_{st})$.[10] Storage is not possible, so that goods and services must be consumed as they are produced. Producer prices $p_t \equiv (p_{1t}, ..., p_{nt}, p_{st})$ are expressed in terms of the numeraire. Goods and services are taxed at unit levels $\theta_t \equiv (\theta_{1t}, ..., \theta_{nt}, \theta_{st})$ so that the representative consumer faces prices $q_t = p_t + \theta_t$. In this autarkic economy, as in any situation where production equals consumption, taxes may indifferently be interpreted as falling on consumers or producers, but must be such that they leave non-negative profits to producers. In the case of the non-renewable resource, this requires that, at any date, the discounted profits accruing to producers over the remaining life of the mine be non negative. Taxes that meet these conditions will be called feasible.

Since the resource is non renewable it must be true that

$$\int_0^{+\infty} x_{st}\, dt \leq S_0, \tag{1}$$

where S_0 is the initial size of the depletable stock.

In the rest of the paper, a "$\sim$" on top of a variable means that the variable is evaluated at the competitive market equilibrium. For given feasible taxes $\Theta \equiv \{\theta_t\}_{t\geq 0}$, competitive markets lead to the equilibrium allocation $\{\widetilde{x}_t\}_{t\geq 0}$ where $\widetilde{x}_t = (\widetilde{x}_{1t}, ..., \widetilde{x}_{nt}, \widetilde{x}_{st})$. Under the set of taxes Θ, this intertemporal allocation is second-best efficient.

Defining social welfare as the cumulative discounted sum of instantaneous utilities $\widetilde{W}_t$, the OCT problem consists in choosing a feasible set of taxes Θ in such a way as to maximise welfare while raising a given level of discounted revenue $R_0 \geq 0$:

$$\max_{\Theta} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt \tag{2}$$

[10] Production processes by which resources are transformed into final products are linear in the quantity of pre-transformed resource; hence, there is no need to distinguish the raw extracted resource from its transformed derivative.

$$\text{subject to} \int_0^{+\infty} \theta_t \widetilde{x}_t e^{-rt}\, dt \geq R_0. \tag{3}$$

It is assumed that the set of feasible taxes capable of collecting R_0 is not empty.

The tax revenue constraint (3) does not bind the government at any particular date because financial markets allow expenditures to be disconnected from revenues. The government accumulates an asset a_t over time by saving tax revenues:

$$\dot{a}_t = ra_t + \theta_t \widetilde{x}_t, \tag{4}$$

where the initial amount of asset is normalised to zero and

$$\lim_{t \to +\infty} a_t e^{-rt} = R_0. \tag{5}$$

Thus the problem of maximising (2) subject to (3) can be replaced with the maximisation of (2) subject to (4) and (5), by choice of a feasible set of taxes.

As in Ramsey (1927, p. 55) and Baumol and Bradford (1970), we assume that the demand $D_i(q_{it})$ for each commodity i or s depends only on its own price, with $D_i'(.) < 0$, implying that consumer preferences are quasi linear in the numeraire.[11] Moreover, following Baumol and Bradford (1970) and many others, we assume in this section that the supply of each commodity is perfectly elastic, *i.e.* that marginal costs of production are constant in terms of the numeraire. Let $c_i \geq 0$ be the marginal cost of producing good $i = 1, ..., n$.

In the case of the non-renewable resource, the supply is determined by Hotelling's rule under conditions of competitive extraction. Consistently with our assumption of constant marginal costs of production, we assume that the unit cost of extracting the resource is constant, equal to $c_s \geq 0$.

However, this does not imply that the producer price of the NRR reduces to this marginal cost; Hotelling's analysis shows supply to be determined in competitive equilibrium by the so-called "augmented marginal cost" condition:

$$\widetilde{p}_{st} = c_s + \widetilde{\eta}_t, \tag{6}$$

where $\widetilde{\eta}_t$ is the current-value unit Hotelling rent accruing to producers; it depends on the tax and the level of initial reserves, and must grow at the rate of discount over time. In

[11] In Section 3, we introduce non-zero cross-price demand elasticities.

competitive Hotelling equilibrium,

$$\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}. \tag{7}$$

At any date, the net consumer surplus, producer surplus, and resource rents in competitive equilibrium are respectively

$$\widetilde{CS}_t = \sum_{i=1,\ldots,n,s} \int_0^{\widetilde{x}_{it}} D_i^{-1}(u)\, du - \sum_{i=1,\ldots,n,s} (\widetilde{p}_{it} + \theta_{it})\widetilde{x}_{it}, \tag{8}$$

$$\widetilde{PS}_t = \sum_{i=1,\ldots,n,s} \widetilde{p}_{it}\widetilde{x}_{it} - \sum_{i=1,\ldots,n,s} c_i \widetilde{x}_{it} - \widetilde{\eta}_t \widetilde{x}_{st} \tag{9}$$

and

$$\widetilde{\phi}_t = \widetilde{\eta}_t \widetilde{x}_{st}. \tag{10}$$

Define $\widetilde{W}_t$ in problem (2) as the sum of net consumer surplus, net producer surplus, and resource rents accruing to resource owners[12,13]. The present-value Hamiltonian associated with the problem of maximising cumulative discounted social welfare (2) under constraints (4) and (5) resulting from the budget requirement of the government is

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t \widetilde{x}_t), \tag{11}$$

where λ_t is the co-state variable associated with a_t while θ_t is the vector of control variables. λ_t can be interpreted as the current unit cost of levying one dollar of present-value revenues through taxes. From the maximum principle, $\dot{\lambda}_t = -\frac{\partial \mathcal{H}}{\partial a_t}$, so that $\lambda_t = \lambda e^{-rt}$, where λ is the present-value unit cost of levying tax revenues. Indeed tax revenues must be discounted according to the date at which they are collected. λ is equal to unity when there is no deadweight loss associated with taxation; it is higher than unity otherwise.

[12] Although changes in current taxes may affect current tax revenues, the budget constraint of the government applies only over the entire optimisation period. The revenue requirements being treated as given over that period, they enter the general problem as a constant and thus no amount of redistributed taxes needs to enter the objective.

[13] This formulation has the advantage of making the value of the resource as a scarce input explicit; it would also apply if producers were not owners of the resource but were buying the resource from its owners at its scarcity price $\widetilde{\eta}_t$.

2.1 Optimal taxation of conventional goods

Assuming that there exist feasible taxes that yield an interior solution to the problem, the first-order condition for the choice of the tax θ_{it} on good $i = 1, ..., n$ is

$$[D_i^{-1}(\widetilde{x}_{it}) - \theta_{it} - c_i]\frac{d\widetilde{x}_{it}}{d\theta_{it}} - \widetilde{x}_{it} + \lambda(\widetilde{x}_{it} + \theta_{it}\frac{d\widetilde{x}_{it}}{d\theta_{it}}) = 0. \tag{12}$$

Since the competitive equilibrium allocation $\widetilde{x}_t$ satisfies $D_i^{-1}(\widetilde{x}_{it}) = c_i + \theta_{it}$, it is the case that $\frac{d\widetilde{x}_{it}}{d\theta_{it}} = \frac{1}{D_i^{-1\prime}(.)}$. The optimum tax is thus $\theta_{it}^* = \frac{1-\lambda}{\lambda}\widetilde{x}_{it}D_i^{-1\prime}(.)$ and the optimum tax rate is

$$\frac{\theta_{it}^*}{\widetilde{q}_{it}} = \frac{\lambda - 1}{\lambda}\frac{1}{-\widetilde{\varepsilon}_i}. \tag{13}$$

In this formula, the elasticity of demand $\varepsilon_i \equiv \frac{D_i^{-1}(.)}{x_{it}D_i^{-1\prime}(.)}$ is negative and decreasing in x_{it}; this standard monotonicity property guarantees that the optimal tax in (13) is unique. As $\lambda \geq 1$, the optimal tax rates on conventional goods $i = 1, ..., n$ are positive in general, lower than unity, and vanish if $\lambda = 1$.

Formula (13) is Ramsey's formula for the optimal commodity tax rate. It provides an inverse elasticity rule for the case of perfectly-elastic supplies. Since market conditions are unchanged from one date to the other, the taxes and the induced tax rates are constant over time.

2.2 Optimal taxation of the non-renewable resource

The first-order condition for an interior solution to the choice of the resource tax is

$$[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = 0. \tag{14}$$

However, since resource supply is determined by condition (6), it follows that $D_s^{-1}(\widetilde{x}_{st}) - c_s - \theta_{st} = \widetilde{\eta}_t$, which is different from zero unlike the corresponding expression in (12). Consequently the Ramsey-type formula obtained for conventional goods does not apply.

If $\lambda = 1$, (14) reduces to $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = 0$. This means that the tax should not distort the Hotelling extraction path. Such a non-distortionary resource tax exists (Burness, 1976; Dasgupta *et al.*, 1981); it must grow at the rate of interest to keep the path of consumer prices unchanged[14]:

[14] Their proof goes as follows. Assume $\theta_{st} = \theta_{s0}e^{rt}$, for any θ_{s0} lower than the consumer price exclusive of the marginal cost in the absence of any resource tax. Then $\widetilde{q}_{st} = \widetilde{p}_{st} + \theta_{st} = c_s + \widetilde{\eta}_t + \theta_{st} = c_s + (\widetilde{\eta}_0 + \theta_{0t})e^{rt}$. Therefore, the price with the tax satisfies the Hotelling rule. The exhaustibility constraint must also be satisfied with equality: $\int_0^{+\infty} D_s(\widetilde{q}_{st})\, dt = S_0$. As a result, the extraction path under this tax is the same as

$\theta^*_{st} = \theta^*_{s0} e^{rt}$. Since θ^*_{st} grows at the rate of interest and the resulting $\widetilde{q}_{st}$ generally grows at a lower rate, the neutral tax rate is rising over time. The only exception is when the marginal cost of extraction is zero so that $\widetilde{q}_{st}$ grows at the rate of interest and the resulting optimal tax rate is constant.

As shown earlier, when $\lambda = 1$, commodity taxes on conventional goods are zero. Hence the totality of the tax burden falls on the non-renewable resource. Since the tax on the resource is neutral in that case, then a value of unity for λ is indeed compatible with taxing the natural resource exclusively. Consequently, provided the tax on the non-renewable resource brings sufficient cumulative revenues, the government should tax the resource exclusively, and should do so while taxing a proportion of the resource rent that remains constant over time.

The maximum revenue such a neutral resource tax can extract is the totality of gross cumulative scarcity rents that would accrue to producers in the absence of a resource tax. Since unit rents are constant in present value, any reserve unit fetches the same rent, whatever the date at which it is extracted. The present value of total cumulative exhaustibility rents is thus $\widetilde{\eta}_0 S_0$ and its maximum possible value $\overline{\eta}_0 S_0$ corresponds to the absence of taxation; the maximum tax revenue that can be raised by a neutral resource tax is thus

$$\overline{R}_0 = \overline{\eta}_0 S_0.$$

This maximum is implemented with a tax equal to the unit rent in the absence of taxation: $\theta^*_{st} = \overline{\eta}_0 e^{rt}$. Both $\widetilde{\eta}_0$ and $\overline{\eta}_0$ are determined in Appendix A. If the tax revenues needed by the government are lower than $\overline{R}_0$, the level of the neutral resource tax θ^*_{st} is set in such a way as to exactly raise the required revenue: $\theta^*_{st} = \theta^*_{s0} e^{rt}$ with

$$\theta^*_{s0} = \overline{\eta}_0 - \widetilde{\eta}_0 = \frac{R_0}{S_0}. \tag{15}$$

If $R_0 > \overline{R}_0$, revenue needs cannot be met by neutral taxation of the resource sector and $\lambda > 1$; this case will be discussed further below. The following proposition summarises our findings when government revenue needs are low in the sense that $\lambda = 1$.

in the absence of tax.

Proposition 1 *(Low government revenue needs) The maximum tax revenue that can be raised neutrally from the non-renewable resource sector is* $\overline{R}_0 = \overline{\eta}_0 S_0$ *where* $\overline{\eta}_0$ *is the unit present-value Hotelling rent under perfect competition and in the absence of taxation.*

1. *If and only if* $R_0 \leq \overline{R}_0$*, government revenue needs are said to be low and* $\lambda = 1$*; if and only if* $R_0 > \overline{R}_0$*, government revenue needs are said to be high and* $\lambda > 1$*;*
2. *When* $R_0 \leq \overline{R}_0$*, the optimum unit tax on the non-renewable resource is positive and independent of demand elasticity while the optimum unit tax on produced goods is zero. The resource tax raises exactly* R_0 *over the extraction period.*

As long as the government's revenue needs are low, Proposition 1 indicates that the archetypical distortionary tax of the OCT literature should not be applied to conventional commodities; taxation should be applied to the sole resource according to a rule that has nothing to do with Ramsey's rule, is independent of the elasticity of demand and does not induce any distortion.[15] Except for a few resource rich economies, this situation of low revenue needs is less realistic than the second-best case studied below.

If the government revenue needs are high in the sense that $R_0 > \overline{R}_0$ and $\lambda > 1$, revenue needs cannot be met by neutral taxation; then we have shown that both the resource and the conventional goods should be taxed. Furthermore, the question arises whether the government can and should collect more resource revenues by departing from neutral taxation of the resource sector[16]. This possibility was not addressed by Dasgupta, Heal and Stiglitz (1981), nor by followers in the NRR taxation literature. Barrage (2014) came closest by introducing a NRR sector in her study of OCT with carbon pollution (p. 50) but she left Ramsey dynamic distortions to this sector unexplored.

The neutral tax that maximises tax revenues does not leave any resource rent to producers: $\widetilde{q}_{st} = c_s + \theta_{st}$. Assume, as will be seen to be true later on, that the government can maintain its complete appropriation of producers' resource rents while further increasing tax revenues: the condition $\widetilde{q}_{st} = c_s + \theta_{st}$ remains true while θ_{st} is set so as to further extract some of the consumer surplus. This implies that, when $\lambda > 1$, $\widetilde{p}_{st} = c_s$, $\widetilde{\eta}_t = 0$, $\widetilde{x}_{st} = D_s(c_s + \theta_{st})$.

[15] The fact that neutral taxation of the Hotelling commodity is possible does not mean that neutral profits taxation à la Stiglitz and Dasgupta (1971) or capital levy à la Lucas (1990), or some form of resource rent tax à la Boadway and Keen (2010) have been allowed into the model. It should be clear from the formulation that the result is reached by commodity taxation only.

[16] Clearly, at each date, a non-linear tax on the resource extraction rate reaching the level of the maximum constant neutral tax at the Pareto-optimal extraction rate, would achieve such a goal. However such non-distortionary tax is ruled out in the conventional Ramsey-Pigou optimal taxation analysis. If it was feasible the Ramsey-Pigou problem would be meaningless.

With $\widetilde{\eta}_t = 0$, resource extraction is no longer determined by the Hotelling supply condition (6): since $\widetilde{p}_{st} = c_s$, producers are indifferent to the quantity they supply so that quantity is determined on the demand side by the condition $\widetilde{q}_{st} = c_s + \theta_{st}$. Consequently, the choice of θ_{st} by the government determines extraction so that the finiteness of reserves, if it turns out to be binding, comes as a constraint faced by the government in its attempt to increase cumulative tax revenues rather than as a constraint faced by producers in maximising cumulative profits. Thus the government's problem is now to maximise (2), not only subject to (4) and (5), but also subject to

$$\dot{S}_t = -\widetilde{x}_{st}, \tag{16}$$

where S_t denotes the size of the remaining depletable stock at date t.

The Hamiltonian is modified to

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t, \mu_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t\widetilde{x}_t) - \mu_t\widetilde{x}_{st}, \tag{17}$$

where $\widetilde{CS}_t$, $\widetilde{PS}_t$ and $\widetilde{\phi}_t$ are defined as before but with $\widetilde{\eta}_t = 0$, and μ_t is the co-state variable associated with the exhaustibility constraint (16). From the maximum principle, $\lambda_t = \lambda e^{-rt}$, as above, and $\mu_t = \mu \geq 0$. If the exhaustibility constraint is binding, that is to say if optimal taxation induces complete exhaustion of the reserves, $\mu > 0$; if optimal taxation leads to incomplete exhaustion, then $\mu = 0$.

The first-order condition for the choice of the tax on the resource becomes

$$[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}. \tag{18}$$

Since no resource rent is left to producers above the marginal cost of extraction, $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s = 0$, $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$, and the optimum tax on the resource is thus

$$\theta_{st}^{*} = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}, \tag{19}$$

where the elasticity of NRR demand $\varepsilon_s \equiv \frac{D_s^{-1}(.)}{x_{st}D_s^{-1\prime}(.)}$ is negative and decreasing as in the case of conventional commodities.

Provided the resource is scarce ($\mu > 0$) from the government's point of view, (19) implies that the resource is taxed at a higher rate than would be the case according to (13) for a conventional commodity having the same demand elasticity. Furthermore, while the first term on the right-hand side of (19) is neutral as it rises at the rate of discount, the presence of the second term implies that the tax is not constant in present value, so that it is distortionary in general.

Can the tax revenue collection motive cause the government to assign no scarcity value to a resource that would otherwise be extracted until exhaustion? The answer is negative. For suppose that $\mu = 0$ in (19). This implies that the tax rate is constant over time, so that the extraction rate is also constant and strictly positive, which in turn implies that the exhaustibility constraint must be violated in finite time.

The following proposition summarises the results on the optimum taxation of the resource when neutral taxation is not sufficient to collect the revenue needs.

Proposition 2 *(High government revenue needs) If $R_0 > \overline{R}_0$, then commodity taxation is distortionary ($\lambda > 1$) and both the NRR sector and conventional sectors are subject to taxation. In that case:*

1. *Taxes on conventional commodities are given by Ramsey's rule (13) and the tax on the NRR is given by (19), where λ is determined by the condition that total tax revenues levied from the non-resource and resource sectors equal R_0;*

2. *The NRR is taxed at a higher rate than a conventional commodity having the same demand elasticity;*

3. *The after-tax resource rent to producers is nil: $\widetilde{\eta}_t = \widetilde{\eta}_0 = 0$;*

4. *The optimal distortion to the NRR extraction is determined by the time path of taxes (19). Optimal taxation of the NRR never causes the duration of extraction to diminish nor reserves to be left unexploited.*

Propositions 1 and 2 also have implications on the evolution of the total flow of tax revenues over time. When the government's revenue needs are low, the total flow of tax revenues decreases in present value as the resource unit tax is constant in present value while extraction diminishes. Tax revenues from conventional sectors being nil, total tax revenues decrease in present value and vanish entirely if the resource is exhausted in finite time. When government revenue needs are high, the flow of tax revenues from conventional sectors is constant in current value. If the resource is exhausted in finite time, the total tax revenue flow is thus lower at and after the date of exhaustion than before exhaustion. In either case, the government's assets accumulated at resource exhaustion must be sufficient to ensure that expenditures taking place after exhaustion can be financed.

When the government cannot avoid the introduction of distortions, as when revenue needs are high, its problem acquires a revenue-maximising dimension. This confers OCT a resemblance with monopoly pricing as the term $\frac{1}{-\tilde{\varepsilon}_s}$ in (13) is nothing but a monopoly mark-up (for details see Appendix D). The resource monopoly literature has shown that the exercise of market power by a Hotelling resource monopoly is constrained by exhaustibility. The sharpest example is Stiglitz (1976) who showed that a resource monopoly facing a constant-elasticity demand and zero extraction costs must adopt the same behaviour as a competitive firm; such a monopoly cannot increase its profits above the value of the mine under competition by distorting the extraction path. This limitation also applies to the OCT problem. With zero extraction cost and isoelastic demand, the tax defined by (19) is neutral and rises at the discount rate. In that case, OCT requires that no distortion be imposed to the NRR extraction. We prove that result and make use of it in Section 4, where initial reserves are treated as endogenous.

From Propositions 1 and 2, the resource should be taxed in priority whatever its demand elasticity and whatever the demand elasticity of regular commodities. This irrelevance of demand elasticities contrasts sharply with the standard rationalisation of OCT but not with Ramsey's original message. The message is "tax inelastic sectors" whether the source of inelasticity is demand or supply. Once it is realised that long-run reserve supply fixity results in reduced short-run resource supply elasticity, it becomes clear that the emphasis should shift from demand to supply in the case of a Hotelling resource.

In Daubanes and Lasserre (2015, Appendix E), we extend the analysis to the case of increasing marginal costs of production and increasing marginal costs of extraction, so that supply elasticity is no longer infinite. While the inverse elasticity rule then acquires a supply elasticity component, the finiteness of ultimate reserves implies that NRRs should be taxed in priority and at higher rates than otherwise identical conventional commodities. The inelasticity of long-run resource supply dominates other considerations. We also examine the role of resource heterogeneity. Again, the results are altered but not modified in any fundamental way.

In Section 3, we do away with the assumption that demands are independent from each other; a standard assumption under which the inverse elasticity rule is usually derived. This allows us to examine the specific tax treatment that resource substitutes and complements should receive from an OCT perspective.

In Section 4, it is the Hotelling assumption that reserves are exogenously given that is relaxed. Doing away with this assumption introduces the long-run supply elasticity of the resource and also allows us to highlight the distinction between a NRR and conventional capital.

3 OCT with resource substitutes or complements

Assume that conventional goods may be substitutes or complements for the NRR or for each other. For the reasons explained in the previous section, when government revenue needs are low in the sense of Proposition 1, substitutes for, or complements to, the NRR may be left untaxed while the resource alone is taxed. However, high government revenue needs warrant that NRR substitutes and complements be given specific tax treatments.

Assume that the demand $D_j(q_{jt}, q_{kt})$ for a conventional commodity $j \in \{1, ..., n\}$ not only depends on its own price, but also on the price of another commodity $k \in \{1, ..., j-1, j+1, ..., n, s\}$, with $\frac{\partial D_j(.)}{\partial q_j} < 0$, $\frac{\partial D_k(.)}{\partial q_k} < 0$, and $\frac{\partial D_j(.)}{\partial q_k}, \frac{\partial D_k(.)}{\partial q_j} > 0$ (< 0) if the goods are substitutes (complements). The joint consumer surplus arising from that pair of goods is given by the concave money-metric

$$\psi(\widetilde{x}_{jt}, \widetilde{x}_{kt}), \text{ with } \frac{\partial \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_j} = \widetilde{q}_j \text{ and } \frac{\partial \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_k} = \widetilde{q}_k. \tag{20}$$

Redefining the consumer surplus (8) accordingly, the constrained maximisation of (2) gives first-order conditions that take account of the effect of any tax θ_{jt} on the tax income raised in sector k.

Consider two conventional commodities $j \in \{1, ..., n\}$ and $k \in \{1, ..., n\}$ that are substitutes for or complements to each other. The first-order condition for the tax on j is[17]

$$\left[\frac{\partial \psi(.)}{\partial x_{jt}} - \theta_{jt} - c_j\right] \frac{d\widetilde{x}_{jt}}{d\theta_{jt}} + \left[\frac{\partial \psi(.)}{\partial x_{kt}} - \theta_{kt} - c_k\right] \frac{d\widetilde{x}_{kt}}{d\theta_{jt}} - \widetilde{x}_{jt} + \lambda \left(\widetilde{x}_{jt} + \theta_{jt} \frac{d\widetilde{x}_{jt}}{d\theta_{jt}} + \theta_{kt} \frac{d\widetilde{x}_{kt}}{d\theta_{jt}}\right) = 0, \qquad k \neq s, \tag{21}$$

[17] The present-value Hamiltonian associated with the maximisation of (2) subject to (4) and (5) takes the same form as in Section 2:

$$\mathcal{H}(a_t, \theta_t, \lambda_t) = \left(\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\Phi}_t\right) e^{-rt} + \lambda_t (r a_t + \theta_t \widetilde{x}_t).$$

However, the consumer surplus is adjusted to comprise the relation (20):

$$\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\Phi}_t = \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt}) + \sum_{i=1,...,n,s \ i \neq j,k} \int_0^{\widetilde{x}_{it}} D_i^{-1}(u)\, du - \sum_{i=1,...,n,s} (c_i + \theta_{it}) \widetilde{x}_{it},$$

where $D_i^{-1}(\widetilde{x}_{it})) - c_i - \theta_{it} = 0$ for conventional goods $i = 1, ..., n$, and $D_s^{-1}(\widetilde{x}_{st})) - c_s - \theta_{st} = \widetilde{\eta}_t$ for the NRR. The maximum principle implies $\lambda_t = \lambda e^{-rt}$ as in Section 2.

where $\frac{\partial\psi(\widetilde{x}_{jt},\widetilde{x}_{kt})}{\partial x_{jt}} = c_j + \theta_{jt}$ and $\frac{\partial\psi(\widetilde{x}_{jt},\widetilde{x}_{kt})}{\partial x_{kt}} = c_k + \theta_{kt}$. Moreover, $\widetilde{x}_{jt} = D_j(\widetilde{q}_{jt}, \widetilde{q}_{kt})$ and $\widetilde{x}_{kt} = D_k(\widetilde{q}_{kt}, \widetilde{q}_{jt})$ so that $\frac{d\widetilde{x}_{jt}}{d\theta_{jt}} = \frac{\partial D_j(.)}{\partial q_{jt}}$ and $\frac{d\widetilde{x}_{kt}}{d\theta_{jt}} = \frac{\partial D_k(.)}{\partial q_{jt}}$. It follows that the optimum tax on conventional commodity j when its complement or substitute k is not a NRR is

$$\theta^*_{jt} = \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{jt}}{-\widetilde{\varepsilon}_{jj}} + \theta_{kt}\frac{\widetilde{x}_{kt}\widetilde{\varepsilon}_{kj}}{-\widetilde{x}_{jt}\widetilde{\varepsilon}_{jj}},\ k,\ j \neq s. \tag{22}$$

Obviously, the optimal tax on good $k \neq s$ is given by the same expression where k and j are interchanged. Consequently time does not enter the above tax formula either directly or through θ_{kt}. Therefore, the optimal taxes and induced tax rates for commodities that are neither NRRs, nor substitutes for (or complements to) NRRs, are constant over time.

A similar derivation for a conventional good j when its substitute or complement is a NRR ($k = s$) yields a time dependent optimal tax

$$\theta^*_{jt} = \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{jt}}{-\widetilde{\varepsilon}_{jj}} + \left(\theta_{st} - \frac{1}{\lambda}\mu e^{rt}\right)\frac{\widetilde{x}_{st}\widetilde{\varepsilon}_{sj}}{-\widetilde{x}_{jt}\widetilde{\varepsilon}_{jj}},\ j \neq s,\ k = s. \tag{23}$$

In both formulas (22) and (23), the own-price elasticity of the demand for good j is now denoted by $\varepsilon_{jj} = \frac{q_{jt}\frac{\partial D_j(.)}{\partial q_j}}{x_{jt}}$ while $\varepsilon_{kj} = \frac{q_{jt}\frac{\partial D_k(.)}{\partial q_j}}{x_{kt}}$ is the cross-price elasticity of the demand for commodity k with respect to the price of commodity j. The former is negative as in Section 2; the latter is positive for substitutes, and negative for complements.

Finally consider the taxation of the NRR sector. When the resource admits conventional commodity j as a substitute or complement, the first-order condition for the choice of the NRR tax is the same as (21) except that s and j must be interchanged on the left-hand side and that the right-hand side is $\mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}$ rather than zero. Thus the optimum tax on a NRR that has a substitute or complement j is

$$\theta^*_{st} = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_{ss}} + \theta_{jt}\frac{\widetilde{x}_{jt}\widetilde{\varepsilon}_{js}}{-\widetilde{x}_{st}\widetilde{\varepsilon}_{ss}}. \tag{24}$$

All three tax formulae (22)-(24) are identical to their independent-demand counterparts in Section 2 (see (13) for conventional commodities and (19) for the NRR) except for the last term on the right-hand side of each formula. This new term reflects the change in the fiscal revenues levied on the sector indirectly affected by the tax. In fact taxes θ_{it} can be interpreted as producer mark-ups. When the commodity indirectly impacted is a conventional good, the new term is the same as in the formula giving the price chosen by a firm that holds monopoly power on two commodities with interrelated demands and separable costs (*e.g.* Tirole, 1988,

p. 70). The adjustment to the markup (to the tax) is positive (negative) when commodities j and k are substitutes (complements) and does not depend on time directly. This applies whether the sector indirectly affected by the tax is a conventional sector as in (22) or a NRR sector as in (24).

When the commodity indirectly impacted by the tax is a resource, the additional term must also be interpreted as a correction accounting for monopoly power on two markets. However the correction in (23) is reduced by the time-dependent term $\frac{1}{\lambda}\mu e^r t$ that reflects the scarcity of the resource. Proposition 3 summarises the results.

Proposition 3 *(Resource substitutes and complements) Assume that the NRR admits substitutes and complements.*

1. *If $R_0 \leq \overline{R}_0$, resource substitutes and complements should be left untaxed, while the resource should be taxed positively;*
2. *If $R_0 > \overline{R}_0$,*
   (a) *Other things equal, conventional goods that are substitutes for (complements to) a NRR should be taxed at lower (higher) rates than substitutes for (complements to) a conventional good;*
   (b) *The optimal unit tax on resource substitutes and complements depends on time.*

The rationale behind the special tax treatment of substitutes or complements is clear. On the one hand, a higher mark-up on any good positively (negatively) affects the demand for substitutes (complements), thus their tax base. On the other hand, this effect is less pronounced when the impacted substitute or complement is a NRR (compare the right-hand sides of (23) and (22)). The reason is that the tax on a NRR substitute (complement) shifts demand towards (away from) a sector with an inelastic supply, unlike other (conventional) goods.

This specific treatment sharply differs from the treatment of substitutes for, or complements of, externality generating commodities. In Sandmo (1975), the "marginal social damage (...) does not enter the formulas for the other [non externality-creating] commodities, regardless of the pattern of complementarity and substitutability." (p. 92). Indeed in his setup, the government's ability to draw income from other sectors is not affected by the presence of externalities.

Proposition 3 further indicates that the optimal tax on the substitutes for, or complements to, a NRR depends on time, unlike the optimal tax on conventional commodities without links to a NRR. The link with the NRR confers a dynamic dimension to the tax on its substitutes and complements, as can be seen by comparing (23) with (22). For a given NRR tax level, the term in μ indicates that the tax on a NRR substitute should fall as the resource scarcity increases; *vice versa* the tax on a NRR complement should rise. The tax on NRR substitutes and complements is also compounded by the motion of the NRR tax θ_{st} itself: the tax on NRR substitutes should rise as the resource tax increases; *vice versa* for NRR complements.

4 Endogenous reserves

In order to focus on the role of the long-run supply of reserves, we assume in this section, as in Section 2, that marginal extraction costs are constant, equal to $c_s \geq 0$. This means that the supply of the natural resource is only limited by the availability of reserves. Consistently, we assume that marginal costs of production are constant, equal to $c_i \geq 0$.[18]

The stock of reserves exploited by a mine does not become available without some prior exploration and development investment. Although exploration for new reserves and exploitation of current reserves often take place simultaneously at the industry level (*e.g.* Pindyck, 1978, and Quyen, 1988), it is convenient and meaningful to adopt the micro-economic view that they take place in a sequence, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). This way to model the supply of reserves is particularly adapted to the OCT problem under study because it provides a simple and natural way to distinguish short-run supply elasticity from long-run supply elasticity. It also raises the issue of the government's ability to tax and subsidise, as well as its ability to commit.[19]

Most commonly observed extractive resource tax systems feature royalties and levies based on extraction revenues or quantities, often combined with tax incentives to exploration and development. During the extraction phase, *i.e.* once reserves are established, these systems let some Hotelling rents accrue to producers, perhaps to compensate firms for the prior production of reserves.[20] On the other hand, state-owned extraction sectors are common. A nationalised industry means that no extraction rents are left to private producers.

[18]See Daubanes and Lasserre (2015, Appendix E) for the case of rising marginal costs of production.

[19]On issues of commitment and regime changes in resource taxation, see Daniel *et al.* (2010).

[20]Clearly, Ramsey's tax setup rules out the direct taxation of rents, but not their indirect taxation by commodity taxes.

Thus two situations are common empirically: in the first instance extraction is taxed in such a way that strictly positive rents are left to firms; in the second instance no extraction rents are left to firms. The results from the previous section point to the importance of that distinction. Indeed, when S_0 is given as in Section 2, if the government has high revenue needs in the sense of Proposition 2, it should use the NRR commodity tax to take the totality of extraction rents away from producers. If it did so when S_0 were endogenous, it would tax quasi-rents together with scarcity rents, thus removing incentives for producers to generate reserves in the first place. If the government wants to create a tax environment allowing net extraction profits to compensate firms for the cost of reserve production, it must be able to commit, prior to extraction, to a system of *ex post* extraction taxation that leaves enough rents to producers. Alternatively, if the government taxes away extraction rents, including quasi-rents sunk into them, it must compensate firms by subsidies prior to extraction.[21] In fact we will show that there exists a continuum of mixed systems, combining subsidies (negative commodity taxes) on reserve production with positive taxes on extraction, that leave some rents in the hands of firms while meeting the government's revenue needs.[22]

For simplicity assume that *ex ante* reserve producers (explorers) are the same firms as *ex post* extractors. Assume that the stock of reserves to be exploited is determined prior to extraction by a supply process that reacts to the sum of the subsidies obtained by the firms for reserve production and the cumulative net present-value rents accruing to resource producers during the exploitation stage; also for simplicity, assume that reserve production is instantaneous.

Express total cumulative present-value rents from extraction as $\eta_0 S_0$. Suppose further that a negative linear tax $-\rho$ may be applied to the production of reserves, for a total subsidy of ρS_0. Then the initial stock of reserves may be written as a function of $\eta_0+\rho$. This function $\mathcal{S}(\eta_0+\rho)$ can be interpreted as the long-run after-tax supply of reserves as follows. Suppose that reserves can be obtained, via exploration or purchase, at a cost $E(S_0)$. As not only known reserves but also exploration prospects are finite, the long-run supply of reserves is subject to decreasing returns, so that $E'(S_0) > 0$ for any $S_0 > 0$, and $E''(.) > 0$. Then the profit from the production of a stock S_0 of initial reserves is $(\widetilde{\eta}_0+\rho) S_0 - E(S_0)$. Given ρ

[21]Subsidies are widespread. For example, in the case of conventional oil and natural gas, the Albertan system commits to royalty rate reductions that depend on a well's discovery date; those reductions are thus linear in discovered quantities, irrespective of expenditures and irrespective of the fact that they depend on wells' discovery dates. They amount to linear exploration subsidies whose payment is postponed until extraction.

[22]These mixed systems are feasible if the government is able to commit to leave firms the prescribed after-tax extraction rent; otherwise, an optimal system relying on reserve supply subsidies while not leaving the firms any extraction surplus can also achieve the same objective.

and $\widetilde{\eta}_0$, its maximisation requires $\widetilde{\eta}_0 + \rho = E'(S_0)$. We define $\mathcal{S}(\widetilde{\eta}_0 + \rho) \equiv E'^{-1}(\widetilde{\eta}_0 + \rho)$, making the following assumption.

Assumption 1 *(Long-run supply) The supply of initial reserves $\mathcal{S}(.)$ is continuously differentiable and such that $\mathcal{S}(0) = 0$, $\mathcal{S}(\eta_0 + \rho) > 0$ for any strictly positive value of $\eta_0 + \rho$, and $\mathcal{S}'(\eta_0 + \rho) > 0$.*

The property $\mathcal{S}(\eta_0 + \rho) > 0$ for any strictly positive value of $\eta_0 + \rho$ is introduced because it is sufficient to rule out the uninteresting situation where the demand for the NRR does not warrant the production of any reserves.

4.1 Optimal resource taxation with a strictly positive producer rent

Even when the government can subsidise exploration, *i.e.* when $\rho > 0$, leaving some positive after-tax extraction rent to producers may be desirable for the government. Two reasons make it interesting to analyse situations where the government leaves positive extraction rents to producers. First, they are empirically relevant. Second, they will be shown to constitute a general case that includes no-commitment as a limiting case. In this subsection, we assume that ρ is given and is not high enough to remove the need for the government to leave producers positive after-tax extraction rents. Later on, we will analyse the choice of ρ and study whether it is desirable for the government to leave positive extraction rents to producers at all.

Ex post, once reserves have been established, producers face a standard Hotelling extraction problem and the government chooses taxes. Furthermore we assume that the government is committed to leaving the producers a Hotelling rent $\widetilde{\eta}_t > 0$, with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$, as defined in (6) and (7), for a total rent commitment of $\widetilde{\eta}_0 S_0$. Clearly, given ρ, the level of initial reserves will be determined *ex ante* by that commitment; it will be denoted $\widetilde{S}_0$, with

$$\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho), \tag{25}$$

and discussed further below.

Thus the government chooses optimal taxes on extraction given $\widetilde{\eta}_0$, or, equivalently, given any positive $\widetilde{S}_0$. The problem is thus identical to the problem with exogenous reserves analysed in Section 2, except that the government is now subject to its *ex ante* rent commitment. The Hamiltonian is thus (17), with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$ rather than $\widetilde{\eta}_t = 0$:

$$\mathcal{H}(a_t, \theta_t, \lambda_t, \mu_t) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t \widetilde{x}_t) - \mu_t \widetilde{x}_{st}, \tag{26}$$

where $\widetilde{CS}_t$, $\widetilde{PS}_t$ and $\widetilde{\phi}_t$ are respectively defined by (8), (9), and (10), with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$. The control variables are the taxes θ_t.

Suppose, as an assumption to be contradicted, that revenue needs are low ($\lambda = 1$); then, according to Proposition 1, conventional goods are not taxed and a tax is imposed on the NRR during the extraction phase to satisfy revenue needs. This reduces the rent accruing to extracting firms and, by (25), reduces the initial amount of reserves relative to the no-tax situation. Consequently, any attempt to satisfy revenue needs by taxing the resource extraction sector results in a distortion, so that, in contradiction with the initial assumption, λ is strictly higher than unity whatever the revenue needs. It follows that the tax on conventional goods is given by (13) with $\lambda > 1$.

Consider the taxation of the resource sector now, with $\lambda > 1$. In Appendix F, we show that the optimal extraction tax differs from its value when reserves are exogenous, in that it now depends on the rent that the government is committed to as follows:

$$\theta^*_{st} = \frac{1}{\lambda}(\mu - \widetilde{\eta}_0)e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}. \tag{27}$$

The second term on the right-hand side of that expression is the familiar inverse elasticity rule; it appears in the same form as in Formula (19) describing the resource tax when reserves are exogenous. As in that case, the tax rate on the resource thus exceeds the tax rate on a conventional good of identical demand elasticity if and only if the first term is non negative. Such is clearly the case with exogenous reserves when the first term on the right-hand side is $\frac{1}{\lambda}\mu e^{rt}$ but not so with endogenous reserves as the sign of the first term on the right-hand side of (27) depends on the sign of $(\mu - \widetilde{\eta}_0)$. Intuition suggests that the government would not commit *ex ante* to leaving a unit after-tax rent of $\widetilde{\eta}_0$ to firms if this was not at least equal to its *ex post* implicit valuation μ of a reserve unit. One can validate this intuition by analysing the choice of $\widetilde{\eta}_0$, which we now turn to.

Let us characterise the *ex ante* choice of the rent $\widetilde{\eta}_0$ left to firms after payment of the extraction taxes, for a given level of ρ.[23] The choice of $\widetilde{\eta}_0$ is dual to the choice of reserves $\widetilde{S}_0$ since (25) must hold. The marginal cost of establishing reserves at a level S_0 is $E'(S_0) = \mathcal{S}^{-1}(S_0)$ implying a total cost of reserves $\int_0^{S_0} \mathcal{S}^{-1}(S)\, dS$. This cost should be deducted from the *ex ante* objective of the government which is given by (2) when reserves are exogenous. The objective should also include as benefit the total subsidy payment to producers ρS_0.

[23]Clearly the subsidy must be low enough to necessitate the presence of after-tax rents at the extraction stage. This will be addressed further below.

The *ex ante* problem of the government is thus

$$\max_{\widetilde{\eta}_0,\ \Theta} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt + \rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS \tag{28}$$

subject to (25) and subject to the tax revenue constraint, adapted to take account of the additional liability associated with the reserve subsidy:

$$\int_0^{+\infty} \theta_t \widetilde{x}_t e^{-rt}\, dt \geq R_0 + \rho \widetilde{S}_0 \equiv R. \tag{29}$$

Denote by $V^*\left(\widetilde{S}_0, R; \rho\right)$ the value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt$ maximised under (29) with respect to Θ given $\widetilde{\eta}_0$; because (25) holds, this value function may be defined indifferently as a function of $\widetilde{S}_0$ or $\widetilde{\eta}_0$. Thus, by definition, $V^*\left(\widetilde{S}_0, R; \rho\right)$ is the value function for the *ex post* problem just analysed, whose Hamiltonian is (26) and which requires that the optimal tax satisfies (27). The constant co-state variable μ in (26) gives the value $\frac{\partial V^*}{\partial \widetilde{S}_0}$ of a marginal unit of reserves, while $-\lambda$ gives the marginal impact $\frac{\partial V^*}{\partial R}$ of a tightening of the budget constraint. Define $\mathcal{V}\left(\widetilde{S}_0; R_0, \rho\right) \equiv V^*\left(\widetilde{S}_0, R; \rho\right)$, making use of the definition $R = R_0 + \rho \widetilde{S}_0$; we have $\frac{\partial \mathcal{V}}{\partial \widetilde{S}_0} = \frac{\partial V^*}{\partial \widetilde{S}_0} + \rho \frac{\partial V^*}{\partial R} = \mu - \rho\lambda$.

Problem (28) can thus be written as that of maximising $\mathcal{V}\left(\widetilde{S}_0; R_0, \rho\right) + \rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS$ with respect to $\widetilde{S}_0$. The first-order condition is $\frac{\partial \mathcal{V}}{\partial \widetilde{S}_0} + \frac{\partial\left(\rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS\right)}{\partial \widetilde{S}_0} = 0$, i.e. $\mu - \rho\lambda + \rho - \mathcal{S}^{-1}(\widetilde{S}_0) = 0$ so that, using (25),

$$\mu = \lambda\rho + \widetilde{\eta}_0. \tag{30}$$

Indeed, as hinted earlier, the marginal unit value of reserves for the government in its taxation exercise exceeds the private marginal cost $\rho + \widetilde{\eta}_0$ of developing those reserves by a factor reflecting the cost of raising funds ($\lambda > 1$) to finance the subsidy payment.

With $\mu - \widetilde{\eta}_0 \geq 0$, it thus follows from (27) and (13) that the tax rate on the resource is higher than the tax rate on a conventional good with the same demand elasticity. Precisely, the unit tax θ^*_{st} on the resource exceeds the common inverse-elasticity term by ρe^{rt}. This component of the unit tax grows at the discount rate so that, alone, it would leave the extraction profile unchanged. In contrast, the component that is common to the resource tax and the tax on the conventional good[24] causes a distortion to the extraction profile; its

[24]As already mentioned, an exception arises when the demand has constant elasticity and the extraction

value is $\frac{\lambda-1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}$, exactly that of a conventional Ramsey tax. This is stated in Proposition 4.

Proposition 4 *(Optimal extraction taxes; endogenous reserves) When the supply of reserves is elastic and is subsidised at the unit rate $\rho \geq 0$ while the supply of conventional goods or services is infinitely elastic,*

1. *The NRR is taxed at a strictly higher rate than a conventional good or service having the same demand elasticity if $\rho > 0$; it is taxed at the same rate if $\rho = 0$;*
2. *The resource tax rate is given by* (31)*; it is made up of a non-distortionary component complemented by a generally distortionary Ramsey inverse-elasticity component.*

Substituting (30) into (27) implies

$$\frac{\theta^*_{st}}{\widetilde{q}_{st}} = \frac{\rho e^{rt}}{\widetilde{q}_{st}} + \frac{\lambda - 1}{\lambda}\frac{1}{-\widetilde{\varepsilon}_s}, \tag{31}$$

where $\widetilde{q}_{st} = c_s + \widetilde{\eta}_0 e^{rt} + \theta^*_{st}$. This expression identifies the role of the reserve subsidy on the tax rate at any extraction date explicitly. Any parametric change $\Delta\rho$ exactly compensated by a one-to-one change $\Delta\widetilde{\eta}_0 = -\Delta\rho$ and by a change $\Delta\theta^*_{st} = -\Delta\widetilde{\eta}_0 e^{rt}$ not only leaves $\widetilde{q}_{st}$ unchanged but ensures that (31) remains satisfied without any further adjustment. This is because $\widetilde{\eta}_0 + \rho$ is then unchanged so that the new combination of subsidy and after-tax rent commands the same reserves level; as $\widetilde{q}_{st}$ is unchanged it generates the same extraction path; all constraints remain satisfied. In other words the optimum after-tax rent depends on the *ex ante* subsidy: $\widetilde{\eta}_0 = \widetilde{\eta}_0(\rho)$; similarly $\theta^*_{st} = \theta^*_{st}(\rho)$, with $\frac{d\widetilde{\eta}_0(\rho)}{d\rho} = -1$ and $\frac{d\theta^*_{st}(\rho)}{d\rho} = e^{rt}$. However the optimum level of reserves $\widetilde{S}_0$ and the equilibrium price profile are independent of ρ.

This is true within an admissible range for ρ. Indeed the subsidy must not exceed the threshold level above which it would not be necessary for the government to leave firms a rent during the extraction phase. That threshold can be determined as follows. The unit after-tax extraction rent induced by the optimal policy is $\widetilde{\eta}_0(\rho) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(\rho) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0) - \rho$. Therefore, the condition ensuring that the after-tax rent $\widetilde{\eta}_0$ remains strictly positive is

$$\rho < \bar{\rho} \equiv \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0), \tag{32}$$

where $\widetilde{S}_0$ must satisfy (25), or $\mathcal{S}^{-1}(\widetilde{S}_0) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0) = \bar{\rho}$.

cost is zero (Stiglitz, 1976), in which case the tax has no effect on extraction, given reserves.

Proposition 5 *(Tax-subsidy mix) For $0 \leq \rho \leq \bar{\rho}$, the optimum initial reserve level and the optimum extraction profile are independent of the combination of tax and subsidy by which they are induced.*

An immediate corollary is that subsidies are not necessary to achieve the optimum if the government can commit to extraction taxes that leave sufficient rents to extractors; *vice versa* commitment is not necessary if the government is willing to subsidise sufficiently, at $\rho = \bar{\rho}$. This subsidy level corresponds to the special case of Section 2 taken with initial reserves at $\widetilde{S}_0$. By Proposition 2, the tax is then given by (19) where $\mu = \lambda\bar{\rho}$ according to (30). Thus the observed variety in NRR taxation systems is compatible with optimum Ramsey taxation. To the extent that commitment is not costly, the government is financially indifferent between the proportion of *ex ante* subsidies and *ex post* rents left to extracting firms in order to finance exploration and development expenditures.

4.2 Distortion to reserve production under optimum NRR taxes

There is another peculiarity in (31). The usual interpretation of Ramsey's inverse elasticity rule is that goods or services whose demand is relatively less elastic should be taxed at a relatively higher rate because this keeps quantities demanded as close as possible to the Pareto optimum, thus balancing the distortions across sectors in the socially least costly way. Here, this interpretation does not apply. As a matter of fact, a NRR tax may leave the extraction path of a given stock of reserves undisturbed. In that case the distortion affects the initial stock of reserves but not their extraction profile over time. In fact, as underlined by Stiglitz (1976) in his analysis of monopoly pricing in the Hotelling model, confronted with the dilemma of raising the price at some date while increasing supply at some other date, a zero-cost monopoly facing an isoelastic demand ends up choosing the same price as would prevail under competitive equilibrium. Under the same cost and demand conditions the Ramsey tax will be neutral for the same reason. More generally, even when it affects current extraction, a Ramsey tax cannot be given the standard interpretation in terms of distortion to current extraction. It must simultaneously be appraised in terms of the distortion that it causes to initial reserves.

Given the subsidy, initial reserves are determined by the optimum level of the unit after-tax rent $\widetilde{\eta}_0$ via (25). Since $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ and $\widetilde{q}_{st} = c_s + \widetilde{\eta}_t + \theta^*_{st}$, $\widetilde{\eta}_0$ also affects the optimum tax rate given by (31). However, it is very difficult in general to isolate its effect because there is a continuum of relationships such as (31) – one at each date – and it is their combined influence over the whole extraction period that determines initial reserves. An exception is the special case just discussed. With an isoelastic demand and zero extraction cost, the optimal tax does not cause any distortion to the extraction profile of a given stock of reserves; this provides the ideal laboratory for the analysis of the distortion to initial reserves.

When the tax is neutral at given initial reserves, it grows at the rate of discount, so that the optimal tax can be characterised at any date by its initial level and alternative tax profiles can be compared by comparing initial levels. A higher initial tax level implies a lower after-tax rent to firms which implies lower initial reserves by (25). In the spirit of Ramsey taxation, one would then expect the optimal initial tax to be inversely related to supply elasticity. This is precisely the message of the following expression established in Appendix I for the optimum long-run resource tax rate:

$$\frac{\theta^*_{s0}}{\widetilde{q}_{s0}} = \frac{\rho}{\widetilde{q}_{s0}} + \frac{\lambda - 1}{\lambda}\left[\frac{1 - \frac{\theta^*_{s0}}{\widetilde{q}_{s0}}}{\widetilde{\zeta}} - \frac{1}{\widetilde{\xi}}\right], \tag{33}$$

where $\widetilde{\zeta} \equiv \frac{\widetilde{\eta}_0}{\widetilde{S}_0 \mathcal{S}^{-1\prime}(.)}$ is the *long-term* elasticity of reserve supply measured using (25) at the resource scarcity rent induced by the tax at the beginning of extraction; and where $\widetilde{\xi} \equiv \left(\frac{d\widetilde{\mathcal{D}}}{dq_{s0}}\right)\frac{\widetilde{q}_{s0}}{\widetilde{\mathcal{D}}}$ is the elasticity of the *cumulative* demand for the resource $\widetilde{\mathcal{D}} \equiv \int_0^{+\infty} D_s(\widetilde{q}_{st})\, dt$ with respect to the initial price q_{s0}, measured over the path of equilibrium prices $\{\widetilde{q}_{st}\}_{t\geq 0}$ induced by the optimal tax.

Keeping in mind that, by Proposition 5, the optimal NRR tax adjusts to changes in the reserve subsidy in such a way that optimal initial reserves are the same for any admissible value of ρ, let us again assume that $\rho = 0$. Then (33) is identical to the expression for the optimum rate of tax that applies to conventional goods whose supply is not perfectly elastic.[25] Its interpretation is also standard: tax more when elasticity is lower, whether the source of elasticity is on the supply or the demand side. Hence, to the extent that the supply of conventional commodities is more elastic than the supply of reserves $\left(\widetilde{\epsilon}_i > \widetilde{\zeta}\right)$, (33) implies that the resource is taxed at a higher rate than commodities of identical demand elasticity. There is an important difference between the NRR and conventional goods or services though, having to do with the notions of elasticities involved.

In (33), the supply elasticity $\widetilde{\zeta}$ measures the long-run adjustment of the stock of initial reserves relative to the percentage change in the unit producer rent. This stock elasticity depends on how sensitive exploration is to the rent. In usual formulas of the inverse elasticity

[25] This expression involves the sum of the reciprocals of demand and supply elasticities. See for instance Expression (11) in Ramsey (1927); for a formula derived under the same notations as in this paper, see (E.8) in Daubanes and Lasserre (2015).

rule applying to commodities whose supply is not perfectly elastic, the concept of supply elasticity is standard; it measures the instantaneous percentage change in production (a flow) relative to the percentage change in the unit producer price.[26]

Similarly, while the elasticity of demand is the standard flow notion in (13), its counterpart in (33) is defined as the elasticity of cumulative resource demand – over the whole extraction period – with respect to the initial resource price. In the current special case, the long-run elasticity of cumulative demand is the same as the standard flow demand elasticity: $\widetilde{\xi} = \widetilde{\varepsilon}_s$.

The results are gathered in the following proposition.

Proposition 6 *(Time profile and initial reserves) When the supply of reserves is elastic and is subsidised at the unit rate $\rho \geq 0$,*

1. *The Ramsey tax profile described by (31) implies distortions in both the time profile of extraction and the level of initial reserves;*

2. *When the demand for the NRR is isoelastic and the extraction cost is zero, the optimal extraction tax is neutral with respect to the time profile of extraction and only affects the level of initial reserves. In that case, the optimal tax rate is given by (33), a rule resembling that for conventional goods and services whose supply is not perfectly elastic.*

The analogy underlined in Section 2 between Ramsey taxation and monopoly pricing when reserves are exogenous is thus even more pronounced when reserves are endogenous. Whether one considers the tax on the production flow of conventional goods or the extraction flow of NRR as in (13) and (31), or the long-run formula (33), the optimal tax rate approaches a monopoly mark-up as the factor $\frac{\lambda-1}{\lambda}$ approaches unity, *i.e.* when the government's revenue needs are at their highest.

5 Conclusion and final remarks

The standard Ramsey-Pigou framework used in this paper considers indirect, linear taxes or subsidies on any commodity or service. This includes linear subsidies to the production of natural resource reserves (exploration) as well as linear taxes on extraction and on consumption of the natural resource. In that framework, the objective of the government is to maximise the welfare of producers and consumers while securing a given level of revenues

[26]If the supply elasticity of a conventional good is finite, it must be the case that some input, *e.g.* the stock of capital, does not fully adjust to price and tax changes, which implies decreasing returns to scale. For a NRR the increasing scarcity of exploration prospects makes decreasing returns unavoidable in the long run.

for the production of public goods. The need to secure revenues confers a profit-maximising dimension to government taxation decisions. Optimum taxes distort consumer prices away from the Pareto optimum towards the monopoly price. For the Hotelling resource, this means that results from the resource monopoly literature are relevant to Ramsey taxation.

In the closed-economy context of this paper, when initial reserves are exogenous, the NRR must be taxed in priority, however elastic the demands for the conventional goods and for the non-renewable resource. Precisely, the resource should be the sole taxed commodity unless the required tax revenue exceeds the totality of the rents that would be generated by the untaxed resource. When the required tax revenue is higher than the maximum that can be generated by neutral resource taxation, conventional producible goods and services should contribute to government revenues, but the resource should be taxed at a higher rate than conventional producible goods having identical elasticities.

When the supply of initial reserves is elastic and determined by the combination of after-tax rents to extraction and *ex ante* subsidies to reserve production, all sectors should be taxed simultaneously whatever the tax revenue needs of the government. In the absence of any subsidies to the production of reserves and provided the government can commit to leaving after-tax rents to firms, the optimum tax rate on resource extraction is determined according to the inverse elasticity rule applying to any conventional good whose supply elasticity is infinite. However, this formal similarity hides a crucial difference: due to the dynamic nature of the extraction problem, a similar rule must hold at all dates during the extraction period. As a result, the distortion to extraction cannot be measured simply according to the tax applying at any particular date, however determined, but also depends on the tax applied at all other dates. If the demand for the non-renewable resource is isoelastic and the marginal extraction cost is zero, this goes as far as implying that the optimal tax, although set according to a standard inverse elasticity rule, does not cause any distortion to the extraction path. The distortion imposed on the industry then materialises at the level of reserve production rather than the extraction profile. It can be expressed by the standard inverse elasticity rule applying to elastically supplied conventional goods and services, where the elasticities are the long-run notions defined in the paper: both the supply and demand elasticities relevant to the Hotelling resource are elasticities of a stock in response to an after-tax asset price, rather than the flow elasticities encountered in usual Ramsey formulae.

Another remarkable result arising with endogenous reserves is that, although the optimal extraction tax varies according to the reserve subsidy, the optimal amount of initial reserves and the optimal extraction path of these reserves, do not depend on the extraction-tax reserve-subsidy combination. As a result, all the tax-induced distortions just described when subsidies are absent, are insensitive at the optimum to the tax-subsidy combination adopted by the government. In particular, a government that were unable to commit to leaving positive after-tax rents to firms during the extraction period, could finance reserve production by subsidies exclusively and achieve the same objective as a government that were able to commit. Similarly, a government that could not devote subsidies to reserve production could give the same incentives by committing to limit extraction taxes appropriately. As a result, Ramsey taxation is compatible with institutional forms ranging from a nationalised industry, where the entire reserve production effort is subsidised while the total surplus from extraction is taxed away, to a system where firms finance reserve production and are paid back by future extraction rents.

The rest of this concluding section connects the above findings with major results in two related literatures. The first one is the literature on the taxation of capital income. The second one is the literature on the capture of foreign NRR rents.

5.1 Ramsey extraction taxes as a capital income tax

Natural resource reserves are a form of capital while discoveries and extraction are forms of positive and negative investments. While Ramsey taxation rules out the direct taxation of capital and profits, the linear indirect commodity taxes considered in this paper have the ability to tax natural resource rents. We found that resource rents should be taxed prior to introducing distortionary commodity taxes when the initial amount of reserves is exogenous, as anticipated by Stiglitz and Dasgupta (1971). When reserves are endogenous and resource rents include quasi-rents, the situation is close to that analysed by Chamley (1986) in that the question whether capital should be taxed in the long run arises in a similar fashion. Chamley identified two aspects of capital revenue taxation. In the short run, capital is rigid; this makes it an attractive target for taxation if the objective is to obtain revenues while avoiding distortions. However, when capital is not rigid, its constitution relies on investment, and investment becomes less profitable, the more capital is taxed. In the long run, Chamley finds that the latter effect becomes dominant and that the revenue from capital should not be taxed at all if the horizon of the government is long enough. Chamley's famous result obeys the standard OCT logic: the social cost of capital taxation over the long run is so high that it is impossible to evenly spread distortions across sectors while imposing a positive capital tax.

Our OCT analysis, however, yields a very different result when capital is a non-renewable natural resource rather than Chamley's conventional capital. As *per* Proposition 4, the natural resource should be taxed whatever the horizon of the government, despite the fact that the supply of reserves is responsive to the tax.

The reason is resource scarcity. While Chamley's capital can be constituted (by production and investment) without limit under constant returns to scale, reserves are endogenously produced (by exploration) under conditions of decreasing returns because exploration prospects are not unlimited. Unlike Chamley's capital, the supply of a natural NRR is not infinitely elastic in the very long run. It follows that the distortion associated with the taxation of NRR capital is finite and can be weighted against other distortions. In that sense, the intuition is comparable with other results contradicting Chamley. For instance, Piketty and Saez (2013) departed from Chamley's treatment by introducing inter-generation inheritance in the capital accumulation process, in the spirit of Cremer, Pestieau and Rochet (2003). This modification limits the elasticity of capital supply in a way similar to our framework, and yields the same conclusion. When the supply of a capital is not infinitely elastic, be it a resource or not, capital should be taxed in the long run.

5.2 Ramsey taxes and the capture of foreign resource rents

More often than not, Ramsey's commodity taxes are applied domestically to an open economy. This has two immediate implications for OCT. First, since domestic supply need not meet domestic demand, taxes applied on demand and supply are not equivalent; consequently distinct optimum taxes must be chosen for the domestic supply and the domestic demand of each traded good or service, rather than one tax applying indifferently to demand or supply. Second, prices are formed on international markets, so that the effects of commodity taxes on prices are diluted.[27] The imperfect elasticity of world NRR supply makes these effects all the more relevant.

It is well known that the combination of domestic demand and supply taxes plays the role of a tariff (Friedlander and Vandendorpe, 1968; Dornbusch, 1971). When they are susceptible of affecting international prices, commodity taxes are then capable of pursuing both a tax revenue objective and a rent capture objective as optimum tariffs do.[28] While

[27] The literature on resource oligopolies and oligopsonies is relevant to the problem of OCT in an open economy. According to Karp and Newbery (1991) "the evidence for potential market power on the side of importers is arguably as strong as for oil exporters" (p. 305); the more so when suppliers and/or buyers act in concert as suggested by Bergstrom (1982). On market power on the demand side, see also Liski and Montero (2011).

[28] The OCT problem and the optimum-tariff problem differ only by the constraint for the OCT government to collect a minimum revenue. The latter characterises an unconstrained optimum while optimum commodity taxes are distortionary: as Boadway *et al.* (1973) put it "domestic commodity taxes *introduce* a distortion

the rent capture objective has been addressed in a rich literature on optimum tariffs, it has resonated especially loudly in the NRR taxation literature with the famous paper by Bersgtrom (1982) on the capture of foreign NRR rents.

In Daubanes and Lasserre (2015, pp. 26-35), we extend the previous section's analysis of OCT with endogenous NRR reserves to the case of an open economy. We formally derive in that case the open-economy counterparts of the tax formulae (31) and (33), and we reexamine the optimal distortions to the NRR extraction and to reserve supply.

The main results are the following. Because of the ability of the Ramsey commodity taxes to pursue both a minimum revenue objective and the objective to capture foreign rents by manipulating prices, the distinction between low and high revenue needs emerges in an open economy with endogenous reserves as it does in a closed economy with exogenous reserves (Section 2). Revenue needs are low or high according to whether government needs are covered or not by the amount raised when NRR taxes are set so as to maximise welfare in absence of tax-revenue constraint, that is with the sole purpose of capturing foreign NRR rents.

When revenue needs are low in the sense defined above, our open-economy OCT rule for NRR extends Bergstrom's (1982) analysis to the case of endogenous reserves but does not modify his message that importing countries should tax NRR consumption and that exporting countries should subsidise NRR consumption. As he showed for fixed reserves, the two-country Nash equilibrium is then such that importing countries capture some foreign NRR rents while exporting countries limit the cut to their rents by subsidising.

When revenue needs are high, the revenue collection constraint becomes active, and the Bergstromian rent capture component is complemented by the two components described in the inverse elasticity formula (33) for the closed economy with endogenous reserves.[29] For an importing country, the result that the NRR should be taxed at a higher rate than commodities of identical demand elasticity comes reinforced because the three terms of the open-economy inverse elasticity formula push in the same direction. In the case of exporters, the Bergstromian component is negative and counteracts the traditional Ramsey components; exporting countries may even subsidise resource consumption despite a pressing fiscal-revenue constraint.

while optimum tariffs *eliminate* a distortion" (p. 397, their italics).

[29]The definitions of the demand and supply elasticities must be adjusted to reflect the fact that there is a home and a foreign sector.

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A The Hotelling rent and the neutral tax

A Hotelling resource is a homogenous non-renewable natural asset, such as an oil deposit. As an asset it should provide the same return as any traded asset if it is to be detained. Since a unit of oil underground does not provide any return other than the value realised upon extraction, its return consists of capital gains over time. If oil was traded underground, absent any uncertainty, non arbitrage would thus require its current price to rise at the risk-free rate of interest. The value of such a non-traded asset is known as Hotelling rent and the non-arbitrage rule that it should satisfy is known as Hotelling's rule (Hotelling, 1931; Dasgupta and Heal, 1979, pp. 153-156; Gaudet, 2007).

This appendix defines the Hotelling rent with tax $\widetilde{\eta}_0$ and the Hotelling rent without tax $\overline{\eta}_0$ in competitive equilibrium. In competitive equilibrium with linear taxation, Hotelling's current-value unit rent to producers equals producer price minus marginal cost. At time zero, with constant unit extraction cost, this is $\widetilde{\eta}_0 = \widetilde{q}_{s0} - \theta_{s0} - c_s$. By Hotelling's rule the rent is constant in present value so that, at any date, its present value is $\widetilde{\eta}_0$; it can be computed as follows.

If there exists a finite choke price $\overline{q} = D_s^{-1}(0)$ for the resource, the resource will be depleted in finite time, at a date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$, where $\widetilde{T}$ is defined by the condition that reserves are exactly exhausted over the period $\left[0, \widetilde{T}\right]$: $\int_0^{\widetilde{T}} D_s(\widetilde{q}_{st})dt = S_0$, with $\widetilde{q}_{st} - \theta_{st} - c_s = (\overline{q} - \theta_{s\widetilde{T}} - c_s)e^{-r(\widetilde{T}-t)}$. At time zero, the rent is thus $\widetilde{\eta}_0\left(S_0\right) = \widetilde{q}_{s0} - \theta_{s0} - c_s = (\overline{q} - \theta_{s\widetilde{T}} - c_s)e^{-r\widetilde{T}}$. If there is no finite choke price for the resource and the resource is not exhausted in finite time, then similar conditions must hold in the limit and define the present-value rent $\widetilde{\eta}_0\left(S_0\right)$ implicitly: $\lim_{T\to+\infty} \int_0^T D_s(\widetilde{\eta}_t + \theta_{st} + c_s)dt = S_0$, where $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$. It can be shown that $\widetilde{\eta}_0$ is a positive and decreasing function of S_0.

The maximum value that can be raised from the mine by non-distortionary taxation is its discounted cumulative rent under competitive extraction and in the absence of taxation. That is $\overline{\eta}_0\left(S_0\right) = \widetilde{\eta}_0\left(S_0\right)$, where $\widetilde{\eta}_0$ is computed as above for the values of $\widetilde{q}_{st}$ implied by $\theta_{st} = 0$, $\forall\, t$. The present value of the mine in the absence of tax is thus $\overline{\eta}_0\left(S_0\right) S_0$.

If taxes are neutral, $\theta_{st} = \theta_{s0}e^{rt}$ and part of the unit scarcity rent is captured. The present value of the net-of-tax unit rent earned by the owner of the mine is thus $\widetilde{\eta}_0\left(S_0\right) = \overline{\eta}_0\left(S_0\right) - \theta_{s0}$ and the after-tax present value of the mine is $\widetilde{\eta}_0\left(S_0\right) S_0$.

B Proof of Proposition 1

1. We have shown in the main text that $\lambda = 1$ implies $\theta_i^* = 0$, $i = 1, ..., n$, and $\theta_{st}^* = \theta_{s0}^* e^{rt}$, so that the totality of tax revenues is raised from the resource sector. Moreover, we have argued that, if $\lambda = 1$, it must be the case that $R_0 \leq \overline{\eta}_0 S_0$. The contrapositive of that statement is that if $R_0 > \overline{\eta}_0 S_0$, then $\lambda > 1$. In that case, we have shown in the main text that $\theta_i^* > 0$, $i = 1, ..., n$, and that θ_{st}^* must be set in such a way as to raise more than $\overline{\eta}_0 S_0$ from the resource sector.

There remains to show that $R_0 \leq \overline{\eta}_0 S_0$ implies $\lambda = 1$. Assume $R_0 \leq \overline{\eta}_0 S_0$ and $\lambda > 1$. Then taxes on conventional goods θ_i^*, $i = 1, ..., n$, raise a strictly positive revenue, causing distortions. Since it is possible to generate $\overline{\eta}_0 S_0 \geq R_0$ without imposing any distortions by taxing the natural resource, this cannot be optimal. Hence, $R_0 \leq \overline{\eta}_0 S_0$ implies $\lambda = 1$.
2. Shown in the main text. ■

C Proof of Proposition 2

1. As shown in the main text, when $\lambda > 1$, the optimum tax rate on conventional good $i = 1, ..., n$ is θ_{it}^* as given in (13) and depends on λ. The optimum tax on the resource is given by (19), where $\mu > 0$ is determined to satisfy (1) with equality. Together, taxes on conventional goods and the tax on the resource must exactly raise $R_0 > \overline{\eta}_0 S_0$, which requires that $\sum\limits_{i=1,...,n,s} \int_0^{+\infty} \theta_{it}^* \widetilde{x}_{it} e^{-rt}\, dt = R_0$. Substituting for θ_{it}^* implicitly defines λ.
$2 - 4$. Shown in the main text.
5. The result that OCT of the NRR never induces reserves to be left unexploited is shown in the main text. There remains to show that the optimal NRR tax does not induce a more rapid extraction than in the non-distortive case.

In any non-distotive equilibrium – *a fortiori* in absence of resource tax – the resource price at any date of the exploitation period is

$$\widetilde{q}_{st} = c_s + \overline{\eta}_0 e^{rt}, \tag{C.1}$$

where $\overline{\eta}_0$ is defined in Appendix A. If there exists a finite choke price $\overline{q} = D_s^{-1}(0)$ for the resource, the resource is depleted in finite time, at a date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$, where $\widetilde{T}$ is defined by the condition that reserves are exactly exhausted over the period $\left[0, \widetilde{T}\right]$:

$$\int_0^{\widetilde{T}} D_s(\widetilde{q}_{st}) dt = S_0. \tag{C.2}$$

If there is no finite choke price for the resource and the resource is not exhausted in finite time, then similar conditions must hold in the limit and define $\overline{\eta}_0$ implicitly: $\lim\limits_{T \to +\infty} \int_0^T D_s(\widetilde{q}_{st}) dt = S_0$.

In the second-best equilibrium where high revenue needs imply that the resource should by taxed at the rate θ_{st}^* given by (19) with $\lambda > 1$, the resource price at any date of the exploitation period is $\widetilde{q}_{st} = c_s + \theta_{st}^*$; indeed, the producer rent $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ must be zero in this case, as explained in the main text. Making use of the presence of $\widetilde{q}_{st}$ on the right-hand side of (19), the resource price may be written as follows:

$$\widetilde{q}_{st} = \frac{c_s + \frac{1}{\lambda} \mu e^{rt}}{1 + \frac{\lambda - 1}{\lambda} \frac{1}{\widetilde{\varepsilon}_s}}. \tag{C.3}$$

In this expression, $\lambda > 1$, and $\mu > 0$ is determined in a way similar as $\overline{\eta}_0$ in the non-distortive case. It can easily be verified that the denominator on the right-hand side of (C.3) is strictly positive. Indeed, rearranging (19) immediately yields

$$\frac{\lambda - 1}{\lambda} \frac{1}{\widetilde{\varepsilon}_s} = \frac{\theta^*_{st} - \frac{1}{\lambda}\mu e^{rt}}{\theta^*_{st} + c_s} < 1.$$

When $c_s = 0$ and the ε_s is constant, as in Stiglitz's particular case, both the second-best equilibrium price (C.3) and the non-distortive equilibrium price (C.1) reduce to a single term that rises at the rate of discount. Their levels are also identical as they are both determined in such a way that (C.2) holds. Indeed, we have established that the optimal NRR tax induces reserves to be entirely depleted. In this case, the second-best equilibrium with $\lambda > 1$ implies the same extraction profile as prevails when no distortions are needed at all.

In all other cases, extraction cost c_s is strictly positive or the NRR demand elasticity $\varepsilon_s < 0$ decreases with q_{st} along the demand (increases in absolute value), and the second-best extraction profile strictly differs from the non-distortive case. In (C.3), the numerator takes the same form as (C.1); it consists of the same constant c_s and of a term rising at the rate of discount. However, as time t goes and price $\widetilde{q}_{st}$ increases, the denominator in (C.3) increases. Clearly, price $\widetilde{q}_{st}$ increases less rapidly in the second-best equilibrium. Since (C.2) must hold, the NRR price must be higher at early dates and lower at more distant dates relative to the non-distortive case.

When there exists a finite choke price $\overline{q}$ for the resource, the resource is depleted at a finite date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$. Since the resource price rises less rapidly in the second-best case, it is immediate that the exhaustion date $\widetilde{T}$ is postponed relative to the non-distortive case. ■

■

D OCT and monopoly pricing

If the need of tax revenues was extreme, that is to say if λ tended towards infinity, the optimum tax rate implied by (19) would be[30] $\frac{\theta^*_{st}}{\widetilde{q}_{st}} = \frac{1}{-\widetilde{\varepsilon}_s}$, corresponding to static monopoly pricing; indeed, $\frac{\theta_{st}}{\widetilde{q}_{st}} = \frac{\widetilde{q}_{st} - c_s}{\widetilde{q}_{st}}$ is the static Lerner index for the resource industry. Under such extreme condition the optimum resource tax rate would be determined by the same inverse elasticity rule as the tax rate applying to other commodities according to (13).

When revenue needs equal total rents ($\lambda = 1$), the second term in the right-hand side of (19) vanishes so that the optimal extraction tax is neutral.

Since $\frac{1}{\lambda}$ and $\frac{\lambda-1}{\lambda}$ sum to unity, the optimum tax on the resource industry given by (19) is a weighted sum of two elements. The first element μe^{rt} can be interpreted as the neutral component of the tax since it rises at the rate of discount as does a neutral Hotelling tax. The second element was just seen to correspond to monopoly pricing.

[30] Although μ varies as λ changes, this scarcity rent cannot become infinite as $\lambda \to \infty$ so that the first term on the right-hand side of (19) indeed vanishes as required for this statement to be true.

E Proof of Proposition 3

1. Shown in the main text.
2.(a) The result follows from the fact that the tax in (23) is adjusted by a positive term when $\widetilde{\varepsilon}_{sj}, \widetilde{\varepsilon}_{js} > 0$ and by a negative term otherwise.
2.(b) Shown in the main text. ■

F Proof of Expression (27)

The Hamiltonian (26) associated with the *ex post* problem is identical to (17). Hence, the application of the maximum principle also gives $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu$. The first-order condition for the choice of the tax is also (18). However, unlike in Section 2, the first term on the left-hand side is not zero since the government is subject to its *ex ante* commitment, which determines $\widetilde{\eta}_0$ at this stage: $D_s^{-1}(\widetilde{x}_{st}) - \theta^*_{st} - c_s = \widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$. Therefore, $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$. Substituting into the first-order condition and rearranging gives (27), where $\varepsilon_s \equiv \frac{q_{st}}{x_{st} D_s^{-1\prime}(.)}$.

G Proof of Proposition 4

1. Shown in the main text.
2. This is a restatement of (31), which is immediately obtained by substituting (30), shown in the main text, into (27), proven in Appendix F. The rest of the proposition summarises findings established in the text preceding it. ■

H Proof of Proposition 5

The proof is shown in the main text. ■

I Proof of Expression (33)

Expression (33) is established under the assumption that extraction cost is zero, $c_s = 0$, and that the demand for the resource is isoelastic, $\varepsilon_s(q_{st}) = \varepsilon_s$. As mentioned in the main text, substituting $\widetilde{q}_{st} = \widetilde{\eta}_0 e^{rt} + \theta^*_{st}$ into (19) with $\widetilde{\eta}_0 = 0$, or into (27) and into (31) with $\widetilde{\eta}_0 \geq 0$, while using the constancy of $\widetilde{\varepsilon}_s$, immediately shows that the optimal extraction unit tax then grows at the rate of interest:

$$\theta^*_{st} = \theta^*_{s0} e^{rt}, \tag{I.1}$$

where θ^*_{s0} is to be determined.

For a given ρ, the *ex ante* choice of θ^*_{s0} is equivalent to the choice of the unit rent $\widetilde{\eta}_0$ it induces, account being taken of (25). The first-order condition for the *ex ante* static maximisation of (28) with respect to θ^*_{s0} subject to (29), taking the *ex post* solution (I.1) into account is

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt}\, dt + \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} - \mathcal{S}^{-1}(.) \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0,$$

where $\frac{d\widetilde{W}_t}{d\theta_{s0}}e^{-rt} = \left(D_s^{-1}(\widetilde{x}_{st}) - \theta_{s0}^* e^{rt}\right)\frac{d\widetilde{x}_{st}}{d\theta_{s0}}e^{-rt} - \widetilde{x}_{st} = \widetilde{\eta}_0 \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st}$ and where $\mathcal{S}^{-1}(.) = \widetilde{\eta}_0 + \rho$. Substituting, one has

$$\int_0^{+\infty} \left(\widetilde{\eta}_0 \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st}\right) dt - \widetilde{\eta}_0 \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta_{s0}^* \frac{d\widetilde{x}_{st}}{d\theta_{s0}}\right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}}\right) = 0.$$

Integrating with $\int_0^{+\infty} \widetilde{x}_{st}\, dt = \widetilde{S}_0$ and $\int_0^{+\infty} \frac{d\widetilde{x}_{st}}{d\theta_{s0}}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$ gives

$$\theta_{s0}^* = \rho - \frac{(\lambda - 1)}{\lambda} \frac{\widetilde{S}_0}{\frac{d\widetilde{S}_0}{d\theta_{s0}}}. \tag{I.2}$$

In long-run market equilibrium $\mathcal{S}^{-1}(\widetilde{S}_0) = \widetilde{\eta}_0 + \rho$ and $\int_0^{+\infty} D_s(\widetilde{\eta}_t + \theta_{st}^*)\, dt = \int_0^{+\infty} D_s\big((\widetilde{\eta}_0 + \theta_{s0}^*)e^{rt}\big)\, dt = \widetilde{S}_0$. It follows by differentiation with respect to θ_{s0} that $\mathcal{S}^{-1\prime}(.)\frac{d\widetilde{S}_0}{d\theta_{s0}} = \frac{d\widetilde{\eta}_0}{d\theta_{s0}}$ and $\left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}} + 1\right)\int_0^{+\infty} D_s'(.)e^{rt}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$. Substituting in $\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$, one obtains $\frac{d\widetilde{S}_0}{d\theta_{s0}} = \frac{\int_0^{+\infty} D_s'(.)e^{rt}\, dt}{1 - \mathcal{S}^{-1\prime}(.)\int_0^{+\infty} D_s'(.)e^{rt}\, dt}$. Introducing this expression into (I.2) yields

$$\theta_{s0}^* = \rho + \frac{\lambda - 1}{\lambda}\left[\widetilde{S}_0 \mathcal{S}^{-1\prime}(.) - \frac{\widetilde{S}_0}{\int_0^{+\infty} D_s'(.)e^{rt} dt}\right], \tag{I.3}$$

from which (33) is derived after substituting the expressions for $\widetilde{\zeta}$ and $\widetilde{\xi}$ defined in the main text and using the fact that $\widetilde{q}_{st} = (\widetilde{\eta}_0 + \theta_0^*)e^{rt} = \widetilde{q}_{s0}e^{rt}$ under (I.1) so that $\frac{d\widetilde{\mathcal{D}}}{dq_{s0}} = \int_0^{+\infty} D_s'(.)e^{rt}\, dt$. Furthermore, the constancy of ε_s implies $\widetilde{\xi} = \varepsilon_s$.

J Proof of Proposition 6

The proposition summarises findings established in the main text. ■