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Wood-Based Bioenergy

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Abstract

During recent years increased attention has been given to second-generation wood-based bioenergy. The carbon stored in the forest is highest when there is little or no harvest from the forest. Increasing the harvest from a forest, in order to produce more bioenergy, may thus conflict with the direct benefit of the forest as a carbon sink. We analyze this conflict using a simple model where bioenergy and fossil energy are perfect substitutes. Our analysis shows how the social optimum will depend on the social cost of carbon, and how the social optimum may be obtained by suitable taxes and subsidies.

JEL-Code: Q300, Q420, Q540, Q580.

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1 Introduction

Due to an increasing concern for future climate change, biofuel and other forms of bioenergy have by many countries been considered as an important alternative to fossil energy. However, concerns have been raised about the use of bioenergy, at least of first-generation food-crop-based biofuels. The critique has partly been due to the upward pressure such biofuel production has put on food prices [2]. This type of biofuel has also been criticized for the greenhouse gas emissions related to growing and processing. Obvious sources of emissions from biofuel production include the use of fertilizer when growing energy crops [3], as well as the use of fossil energy in the harvesting and processing of the crops [4]. Moreover, direct and indirect land use changes can lead to additional greenhouse gas emissions, if the area of arable land is increased to accommodate increasing use of biofuels [5]. Partly due to the critique of food-crop-based bioefuels, increased attention has been given to second-generation, wood-based biofuels. In particular, the possibility of producing liquid biofuel from cellulosic biomass may be a promising alternative to using food crops [6]. However, biofuel and other forms of bioenergy from forests are not unproblematic from a climatic point of view. The carbon stored in the forest is highest when there is little or no harvest from the forest. Hence, increasing the harvest from a forest in order to produce more biofuel may conflict with the direct benefit of the forest as a sink of carbon.

Bioenergy is a broad product group ranging from burning wood in a fireplace to biogas and second generation biofuel. The common denominator is that there is an underlying biological process, which will remove carbon from the atmosphere and store it in biological materials. To analyze the climatic effects of wood-based bioenergy in more detail we present a simple but general model of this biological process and the interactions between the gradual crop growth inducing depletion of atmospheric carbon and the instantaneous emission from energy consumption.

We present our model in section 2. In this model bioenergy and fossil energy are assumed perfect substitutes. The cost of producing fossil energy is assumed increasing in cumulative extraction, so that in the long run fossil energy production will tend to zero. We derive the properties of the social optimum, in which there will exist a phase prior to the non-fossil era when bioenergy and fossil energy will both be produced. Our analysis shows how the social optimum will depend on the social cost of carbon. In particular, we show that the long-run carbon stock contained in the forest is higher the higher is the social cost of carbon. The long-run output level of bioenergy may be either increasing or declining in the social cost of carbon, depending both on the size of this cost and on the cost of producing bioenergy.

In section 3 we briefly describe the unregulated market economy, and show how the equilibrium in such an economy differs from the social optimum. In section 4 we show that the equilibrium of the market economy will coincide with the social optimum if all carbon emissions to the atmosphere are taxed at a rate equal to the social cost of carbon, and carbon sequestration through forest growth is subsidized at the same rate. If policy is restricted to taxes on the two types of fuels, the first-best may nevertheless be achieved in our simple model. The tax rate on fossil fuel should in this case be equal to the social cost of carbon, while the tax rate on bioenergy will generally differ from the fossil fuel tax rate.

2 The model

In this model only two different types of energy are explicitly included: energy produced using fossil materials and energy produced with biological material, denoted fossil energy and bioenergy, respectively. The two energy types are perfect substitutes, but differ in production costs and environmental impact. Fossil energy, R_t, is produced from a non-renewable stock, $S_t \geq 0$, such that the change in the resource stock is given by the gross production,

$$\dot{S}_t = -R_t. \tag{1}$$

As the fossil energy source gets depleted it is necessary to utilize less accessible sources, like deep water oil drilling, or use unconventional techniques as extraction of oil from oil sand. The technology constraint is captured in the stock dependent unit cost of production, increasing as the stock gets depleted: $c = c(S_t)$: $c'(S_t) < 0$, $c''(S_t) > 0$ and $c(S_t) \to \infty$ as $S_t \to 0$. The increasing production cost will be the binding constraint, and will reduce the production rate to zero before the resource is completely exhausted.

Bioenergy production is modeled as harvest of available vegetation. We use a generalized growth function, which is representing any type of growing crop, for example sugar canes or corn, but more interesting are the slower growing crops like boreal forest or palm trees. Alternative land use will not be included in the model, but we assume that suitable areas for bioenergy production are used, so that the climate effects from land conversions are negligible.

We will use a logistic growth model [7], which gives a suitable description of the growth of both trees and plants. We will not constrain the model to a specific function, but assume the following more general properties: $f(V_t)$: $f'(V_t) > 0$ whenever $V < V_{MSY}$, where V_{MSY} is the maximum sustainable yield, i.e. $f'(V_{MSY}) = 0$ and $f(V_{MSY}) = max(f(V))$. For $V > V_{MSY}$ we assume $f'(V_t) < 0$. In both cases $f''(V_t) < 0$ for all V_t. In addition $f(0) = f(\bar{V_t}) = 0$, which means that without harvest the crops volume will stabilize at the level $\bar{V}$, corresponding to the maximum volume. A simple sketch of a possible growth function shown in figure 1. The change in the volume of the crops is given by:

$$\dot{V_t} = f(V_t) - H_t, \tag{2}$$

where H_t represent the harvest of the crops at time t. The cost of harvesting the crops is given by the cost function $b = b(H_t) : \ b'(H_t) > 0, \ b''(H_t) \geq 0$. Total energy production and consumption is denoted $E_t = R_t + H_t$, and will equal the total emissions from energy consumption with appropriate adjustment of the units. The net amount of carbon released into the atmosphere equals these emissions minus the carbon which is removed from the atmosphere due to the crop growth, $f(V_t)$. Hence, at time t, $E_t - f(V_t)$ is released to the atmosphere.

We follow the recommendations by David Archer when modeling atmospheric carbon and its decay. In his article "Fate of fossil fuel CO2 in geologic time", he states that "A better approximation of the lifetime of fossil fuel CO2 for public discussion might be "300 years, plus 25% that lasts forever."" [8]. We will capture this by dividing the atmospheric carbon into two repositories, A_1 and A_2, as done by Farzin and Tahvonen [9]. 75% of the emissions will go into

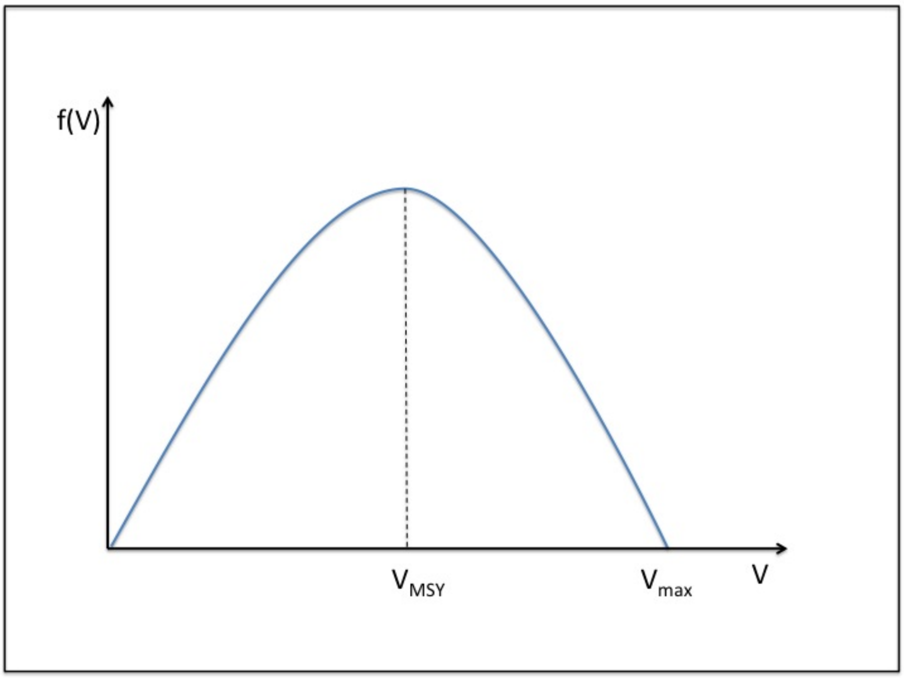

Figure 1: Sketch of crop growth as a function of volume.

A_1, which has a corresponding depletion rate α. The other 25% will end up in reservoir A_2, which has no intrinsic depletion rate. With total emissions given by E_t, the atmospheric carbon changes according to the equations below.

$$\dot{A}_t = \dot{A}_t^1 + \dot{A}_t^2 \quad \text{where} \tag{3}$$

$$\dot{A}_t^1 = \frac{3}{4}(E_t - f(V_t)) - \alpha A_t^1 \tag{4}$$

$$\dot{A}_t^2 = \frac{1}{4}(E_t - f(V_t)). \tag{5}$$

The social benefit of energy consumption meets the standard conditions for utility functions: $B = B(E_t) :\ B'(E_t) > 0$ and $B''(E_t) < 0$. The environmental damage from atmospheric carbon is assumed to be strictly increasing: $D = D(A_t)$: $D'(A_t) > 0$ and $D''(A_t) \geq 0$.

2.1 The social planner problem

The net social welfare is given by the benefits from energy consumption[1], subtracted production costs and the damage of atmospheric carbon,

$$U_t = B(E_t) - c(S_t)R_t - b(H_t) - D(A_t). \tag{6}$$

The social planner is seeking to find the production of fossil energy and bioenergy that maximizes the discounted social welfare across all time periods.

$$\max_{\{R_t\},\{H_t\}} \int_0^\infty U_t e^{-\rho t} dt \tag{7}$$
$$\text{subject to } (1), (2), (4) \text{ and } (5).$$

Using standard optimal control theory, we construct the current value Hamiltonian and derive the corresponding first order conditions for an interior optimum:

$$\begin{aligned} \mathcal{H}_t =& B(E_t) - c(S_t)R_t - b(H_t) - D(A_t) + \kappa_t[-R_t]+ \\ & \eta_t[f(V_t) - H_t] + \upsilon_t^1[\frac{3}{4}(R_t + H_t - f(V_t)) - \alpha A_t^1] \\ & \upsilon_t^2[\frac{1}{4}(R_t + H_t - f(V_t))], \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial R_t} =& B'(E_t) - c(S_t) - \kappa_t + \upsilon_t^1 \frac{3}{4} + \upsilon_t^2 \frac{1}{4} = 0 \\ \frac{\partial \mathcal{H}}{\partial H_t} =& B'(E_t) - b'(H_t) - \eta_t + \upsilon_t^1 \frac{3}{4} + \upsilon_t^2 \frac{1}{4} = 0 \end{aligned}$$

The υ_t^j-values will always be negative as they represent the value of adding more carbon into the atmosphere. The negative, weighted sum of the two υ_t^j terms will represent the social cost of increasing the level of carbon in the atmosphere, and thus the pigovian tax of atmospheric carbon. We will denote this tax $\tau_t =$

[1] We can interpret $B(E)$ as a reduced form function giving utility as a function of fossil energy plus bioenergy when other energy (nuclear and renewable) are optimally chosen, assuming these are either imperfect substitutes to E or have increasing marginal costs of production.

$-(\frac{3}{4}\upsilon_t^1 + \frac{1}{4}\upsilon_t^2)$, giving a more compact version of the first order conditions,

$$B'(E_t) = c(S_t) + \kappa_t + \tau_t \tag{9}$$

$$B'(E_t) = b'(H_t) + \eta_t + \tau_t \tag{10}$$

The first order conditions have a well known economic interpretation: The marginal benefit of increasing energy consumption must equal the marginal cost of increasing production of any of the two energy types. The cost of fossil energy depends on the real unit cost, $c(S_t)$, the resource rent, κ_t, and the carbon tax τ_t . The social cost of producing bioenergy depends on the real marginal cost, $b'(H_t)$, the shadow price of the standing crops, η_t, in addition to the cost of carbon. Since the two energy types are perfect substitutes, their marginal benefits are equal. To ensure efficiency, the volume consumed of each energy type must also be such that the marginal costs are equal, described by $c(S_t) + \lambda_t = b'(H_t) + \eta_t$.

The time development of the system is governed by the equations of motion, that is, the time development of the shadow prices:

$$\dot{\kappa}_t - \rho\kappa_t = c'(S_t)R_t \tag{11}$$

$$\dot{\eta}_t - (\rho - f'(V_t))\eta_t = -\tau_t f'(V_t) \tag{12}$$

$$\dot{\upsilon}_t^1 - (\rho + \alpha)\upsilon_t^1 = D'(A_t) \tag{13}$$

$$\dot{\upsilon}_t^2 - \rho\upsilon_t^2 = D'(A_t). \tag{14}$$

The corresponding transversality conditions are necessary to ensure an internal solution of the system,

$$\lim_{t\to\infty} e^{-\rho t}\kappa_t = 0$$

$$\lim_{t\to\infty} e^{-\rho t}\eta_t = 0$$

$$\lim_{t\to\infty} e^{-\rho t}\upsilon_t^j = 0.$$

Combining the equations above yields

$$\tau_t = -(\frac{3}{4}\upsilon_t^1 + \frac{1}{4}\upsilon_t^2) = \int_t^\infty (1 + 3e^{\alpha(t-t')})\frac{1}{4}e^{\rho(t-t')}D'(A'_t)dt' \tag{15}$$

$$\kappa_t = -\int_t^\infty e^{\rho(t-t')}c'(S_t)R_t dt'. \tag{16}$$

The carbon tax (15) depends only on the marginal damage of carbon, which is positive by assumption. This leads to the conclusion that all carbon emitted into the atmosphere gives the same environmental costs, regardless of whether the carbon source is fossil energy or bioenergy. The social cost of carbon reflects the damage today, as well as all future damages, of adding one more unit of carbon into the atmosphere. If one unit is emitted at time t, the direct damage is given by $D'(A_t)$. If no more carbon is emitted in the future, the part stored in repository 2 will give a future discounted damage of $\frac{1}{4}e^{\rho(t-t')}D'(A_t)$, for all future times t'. In repository 1, there is also a depletion rate, so the future damage will be $\frac{3}{4}e^{(\rho+\alpha)(t-t')}D'(A_t)$. Summing up the combined damage over all times $\tau \geq t$ yields the expression in equation (15).

The resource rent κ_t is a reflection of the added cost of producing fossil energy, due to the scarcity of the resource. The direct effect of extracting one unit of fossil energy today is that the stock of fossil energy decreases. This will lead to an increase in the unit cost of production and thus make all future extractions more costly. The total effect of one unit extraction today is more complex, as it depends on the entire extraction future path. The main effect is still that the efficient marginal cost of extraction becomes higher than the real marginal cost, when scarcity is taken into account.

It is not possible to obtain an analytical expression for η_t, as the effective discount rate $\rho - f'(V_t)$ is not constant. But by studying (12) closer, it is still possible to give this shadow price a meaningful interpretation. Keeping in mind that η_t is the value of adding one more unit of standing crops, i.e. increasing V_t, one can divide $\dot{\eta}_t$ into three terms to easier be able to identify the different effects in play. The first term, $\rho\eta_t$, represents the necessary adjustment in η_t to keep up with peoples impatience or discounting. The second term, $-f'(V_t)\eta_t$, takes into account that when the growth rate changes due to present volume changes, this will influence the volume in the subsequent periods. The last term, $-\tau_t f'(V_t)$, includes the environmental impact, which arises due to changes in the efficient depletion rate.

2.1.1 Steady state solution

In order to simplify the dynamics of the solution, we make the assumption that $D'' = 0$ and denote D' by a. In other words, the damage of adding one more unit of atmospheric carbon is independent of the current level of carbon in the atmosphere. It immediately follows from (15) that the carbon tax takes the form

$$\tau = D'(A)(\frac{3}{4}\frac{1}{\rho+\alpha} + \frac{1}{4}\frac{1}{\rho}) \tag{17}$$

$$= \frac{3}{4}\frac{a}{\rho+\alpha} + \frac{1}{4}\frac{a}{\rho}, \tag{18}$$

and is constant for all times t. The carbon tax depends on the social discount rate ρ, the depletion rate α, and the damage of atmospheric carbon, a. The first part $\frac{a}{\rho+\alpha}$ accounts for the damage of adding one more unit of carbon into repository 1, while $\frac{a}{\rho}$ embodies the cost of adding carbon to repository 2. The cost of carbon in repository 2 is higher than the cost of carbon in repository 1, because the depletion rate reduces the damage over time, having a similar effect on the tax as the discount rate. The weighted sum of these costs reflects the fact that when you increase emissions by one unit, 75% ends up in repository 1 and 25% ends up in repository 2, yielding a total cost of τ.

The steady state solution is characterized by $\dot{S}_t = \dot{V}_t = \dot{A}_t^1 = \dot{A}_t^2 = \dot{\kappa}_t = \dot{\eta}_t = 0$. This removes all time-dependence, and we get the long-run or steady state values of the variables. The equation set describing the steady state is given by:

$$R_* = 0 \tag{19}$$
$$A^1_* = 0 \tag{20}$$
$$A^2_* = A^2_0 + \frac{1}{4}\left[(S_0 - S_*) + (V_0 - V_*)\right] \tag{21}$$
$$A_* = A^2_* \tag{22}$$
$$H_* = f(V_*) \tag{23}$$
$$B'(R_* + H_*) = c(S_*) + \kappa_* + \tau \tag{24}$$
$$B'(R_* + H_*) = b'(H_*) + \eta_* + \tau \tag{25}$$
$$\rho\kappa_* = -c'(S_*)R_* \tag{26}$$
$$(\rho - f'(V_*))\eta_* = f'(V_*)\tau, \tag{27}$$

where "$*$" indicates a steady state value and initial values are marked with "$_0$". The 9 equations (19)-(27) give the steady state solutions for the 9 endogenous variables $S_*,\ R_*,\ V_*,\ H_*,\ A^1_*,\ A^2_*,\ A_*,\ \kappa_*$ and η_*. These equations follow immediately from our dynamic equations, with the exception of (21), which is derived as follows:

$$\begin{aligned}
A^2_* &= A^2_0 + \int_0^\infty \dot{A}^2_t dt \\
&= A^2_0 + \frac{1}{4}\int_0^\infty E_t - f(V_t)dt \\
&= A^2_0 + \frac{1}{4}\int_0^\infty R_t + H_t - (\dot{V}_t + H_t)dt \\
&= A^2_0 - \frac{1}{4}\int_0^\infty (\dot{S}_t + \dot{V}_t)dt \\
&= A^2_0 + \frac{1}{4}\left[(S_0 - S_*) + (V_0 - V_*)\right]
\end{aligned}$$

The interpretation of (21) is that in addition to the initial carbon in repository 2, 1/4 of the net emissions remain in the atmosphere for ever. Total emissions from $t = 0$ to infinity are $S_0 - S_*$ from fossil energy extraction and $V_0 - V_*$ from the change in carbon contained in the biomass. If $V_0 - V_*$ is less than zero, the net "emissions" from the production and management of energy crops will be negative. This implies that there has been planted more crops than has been harvested, which means that the production (or lack of production) of bioenergy in it self has reduced the amount of atmospheric carbon.

The production of fossil energy will necessarily tend to zero in the long run caused by the continued increase in production costs, as the resource gradually gets depleted. The resource rent will then decrease to zero (as seen from (26)), as the remaining stock of the fossil resource no longer has any value. Hence, the total energy production in the long run will solely be given by the bioenergy production, when the production of fossil energy ceases. Equation (23) shows that the steady state production of bioenergy will equal the long-term natural growth of the crops. This means that for any positive volume V_*(less than V_{max}), it is possible to have a positive energy production in the long run.

Even though the production of fossil energy will tend asymptotically towards zero, this is not the case for the *stock* of fossil energy, S. Standard Hotelling models without increasing extraction costs will always yield complete exhaustion of a scarce resource. However, when environmental damages and increasing marginal costs of extraction are included, we have two strong effects pulling towards zero extraction before the resource is depleted. We know that the extraction costs tend to infinity, as the stock gets depleted ($c(S) \to \infty$ as $S \to 0$). This imply that the steady state level S_* must be strictly positive, even without taking the environmental damage into account. Including the environmental effects, represented by the cost of carbon, τ, will increase the amount of unutilized fossil energy. The steady state level of the stock will also be linked to the steady state bioenergy production, and thus the volume of standing crops. By combining (23) - (25) we see that the remaining stock of the fossil energy source will be given by $c(S_*) = b'(H_*) + \eta_* = b'(f(V_*)) + \eta_*$. That is, the marginal cost of producing the last unit of fossil energy will equal the marginal cost of the steady state production of bioenergy plus the shadow price of the standing crops.

The steady state described by (19)-(27) will only be reached asymptotically. To see this assume that the steady state is reached at some finite date T. The dynamics of the system imply that all variables remain constant from T and onwards. Moreover, the same dynamics imply that all variables remain constant also when we move *backwards* in time from T. But this can only be a solution to our equations if $S_0 = S_*$ and $V_0 = V_*$, i.e. if we already are at the steady state initially.

There are several different possibilities for the steady state solutions for η_* and V_*, depending on the underlying assumptions and the specific functions involved. This will be discussed in the next subsections.

2.2 The dynamics toward the steady state

This section gives a more detailed picture of the properties of the steady state and the dynamics toward the steady state. To be able to discuss the dynamic properties, we have made some simplifications. When the system has reached the steady state, fossil energy production will be zero, so only the bioenergy production will affect the marginal benefit B'. However, when discussing the saddle path, this will generally not be the case. To be able to clearly display the interactions between the shadow price and the volume of crops, we will disregard the interaction with fossil energy in the discussions below. The arguments will still be valid for any constant level of fossil energy production. (In the end of section 2.3 we briefly return to the consequences of a declining output of fossil energy.)

To construct a phase diagrams we need to find the conditions ensuring $\dot{\eta} = 0$ and $\dot{V} = 0$. The price equation can be found directly from the steady state solution (27), while the steady state volume is characterized by the harvest of the crops being equal to the growth rate of the crops (23). Combining this with the first order conditions (25), we have both the $\dot{\eta} = 0$ and the $\dot{V} = 0$ loci:

$$\dot{\eta} = 0 \rightarrow \quad \eta = \frac{\tau f'(V)}{\rho - f'(V)} \text{ for } f'(V) \neq \rho \tag{28}$$

$$\dot{V} = 0 \rightarrow \quad \eta = B'(f(V)) - b'(f(V)) - \tau. \tag{29}$$

Consider first the curve for $\dot{\eta} = 0$, i.e. (28). As seen from this equation, η is not defined for $\rho = f'(V)$, and we will denote this limit volume V^ρ. Looking at the derivative of the $\dot{\eta} = 0$ curve we find $\frac{\partial}{\partial V}(\frac{\tau f'(V)}{\rho - f'(V)}) = (\frac{\rho\tau}{(\rho - f')^2})f''(V) < 0$ since $f''(V) < 0$; hence the $\dot{\eta} = 0$ locus is a decreasing function of V. For $V < V^\rho$ we have $\eta < 0$, since $f'(V) > 0$ for $V < V^\rho$. For $V^\rho < V < V_{MSY}$ we have $\eta > 0$, since $0 < f'(V) < \rho$ for these values of V. Finally, for $V > V_{MSY}$ we have $\eta < 0$, as $f'(V) < 0$ for $V > V_{MSY}$. The curve giving $\dot{\eta} = 0$ is thus discontinuous in $V = V^\rho$, as the curve tends to $+\infty$ when approaching V^ρ from above and $-\infty$ when approaching from below. Moreover, for $V > V^\rho$ this curve is is downward sloping and cuts the horizontal axis at V_{MSY}.

Rewriting equation (12) to: $\dot{\eta} = \rho\eta - f'(V)(\tau + \eta)$ makes it easier to find the regions where η increases or declines. In the region $V < V^\rho$ we have: $f'(V) > \rho > 0$, $\eta < 0$ and $(\eta + \tau) < 0$ along the $\dot{\eta} = 0$ curve. Increasing V marginally, while holding η constant then yields: $\frac{\partial \dot{\eta}}{\partial V} = -f''(V)(\tau + \eta) < 0$. This implies that η is declining to the right of the $\dot{\eta} = 0$ curve, and increasing to the left, for all volumes less than V^ρ. For volumes between V^ρ and V_{MSY} we have: $0 < f'(V) < \rho$ and $\eta < 0$ along the $\dot{\eta} = 0$ curve. In this area $-f''(V)(\tau + \eta) > 0$, which means that η is increasing to the right of the $\dot{\eta} = 0$ locus. The last region we need to examine is $V > V_{MSY}$, where $f'(V) < 0$, $\eta < 0$ and consequently $(\tau + \eta) > 0$. From this we see that $-f''(V)(\tau + \eta) > 0$, also in this region.

Next, consider the curve for $\dot{V} = 0$. The bioenergy cost function is by assumption monotonically increasing in bioenergy production, that is, $b' > 0$ for all levels of H. The bioenergy production is uniquely determined by the steady state volume, $H = f(V)$. Since $b'' \geq 0$ and $B'' \leq 0$, the slope $\frac{\partial}{\partial V}(B'(f(V)) - b'(f(V)) - \tau) = (B'' - b'')f'(V)$ will always have the opposite sign of $f'(V)$, and the minimum value of η will coincide with the maximum of $f(V)$ at V_{MSY}.

To find in what regions the volume grows and declines, it is useful to start with the first order condition determining the bioenergy production, $B'(H) = b'(H) + \eta + \tau$. Rewriting the first order condition gives $H = H(\eta + \tau)$, where $H' < 0$ due to $B'' < 0$ and $b'' > 0$. Using this equation in the growth equation for the volume yields $\dot{V} = f(V) - H(\eta + \tau)$. Starting from the $\dot{V} = 0$ curve and increasing η marginally will lead to a decrease in the bioenergy production and thus an increase in the growth rate of the crops. Thus V is increasing above the $\dot{V} = 0$ locus, and decreasing below.

The above properties of the curve for $\dot{\eta} = 0$ and $\dot{V} = 0$ are used in the phase diagrams below.

2.3 Low-cost bioenergy

In this section we consider low-cost bioenergy. More precisely, we make the following assumption:

$$B'(f(V_{MSY})) - b'(f(V_{MSY})) > 0 \tag{30}$$

This means that in the absence of any fossil energy production and climate costs, short-run maximization of U, implying $B'(H) - b'(H) = 0$, would given an unsustainable value of H (i.e. $H > f(V_{MSY})$). Hence, in this case the biological dynamics of the crops (given by (2)) is a restriction that reduces social welfare in the absence of climate costs, since optimal fossil energy production must approach zero in the long run. As explained above, the minimum value of η in figures 2 and 3 is given by $\eta = B'(f(V_{MSY})) - b'(f(V_{MSY})) - \tau$. Due to assumption (30), this minimum value of η is positive if τ is sufficiently small (figure 2), but may be negative if τ is sufficiently large (figure 3).

We will start by looking at the case where the carbon tax τ is "low", meaning that $B' > b' + \tau$ for all volumes V (figure 2). In this case it will be optimal to choose a steady state volume that ensures a high production volume of bioenergy. The highest possible steady state bioenergy production is obtained when $V = V_{MSY}$, but due to discounting, the steady state volume will end up strictly less than V_{MSY}. If $V_0 < V_*$ the value of standing crops starts at a higher level than the marginal *social* profit ($\eta_0 > B'(f(V_0)) - b'(f(V_0)) - \tau_0$). It will then be optimal to harvest below the growth rate, as this will lead to an increase in both the volume and the growth rate of the standing crops. Along the saddle path η is decreasing, and the volume will continue to increase until the steady state is reached. A similar argument can be used when $V_0 > V^*$, but then we have the opposite movements in the variables. In both cases, the closer the minimum point of the $\dot{V} = 0$ curve is to zero, the closer the steady state volume gets to V_{MSY}. Thus with a low carbon tax η_* will be positive, as the $\dot{\eta} = 0$ locus is above zero between V^ρ and V_{MSY}.

A special case of a low carbon tax is that $\tau = 0$. This case of no climate externality is of particular interest, as the unregulated market outcome in the absence of any externality will coincide with the social optimum. The curve for $\dot{V} = 0$ will be as drawn in figure 2. When $\tau = 0$ it is clear from (12) that $\dot{\eta} = 0$ only if $\eta = 0$ or $f(V) = 0$ (i.e. $V = V^{\rho}$). Hence, the downward sloping curve for $\dot{\eta} = 0$ in figure 2 takes the limiting upside-down T-shaped form as illustrated in figure 4. In this case the steady state value of V is V^{ρ}.

Consider next the case for which the carbon tax is so high that $B'(f(V_{MSY})) - b'(f(V_{MSY})) - \tau < 0$. In this case the minimum value of η is negative, as illustrated in figure 3[2]. The difference from the low-tax case is that the higher environmental cost makes it less profitable to produce bioenergy, and the system is then pushed towards a higher steady state volume than in the low tax case. The steady state volume will now be to the right of V_{MSY}, and the corresponding η will be negative.[3] To investigate how the steady state value of the volume of the crop depends on the carbon tax, we differentiate (25) and (27) with respect to τ (after inserting $R_* = 0$ and $H_* = f(V_*)$). This gives

$$\begin{pmatrix} (B'' - b'')f' & -1 \\ -(\eta_* + \tau)f'' & (\rho - f') \end{pmatrix} \begin{pmatrix} \frac{dV_*}{d\tau} \\ \frac{d\eta_*}{d\tau} \end{pmatrix} = \begin{pmatrix} 1 \\ f' \end{pmatrix}$$

implying

$$\frac{dV_*}{d\tau} = \frac{\rho}{C} \tag{31}$$

$$\frac{d\eta_*}{d\tau} = \frac{1}{C}\left[(B'' - b'')(f')^2 + (\eta_* + \tau)f''\right] \tag{32}$$

where

$$C = (B'' - b'')f'(\rho - f') - (\eta_* + \tau)f'' \tag{33}$$

In Appendix 1 we show that $C > 0$ for a saddle-point equilibrium of the type illustrated in figures 2-4. Hence, the steady state value of V is increasing in τ. Due to the property of the growth function f, this implies that as τ increases

[2] Assuming for now that there is only one equilibrium point in this case, even though it is possible to get multiple equilibria if the $\dot{V} = 0$ curve cross the $\dot{\eta} = 0$ curve on the left hand side of V^{ρ}. More on multiple equilibria in section 2.4.1

[3] We disregard cases where τ is so large that the $\dot{\eta} = 0$ and $\dot{V} = 0$ do not intersect for an interior value of V

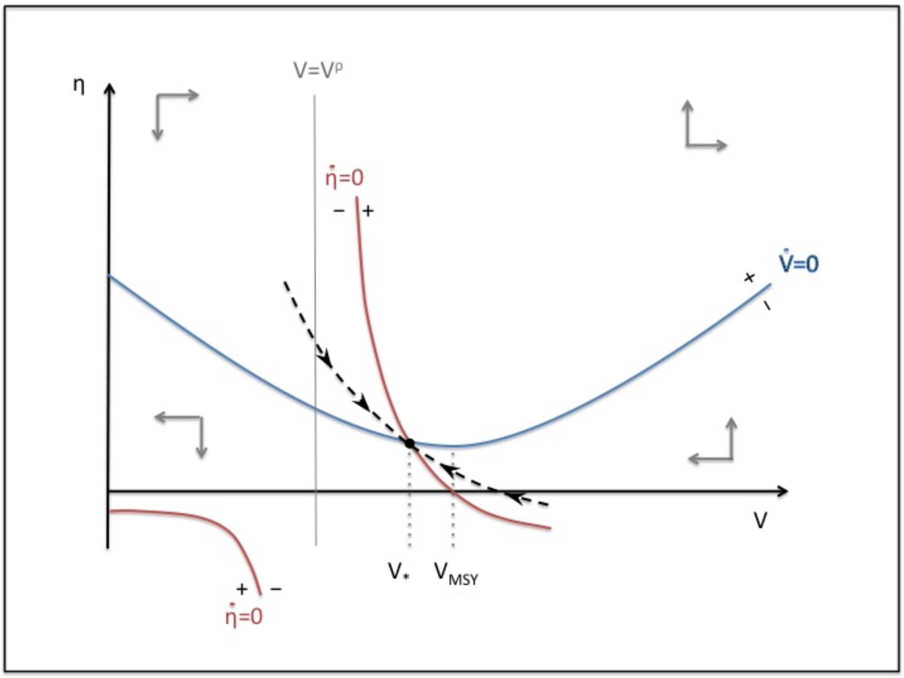

Figure 2: Phase diagram for low cost bioenergy with low carbon tax.

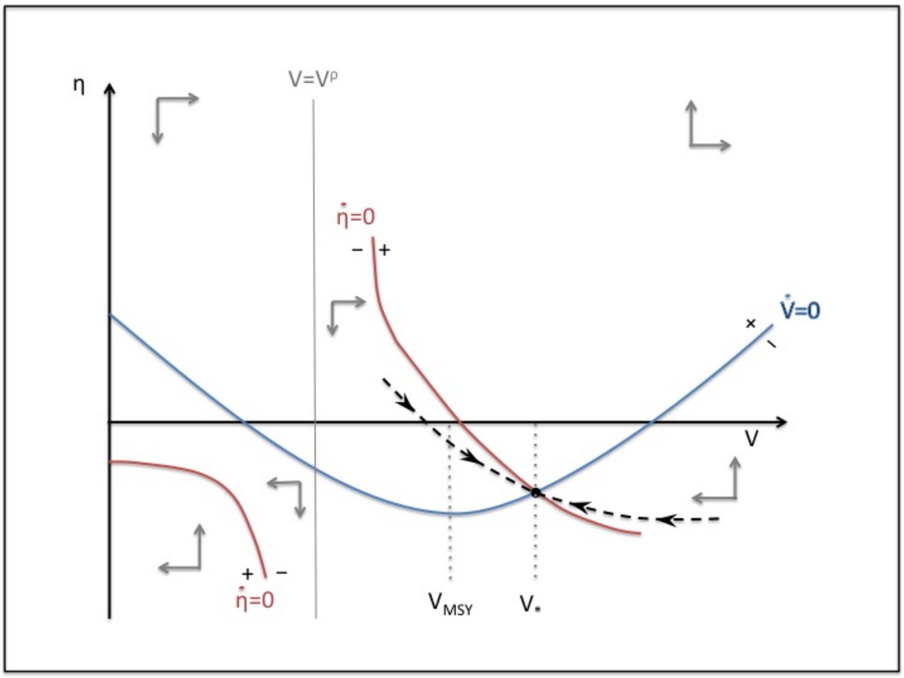

Figure 3: Phase diagram for low cost bioenergy with medium to high carbon tax.

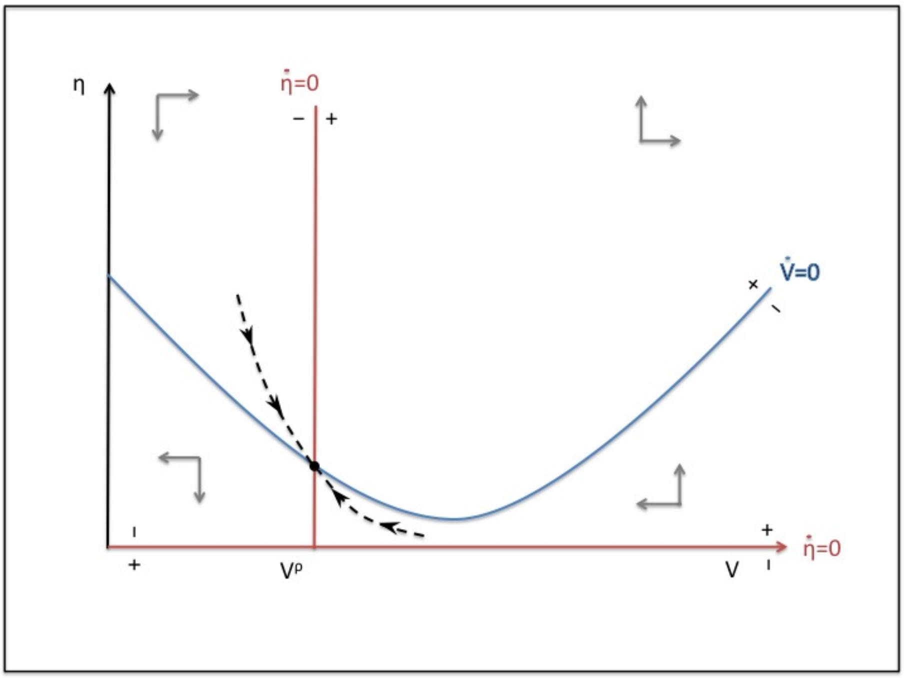

Figure 4: Phase diagram for low cost bioenergy with no carbon tax.

from 0 to $B'(f(V_{MSY})) - b'(f(V_{MSY}))$, the steady state bioenergy production H_* increases. However, as τ increases further, f' becomes negative, so H_* declines with a rising τ. This implies that a higher carbon tax will only result in a higher steady state level of bioenergy production, if the steady state volume is sufficiently low, that is, below V_{MSY}

For low-cost bioenergy we have $B' - b' > 0$ for any bioenergy production $H \leq f(V_{MSY})$, and (29) then implies that $\tau + \eta_* > 0$. From this we can conclude that the square bracket in (32) is negative for low-cost bioenergy, which then tells us that the steady state shadow price η_* declines as τ increases. The formal analysis above was done assuming $R = 0$, which is unproblematic with regards to the $\dot{\eta} = 0$ loci, as equation (28) is independent of the value of R. However, the position of the curve for $\dot{V} = 0$, given by $\eta = B'(f(V) + R) - b'(f(V)) - \tau$, is lower the higher is R, since $B'' < 0$. As R gradually declines as we approach the steady state, this means that this curve is gradually moving upwards. This will make the detailed dynamics of V and η slightly different than what we have illustrated in figures 2-4. In particular, the approach of η towards its steady state value may no longer be monotonic. This may in turn have implications for the detailed time path of the bioenergy production towards its steady state value $f(V_*)$.

2.4 High-cost bioenergy

Assume now that (30) doesn't hold, that is, $B'(f(V_{MSY})) - b'(f(V_{MSY})) \leq 0$. This means that in the absence of any fossil energy production and climate costs, short-run maximization of U, implying $B'(H+R) - b'(H) = 0$, would given a sustainable value of H (i.e. $H \leq f(V_{MSY})$). In this case the minimum value of η is negative for all $\tau \geq 0$. Figure 3 is an example of this case for a $\tau > 0$.

If cost of bioenergy production is sufficently high, we could have $B'(0) - b'(0) \leq 0$, in which case the U-shaped curve for $\dot{V} = 0$ is below the horizontal axis for all $V \in [0, \bar{V}]$. It is then optimal to have no bioenergy production even if fossil energy production is zero, no matter what non-negative carbon tax we have. Whenever $B'(0) - b'(0) > 0$ and $\tau > 0$ the U-shaped curve for $\dot{V} = 0$ will intersect the $\dot{\eta} = 0$ locus for some $V < \bar{V}$. The long-run equilibrium will then be characterized by $V_{MSY} < V_* < \bar{V}$ and $\eta_* \leq 0$, for any strictly positive value of τ. We will take a closer look at the limiting case of $\tau = 0$ in section 3. Equations (31) and (32) are still valid in the present case, implying that V_* is higher the higher is τ. However, H_* will in this case be lower the higher V_* is. This indicates that for high-cost bioenergy a high carbon tax will result in a lower long-run bioenergy production.

2.4.1 Multiple steady states

In the previous cases we assumed that there was only one steady state solution in each case. In this section we will take a closer look at some situations where multiple equilibria can arise. Figure 5 displays a case with either a high carbon tax or a low carbon tax coupled with high-cost bioenergy. Here one can see that two new equilibria (V_{low} and V_{med}) is introduced, where one is unstable. This means that there are two possible time paths that solve the maximization problem 7. The low steady state volume, V_{low}, will be realized if $V_0 < V_{med}$, that is, the initial volume is less than the volume corresponding to the unstable

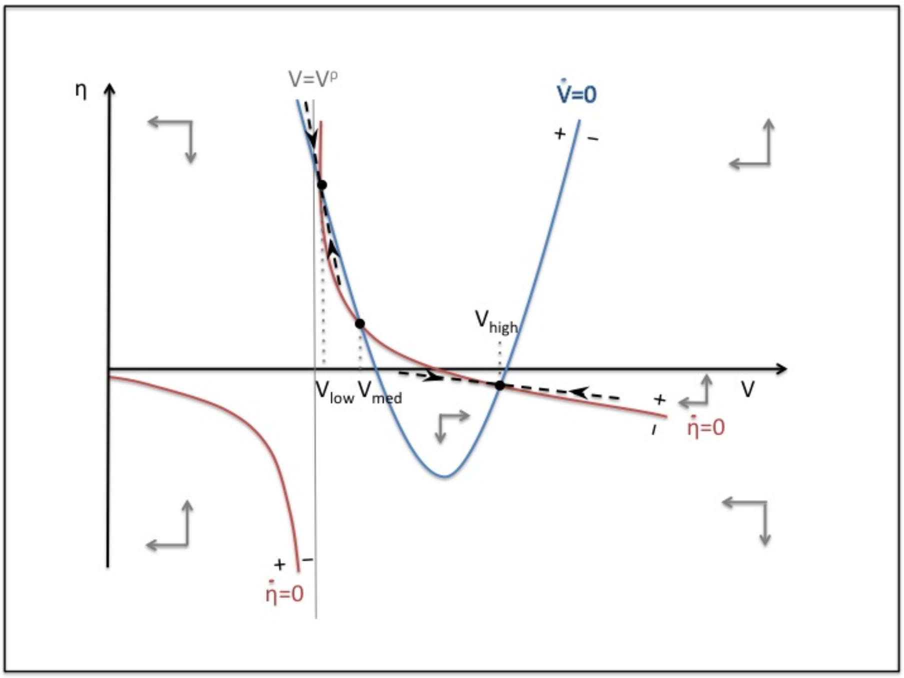

Figure 5: Phase diagram with multiple equilibria for high cost bioenergy and/or high carbon tax.

equilibrium. If $V_0 > V_{med}$ the steady state volume will be V_{high}.

When there is no carbon tax, multiple equilibria can only arise if the cost of producing bioenergy is high, i.e. $B'(f(V_{MSY})) - b'(f(V_{MSY})) \leq 0$. This case is shown in figure 6. The two cases are very similar, but the main difference is that in the no tax case the steady state shadow price of the standing crops will never be negative. Common for all stable, long term solutions is that the unregulated market (no carbon tax) will always have a too low level of standing crops.

Two other multiple equilibria cases are displayed in appendix 2.

3 The market outcome

The market outcome maximizes consumer plus and producer surplus, in the absence of externalities and regulations. This means that the market outcome is identical to what the social optimum would be without any environmental costs. The steady state properties of this outcome were illustrated by figure

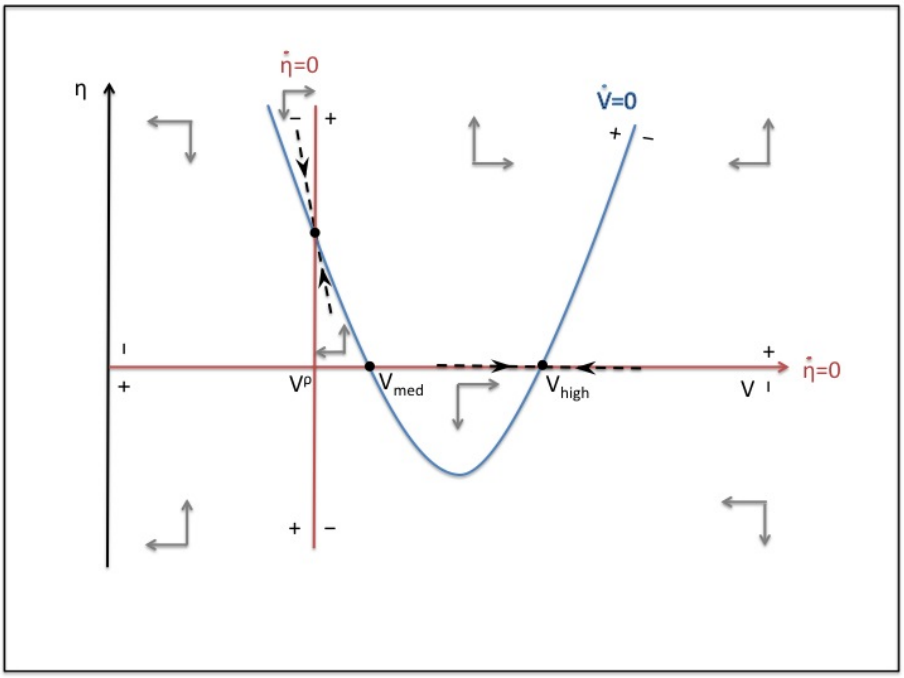

Figure 6: Phase diagram with multiple equilibria for high cost bioenergy with no carbon tax.

3 for the case of low-cost bioenergy and by figure 4 for the case of high-cost bioenergy. We have shown that for both cases the socially optimal steady state volume of bioenergy crops is higher the higher is the carbon tax. It follows that the steady state volume of bioenergy crops in an unregulated market is lower than what is socially optimal in the presence of climate costs. For the case of high-cost bioenergy (implying $V_* > V_{MSY}$ for all τ and hence H_* lower the higher is V_*), this implies that the steady state bioenergy production in an unregulated market is higher than what is socially optimal in the presence of climate costs. For low-cost bioenergy the reverse is true, that is, the bioenergy production in an unregulated market is too low compared to what is socially optimal.

Even if the socially optimal long-run bioenergy production could be lower than the long-run bioenergy production in an unregulated market, the relationship could be the opposite in the short run.This is illustrated in figure 7. The time paths for the unregulated marked (zero carbon tax) and social optimum (positive carbon tax) are denoted by UM and SO, respectively. Since the UM-curve lies above the SO-curve in the long run, we must have $V_* > V_{MSY}$, as illustrated in figure 3. Before oil is depleted, biofuel production is determined by (10), i.e. $B'(R_t + H_t) - b'(H_t) = \eta_t + \tau$. A socially optimal carbon tax will reduce R_t in the short run compared with the case without a carbon tax. From the equation above we therefore see that the direct effect of increasing τ is to reduce H_t, but that the indirect effect through reduced R_t is to *increase* H_t. If this indirect effect dominates the direct effect, bioenergy production will be higher in the near term with a carbon tax than without, as illustrated in figure 7.[4]

This figure is drawn for $R = 0$, but is valid for any constant R. The curve for $\eta = 0$ is independent of R, while the curve giving $\dot{V} = 0$ is higher the lower is R. A lower *constant* value of R hence gives a higher (less negative) steady state value of η, and therefore also a higher path of A socially optimal carbon tax will reduce fossil energy use in the short run(t) toward the steady state.

A socially optimal carbon tax will reduce fossil energy use in the short run, and the total energy produced will also be lower with a tax than without. Although we have not given a formal analysis of the effect of changing the level of the $R(t)$-path over some time interval when $R(t)$ is *not* constant, it seems plausible from the discussion above that a lower level of the $R(t)$-path should tend to increase the time path of $\eta(t)$ along this time interval. This would then imply that bioenergy production is lower in the near term with a carbon tax than without, as illustrated in figure 7.

A socially optimal carbon tax will reduce fossil energy use in the short run, and the total energy produced will also be lower with a tax than without. The lower is the production of fossil energy, the higher lies the curve giving $\dot{V} = 0$. Moreover, the lower a *constant* value of R is the the higher will the path of $\eta(t)$ toward the steady state lie (in figures 1 and 2). Although we have not given a formal analysis of the effect of changing the level of the $R(t)$-path over some time interval when $R(t)$ is *not* constant, it seems plausible that a lower level of the $R(t)$-path should tend to increase the time path of $\eta(t)$ along this time interval. This would then imply that bioenergy production is lower in the near

[4] A complete analysis of this issue would have to take into consideration that the time path of η_t would also be affected by τ.

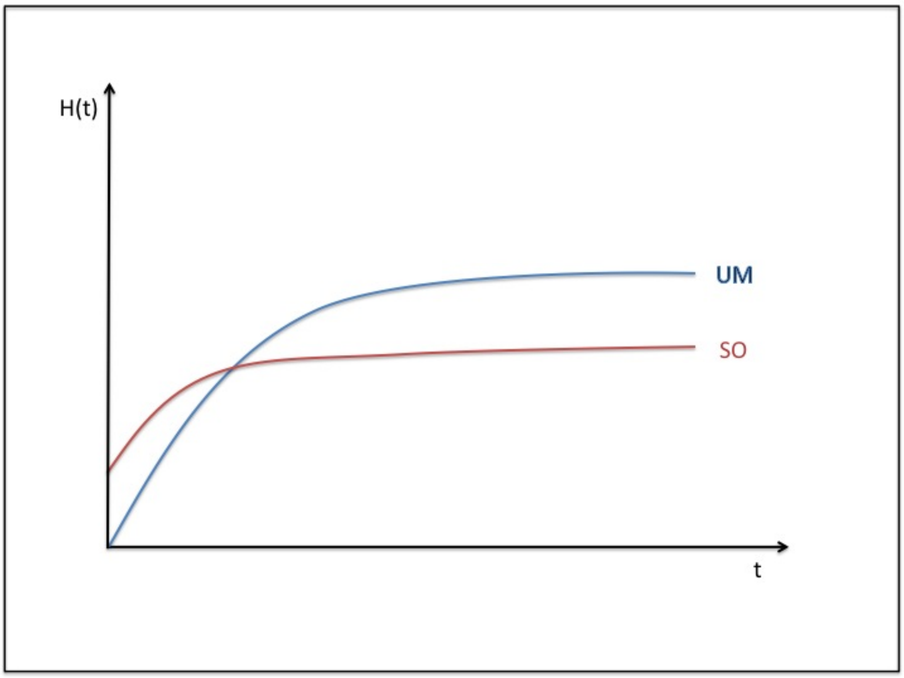

Figure 7: Sketch of possible time paths of bioenergy production with a carbon tax (SO) and without a carbon tax (UM).

term with a carbon tax than without, as illustrated in figure 7.

4 Policy options

Tax on carbon emissions

As usual, the social optimum may be achieved by setting a Pigovian tax on net carbon emissions to the atmosphere. This tax should be equal to the value given by (15), and should be applied both to the emission from fossil energy use and to *net* emissions from consumption (emission from use of bioenergy minus growth of the bioenergy crops). With such a tax scheme the market outcome will coincide with the solution to the problem described by (7), with a slightly modified "utility function" given by:

$$U_t^{market} = B(H_t + R_t) - c(S_t)R_t - b(H_t)H_t - \tau_t(R_t - \dot{V}_t) \qquad (34)$$

where τ_t is given by (15). It is straightforward to verify that the solution to this problem is given by (9)-(12).

Differentiated tax on energy consumption

If the government lacks information about the crop growth, it will not be possible to reach the first-best solution using only a tax on net carbon emission. Another policy option can be to impose a tax or subsidy on the different energy types. As shown below, this policy option will also reproduce the first-best solution.

The government's problem can be constructed as a Stackelberg game, where the government is the leader and announces the tax paths that it will commit to. The producer is the follower, and will maximize profits taking the announced tax plans as given. The idea behind this game is that the government can calculate how the producer will respond to the different tax paths, and based on this, choose the tax paths yielding the highest net social benefits. The government's control variables are the tax paths, while the producer's control variables are the production of fossil energy and bioenergy as before.

In the present model the solution to the government's optimization problem is in principle simple: The first-best social optimum is achieved if fossil energy consumption is taxed the rate τ_t given by (15) and bioenergy consumption is taxed at the rate $\tau_t + \eta_t$, where η_t is given by the social optimum. With these taxes the market outcome solves the following problem:

$$\max_{\{R_t\},\{H_t\}} \int_0^\infty \left[B(R_t + H_t) - c(S_t)R_t - b(H_t) - \tau_t R_t - (\tau_t + \eta_t)H_t\right] e^{-\rho t} dt \qquad (35)$$

subject to (1) and (2).

The determination of R_t is given by the same equations as in the social optimum. Moreover, at each t the value of H_t that maximizes the square brackets in (35) is given by (10). We know that this time path of H_t satisfies the constraint (2), which hence is redundent in the optimization problem above. The social optimum therefore solves the optimization problem above. Under the assumption that climate damages are linear with respect to the carbon in the atmosphere, the optimal tax on fossil energy (τ) is constant. The optimal tax on bioenergy ($\tau + \eta_t$) will generally vary over time. The long-run value of this tax may be higher (figure 2; $\eta_* > 0$) or lower (figure 3; $\eta_* < 0$) than the tax on fossil energy. As explained in the end of section 3, the time path of the difference between these tax rates may be non-monotonic.

The motivation for using energy taxes instead of targeting carbon emission and mitigation was information problems with regards to measuring the crop growth. Unfortunately, the government will meet information problems when using energy taxes as well. This analysis relied on the assumption that the government was informed about the market response functions, giveven implicitly by (9) and (10). This is clearly a very strong assumption, and in real life there is no reason to believe that the government would know the exact function forms of the market response functions. One can therefore not expect the implemented policies to fully replicate the social optimum.

The reason why we in principle can achieve the first-best social optimum without directly observing the growth of the crops is that the growth of the crops only depend on the bioenergy production in our model. In reality, farmers may be able to influence net carbon emissions also via other channels, e.g. through type of crop and in the way the crops are managed. A tax only on bioenergy consumption will in this case generally not make the market outcome coincide with the social optimum.

5 Conclusions

The analysis has derived properties of the socially optimal combination of fossil energy and bioenergy, and demonstrated the optimal solution's dependance on production costs and the social cost of carbon. The social optimum may in principle be obtained as a competitive equilibrium, provided that the taxes and subsidies are designed correctly.

With no taxes or subsidies directed towards the use or production of bioenergy, the long-term volume of bioenergy crops will be too low, regardless of the production costs of bioenergy. However, this does not mean that the market supply of bioenergy will be too low. Our analysis shows that without a tax on carbon, the steady state production of bioenergy will in an unregulated market economy be too high when the production costs of bioenergy are high. The converse is true for low-cost bioenergy. These results are a direct result of the biological dynamics of the crops in combination with the atmospheric depletion process.

6 Appendix 1

The steepeness of the curves for $\dot{\eta} = 0$ and $\dot{V} = 0$ follow from (28) and (29):

$$\begin{aligned} \left(\frac{\partial \eta}{\partial V}\right)_{\dot{\eta}=0} &= \frac{\rho\tau}{(\rho - f')^2} f'' \\ \left(\frac{\partial \eta}{\partial V}\right)_{\dot{V}=0} &= (B'' - b'') f' \end{aligned}$$

In a saddlepoint equilibrium the curve for $\dot{V} = 0$ must be steeper (including the sign) than the curve for $\dot{\eta} = 0$. From the equations above this implies that

$$(B'' - b'') f' - \frac{\rho\tau}{(\rho - f')^2} f'' > 0 \tag{36}$$

Inserting (36) into the expression (33) for C gives

$$\begin{aligned} C &= (B'' - b'') f'(\rho - f') - \frac{\tau\rho}{\rho - f'(V_*)} f'' \\ &= (\rho - f') \left[(B'' - b'') f' - \frac{\rho\tau}{(\rho - f')^2} f'' \right] \end{aligned}$$

From (36) we know that the term in square brackets is positive. Moreover, for $\tau > 0$ we have $\rho - f'(V_*) > 0$. Hence, $C > 0$.

7 Appendix 2

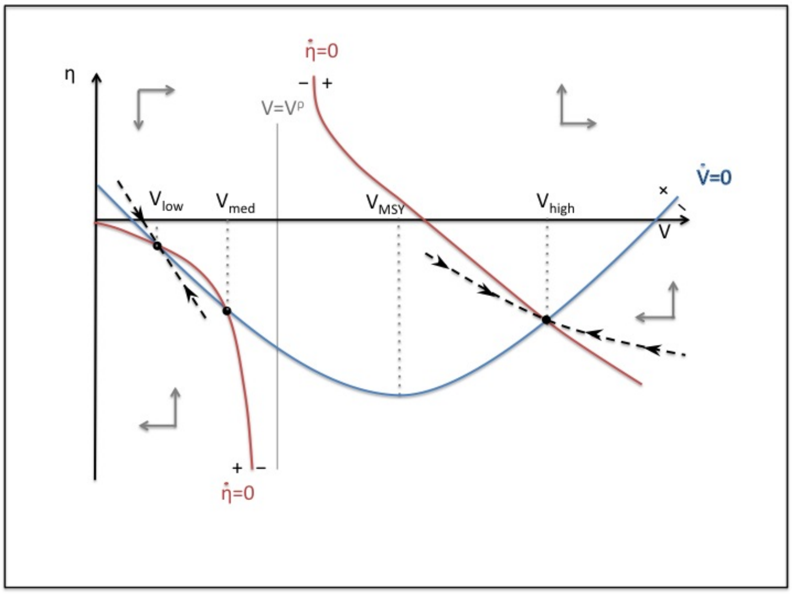

Figure 8: Phase diagram for high cost bioenergy and/or high carbon tax with multiple equilibria.

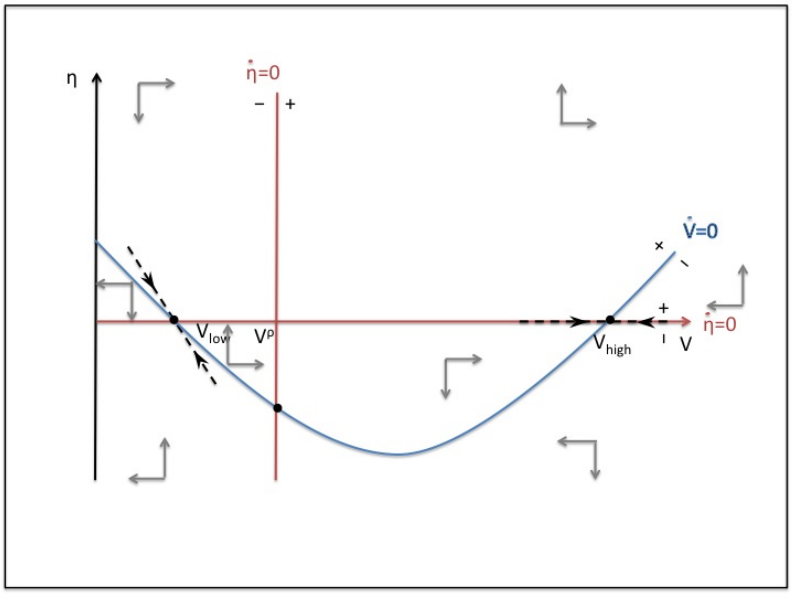

Figure 9: Phase diagram for high cost bioenergy with no carbon taxt with multiple equilibria.

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