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Equitable Representation in the Councils of the United Nations: Theory and Application

Abstract

We develop a theoretical framework for equity in *council voting games* (CVGs). In a CVG, a fully representative voting body delegates decision-making to a subset of the members, as describes, e.g., the United Nations Security Council (UNSC). Three equity concepts are proposed: ex-ante (procedural) equity, ex-post (outcome) equity and regional equity. The last two concepts are consistent with a new square-root rule on the probability of council membership, but no CVG can meet all three concepts. We apply our framework to evaluate the equitability of the UNSC, and the claims of those who seek to reform it.

JEL-Code: D720, D710, C710, C630.

Keywords: United Nations Security Council, United Nations, voting power, councils, square-root rule, equity.

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1. Introduction

Decision-making within international organisations is sometimes made by voting bodies that comprise a proper subset of the membership (a “council”). The pre-eminent such council, and the primary motivator of this paper, is the United Nations Security Council (UNSC), the only international body with the power to authorise the use of armed force. At any one time, the UNSC contains only 15 members from a total United Nations (UN) membership of 193. A second council operating within the UN, the United Nations Economic and Social Council (ECOSOC), contains 54 elected member countries at any one time. It is responsible for coordinating the economic, social and related work of 14 UN specialised agencies including the World Bank and International Monetary Fund.¹

In this paper we develop a theoretical framework for analysing democratic equitability in such Councils. We then apply the theory to the UNSC. The standard approach to formal quantitative appraisal of democratic equitability in international voting bodies is based upon Penrose’s (1946) square-root rule. Applications include Felsenthal and Machover (1997a, 1997b, 2001, 2004, 2007), Laruelle and Widgrén (1998) and Leech (2002a) to the Council of the European Union; Napel and Widgrén (2006) to the European Parliament; Manno (1966), Newcombe, Wert and Newcombe (1971), and Dixon (1983) to the UN General Assembly (UNGA); Leech (2002b), Leech and Leech (2013), and Rapkin and Strand (2006) to the IMF Executive Board; and Leech and Leech (2005) to the World Bank Executive Boards.² Note that these bodies are all fully representative: the UNSC stands out as the only major international body not to have been addressed by this literature.³

What lies behind this lacuna? The square-root rule is predicated on a two-stage voting model – first a national vote, second an international vote – which anticipates that all members vote in the second stage. When international decision-making is by a council, this framework

¹ Why do councils exist? In the case of military or emergency action, the lengthy deliberations of a fully representative body are thought to prevent such a body from being able to react with sufficient speed to developing security threats. Alternatively, councils may function in domains deemed to require detailed or specialised analysis (ECOSOC being an example). Councils can also arise at the national level. For instance, some countries have “Privy” or “Executive” Councils with the right to enact legislation during states of emergency, and/or committees that perform detailed tasks such as voting over proposed new legislation on a clause-by-clause basis prior to final approval by the parliament.

² Applications to national legislatures include Miller (2009) and Banzhaf (1968) to the U.S. Electoral College; and McLean, McMillan and Leech (2005) and Dunleavy (2010) to the UK Parliament. Again, these bodies are fully representative.

³ The IMF Executive Board is composed of only 24 Executive Directors, and the World Bank Executive Board of only 25 Executive Directors. In both bodies, however, the Executive Directors represent all 188 member-countries, so each can be interpreted as fully representative.

cannot be applied directly for at least two reasons. First, only a subset of members votes in the second stage. Second, this subset is not constant over time (the membership of the UNSC changes each year, for instance).

The paper contributes to both the theory and application of democratic equitability in voting bodies. In respect of theory, our first contribution is to formally define a council voting game (CVG), to describe the Councils empirically observed in the UN. In particular, we consider a setting in which a fully representative “assembly” allocates (by election or otherwise) members to a “council”. Existing measures of democratic equity with respect to fully representative bodies at time t require the triple (A_t, Q_t, V_t) , where A_t is the set of members, Q_t is the set of member populations, and V_t is the set of member relative voting powers. By contrast equity measures for CVGs are defined at time t with respect to a 7-tuple, C , that nests (A_t, Q_t) , but in which V_t derives from two primitives – one that determines the total relative voting power of the countries belonging to a region, and a second that divides this total between countries. In addition, C also details (i) the regional partition on the members of the fully representative body (the UN membership is divided into five regions) (ii) the division of council seats to regions; and (iii) the stochastic process that determines the allocation of countries to the council.

Second, we define three equity concepts for CVGs that distinguish between ex-ante and ex-post notions of equity, and between equity at the country and regional levels. Ex-ante equity requires that the democratic principle of one person one expected vote (OPOEV) applies among all world citizens before the allocation of countries to the council is known. Ex-ante equity, therefore, depends upon both a country’s voting power when a council member, and *how often* they are a council member. Ex-post equity requires that, once allocation to the council is known, the democratic principle of one person one vote (OPOV) applies among the citizens of member countries of the council. It is applicable if council members are viewed as representing only their own citizens, rather than the region they belong to as a whole. Region equity requires that the total voting power of a region on the council (if its members vote independently) corresponds to that which it would obtain in a fully representative body under the OPOV principle. It is applicable if members on the council are viewed as representing their region as a whole, rather than solely their own population.

We show there is an equity rule that implements each equity concept. The ex-post equity concept is satisfied by the implementation among the members of the council of Penrose’s (1946) square-root rule for voting power. Ex-ante equity also requires a square-root rule to

hold, but on the expected voting powers before the allocation of countries to the council is known. Regional equity does not require any property of the individual voting powers, but requires that a form of aggregated square-root rule for voting power hold at the regional level. We establish an impossibility result among our notions of equity: no CVG can be both ex-post equitable and also region equitable. As such, a CVG that is equitable if council members only represent themselves (as opposed to their region) will necessarily fail to achieve the desired level of regional representation, and the reverse also holds.

Given that no CVG can attain all three of our equity concepts, we look for (first-best) equity rules that implement two of the three concepts. We define a CVG as *country first-best* if it attains both the ex-post equity and ex-ante equity concepts and as *regional first-best* if it attains both the ex-ante equity and regional equity concepts. We find that attainment of the country first-best requires, first, that voting power on the council satisfies Penrose's square-root rule, and, second, that every country have an equal probability of allocation to the council. The latter condition implies that the number of council seats allocated to each region must be in proportion to the number of countries within each region. The regional first-best, however, is consistent with a range of rules. These include, as a special case, a square-root rule on the allocation probabilities (rather than on the voting powers), together with a flat rule for voting power.

With respect to application, our paper is the first we are aware of to present a quantitative assessment against formal equity concepts of the equitability of the UNSC for both individual countries and regions. As we discuss in more detail in Section 3, the UNSC is witnessing a protracted reform debate that centres on national and regional representation (see, e.g., Franck, 2003). At the regional level, reformers argue that Africa and Asia have too little power, and there is a claimed north-south divide. At the national level, countries such as Germany and Japan – who are only eligible for Non-Permanent Member (NPM) status on the UNSC – claim to be severely under-represented, and the Permanent Members (PMs) – who wield an individual veto – are argued to have too much representation.

Our analysis presents more nuanced conclusions. Our regional equity concept indeed shows that Africa, Asia and Latin America are under-represented as a whole, and that substantial north/south inequity exists. Within this picture, however, some countries in these regions actually receive too much voting power.

We do not find that the PMs receive too much voting power according to our ex-post equity

concept – indeed these countries are in some cases substantially under-represented. We do, however, find that the combination of preferential voting power *and* the right to be ever-present on the UNSC makes PMs substantially over-represented in the metric of *expected* voting power. Japan is under-represented from both an ex-ante and ex-post perspective, but Germany is, we find, over-represented according to our ex-ante equity concept.

Our first-best concepts provide little support for the notion that the power of veto should be abolished. Both concepts are consistent with the existence of veto players, and our country first-best entails some countries receiving substantially higher voting power than do PMs under the present arrangements. We find, however, that no country is a veto player when a member of the UNSC *and* ever-present on the UNSC under either first-best concept. The analysis also suggests a case for allocating the right of veto to a different set of countries.

The plan of the paper is as follows: Section 2 develops a theoretical framework for the analysis of democratic equity in councils; Section 3 presents an application of the theory to the UNSC; and Section 4 concludes. All proofs are located in Appendix 1.

2. Theory

In this section we consider a setting in which a fully representative “assembly” allocates members to a “council”. As with other aspects of the model, this setting is intended to mirror the structure observed within the UN, in which context the assembly should be interpreted as the UNGA, the main deliberative body of the UN containing all 193 of its members, and the council could refer either to the UNSC or to ECOSOC. As in the UNGA, we partition the assembly membership into regional groups. Countries are then allocated to the council in fixed proportions from each of the regions.

2.1 Council Voting Games

In this section we formally develop a class of voting game we term a *council voting game* (CVG). We begin by describing the elements of a CVG at a given time t .

Let the (fully-representative) assembly be denoted as the finite set A_t . We write $A_t = \cup_j R_{jt}$, where R_{jt} is the j^{th} region, $j \in J$. Let $R_t = \{R_{jt}\}_{j \in J}$ denote the set of regional partitions of A_t . Each region is a set of countries and we define a_{ij} as the i^{th} country within R_{jt} . Each country

possesses a population denoted $q_{ijt} \in \mathbb{N}$, and let $Q_t = \{q_{ijt}\}_{a_{ij} \in A_t}$ denote the set of member populations.

The council is formed of a set of members $M_t \subset A_t$. The number of seats belonging to the members of each region j is given by the vector $\mathbf{n}_t = (n_{1t}, n_{2t}, \dots, n_{|J|t})^T$, where it is assumed that the number of seats for each region is always smaller than the size of the region, $|R_{jt}| > n_{jt}$. The size of the council is denoted by $N_t \equiv |M_t| = \mathbf{1} \cdot \mathbf{n}_t < |A_t|$.

Rather than specify a method of allocation of countries to the council, here we adopt a reduced form representation that allows for essentially any allocation method. An *allocation process* P is a stochastic process that induces, at every time t , a probability $p_{ijt} \equiv \Pr(a_{ij} \in M_t)$ of council membership. By definition, the p_{ijt} must satisfy $\sum_{a_{ij} \in R_{jt}} p_{ijt} = n_{jt}$. The allocation process applying at time t is denoted P_t .

We assume that each country possesses a score $s_{ijt} \geq 0$, where the set of scores is denoted as $S_t = \{s_{ijt}\}_{a_{ij} \in A_t}$. If allocated to the council, s_{ijt} translates into a realised voting power, β_{ijt} , according to $\beta_{ijt} = w_{jt}s_{ijt}$, where $\sum_{a_{ij} \in M_t} w_{jt}s_{ijt} = 1$. The w_{jt} are regional weights of the form $w_{jt} = w_t \left(\sum_{a_{ij} \in M_t \cap R_{jt}} s_{ijt} \right)$. Note that the s_{ijt} are specified ex-ante (before M_t is realised), but the w_{jt} are not, in general determined until M_t is realised.⁴ Thus, once M_t is known, the w_{jt} may be employed to choose the sum of the voting powers of the countries belonging to a given region.

We may now define a CVG in two parts:

Definition 1

- (i) A council voting game at time t is a tuple $C_t = (A_t, \mathbf{n}_t, R_t, Q_t, P_t, S_t, w_t)$.
- (ii) A council voting game is the set $C = \{C_t\}_{t \in T}$.

2.2 Equity in CVGs

In order to derive equity concepts for a CVG at a point in time t_0 we conduct a thought experiment in which we hold constant the elements of C as at time t_0 , and examine the CVG formed by the resulting infinite repetition of C_{t_0} .

⁴ In practice, once the β_{ijt} are known, they must be induced by an appropriate decision rule – a mapping from the space of voting outcomes of the council members to an outcome space, satisfying appropriate monotonicity conditions. See, e.g., Freixas and Zwicker (2003) for a general class of (j,k) decision rules.

This approach may be interpreted as yielding *a-priori* measures of equity, for the assessment of equitability is based on the hypothetical continuation of a fixed state that is based only on information known up to time t_0 .⁵ This a-priori approach implies that to determine equity concepts for C at time t_0 , and regardless of the actual evolution of C beyond time t_0 , we may consider the CVG C' described at every time $t \in T'$ by $C'_t = C_{t_0}$, where $T' = 1, 2, \dots$ is an infinite set. For notational emphasis, we drop all time subscripts with respect to C_{t_0} , such that $C_{t_0} = (A, \mathbf{n}, R, Q, P, S, w)$. Accordingly, we write $w_{t_0}(\cdot) = w(\cdot)$, $q_{ijt_0} = q_{ij}$, $s_{ijt_0} = s_{ij}$, and so on.

With respect to C' , let the *expected voting power* under P of country a_{ij} at time t (before the allocation to the council is realised) be given by $\alpha_{ijt} = p_{ijt}s_{ij}\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt})$, where $\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt})$ denotes the expectation of w_{jt} over \mathcal{M}_t , where $M_t \in \mathcal{M}_t$ if and only if $|M_t \cap R_j| = n_{jt}$ and $p_{ijt} > 0$ for all $a_{ij} \in M_t$. Between the elements of two vectors \mathbf{x} and \mathbf{y} containing country-level statistics x_{ij} and y_{ij} , we write $x_{ij} \propto_M y_{ij}$ to denote that there exists a $\lambda_t > 0$ at every time t such that $x_{ij} = \lambda_t y_{ij}$ for all $a_{ij} \in M_t$ such that $M_t \in \mathcal{M}_t$. We define \propto_R and \propto_A analogously, but where \propto_R requires proportionality to hold for all $a_{ij} \in R_j$; and \propto_A requires proportionality to hold for all $a_{ij} \in A$. Likewise, we define \propto_J , which requires proportionality to hold for all $j \in J$ between the elements of two vectors containing region-level statistics.

To focus on empirically relevant cases, we henceforth impose three further assumptions. First, as no two UN members have identical recorded populations, we assume that more than N of the q_{ij} are distinct. Second, we assume that $p_{ijt} = 0$ for less than $|A| - N$ countries at any time t , to rule out degenerate cases in which the same N countries are allocated to the council every time an allocation is made. Together, these assumptions are sufficient to imply that $\sum_{a_{ij} \in M_t} \sqrt{q_{ij}}$ is a random variable before M_t is realised. Third, we assume that $p_{ijt} \neq 0$ on a non-empty subset of T' for every $a_{ij} \in A$. This assumption rules out the existence of countries that can never be allocated to the council, hence $s_{ij} \propto_M \beta_{ijt}$ implies $s_{ij} \propto_A \beta_{ijt}$.

2.2.1 Equity concepts

We base our concepts of democratic equity upon an idealised three-stage decision-making process. In Stage 1, a national ballot is held in each country $a_{ij} \in A$ under a simple majority decision rule. In Stage 2 a subset $M_t \subset A$ of countries are allocated to the council. In Stage 3, countries $a_{ij} \in M_t$ cast their vote according to the outcome of their national ballot in Stage 1.

⁵ The alternative *a-posteriori* approach is instead backwards looking, using historical data to estimate realised equitability over a period of time.

We develop three distinct equity concepts. Our first notion of equity we term ex-ante equity (AE). According to AE, it is desirable that the expected voting power of every world citizen is equal before the allocation of countries to the council is made in Stage 2, i.e., OPOEV. The ex-ante perspective acknowledges that the power of a world citizen in the council depends not only on the voting power of his or her country when it is a member of the council, but also on how frequently his or her country is a member of the council.

We develop two alternative concepts of ex-post equity. The first, which we term simply ex-post equity (PE), is that it is desirable that the democratic principle of OPOV hold among the citizens of council member countries, once these are known. This concept is of particular relevance if council members are viewed as representing their own populations, rather than their region at large.

Our final concept of equity, regional equity (RE), is also ex-post in nature. According to RE, it is desirable that the combined voting power of the council members from each region be proportionate with the level of representation that each region would obtain in a fully representative voting body satisfying OPOV. This concept is of particular relevance if council members are viewed as representing their region, rather than only themselves, for it entails that voting power be commensurate with the population of the region a country belongs to (rather than with its own country-specific population).⁶

The distinction between our ex-ante and ex-post concepts of equity is analogous to the distinction made by scholars of law between “procedural” and “distributive” justice (e.g., Konovsky, 2000); and by scholars of psychology between “procedural” and “outcome” fairness (e.g., De Cremer *et al.*, 2010). The AE concept requires procedural equitability, but not outcome equitability, whereas PE and RE require outcome equitability, but not procedural equitability.

All three equity concepts need not apply to every council, or in equal degrees. In some contexts, country concerns may prevail over regional, while the opposite could apply in other contexts. The applicability of RE relative to PE, for instance, depends largely on whether countries are viewed as representing only themselves, or the whole of the region to which they belong, when a member of the council. In the case of the UNSC, we appear to observe

⁶ The logical ex-ante counterpart to RE is that the combined *expected* voting power of the council members from each region be proportional to the level of representation that each region would obtain in a fully representative voting body satisfying OPOV. We do not consider this equity concept separately, however, for if a CVG satisfies AE at time t , then the ex-ante counterpart to RE is necessarily satisfied too. See footnote 11 for further details.

evidence of a concern for each equity notion. As we detail later, both country- and regional-level equity concepts are frequently cited by those that propose reform. In respect of ex-ante equity, it is notable that several of the proposals for reform of the UNSC detailed in Cox (2009) leave the country voting powers unchanged, but modify the allocation probabilities, suggesting that world leaders understand (at least intuitively) the importance of allocation probability as well as voting rights.⁷

To help formalise each of these equity concepts we make the following assumption on voting in Stage 1:

Assumption 1 *Voting in Stage 1 is assumed, a-priori, to be independent within and across countries.*

As argued by, e.g., Felsenthal and Machover (1997c, 2003), Assumption 1 should be understood as reflecting Bernoulli's Principle of Insufficient Reason: a-priori we do not know how countries will actually vote. An alternative a-priori assumption might be that voting is perfectly correlated within regions, but independent across regions, such that the regions act as unitary blocs. We note, however, that if regions become unitary players the council becomes a fully-representative body that may be analysed with existing theory. In this sense, the analysis of equity in CVGs differs from that in fully representative bodies only if individual countries are a-priori distinct from regions.⁸

Proposition 1 *Under Assumption 1:*

- (i) *a CVG is AE at time t_0 if and only if C' satisfies $\alpha_{ijt} \propto_A \sqrt{q_{ij}}$;*
- (ii) *a CVG is PE at time t_0 if and only if C' satisfies $\beta_{ijt} \propto_M \sqrt{q_{ij}}$;*
- (iii) *a CVG is RE at time t_0 if and only if C' satisfies $\sum_{a_{ij} \in M_t \cap R_j} \beta_{ijt} \propto_J \sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$.*

In interpreting Proposition 1, it is most straightforward to begin with part (ii), which restates the inspiration of Penrose (1946) that, under Assumption 1, OPOV is achieved by a decision rule that sets voting power in the council to be in proportion to the square-root of each

⁷ Examples include proposals that allow some current NPMs to be ever-present members, proposals that would extend the term length of some NPM seats, and proposals that would introduce seats that are allocated on a strict rotation basis within regions.

⁸ Empirically, countries on the UNSC do seem to act as distinct entities within regions. Each council member has full sovereignty over how it votes and countries pour large sums of money into campaigns for election to the UNSC (see, e.g., Malone, 2000), suggesting that they do not perceive membership by another of their regional group to be a perfect substitute for their own membership. Also, the voting behaviour in the UNGA of serving members of the UNSC is no more similar to that of their regional members than to the votes of the remaining UNGA members (Lai and Lefler, 2009).

country's population.⁹ Part (i) may then be understood as simply requiring the same square-root rule to hold, but this time on the *expected* voting powers (before the allocation to the council in Stage 2 is known).

For part (iii), note that, under Assumption 1, it holds that the voting power of a region is the sum of the voting powers of the individual members.¹⁰ Hence, via Penrose's square-root rule, a region's voting power in a fully representative body satisfying OPOV is proportional to $\sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$.¹¹

2.3 Equity Rules

What restrictions are required on the underlying parameters to implement the conditions in Proposition 1? To assist in the analysis of this and later questions we first define some special cases in which either the s_{ij} , β_{ijt} or p_{ijt} are invariant on some dimensions:

Definition 2

- (i) (*regional s-invariance*) C' satisfies regional s -invariance if and only if $s_{ij} = s_{i'j} = s_j$ for all $a_{ij}, a_{i'j} \in R_j$ and for all $j \in J$;
- (ii) (β -invariance) C' satisfies β -invariance if and only if $\beta_{ijt} = \beta_{i'jt} = \beta_{ijt'} = \beta$ for all $a_{ij}, a_{i'j'} \in A$ and for all $t, t' \in T$;
- (iii) (p_t -invariance) C' satisfies p_t -invariance if and only if $p_{ijt} = p_{ij't} = p_{ijt}$ for all $a_{ij} \in A$ and for all $t, t' \in T$;
- (iv) (*regional p-invariance*) C' satisfies regional p -invariance if and only if $p_{ijt} = p_{ij't} = p_{i'jt} = p_j$ for all $a_{ij}, a_{i'j'} \in R_j$, for all $t, t' \in T$ and for all $j \in J$;
- (v) (p -invariance) C' satisfies p -invariance if and only if $p_{ijt} = p_{ij't} = p_{i'jt} = p$ for all $t, t' \in T$ and for all $a_{ij}, a_{i'j'} \in A$.

It is straightforward to observe that p -invariance and regional p -invariance both imply p_t -invariance, and that p -invariance implies regional p -invariance. Less obvious is that β -invariance implies regional s -invariance. As the weights w_{jt} are region specific, they cannot

⁹ For equity rules under alternatives to Assumption 1 see Kirsch and Langner (2011).

¹⁰ If Assumption 1 were replaced with the assumption that voting is correlated across countries within a region, but independent across regions, then a different concept would be required as regions would vote as blocs on the council and, in general, the voting power of a bloc does not equal the sum of the individual voting powers of the members when voting independently.

¹¹ Following on from footnote 6, if the condition for AE $\left(a_{ijt} \propto_A \sqrt{q_{ij}} \right)$ is summed over $a_{ij} \in R_j$ on both sides, then one obtains the natural statement of the ex-ante counterpart to RE.

be used to adjust the s_{ij} of an individual country. Hence, β -invariance is implemented if and only if regional s -invariance holds and $w(z_{jt}) = n_j(Nz_{jt})^{-1}$.

We now state our implementation rules:

Proposition 2 *Under Assumption 1:*

(i) C is AE at time t_0 if and only if C' satisfies (a) p_t -invariance; and (b) $p_{ij}^{\text{AE}} s_{ij}^{\text{AE}} \propto_R \sqrt{q_{ij}}$;

(ii) C is PE at time t_0 if and only if C' satisfies (a) $w(z_{jt}) = \frac{1}{\sum_{j \in J} z_{jt}}$; and (b) $s_{ij}^{\text{PE}} \propto_A \sqrt{q_{ij}}$;

(iii) C is RE at time t_0 if and only if C' satisfies $w(z_{jt}) = \left(\frac{\sum_{A_j \in R_j} \sqrt{q_{ij}}}{\sum_{A_j \in A} \sqrt{q_{ij}}} \right) \frac{1}{z_{jt}}$.

Part (i) of Proposition 2 establishes a rule for AE. Intuitively, it states that, under p_t -invariance, any choice of the s_{ij} and p_{ij} such that $p_{ij}^{\text{AE}} s_{ij}^{\text{AE}} \propto_R \sqrt{q_{ij}}$ holds within region is implied by AE, and implies AE for an appropriate choice of $w(\cdot)$. This result is consistent with a range of rules for the underlying s_{ij} and p_{ij} , including a flat rule for the p_{ij} together with a square-root rule for the s_{ij} , and vice-versa. Part (ii) of Proposition 2 establishes an equity rule that implements PE. Condition (a) requires that $w_{jt} \propto_A \left(\sum_{a_{ij} \in M_t} s_{ij} \right)^{-1}$, for this choice of $w(\cdot)$ implies that, at each time t , the β_{ijt} are obtained by simply normalising the s_{ij} of the council members, such that $s_{ij}^{\text{PE}} \propto_A \beta_{ijt}$. Condition (b) then ensures Penrose's square-root rule by asking the s_{ij}^{PE} to satisfy a square-root rule. As PE does not require procedural equity, however, it imposes no requirements on the p_{ijt} .

Part (iii) establishes an equity rule that implements RE. As RE imposes a requirement upon the sum of the voting powers of the council members from each region, a restriction must be placed on the regional weights in order to guarantee that $\sum_{a_{ij} \in M_t \cap R_j} \beta_{ijt} \propto_J \sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$. Note, however, that RE imposes nothing upon the distribution of voting power within each region, so no restriction upon the s_{ij} is made.

Although we show in the next section that AE is compatible with both PE and RE, we find that no CVG can be both PE and RE:

Proposition 3 *Under Assumption 1 no C is both PE and RE at time t_0 .*

The proof of Proposition 3 shows that for both PE and RE to hold, then $\sum_{a_{ij} \in M_i \cap R_j} \sqrt{q_{ij}}$ must be non-random. The most general way this could occur is if the q_{ij} satisfied the following property:

Definition 3 (*q sub-regional invariance – qSRI*) *C'* satisfies qSRI if and only if $q_{ij} = q_{i'j}$ for all $a_{i'j}, a_{ij} \in r_{k_j}$, where the r_{k_j} partition R_j for each j , and $k_j = 1, \dots, g_j$, where g_j is an integer belonging to $[1, n_j]$.

If n_{k_j} seats are allocated to each sub-region r_{k_j} , where $\sum_{a_{ij} \in r_{k_j}} n_{k_j} = n_j$, then under qSRI we have that $\sum_{a_{ij} \in M_i \cap R_j} \sqrt{q_{ij}} = \sum_{k_j \in g_j} n_{k_j} \sqrt{q_{ij}}$, which is non-random. But we rule out qSRI by assumption in Section 2.2 (as, empirically, it is wholly implausible), from which the Proposition follows. Accordingly, in a world where country populations are distinct, it is necessary for world leaders to accept that a CVG that achieves PE will necessarily entail some degree of regional inequity, and that the reverse also holds.

2.3.1 The first-best

Proposition 3 implies that no equity rule can achieve all three concepts we consider. It is possible, however, to derive equity rules that achieve two of the three concepts. The rule that achieves both AE and PE we term the *country first-best* (CFB), and the rule that achieves both AE and RE we term the *regional first-best* (RFB).

We first prove a helpful Lemma:

Lemma 1 *If a CVG satisfies $p_{ijt} \propto_A f_{ijt}$, then $n_j \propto_J \sum_{a_{ij} \in R_j} f_{ijt}$.*

Lemma 1 follows from the observation that the allocation probabilities must, by definition, satisfy $\sum_{a_{ij} \in R_j} p_{ijt} = n_j$, hence $n_j = \sum_{a_{ij} \in R_j} p_{ijt} \propto_J \sum_{a_{ij} \in R_j} f_{ijt}$.¹²

Proposition 4 *Under Assumption 1:*

(i) (*Country first-best*) *C* is AE and PE at time t_0 if and only if *C'* satisfies (a)

$$w(z_{jt}) = \frac{1}{\sum_{j \in J} z_{jt}}; \text{ (b) } p\text{-invariance} - \text{hence } n_j^{\text{CFB}} \propto_J |R_j|; \text{ and (c) } s_{ij}^{\text{CFB}} \propto_A \sqrt{q_{ij}}.$$

¹² As the n_j are integers, this proportionality condition cannot hold exactly unless $(N \sum_{a_{ij} \in R_j} f_{ijt}) (\sum_{j \in J} \sum_{a_{ij} \in R_j} f_{ijt})^{-1}$ is an integer for every j . We therefore interpret the condition $n_j \propto_J \sum_{a_{ij} \in R_j} f_{ijt}$ to imply that, for given f_{ijt} , the n_j are set to achieve maximal proximity to exact proportionality (under an appropriate metric).

(ii) (Regional first-best) C is AE and RE at time t_0 if and only if C' satisfies (a)

$$w(z_{jt}) = \left(\frac{\sum_{A_j \in R_j} \sqrt{q_{ij}}}{\sum_{A_j \in A} \sqrt{q_{ij}}} \right) \frac{1}{z_{jt}}; \text{ and (b) } p_{ijt}^{\text{RFB}} s_{ij}^{\text{RFB}} \mathbf{E}_{M_t \in \mathcal{M}_t} \left(\frac{1}{\sum_{A_j \in M_t \cap R_j} s_{ij}^{\text{RFB}}} \right) \propto_A \frac{\sqrt{q_{ij}}}{\sum_{A_j \in R_j} \sqrt{q_{ij}}}.$$

According to Proposition 4, the country first-best is achieved by the combination of Penrose's square-root rule for voting power (which is implied by conditions a and c) together with a flat rule for allocation probability. The proof is a straightforward consequence of parts (i) and (ii) of Proposition 2. The result illustrates that, although the AE and PE concepts are compatible, potential tensions exist between them. On the one hand, there is no inherent conflict if all countries share the same allocation probability. On the other hand, if the major world powers desire to be members of bodies such as the UNSC and ECOSOC on a more regular basis than are lesser powers, then unequal allocation probabilities are required. With unequal allocation probabilities, AE implies that a country with a low allocation probability must be compensated for longer expected spells outside the council by the exercise of greater voting power when a member of the council. In this case, AE is in conflict with PE, for it results in systematic deviations from Penrose's square-root rule.

Part (ii) of the Proposition, which is a straightforward consequence of parts (i) and (iii) of Proposition 2, establishes the conditions needed for AE and RE. The Proposition clarifies that $p_{ij}^{\text{AE}} s_{ij}^{\text{AE}} \propto_R \sqrt{q_{ij}}$ is a necessary condition for RFB, but it is not sufficient, for (unlike under AE alone) we can now no longer choose $w(\cdot)$ arbitrarily. As there remains a degree of flexibility over the choice of the underlying s_{ij} and p_{ijt} that satisfy RFB, in the following corollary we highlight some special cases.

Corollary 1 *If C satisfies AE and RE at time t_0 then:*

(i) *If C' satisfies regional s -invariance then it also satisfies (a)*

$$w(s_j^{\text{RFB}}) = \frac{1}{n_j s_j^{\text{RFB}}} \left(\frac{\sum_{A_j \in R_j} \sqrt{q_{ij}}}{\sum_{A_j \in A} \sqrt{q_{ij}}} \right); \text{ (b) } p_r\text{-invariance}; \text{ and (c) } p_{ij}^{\text{RFB}} \propto_A n_j \left(\frac{\sqrt{q_{ij}}}{\sum_{A_j \in R_j} \sqrt{q_{ij}}} \right) \propto_R \sqrt{q_{ij}};$$

(ii) *If C' satisfies β -invariance then it also satisfies (a) $w(s_j^{\text{RFB}}) = \frac{1}{N s_j^{\text{RFB}}}$; (b) p_r -invariance;*

$$\text{and (c) } p_{ij}^{\text{RFB}} \propto_A \sqrt{q_{ij}}.$$

(iii) *If C' satisfies regional p -invariance then it also satisfies*

$$s_{ij}^{\text{RFB}} \left(\sum_{0 \leq h_1 < \dots < h_{n_j} \leq |R_j|} \frac{1}{\sum_{k=1}^{n_j} s_{h_k j}^{\text{RFB}}} \right) \propto_A \frac{1}{p_j^{\text{RFB}}} \left(\frac{|R_j|!}{n_j! (|R_j| - n_j)!} \right) \left(\frac{\sqrt{q_{ij}}}{\sum_{A_{ij} \in R_j} \sqrt{q_{ij}}} \right) \propto_R \sqrt{q_{ij}};$$

(iv) If C' satisfies p -invariance then it also satisfies

$$s_{ij}^{\text{RFB}} \left(\sum_{0 \leq h_1 < \dots < h_{\lfloor N|R_j||A|^{-1} \rfloor} \leq |R_j|} \frac{1}{\sum_{k=1}^{\lfloor N|R_j||A|^{-1} \rfloor} s_{h_k j}^{\text{RFB}}} \right) \propto_A \frac{|R_j|!}{\left(\frac{N|R_j|}{|A|} \right)! \left(\frac{(|A| - N)|R_j|}{|A|} \right)!} \left(\frac{\sqrt{q_{ij}}}{\sum_{A_{ij} \in R_j} \sqrt{q_{ij}}} \right) \propto_R \sqrt{q_{ij}}.$$

If the s_{ij} in part (ii) of Proposition 4 are restricted to satisfy regional s -invariance then we have $\sum_{a_{ij} \in M_i \cap R_j} s_{ij}^{\text{RFB}} = n_j s_j^{\text{RFB}}$. As this is non-random, $\sum_{a_{ij} \in M_i \cap R_j} s_{ij}^{\text{RFB}}$ may be removed from the expectation in condition (b) of part (ii) of Proposition 4 to give part (i) of Corollary 1. Part

(ii) uses Lemma 1 to additionally impose $n_j^{\text{RFB}} = N \left(\sum_{a_{ij} \in R_j} \sqrt{q_{ij}} \right) \left(\sum_{a_{ij} \in A} \sqrt{q_{ij}} \right)^{-1}$. According to part (i), therefore, RFB is consistent with a flat rule for voting power together with a square-root rule for allocation probability (where both rules hold within, but not across, regions). Part (ii) clarifies, that, if the division of seats to regions is made proportional to $\sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$, then RFB is consistent with a flat rule for voting power together with a square-root rule for allocation probability (where both rules hold within and across regions). Thus, the RFB concept is consistent with a square-root rule that – unlike Penrose’s – holds on the allocation probabilities rather than the voting powers.

Part (iii) of the Corollary considers the RFB concept when the allocation probabilities satisfy a flat rule within regions. Under this assumption, the s_{ij} follow a complex rule that relates to (but does not correspond to) a within-region square-root rule. Part (iv) is obtained by further setting $n_j^{\text{RFB}} = N|R_j||A|^{-1}$ in part (iii).¹³ Despite their apparently complex forms, these two rules for s_{ij} have a straightforward intuition. Suppose that we assert the approximation $\mathbf{E}_{M_i \in \mathcal{M}} \left(\left(\sum_{a_{ij} \in M_i \cap R_j} s_{ij} \right)^{-1} \right) \approx \left(\mathbf{E}_{M_i \in \mathcal{M}} \left(\sum_{a_{ij} \in M_i \cap R_j} s_{ij} \right) \right)^{-1}$ and, in violation of Jensen’s inequality, proceed under the basis of equality holding between these two entities. Under regional p -invariance, we then obtain $\mathbf{E}_{M_i \in \mathcal{M}} \left(\sum_{a_{ij} \in M_i \cap R_j} s_{ij} \right) = n_j |R_j|^{-1} \sum_{a_{ij} \in R_j} s_{ij}^{\text{RFB}}$. Substituting this expression into condition (b) in part (ii) of Proposition 4 we then obtain a within-region square-root rule for s_{ij} , which becomes a square-root rule within and across regions under p -

¹³ As discussed in footnote 12, if $N|R_j||A|^{-1}$ is not an integer for some j , then that $N|R_j||A|^{-1}$ must be replaced in part (iv) of Corollary 1 with either $\lfloor N|R_j||A|^{-1} \rfloor$ or $\lceil N|R_j||A|^{-1} \rceil$ as appropriate.

invariance. Therefore, the rules for s_{ij} in parts (iii) and (iv) of the Corollary deviate from square-root rules to the extent that $\mathbf{E}_{M_t \in \mathcal{M}_t} \left(\left(\sum_{a_{ij} \in M_t \cap R_j} s_{ij} \right)^{-1} \right)$ deviates from proportionality with $\left(\mathbf{E}_{M_t \in \mathcal{M}_t} \left(\sum_{a_{ij} \in M_t \cap R_j} s_{ij} \right) \right)^{-1}$.

Note that all the rules in Corollary 1 are monotonic within region in the sense that $s_{ij} \geq s_{i'j} \Leftrightarrow q_{ij} \geq q_{i'j}$ for all $a_{ij}, a_{i'j} \in R_j$ and for all $j \in J$; and that an equivalent condition holds on the p_{ijt} at all times $t \in T'$. If C' satisfies both of these conditions, we say it is *regional monotonic*. Rules that imply regional monotonicity seem the most relevant empirically, and we concentrate upon this class accordingly. We note, however, that AE may be satisfied by a range of less empirically plausible rules in which one of s_{ij} or p_{ij} is a strictly decreasing function of q_{ij} , and the other is an increasing function of q_{ij} that increases faster than $\sqrt{q_{ij}}$.

3. Application

In this section we apply the theory of Section 2 to the case of the UNSC, the most powerful organ within the United Nations, with the authority to make legally binding resolutions to fulfil its mandate of maintaining international peace and security. To that end, it can suspend economic and diplomatic relations between countries, impose blockades, and authorise the use of armed force.

Under the present arrangements – which have been in place since 1965 – the UNSC is comprised of 15 members, of which five – China, France, Russia, the United Kingdom, and the United States – are ever-present and wield a veto on all non-procedural matters. The remaining ten members are elected NPMs who serve time-limited two-year terms. The ten NPM seats are divided between five regional caucusing groups: one country from Eastern Europe (EE); two countries from each of the Western European and Others Group (WEOG), the Latin America and Caribbean Group (GRULAC – *el Grupo Latinoamericano y Caribeño*) and Asia; and three countries from Africa.¹⁴

Both country and regional perspectives on equity are frequently cited by those that propose reform (e.g., Russett, O'Neill and Sutterlin, 1996; Hammer, 2002; Schwartzberg, 2003;

¹⁴ See Appendix 2 for the full membership of each of the regional groups (excluding PMs). Of the PMs, China is a member in Asia, Russia in EE, and France and the UK in the WEOG. Technically, the United States is not a member of any regional group, but it attends meetings of the WEOG as an observer and is considered to be a member of that group for electoral purposes (UN, 2012). For the purposes of this paper, therefore, we give the United States membership in the WEOG.

Annan, 2005; Blum, 2005). From the country perspective, it is commonly argued that the right of veto of the five PMs gives these countries too great an influence; and that other countries are more deserving of PM status than are France and the UK (Germany and Japan both have larger populations and economies, and contribute more to the UN regular budget).¹⁵ Nearly all governments wish to abolish or limit the right of veto, which is viewed as an unfair and anachronistic legacy of the Second World War (Fassbender, 2004; Schwartzberg, 2003).

From the regional perspective it is argued that Africa and Asia are under-represented as together they account for around 75 per cent of the UN population, but are allocated only 20 per cent of the PM seats, and 50 per cent of the NPM seats; and that there exists a broader representational imbalance between the north – defined in Zifcak (2006: footnote 9) as comprising EE, and the WEOG – and south (Africa, Asia and the GRULAC).

In the absence of a formal theoretical framework for measuring the equitability of CVGs, or for addressing issues relating to region- and country-specific notions of equity, existing quantitative analyses are unable to directly assess these claims. Instead, extant studies use the voting power of a PM relative to a NPM as an informal indicator of equitability (see, e.g., Hosli *et al.*, 2011; O’Neill, 1996; Strand and Rapkin, 2011; Straffin, 1993: 180). The theoretical framework of Section 2 permits, for the first time, a formal quantitative assessment of the equitability of the UNSC for both individual countries and regions.

Another contribution with respect to the extant literature is the choice of decision rule to represent the UNSC. The studies above, and many precursors in the literature (e.g., Shapley and Shubik, 1954; Straffin, 1983), model the UNSC decision rule as a binary rule in which members can vote only for or against a resolution. In the UNSC, however, the UN Charter states that decisions over non-procedural matters are made by an affirmative vote of nine or more members, including the *concurring* votes of the PMs. A “concurring” vote has come to be understood, in practice, as either an affirmative vote or an abstention (see Blum, 2005: 636), so a negative vote by a PM is distinct from an abstention. As commented by Felsenthal and Machover (1997c: 348), this feature of the UNSC decision rule implies that it “cannot be faithfully represented” as a binary decision rule.¹⁶ We therefore allow here for a ternary

¹⁵ As of 2012, Japan contributes 12.5 per cent of the UN regular budget, Germany 8.0 per cent, the UK only 6.6 per cent, and France only 6.1 per cent (UN Secretariat, 2011).

¹⁶ The same point is also made in Freixas and Zwicker (2003).

decision rule, which is a map from the set $\{-1, 0, 1\}^N$ to the set $\{-1, 1\}$, satisfying the monotonicity conditions set out in Felsenthal and Machover (1997c: Definition 2.2).

3.1 Measuring Deviations from Equitability

It is desirable to be able to measure, in an objective sense, the proximity of the CVG representing the UNSC, $C_{\text{UNSC}} = \{C_{\text{UNSC},t}\}_{t \in T}$, to each equity concept at time t_0 , where we set $t_0 = 2012$. To this end, we adopt the metric $d(\mathbf{X}, \mathbf{Y}) = \frac{1}{2} \sum |X_i - Y_i|$, where \mathbf{X} and \mathbf{Y} are unit-vectors, which corresponds to the *index of distortion* used in Felsenthal and Machover (2004, 2007), and commonly attributed to Loosemore and Hanby (1971).

Let PM be the set of PMs and OM be the set of the remaining 188 ‘‘ordinary’’ members. We model the UNSC in the framework of the previous section by setting $s_{ij} = \beta_{\text{PM}}$ for $a_{ij} \in PM$, $s_{ij} = \beta_{\text{NPM}}$ for $a_{ij} \in OM$, and $w(z_{jt}) = \left(\sum_{j \in J} z_{jt}\right)^{-1} = 1$. As PMs are guaranteed allocation to the UNSC, we have $p_{ijt} = 1$ for these countries. We then obtain $\alpha_{ijt} = \beta_{\text{PM}}$ for $a_{ij} \in PM$ and $\alpha_{ijt} = p_{ijt}\beta_{\text{NPM}}$ for $a_{ij} \in OM$. From part (i) of Proposition 1 and condition (b) of part (ii) of Proposition 2, we define proximity measures on C'_{UNSC} (as at 2012) with respect to our two country-based equity measures as

$$AE = 1 - \mathbf{E}_{t \in T'}(d(\boldsymbol{\alpha}_t, \boldsymbol{\alpha}^{\text{AE}})); \quad PE = 1 - d(\boldsymbol{\beta}, \boldsymbol{\beta}^{\text{PE}});$$

where $\boldsymbol{\alpha}_t$ is the scaled $|A| \times 1$ unit vector of the α_{ijt} ; $\boldsymbol{\alpha}^{\text{AE}} = \boldsymbol{\beta}^{\text{PE}}$ is the scaled $|A| \times 1$ unit vector of the $\sqrt{q_{ij}}$; and $\boldsymbol{\beta}$ is the scaled $|A| \times 1$ unit vector of the UNSC voting powers.¹⁷ Note that these two measures lie on the unit interval, with unity indicating maximal proximity, and zero indicating the minimum possible proximity.

To define an analogous proximity measure for C'_{UNSC} with respect to RE we note that we may write

$$\sum_{a_{ij} \in M_t \cap R_j} \beta_{ijt} = n_{j,\text{PM}}\beta_{\text{PM}} + (n_j - n_{j,\text{PM}})\beta_{\text{NPM}}. \quad (1)$$

Hence, using part (iii) of Proposition 1, we define

$$RE = 1 - d(\sum \boldsymbol{\beta}, \boldsymbol{\beta}^{\text{RE}}),$$

¹⁷ In the case of the UNSC we have $w(\cdot)$ of the form in condition (a) of part (ii) of Proposition 2. Hence, for PE, it is sufficient to test for proximity to condition (b).

where $\Sigma\boldsymbol{\beta}$ is the scaled $|J| \times 1$ unit vector of the right-side of (1), and $\boldsymbol{\beta}^{\text{RE}}$ is the scaled $|J| \times 1$ unit vector of the $\sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$. To measure proximity to each of our two concepts of the first-best we examine the average proximity to the relevant two equity concepts:

$$CFB = 1 - \frac{1}{2} (\mathbf{E}_{t \in T'} (d(\boldsymbol{\alpha}_t, \boldsymbol{\alpha}^{\text{AE}})) + d(\boldsymbol{\beta}, \boldsymbol{\beta}^{\text{PE}})) = \frac{1}{2} (AE + PE);$$

$$RFB = 1 - \frac{1}{2} (\mathbf{E}_{t \in T'} (d(\boldsymbol{\alpha}_t, \boldsymbol{\alpha}^{\text{AE}})) + d(\Sigma\boldsymbol{\beta}, \boldsymbol{\beta}^{\text{RE}})) = \frac{1}{2} (AE + RE).$$

3.2 Computing equity measures

As is by now conventional in the literature, we adopt the normalised Banzhaf index as our measure of relative a-priori voting power. Note, however, that we adopt the ternary interpretation of the normalised Banzhaf index, as set out by Felsenthal and Machover (1997c), rather than the more conventional binary interpretation. We compute the ternary normalised Banzhaf index using the method of generating functions (see, e.g., Freixas, 2012). In this way we obtain the relative voting power of a PM as $\beta_{\text{PM}} \approx 0.10$ and $\beta_{\text{NPM}} \approx 0.05$, implying that a PM has almost exactly twice the voting power of a NPM.¹⁸

Using country population data for 2012 we compute the vectors $\boldsymbol{\beta}^{\text{PE}}$, $\Sigma\boldsymbol{\beta}$, and $\boldsymbol{\beta}^{\text{RE}}$ defined in Section 3.1.¹⁹ To compute $\boldsymbol{\alpha}^{\text{AE}}$, however, requires knowledge of the p_{ijt} for $a_{ij} \in OM$. We specify the allocation process P_{UNSC} by assigning each country in OM with a probability, $\rho_{ij} \in [0,1]$, where $\sum_{a_{ij} \in R_j} \rho_{ij} = 1$, with which it will be allocated to the UNSC if it is in competition with all members of its region and if only a single seat is being allocated.

We use empirical estimates of the ρ_{ij} that apply in the actual UNSC. These are taken from our earlier analysis, Dreher *et al.* (in press), in which we empirically estimate the systematic determinants of the election of OMs to the UNSC, accounting for the two-stage process by which such members are presently elected.²⁰ There we show that three country characteristics systematically predict UNSC election: population, gross national income per capita, and waiting time since last serving on the UNSC. The estimated co-efficients for these three

¹⁸ Note, in contrast, that if the UNSC decision rule is modeled as binary, then we obtain $\beta_{\text{PM}} \approx 0.167$ and $\beta_{\text{NPM}} \approx 0.017$, which implies that a PM has around ten times as much voting power as a NPM.

¹⁹ Population data are from the CIA World Factbook (<https://www.cia.gov/library/publications/the-world-factbook/index.html#>).

²⁰ In the first stage, the regions make nominations to the UNGA and, in the second stage, the UNGA votes. See, e.g., Dreher *et al.* (in press) for a detailed account.

variables can be used in a straightforward way to compute estimates of the ρ_{ij} .²¹ The resulting estimates are listed in Appendix 2.

In practice, the UNGA simultaneously allocates OMs to the UNSC. For the purposes of developing a tractable simulation model, however, we suppose that when the UNGA must elect more than one NPM from the same region in a given year, countries are elected sequentially, one-by-one. Hence, if there are two seats to be allocated to members of region j , then, in each of two rounds, there is a new realisation of a random variable that, if all countries in the region are competing for the seat, elects country a_{ij} with probability ρ_{ij} . Because, however, UNSC rules prohibit countries from having dual membership, if the same country is elected in both rounds the process is repeated again in full. This continues until distinct countries are elected.

What does this procedure imply for the relationship between the ρ_{ij} and the p_{ijt} ? In a given year, a first set of countries, those half-way through their two-year term, gain automatic renewal of their NPM status in the following year (R_t); a second set of countries, I_t , are those ineligible for election to the UNSC in the following year (UNSC rules prohibit NPMs from seeking immediate re-election); and a final set of countries is eligible for election to the UNSC (E_t). Hence we can write

$$p_{ijt} \begin{cases} = 0 & \text{if } a_{ij} \in I_t; \\ \in (0,1) & \text{if } a_{ij} \in E_t; \\ = 1 & \text{if } a_{ij} \in R_t. \end{cases}$$

For $a_{ij} \in E_t$, let p_{ijt}^n denote p_{ijt} when n NPM seats are being elected in region j , in which case

$$p_{ijt}^0 = 0 ; \quad p_{ijt}^1 = \frac{\rho_{ij}}{\sum_{a_{ij} \in E_j} \rho_{ij}} ; \quad p_{ijt}^2 = \frac{2p_{ijt}^1(1-p_{ijt}^1)}{1 - \sum_{a_{ij} \in E_j} (p_{ijt}^1)^2}.$$

Note that the numerator of p_{ijt}^2 is the binomial probability of observing a distinct country pair containing a_{ij} , and that the denominator corrects for the impossibility of a country obtaining dual UNSC membership.

²¹ Because the Dreher *et al.* dataset ends at 2006, we obtain estimates of country population and gross national income per capita (current US\$) for 2012 from the CIA World Factbook (see footnote 19). We update Dreher *et al.*'s variable measuring waiting time since last serving on the UNSC to 2012 using historical UNSC membership data from the UNSC Web site (<http://www.un.org/Docs/sc>). To produce the estimates in Appendix 2, these data, along with the co-efficient values for population, gross national income per capita, and waiting time since last serving on the UNSC reported in their Table 3a, are fed into their equation (5), where we assume that the sum in the denominator is over all countries in the region (i.e., their " E_{jt} " – the set of countries competing for the seat – is assumed to be R_{jt}).

3.2 Simulating the UNSC

The complexity of P_{UNSC} precludes analytical derivation of $\mathbf{E}_{t \in T'}(d(\alpha_t, \alpha^{\text{AE}}))$. We therefore compute a finite-sample estimate of this statistic from the realisation (via computer simulation) of a CVG, C_{UNSC}^Ω , which is the finite-repetition analogue of C_{UNSC}' on $t \in \Omega$ (Ω a finite set). For the purposes of simulation, we choose Ω to be the set of natural numbers from one to 100,000.²²

We adopt the same pattern of elections as occurs in the present UNSC: the term of the single EE NPM begins in even years. The two NPMs of the WEOG begin their terms in odd years. The terms for the two NPMs of the GRULAC are staggered; one is elected each year. The Asia's two NPM seats are similarly staggered. The three Africa NPM seats are also staggered with two terms beginning in even years and one term beginning in odd years.

3.3 Results

Simulating the UNSC according to the approach of the previous section, our proximity measures are found as

$$AE = 0.45; \quad PE = 0.66; \quad RE = 0.70;$$

$$CFB = 0.56; \quad RFB = 0.68.$$

These measures show that the UNSC is (i) more inequitable from a country perspective than from a regional perspective; and (ii) more inequitable from an ex-ante (procedural) perspective than from an ex-post (outcome) perspective.

To explain these findings, in Appendix 3 we show country-by-country measures for each of our three equity concepts. Whereas our proximity measures above are based upon *absolute* deviations, the measures reported in Appendix 3 report individual *relative* deviations from each of our equity concepts. In particular, we report the measure R where

²² Precisely, we realise marginally more than 100,000 periods, but discard the very earliest periods. This is necessary as we begin with a UNSC containing the five PMs and ten vacant seats. In each period we elect five new NPMs, hence, it is not until the completion of the election in period two that there remain no vacant seats on the elected UNSC. We discard the first four periods, which corresponds to twice the term length of a NPM, as, in all periods beyond the fourth, the elected UNSC contains no vacant seats, and eligibility for election to the UNSC does not depend upon whether a country was elected to the UNSC in either of periods one or two (when, abnormally, $I_t = \emptyset$).

$$R_{ij}^{\text{AE}} = \frac{\mathbf{E}_{t \in \Omega}(\alpha_{ijt})}{\alpha_{ij}^{\text{AE}}}; \quad R_{ij}^{\text{PE}} = \frac{\beta_{ij}}{\beta_{ij}^{\text{PE}}}; \quad R_j^{\text{RE}} = \frac{(\sum \beta)_j}{\beta_j^{\text{RE}}}.$$

We begin with an analysis of the UNSC from a country perspective. The conclusions are, however, sensitive to whether this is done from an ex-ante or ex-post perspective. Taking the ex-post perspective first, the relevant equity concept is PE. The relatively middling score achieved by the UNSC largely reflects two factors. First, within the PM and NPM categories, each country is awarded the same voting power regardless of its population. As may be seen in the first column of the table in Appendix 3, a consequence is that, within each membership category, the most populous countries receive a voting power that is much too low. The most extreme example is India, which receives only 10.7 per cent of its voting power under PE. In spite of receiving the voting power of a PM, China receives only 21 per cent of its voting power under PE. Two further PMs – Russia and the United States – are also substantially underweighted. In the remaining regions, countries such as Brazil (27 per cent), Nigeria (30 per cent), Germany (41 per cent) and Ukraine (56 per cent) also find themselves substantially under-represented according to PE.

An analogous consequence, which principally manifests itself among the NPMs, is that the least populous countries receive a voting power that is much too high. The most extreme example is Tuvalu, which receives 37.9 times its voting power under PE. In the remaining regions, countries such as San Marino (21 times), St. Kitts and Nevis (16 times), The Seychelles (13 times), and Montenegro (5 times) also find themselves substantially over-represented under PE.

The second factor is the division of voting power between the five PMs and the remaining UN membership. It is conventionally assumed that this division is too favourable to the PMs, but we conclude the opposite. Consistent with our comments regarding the underweighting of China, the United States, and Russia above, we calculate the total voting power of the PMs to be only 47 per cent of that consistent with PE. The remaining UN members therefore receive, on average, too much voting power on the UNSC under PE.

When we repeat the same analysis from an ex-ante perspective, however, we observe some important differences. For instance, the balance of power between *PM* and *OM* remains a problem, but now because far too much expected voting power is given to citizens of PM countries. On average, the PMs wield around 4.6 times too much expected voting power: the UK, in particular, wields some 9.6 times more expected voting power than that consistent

with AE, and even China wields around twice too much expected voting power. How can the PMs be simultaneously under-represented according to PE, and over-represented according to AE? The answer lies in the observation that PM countries are always able to cast their vote in Stage 3, whereas all other countries can do so only periodically. It is the combination of their right to be ever-present, together with their right of veto, which gives the PMs a disproportionately large share of the *expected* voting power.

Accordingly, ordinary members suffer a deficit of expected voting power. Therefore, only a small proportion of such countries exceed their expected voting power under AE, and the major individual deviations are for countries that receive too little expected voting power. For instance, under the estimated membership distribution in Appendix 2, Dominica receives just 0.6 per cent of its expected voting power under AE. In other regions, countries such as Chad (11 per cent), Samoa (1 per cent), Montenegro (3 per cent) and Liechtenstein (0.7 per cent) also receive much too little expected voting power.

In summary, the current UNSC deviates significantly from both the AE and PE concepts. The deviations from AE are the more severe, because PMs enjoy the highest voting power *and* the highest allocation probability, whereas these should be traded-off under AE. Whether the PMs are favoured hinges, however, on whether an ex-ante or ex-post perspective is adopted: PMs obtain decisively too much expected voting power ex-ante, but too little realised voting power ex-post. Accordingly, proposals for reform of the UNSC should not seek to erode the voting power of the existing PMs when members of the UNSC (indeed, this should be increased), but should instead focus upon eroding the right of these countries to be ever-present on the UNSC.

We now analyse the UNSC from a regional perspective using the RE concept. We find evidence in the third column of the table in Appendix 3 to support the widely-made claim that Africa and Asia are under-represented. We find that Asia is the most under-represented region, with only 55 per cent of its voting power under RE, closely followed by Africa with just 57 per cent. The GRULAC is also under-represented, but by a smaller margin – it receives 76 per cent of its voting power under RE. Balancing off the under-represented regions are the WEOG – which receives around 2.6 times its voting power under RE, and EE, which is over-represented by 61 per cent. These results imply the existence of a substantial north/south inequity. We find that, overall, the south receives just 60 per cent of its voting power under RE. Accordingly, proposals for reform of the UNSC should principally seek to shift voting power away from the WEOG, and towards Asia and Africa.

Last, we use our theoretical framework to address some of the remaining issues raised by reformers. Should some countries be ever-present on the UNSC? If so, which ones? Under the CFB concept the answer is negative: all countries receive a common allocation probability $p = N|A|^{-1} < 1$. The RFB concept does not preclude ever-present members, but only if the country is sufficiently populous. To see this, note that within the class of regional monotonic C' satisfying RFB, the maximal variation in the p_{ijt} across countries is achieved under a flat rule for voting power. In this case, by Corollary 1, the allocation probabilities are set such that $p_{ij}^{\text{RFB}} \propto_A \sqrt{q_{ij}}$, in which case a country is ever-present if

$$p_{ij}^{\text{RFB}} = N \frac{\sqrt{q_{ij}}}{\sum_{a_j \in A} \sqrt{q_{ij}}} \geq 1.$$

We find, however, that no country is sufficiently populous to meet this condition. Rather, in this case, China would be represented on the UNSC most often, obtaining membership in around 73 out of every 100 years. India would be represented next most often, obtaining membership in around seven years in every ten. The United States would qualify for membership in only around seven years in every twenty, while the UK and France would qualify for membership in only around four in every 25 years.

Our first-best concepts can also shed some light on whether the right of veto should be abolished and, if not, which countries should exercise a veto. A direct analysis is not feasible, however, for, even when we know the first-best voting power of each country, this information is insufficient to determine whether a country exercises a veto. The difficulty is that voting power indices mix two distinct concepts – the power to prevent action by voting against a resolution, and the power to initiate action by voting for a resolution (Coleman, 1971). Hence, the degree of preventative power associated with a given vector of voting powers is a-priori unknown.

By Corollary 1, under RFB and β -invariance council members receive a common voting power $\beta = N^{-1}$. This case is sufficiently simple that we can conclude that either all council members exercise a veto (under a unanimity decision rule), or none do. Hence the RFB concept does not preclude a veto right. Under the CFB concept – in which voting powers are set such that $\beta_{ij} \propto_A \sqrt{q_{ij}}$ – we can instead examine which, if any, countries obtain a voting power that is at least as large as that currently wielded by a PM. Under CFB, countries do not obtain the same voting power every time they are allocated to the UNSC as the w_{jt} are random

variables. To investigate this case we therefore examine each of the 100,000 realised M_t from the simulation of C_{UNSC}^{Ω} under the CFB voting powers. We measure the proportion of a country's appearances on the UNSC for which it votes with a power $\beta_{ij} > \beta_{\text{PM}}$ under CFB. We find that, in our 100,000 realisations, China and India always receive a voting power higher than β_{PM} under CFB. China, in particular, receives, on average, a voting power of around 0.23, which is slightly more than double β_{PM} .²³

In summary, we do not find clear support for the abolition of the veto: RFB does not preclude veto countries, and the CFB concept suggests that some countries warrant more voting power than currently received by a PM. Three points are of note, however: first, our first-best concepts imply that no country should have a veto *and* be ever-present on the UNSC. Second, the CFB concept suggests that if a veto right is to be allocated to five countries it should be the five most populous: China, India, the United States, Indonesia, and Brazil. Thus we agree with reformers that, if the right of veto is to remain, attention should be focused upon its allocation. As, however, we consider democratic ideals rather than economic might, or peacekeeping contributions, our analysis does not suggest that either Germany or Japan should be the recipients of a veto.²⁴ Last, a possible avenue for reform based on our findings may be for the PMs to trade-off extra voting power when a UNSC member with the loss of the right to be ever-present.

The final issue we address is how the 15 UNSC seats should be divided between regions. According to Proposition 4, the CFB concept implies that the n_j should be set proportional to $|R_j|$, i.e., four each to Africa and Asia, three to the GRULAC, and two each to EE and the WEOG. The RFB concept is consistent with a number of rules for n_j . Under p -invariance RFB yields the same rule as under CFB, but, under β -invariance, RFB requires $n_j \propto_I \sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$. Relative to the rule for n_j under CFB, this rule gives one extra seat to Asia (five seats), one less to the GRULAC (two seats), and leaves the entitlements of the remaining regions unchanged. Thus the WEOG, with four seats in the present UNSC, has twice its entitlement under either first-best concept, while Asia and Africa – which both receive three seats – are under-represented by at least one seat (and Asia by two seats under the RFB rule

²³ The remaining countries that sometimes (but not always) receive $\beta_{ij} > \beta_{\text{PM}}$ are the United States (77 per cent of council appearances), Indonesia (33 per cent), Brazil (3.5 per cent), Pakistan (0.6 per cent), Russia (0.06 per cent) and Nigeria (0.04 per cent).

²⁴ As seen in Appendix 3, Japan and Germany – like the PMs – are heavily under-represented according to PE. Both countries have historically achieved election to the UNSC on a regular basis, however, hence their representation under AE tells a different story. Japan's expected voting power is roughly in accordance with AE, while Germany is actually over-represented by 65 per cent under AE.

above).

4. Conclusion

The Councils of the United Nations – the UNSC and ECOSOC – play an important role in global wealth and security. Yet, to our knowledge, no previous analysis has developed formal equity principles for the analysis of such bodies, in which only a subset of member countries may vote at a point in time.

In this paper we develop a new class of voting game we term a council voting game. We then develop three democratic equity concepts for this new class of game. These three concepts differ according to whether equity is in an ex-ante (or procedural) sense, or in an ex-post (or outcome) sense; and whether council members are viewed as regional representatives, or solely as representatives of themselves. Fundamental trade-offs exist between these concepts, for if a CVG is PE then it cannot be RE. The AE concept is consistent with either PE or RE on their own, but even here the conditions required are stringent: either allocation probabilities must be equal across all countries (under the CFB concept) or a more general trade-off between voting power and allocation probability must hold (under the RFB concept).

We demonstrate the utility of our theoretical framework with an application to the UNSC. Significant degrees of inequity exist irrespective of the precise equity concept used, but we find that the UNSC is more inequitable in an ex-ante sense than in an ex-post sense, and more inequitable if countries are viewed as representing themselves, than if they are viewed as representing their region.

What do our findings imply for the ongoing debate on UNSC reform? First, we believe our framework clarifies the nature of the underlying trade-offs. For instance, simultaneous achievement of PE and RE is unfeasible; and, if realpolitik makes giving every country an equal probability of council allocation unfeasible, then some trade-off between AE and PE is unavoidable. Second, our analysis highlights that a successful reform of the UNSC must simultaneously address the distribution of voting power and the distribution of allocation probability, for attempting to achieve our equity concepts by changing only one or the other will lead to extreme outcomes. For instance, if the current allocation probabilities are retained, then our calculations show that attainment of AE requires that (tiny) countries such

as Nauru, Micronesia, Palau, and San Marino receive a voting power well in excess of the current β_{PM} when a member of the UNSC, as compensation for being a member very infrequently.

The apparent tension between realpolitik and the first-best concepts we develop suggests that the latter should be understood as purely theoretical benchmarks against which to assess the equitability of the UNSC and the desirability of alternative reforms. While this in no way undermines the usefulness of these concepts, an avenue for future research might, therefore, be to investigate second-best rules that move a CVG as “close” as possible to our first-best benchmarks under an additional realpolitik constraint. While this idea must await a proper treatment, however, we hope the present contribution at least marks a first step in the analysis of democratic equitability in councils.

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Appendix 1: Proofs

Proof of Proposition 1: For a proof of Penrose's square-root rule (from which the Proposition follows) see Felsenthal and Machover (1998). ■

Proof of Lemma 1: If $p_{ijt} \propto_A f_{ijt}$ then $n_j = \sum_{a_{ij} \in R_j} p_{ijt} \propto_J \sum_{a_{ij} \in R_j} f_{ijt}$. ■

Proof of Proposition 2

(i) Under AE we have $\alpha_{ijt} = p_{ijt}s_{ij}\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt})$ and $\alpha_{ijt} \propto_A \sqrt{q_{ij}}$, so $p_{ijt}s_{ij} \propto_A (\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt}))^{-1}\sqrt{q_{ij}}$. For this condition to hold at all t implies p_t -invariance (for the p_{ijt} have a fixed sum across each region, so increasing one p_{ijt} implies another must fall, leading to a violation of regional proportionality between these two countries). Hence $p_{ijt}s_{ij} \propto_A (\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt}))^{-1}\sqrt{q_{ij}}$. Then $(\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt}))^{-1}\sqrt{q_{ij}} \propto_R \sqrt{q_{ij}}$ so $p_{ijt}s_{ij} \propto_R \sqrt{q_{ij}}$. Now suppose $p_{ijt}s_{ij} \propto_R \sqrt{q_{ij}}$ then there exists a $w(\cdot)$ such that $p_{ijt}s_{ij} \propto_A (\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt}))^{-1}\sqrt{q_{ij}}$, which is AE. ■

(ii) Under (a) we have $w_{jt} \propto_A (\sum_{a_{ij} \in M_t} s_{ij})^{-1}$, hence $s_{ij} \propto_M \beta_{ijt}$. Then, by condition (b), $s_{ij} \propto_M \beta_{ijt} \propto_M \sqrt{q_{ij}}$, which is PE. Now suppose $\beta_{ijt} \propto_M \sqrt{q_{ij}}$ holds, which implies condition (a). Then $\beta_{ijt} \propto_M s_{ij}$, hence $s_{ij} \propto_M \sqrt{q_{ij}}$, which implies $s_{ij} \propto_A \sqrt{q_{ij}}$. ■

(iii) We have $\sum_{a_{ij} \in M_t \cap R_j} \beta_{ijt} \propto_J \sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$ by construction. Conversely, suppose we have $\sum_{a_{ij} \in M_t \cap R_j} \beta_{ijt} \propto_J \sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$, then $\sum_{a_{ij} \in M_t \cap R_j} w_{jt}s_{ij} \propto_J \sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$. As this condition must hold for arbitrary $s_{ij} \geq 0$, it implies that $w(z_{jt})$ is of the form $w(z_{jt}) = x_j(z_{jt})^{-1}$, where $x_j \propto_J \sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$. Then the condition that $\sum_{a_{ij} \in M_t} w_{jt}s_{ij} = 1$ implies $\sum_{a_{ij} \in M_t} x_j = 1$. ■

Proof of Proposition 3: Under RE we have $\sum_{a_{ij} \in M_t \cap R_j} \beta_{ijt} \propto_J \sum_{a_{ij} \in R_j} \sqrt{q_{ij}}$. Under PE we have $\beta_{ijt} \propto_M \sqrt{q_{ij}}$, hence $\sum_{a_{ij} \in M_t \cap R_j} \beta_{ijt} \propto_J \sum_{a_{ij} \in M_t \cap R_j} \sqrt{q_{ij}}$. For both conditions to hold we must have $\sum_{a_{ij} \in R_j} \sqrt{q_{ij}} \propto_J \sum_{a_{ij} \in M_t \cap R_j} \sqrt{q_{ij}}$. For this to hold $\sum_{a_{ij} \in M_t \cap R_j} \sqrt{q_{ij}}$ must be non-random, but this contradicts the assumptions of Section 2.2. ■

Proof of Proposition 4

(i) Conditions (a) and (c) yield PE by part (ii) of Proposition 2. Then, by condition (a) we have $\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt}) \propto_A s_{ij}$, hence $\alpha_{ijt} \propto_A p_{ijt}s_{ij}$. By condition (b) we have $\alpha_{ijt} \propto_A p_{ijt}s_{ij} \propto_A s_{ij}$ so, by (c), we obtain $\alpha_{ijt} \propto_A p_{ijt}s_{ij} \propto_A \sqrt{q_{ij}}$, which is AE. Now suppose AE and PE hold. Then, as (a) must hold for PE, we have $\alpha_{ijt} = p_{ijt}s_{ij}\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt}) \propto_A p_{ijt}s_{ij} \propto_A \sqrt{q_{ij}}$, which implies that $p_{ijt}s_{ij}$ must be time invariant, i.e., $\alpha_{ijt} \propto_A \sqrt{q_{ij}}$. By PE it also holds that $s_{ij} \propto_A \sqrt{q_{ij}}$, which is condition (c): both conditions hold if and only if condition (b) is satisfied. ■

(ii) We have $\alpha_{ijt} = p_{ijt}s_{ij}\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt})$. Under AE $p_{ijt}s_{ij}\mathbf{E}_{M_t \in \mathcal{M}_t}(w_{jt}) \propto_A \sqrt{q_{ij}}$. Then substituting for w_{jt} using condition (a) yields condition (b). ■

Appendix 2: Estimated ρ_{ij} ($t = 2012$)

Africa		Asia		EE		GRULAC		WEOG	
Algeria	0.0860	India	0.47728	Poland	0.3681	Brazil	0.34235	Germany	0.28949
Morocco	0.0501	Japan	0.12114	Ukraine	0.2470	Mexico	0.19389	Turkey	0.16346
Nigeria	0.0497	Pakistan	0.09212	Romania	0.1122	Venezuela	0.16637	Italy	0.11347
Egypt	0.0424	Malaysia	0.04707	Hungary	0.0591	Argentina	0.08081	Spain	0.07454
Ghana	0.0384	Republic of Korea	0.04376	Czech Republic	0.0525	Colombia	0.04673	Austria	0.07041
Tunisia	0.0377	Indonesia	0.03583	Belarus	0.0256	Chile	0.04251	Netherlands	0.04505
Tanzania	0.0376	Bangladesh	0.02381	Serbia	0.0227	Peru	0.02556	Canada	0.03614
South Africa	0.0336	Singapore	0.01899	Bulgaria	0.0191	Ecuador	0.01801	Sweden	0.03072
Zimbabwe	0.0329	Thailand	0.01732	Azerbaijan	0.0178	Uruguay	0.01334	Switzerland	0.02616
Zambia	0.0322	Jordan	0.01521	Slovakia	0.0145	Cuba	0.01019	Ireland	0.02483
Mozambique	0.0319	Philippines	0.01487	Croatia	0.0112	Dominican Republic	0.00765	Denmark	0.02285
Kenya	0.0319	United Arab Emirates	0.01314	Republic of Moldova	0.0078	Honduras	0.00712	Belgium	0.02184
Senegal	0.0309	Sri Lanka	0.01113	Georgia	0.0074	Costa Rica	0.00605	Finland	0.01849
Mali	0.0245	Iran	0.00657	Albania	0.0062	Guatemala	0.00538	Portugal	0.01692
Niger	0.0226	Saudi Arabia	0.00630	Lithuania	0.0060	Panama	0.00484	Norway	0.01666
Cote d'Ivoire	0.0220	Kuwait	0.00579	Slovenia	0.0052	Trinidad and Tobago	0.00458	Australia	0.01228
Guinea	0.0218	Myanmar	0.00518	Bosnia & Herzegovina	0.0047	Guyana	0.00415	New Zealand	0.01027
Congo	0.0218	Nepal	0.00516	Latvia	0.0037	Paraguay	0.00409	Greece	0.00497
Ethiopia	0.0213	Qatar	0.00462	TFYR Macedonia	0.0034	Jamaica	0.00381	Israel	0.00054
Angola	0.0198	Yemen	0.00414	Armenia	0.0034	Nicaragua	0.00371	Malta	0.00039
Libya	0.0189	Iraq	0.00332	Estonia	0.0019	Bolivia	0.00340	Luxembourg	0.00025
Uganda	0.0184	Vietnam	0.00324	Montenegro	0.0005	El Salvador	0.00165	Iceland	0.00007
Burkina Faso	0.0173	Oman	0.00323			Bahamas	0.00149	Monaco	0.00007
Malawi	0.0172	Kazakhstan	0.00257			Belize	0.00070	Andorra	0.00005
Madagascar	0.0170	Fiji	0.00203			Suriname	0.00061	San Marino	0.00004
Mauritania	0.0163	Cyprus	0.00200			Barbados	0.00046	Liechtenstein	0.00003
Sudan	0.0159	Papua New Guinea	0.00196			Haiti	0.00020		
Togo	0.0150	Syrian Arab Republic	0.00168			Antigua and Barbuda	0.00015		
Gabon	0.0147	Bahrain	0.00149			Saint Lucia	0.00013		
Benin	0.0136	Uzbekistan	0.00148			Saint Kitts and Nevis	0.00002		
Namibia	0.0124	Brunei	0.00092			St Vincent & Grenadines	0.00002		
Mauritius	0.0121	Lebanon	0.00082			Grenada	0.00001		
Cameroon	0.0104	DPR Korea	0.00076			Dominica	0.00001		
South Sudan	0.0098	Afghanistan	0.00075						
Botswana	0.0097	Cambodia	0.00062						
Sierra Leone	0.0087	Turkmenistan	0.00060						
Lesotho	0.0087	Mongolia	0.00048						
DR Congo	0.0082	Tajikistan	0.00044						
Eritrea	0.0072	Kyrgyzstan	0.00042						
Djibouti	0.0071	Laos	0.00040						
Gambia	0.0062	Bhutan	0.00033						
Central African Republic	0.0061	Maldives	0.00022						
Burundi	0.0059	Solomon Islands	0.00022						
Rwanda	0.0055	Timor Leste	0.00018						
Swaziland	0.0050	Tonga	0.00008						
Somalia	0.0048	Kiribati	0.00007						
Cape Verde	0.0045	Vanuatu	0.00007						
Comoros	0.0030	Samoa	0.00005						
Chad	0.0029	Micronesia	0.00003						
Guinea-Bissau	0.0024	Nauru	0.00002						
Liberia	0.0023	Marshall Islands	0.00002						
Sao Tome and Principe	0.0019	Tuvalu	0.00002						
Equatorial Guinea	0.0012	Palau	0.00002						
Seychelles	0.0005								

Estimates computed from Table 3a of Dreher *et al.* (in press). Countries are listed in descending order of probability.

Appendix 3: Entitlements and relative deviations

Country	AE		PE		RE	
	α_{ij}^{AE}	R_{ij}^{AE}	β_{ij}^{PE}	R_{ij}^{PE}	β_j^{RE}	R_j^{RE}
<i>Africa</i>	–	–	–	–	0.259	0.573
Algeria	0.0080	0.631	0.0080	1.301	–	–
Angola	0.0059	0.861	0.0059	0.523	–	–
Benin	0.0040	1.264	0.0040	0.556	–	–
Botswana	0.0019	2.654	0.0019	0.811	–	–
Burkina Faso	0.0054	0.927	0.0054	0.500	–	–
Burundi	0.0039	1.299	0.0039	0.253	–	–
Cameroon	0.0059	0.849	0.0059	0.283	–	–
Cape Verde	0.0009	5.339	0.0009	0.789	–	–
Central African Republic	0.0028	1.792	0.0028	0.361	–	–
Chad	0.0045	1.122	0.0045	0.113	–	–
Comoros	0.0012	4.387	0.0012	0.423	–	–
Congo	0.0027	1.870	0.0027	1.256	–	–
Cote d'Ivoire	0.0060	0.846	0.0060	0.574	–	–
Democratic Republic of the Congo	0.0109	0.463	0.0109	0.129	–	–
Djibouti	0.0013	3.989	0.0013	0.889	–	–
Egypt	0.0121	0.418	0.0121	0.507	–	–
Equatorial Guinea	0.0011	4.493	0.0011	0.171	–	–
Eritrea	0.0031	1.641	0.0031	0.398	–	–
Ethiopia	0.0122	0.413	0.0122	0.268	–	–
Gabon	0.0016	3.065	0.0016	1.386	–	–
Gambia	0.0018	2.860	0.0018	0.608	–	–
Ghana	0.0066	0.761	0.0066	0.834	–	–
Guinea	0.0042	1.190	0.0042	0.822	–	–
Guinea-Bissau	0.0017	3.055	0.0017	0.230	–	–
Kenya	0.0085	0.591	0.0085	0.550	–	–
Lesotho	0.0020	2.552	0.0020	0.721	–	–
Liberia	0.0027	1.882	0.0027	0.140	–	–
Libya	0.0034	1.492	0.0034	0.878	–	–
Madagascar	0.0061	0.826	0.0061	0.434	–	–
Malawi	0.0052	0.974	0.0052	0.513	–	–
Mali	0.0053	0.959	0.0053	0.707	–	–
Mauritania	0.0025	2.022	0.0025	1.024	–	–
Mauritius	0.0015	3.299	0.0015	1.286	–	–
Morocco	0.0076	0.665	0.0076	0.895	–	–
Mozambique	0.0065	0.778	0.0065	0.726	–	–
Namibia	0.0020	2.489	0.0020	0.955	–	–
Niger	0.0053	0.955	0.0053	0.653	–	–
Nigeria	0.0169	0.299	0.0169	0.404	–	–
Rwanda	0.0044	1.154	0.0044	0.203	–	–
Sao Tome and Principe	0.0005	9.246	0.0005	0.610	–	–

Senegal	0.0047	1.066	0.0047	0.972	–	–
Seychelles	0.0004	12.784	0.0004	0.241	–	–
Sierra Leone	0.0033	1.552	0.0033	0.442	–	–
Somalia	0.0041	1.231	0.0041	0.193	–	–
South Africa	0.0095	0.531	0.0095	0.517	–	–
South Sudan	0.0042	1.192	0.0042	0.378	–	–
Sudan	0.0078	0.649	0.0078	0.327	–	–
Swaziland	0.0015	3.453	0.0015	0.604	–	–
Togo	0.0033	1.532	0.0033	0.718	–	–
Tunisia	0.0043	1.162	0.0043	1.254	–	–
Uganda	0.0078	0.650	0.0078	0.359	–	–
United Republic of Tanzania	0.0089	0.570	0.0089	0.612	–	–
Zambia	0.0049	1.039	0.0049	0.957	–	–
Zimbabwe	0.0048	1.061	0.0048	1.020	–	–
<i>Asia</i>	–	–	–	–	0.362	0.552
Afghanistan	0.0075	0.671	0.0075	0.016	–	–
Bahrain	0.0015	3.348	0.0015	0.166	–	–
Bangladesh	0.0164	0.308	0.0164	0.225	–	–
Bhutan	0.0011	4.413	0.0011	0.050	–	–
Brunei	0.0008	5.954	0.0008	0.196	–	–
Cambodia	0.0050	1.000	0.0050	0.022	–	–
China	0.0487	0.211	0.0487	2.072	–	–
Cyprus	0.0012	4.180	0.0012	0.278	–	–
DPR Korea	0.0066	0.762	0.0066	0.021	–	–
Fiji	0.0012	4.053	0.0012	0.240	–	–
India	0.0470	0.107	0.0470	0.547	–	–
Indonesia	0.0208	0.243	0.0208	0.261	–	–
Iran	0.0115	0.437	0.0115	0.095	–	–
Iraq	0.0076	0.668	0.0076	0.071	–	–
Japan	0.0151	0.334	0.0151	0.941	–	–
Jordan	0.0033	1.512	0.0033	0.727	–	–
Kazakhstan	0.0054	0.939	0.0054	0.072	–	–
Kiribati	0.0004	11.918	0.0004	0.019	–	–
Kuwait	0.0022	2.273	0.0022	0.434	–	–
Kyrgyzstan	0.0031	1.628	0.0031	0.022	–	–
Laos	0.0033	1.510	0.0033	0.020	–	–
Lebanon	0.0028	1.829	0.0028	0.042	–	–
Malaysia	0.0072	0.706	0.0072	0.938	–	–
Maldives	0.0008	6.691	0.0008	0.054	–	–
Marshall Islands	0.0003	16.176	0.0003	0.016	–	–
Micronesia (Federated States of)	0.0004	11.284	0.0004	0.013	–	–
Mongolia	0.0022	2.265	0.0022	0.043	–	–
Myanmar	0.0093	0.543	0.0093	0.089	–	–
Nauru	0.0001	37.133	0.0001	0.015	–	–
Nepal	0.0073	0.687	0.0073	0.118	–	–

Oman	0.0022	2.254	0.0022	0.232	–	–
Pakistan	0.0177	0.285	0.0177	0.667	–	–
Palau	0.0002	26.282	0.0002	0.026	–	–
Papua New Guinea	0.0035	1.436	0.0035	0.089	–	–
Philippines	0.0130	0.389	0.0130	0.178	–	–
Qatar	0.0018	2.835	0.0018	0.397	–	–
Republic of Korea	0.0093	0.542	0.0093	0.678	–	–
Samoa	0.0006	8.788	0.0006	0.009	–	–
Saudi Arabia	0.0070	0.718	0.0070	0.139	–	–
Singapore	0.0030	1.667	0.0030	0.974	–	–
Solomon Islands	0.0010	5.126	0.0010	0.046	–	–
Sri Lanka	0.0061	0.823	0.0061	0.278	–	–
Syrian Arab Republic	0.0061	0.832	0.0061	0.044	–	–
Tajikistan	0.0035	1.434	0.0035	0.020	–	–
Thailand	0.0112	0.452	0.0112	0.241	–	–
Timor Leste	0.0014	3.546	0.0014	0.022	–	–
Tonga	0.0004	11.657	0.0004	0.027	–	–
Turkmenistan	0.0030	1.675	0.0030	0.029	–	–
Tuvalu	0.0001	37.933	0.0001	0.022	–	–
United Arab Emirates	0.0037	1.372	0.0037	0.576	–	–
Uzbekistan	0.0070	0.718	0.0070	0.034	–	–
Vanuatu	0.0007	7.681	0.0007	0.014	–	–
Vietnam	0.0126	0.401	0.0126	0.042	–	–
Yemen	0.0066	0.767	0.0066	0.101	–	–
EE	–	–	–	–	0.093	1.613
Albania	0.0024	2.101	0.0024	0.160	–	–
Armenia	0.0024	2.138	0.0024	0.086	–	–
Azerbaijan	0.0041	1.241	0.0041	0.267	–	–
Belarus	0.0042	1.214	0.0042	0.375	–	–
Bosnia and Herzegovina	0.0026	1.939	0.0026	0.113	–	–
Bulgaria	0.0037	1.374	0.0037	0.332	–	–
Croatia	0.0028	1.792	0.0028	0.259	–	–
Czech Republic	0.0043	1.161	0.0043	0.727	–	–
Estonia	0.0016	3.247	0.0016	0.080	–	–
Georgia	0.0028	1.802	0.0028	0.173	–	–
Hungary	0.0042	1.190	0.0042	0.821	–	–
Latvia	0.0020	2.506	0.0020	0.110	–	–
Lithuania	0.0024	2.063	0.0024	0.152	–	–
Montenegro	0.0011	4.732	0.0011	0.032	–	–
Poland	0.0083	0.608	0.0083	1.767	–	–
Republic of Moldova	0.0025	1.989	0.0025	0.198	–	–
Romania	0.0062	0.811	0.0062	1.034	–	–
Russian Federation	0.0160	0.641	0.0160	6.292	–	–
Serbia	0.0036	1.386	0.0036	0.380	–	–
Slovakia	0.0031	1.609	0.0031	0.289	–	–

Slovenia	0.0019	2.639	0.0019	0.169	–	–
TFYR Macedonia	0.0019	2.620	0.0019	0.108	–	–
Ukraine	0.0090	0.558	0.0090	1.299	–	–
GRULAC	–	–	–	–	0.130	0.760
Antigua and Barbuda	0.0004	12.625	0.0004	0.047	–	–
Argentina	0.0085	0.592	0.0085	1.174	–	–
Bahamas	0.0008	6.422	0.0008	0.318	–	–
Barbados	0.0007	7.193	0.0007	0.119	–	–
Belize	0.0007	6.736	0.0007	0.130	–	–
Bolivia	0.0042	1.193	0.0042	0.126	–	–
Brazil	0.0187	0.269	0.0187	1.237	–	–
Chile	0.0056	0.909	0.0056	1.064	–	–
Colombia	0.0091	0.553	0.0091	0.695	–	–
Costa Rica	0.0029	1.742	0.0029	0.301	–	–
Cuba	0.0045	1.121	0.0045	0.335	–	–
Dominica	0.0003	14.446	0.0003	0.006	–	–
Dominican Republic	0.0042	1.193	0.0042	0.281	–	–
Ecuador	0.0051	0.989	0.0051	0.508	–	–
El Salvador	0.0033	1.511	0.0033	0.077	–	–
Grenada	0.0004	11.633	0.0004	0.007	–	–
Guatemala	0.0051	0.991	0.0051	0.165	–	–
Guyana	0.0012	4.329	0.0012	0.562	–	–
Haiti	0.0042	1.190	0.0042	0.006	–	–
Honduras	0.0037	1.364	0.0037	0.284	–	–
Jamaica	0.0022	2.271	0.0022	0.260	–	–
Mexico	0.0143	0.353	0.0143	1.273	–	–
Nicaragua	0.0032	1.563	0.0032	0.165	–	–
Panama	0.0025	2.005	0.0025	0.292	–	–
Paraguay	0.0034	1.480	0.0034	0.176	–	–
Peru	0.0072	0.697	0.0072	0.503	–	–
Saint Kitts and Nevis	0.0003	16.427	0.0003	0.006	–	–
Saint Lucia	0.0006	9.008	0.0006	0.034	–	–
Saint Vincent and the Grenadines	0.0004	11.372	0.0004	0.011	–	–
Suriname	0.0010	5.192	0.0010	0.099	–	–
Trinidad and Tobago	0.0016	3.247	0.0016	0.419	–	–
Uruguay	0.0025	2.049	0.0025	0.813	–	–
Venezuela	0.0072	0.699	0.0072	2.294	–	–
WEOG	–	–	–	–	0.155	2.592
Andorra	0.0004	12.908	0.0004	0.020	–	–
Australia	0.0063	0.797	0.0063	0.254	–	–
Austria	0.0039	1.298	0.0039	2.033	–	–
Belgium	0.0044	1.149	0.0044	0.638	–	–
Canada	0.0078	0.645	0.0078	0.570	–	–
Denmark	0.0032	1.596	0.0032	0.920	–	–

Finland	0.0031	1.624	0.0031	0.750	–	–
France	0.0108	0.953	0.0108	9.351	–	–
Germany	0.0122	0.415	0.0122	1.652	–	–
Greece	0.0045	1.116	0.0045	0.149	–	–
Iceland	0.0008	6.646	0.0008	0.008	–	–
Ireland	0.0028	1.779	0.0028	1.088	–	–
Israel	0.0037	1.381	0.0037	0.022	–	–
Italy	0.0104	0.483	0.0104	1.101	–	–
Liechtenstein	0.0003	19.810	0.0003	0.008	–	–
Luxembourg	0.0010	5.279	0.0010	0.032	–	–
Malta	0.0009	5.827	0.0009	0.059	–	–
Monaco	0.0003	19.984	0.0003	0.051	–	–
Netherlands	0.0055	0.923	0.0055	1.001	–	–
New Zealand	0.0028	1.799	0.0028	0.472	–	–
Norway	0.0030	1.702	0.0030	0.720	–	–
Portugal	0.0044	1.151	0.0044	0.508	–	–
San Marino	0.0002	21.176	0.0002	0.021	–	–
Spain	0.0091	0.554	0.0091	0.921	–	–
Sweden	0.0041	1.228	0.0041	0.926	–	–
Switzerland	0.0037	1.358	0.0037	0.878	–	–
Turkey	0.0114	0.441	0.0114	1.299	–	–
United Kingdom	0.0106	0.973	0.0106	9.552	–	–
United States of America	0.0236	0.435	0.0236	4.270	–	–