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# Optimal Fiscal Policy

## Abstract

This paper derives and estimates rules for fiscal policy that prescribe the optimal response to changes in unemployment and debt. We combine the reduced-form model of the economy from a linear VAR with a non-linear welfare function and obtain analytic solutions for optimal policy. The variables in our reduced-form model – growth, unemployment, primary surplus – have a natural rate that cannot be affected by policy. Policy can only reduce fluctuations around these natural rates. Our welfare function contains future GDP and unemployment, the relative weights of which determine the optimal response. The optimal policy rule demands an immediate and large policy response that is procyclical to growth shocks and countercyclical to unemployment shocks. This result holds true when the weight of unemployment in the welfare function is reduced to zero. The rule currently followed by policy makers responds procyclically to both growth and unemployment shocks, and does so much slower than the optimal rule, leading to significant welfare losses.

JEL-Code: E600, H600.

Keywords: optimal control, optimal policy, fiscal policy rules, fiscal consolidation, debt sustainability.

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# 1 Introduction

The demise of Lehman Brothers and the subsequent financial crisis have led to problems in the public finances of many countries. The fall in GDP reduced tax revenues, the financial crisis turned private debt into public debt, and countries implemented fiscal packages to boost effective demand. The immediate danger of a depression has been averted. Now, focus has turned to the question of restoring equilibrium in the budget. On the one hand, postponing austerity measures for too long jeopardizes a country's reputation in financial markets and shifts too much of the burden of the crisis to future generations. On the other hand, a too-early consolidation might slow down the recovery. Although the literature has provided substantial evidence on fiscal multipliers (e.g. Auerbach and Gorodnichenko (2012)), this research does not address the question of what might be the appropriate speed of fiscal consolidation.

Since there is substantial controversy about how to model business cycle fluctuations, this question is not easily addressed. This paper chooses a pragmatic approach, remaining as close as possible to empirical regularities. We apply the estimation results for a VAR model of potential GDP growth, unemployment and the primary surplus, adding a policy variable measuring discretionary fiscal interventions. This VAR model is supplemented by a linearized version of the equation for the debt dynamics and a quadratic intertemporal welfare function with net discounted value of future GDP and unemployment as its arguments. The problem is to find the time path of the fiscal policy variable that maximizes welfare, subject to the constraints imposed by the coefficient estimates of the VAR model. An analysis of optimal policy rules requires non-linear model responses to changes in the policy variable. In a linear model, a policy is either good or bad, which means that the optimal policy would not be an interior solution. What keeps our model tractable is that we construct the variables in the VAR in such a way that its linearity is maintained and all non-linearities are relegated to the welfare function. The advantage of this simple approach is twofold. First, we have set up this model along the lines of a linear-quadratic optimal control problem (Chow, 1975), which allows us to derive a closed-form solution for the optimal policy given the structure of the economy.<sup>1</sup> This extends the traditional use of VARs on fiscal policy from estimation of fiscal multipliers<sup>2</sup> and fiscal reaction functions<sup>3</sup> to normative prescriptions. Second, by sticking as close as possible to estimation results for a fairly standard empirical VAR model we avoid having to take a stance in the theoretical controversies, for example, on the kind of frictions and shocks to enter in the models (Chari *et al.*, 2009). The VAR model considered in the paper is about the minimum needed to capture the main features of business cycle fluctuations.

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<sup>1</sup>For a recent application of this method on monetary policy, see Sack (2000); Martin and Salmon (1999); Polito and Wickens (2012).

<sup>2</sup>See, for example, Blanchard and Perotti (2002); Corsetti *et al.* (2012); Auerbach and Gorodnichenko (2012); Romer and Romer (2010).

<sup>3</sup>See, for example, Bohn (2008).

We consider two non-linearities. The first is the capacity constraint on labour supply, or equivalently, the non-negativity constraint on unemployment. Auerbach and Gorodnichenko (2012) show indeed that fiscal multipliers vary over the cycle. They are low when unemployment is low, and hence there is little spare capacity to increase output. The second non-linearity is the effect of the debt-to-GDP ratio on the growth of GDP. Beyond a certain threshold, investors cease making investments out of fear that the rewards of their investment will be taxed away to service the public debt, thereby further eroding the state's capacity for repayment (see e.g. Reinhart and Rogoff (2010) and Checherita-Westphal and Rother (2012)). The VAR model is characterized by natural rates for potential growth and unemployment, which cannot be affected by changes in the fiscal policy rule. The only thing fiscal policy can achieve is a reduction of the impact of shocks. The non-linearities in the welfare function determine the pay-off to this type of stabilization. The optimal policy strikes a balance between stabilizing unemployment and stabilizing the debt-to-GDP ratio. For low debt and high unemployment, it is welfare-improving to stimulate the economy, as the short-term gains of additional growth and less unemployment outweigh the long-term loss of a higher debt burden. For high debt and low unemployment, the situation is reversed.

The VAR model is estimated using panel data for 17 OECD countries. For fiscal policy, we use data recently collected by the IMF on episodes of fiscal consolidation (Devries *et al.*, 2011). These data are collected from policy documents describing intended consolidation plans.

Hence, this variable does not suffer from the usual causality problem: is it a change in the deficit that causes a change in the economy, or do changes in the economy impact the deficit? Nevertheless, our approach is subject to both the Sims and the Lucas critique. Sims (1980) argues that statistical inference is hampered when the policy plan is correlated to the other (un)observed variables in the model: policy makers respond to information on the state of the economy. Policy is not set in a vacuum. Lucas (1976) carries this argument one step further. Not only is the economy not observed outside the path of the policy rule; a change in the rule will also change the structure of the economy itself. An incidental deviation from the rule reveals nothing about the effect of systematic deviation (that is, a change in the rule). The classical example is the Phillips curve. You can trick people into believing that inflation will be low, so that they accept lower nominal wages, thereby reducing unemployment. But you cannot "fool all of the people all of the time". People will form expectations and adjust their behaviour. The solution to both lines of critique has traditionally been sought in structural modelling, including a theory both for the policy maker's rule setting (countering Sims) and for the behavioural response of agents (countering Lucas). The drawback of this approach is that the structural model provides theoretical guidance about the behaviour of the economy for other policy rules, but we are unable to assess the empirical validity of this prediction. The researcher has imprisoned himself in the cage of his theory, from which there is no escape. Hence, we face a trade-off: either we take the critique by Sims and Lucas literally, thereby depriving ourselves of the possibility of a reality check of our

theoretical beliefs, or we are less strict in the interpretation of this critique, which allows us to test our beliefs empirically. We choose the latter, partly because it is unclear what exactly constitutes a proper "structural" model, and partly because we claim that our model is less sensitive to the Lucas critique, since it exhibits natural rates for growth and unemployment. Hence, by construction, the policy maker is unable to affect these rates by "by fooling all of the people all of the time".

The optimal policy rule that can be calculated from this exercise calls for an activist fiscal policy, with respect to both unemployment and the debt-to-GDP ratio. The optimal response resembles Blanchard and Quah (1989)'s distinction between permanent shocks to GDP and transitory shocks, which mainly affect unemployment. These responses are far more aggressive than the empirical rule that we observe in the data, for both unemployment and public debt. These responses remain when we alter the welfare function such that the policy maker ignores unemployment and focusses solely on maximizing growth. In fact, we can only get to a policy rule that comes anywhere near what we observe empirically when we apply a high rate of time preference, 10% per year or even more. This suggests that the welfare function that is implicitly used by policy makers exhibits a similar high rate of time preference, or equivalently, that politicians have difficulty in committing themselves to policies that benefit future generations. The response to a shock to unemployment is a one-year fiscal stimulus, followed by neutral policy immediately afterwards. This eliminates the initial impact in two years' time, while it would take eight years to half the initial impact under the empirical policy rule. When the policy maker has less information and must use information from the year before last year to set policy for the current year, optimal policy requires a qualitatively similar response. After unemployment shocks, a larger response is warranted. The welfare loss of using the rule that is observed empirically rather than the optimal rule is substantial, 16.6% to 18.4% of GDP.<sup>4</sup> The welfare loss of using the optimal policy with a one-year delay is smaller, 0.5% to 1.3%, as is the welfare loss of a policy rule focussing solely on maximizing growth and ignoring unemployment, 3.1% to 10.3%.

This conclusion differs sharply from that of Lucas (1987), who considers an alternative non-linearity: namely, risk aversion. Fluctuations in GDP cause fluctuations in consumption, which risk-averse people do not like. Lucas shows that this non-linearity is too small to make stabilization of GDP an attractive policy goal to pursue. The lifetime welfare that agents are willing to give up for consumption smoothing is 0.008%. This result has generated a large literature. Barlevy's review (Barlevy, 2005) reports much higher welfare returns for stabilization, up to 6% of lifetime welfare.

The methodology laid out in the paper can easily be extended to larger VAR models that include other shocks. For example, we could include nominal shocks or monetary policy. As it stands, our model does not include these variables; its simple structure serves our illustrative purposes. This is not too much of a restriction

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<sup>4</sup>This implies that empirical policy is either inefficient or maximizes another welfare function (i.e. one with a higher discount rate).

when applied to the euro-area, where individual countries cannot apply monetary policy. The set-up of the paper is as follows. Section 2 derives a general model for determining optimal policy rules from a VAR. Section 3 applies this general model to fiscal policy. Section 4 discusses the identification of discretionary fiscal policy measures and the Lucas critique. In Section 5 we discuss the estimation results for the VAR model. Section 6 discusses the implications of the estimation result for the policy rules. Section 7 concludes.

## 2 Optimal policy with a VAR

Any analysis of optimal policy rules requires a non-linear model. In a linear model, the first-order condition for the policy variable would not depend on the policy variable itself. Hence, there would not be an interior solution for the optimal policy. We apply the simplest form of non-linearity, the parabola. This functional form can be interpreted as a second-order Taylor approximation of a more general non-linear function. Typically, empirical studies have difficulty establishing the magnitudes of higher-order effects, so ignoring higher than second-order effects is unlikely to be an important limitation. Moreover, the first-order conditions that go with a quadratic policy function are linear in the policy variable, which makes them easy to handle. We structure our analysis along the lines of a common linear-quadratic optimal control problem, as described in Chow (1975) and Ljungqvist and Sargent (2004).

Our analysis consists of two building blocks: (i) a VAR model of the economy, and (ii) an intertemporal quadratic welfare function. The optimal policy rule maximizes welfare. This approach enables us to provide an analytical characterization of the optimal policy rule. Consider a reduced-form VAR-type model with one lag:

$$\begin{aligned} z_{t+1} &= a_{0z} + A_z z_t + a_z f_t + v_{z,t+1}, \\ \text{Cov}[v_{z,t+1}] &= V, \end{aligned} \tag{1}$$

where  $z_t$  contains stationary variables describing the economy and  $f_t$  is the discretionary policy variable. We assume that  $I - A_z$  is invertible. The vector  $v_{z,t+1}$  are i.i.d. shocks to the economy. The time subscript of the policy variable  $f_t$  denotes the moment  $t$  for which information is available to set policy  $f$ , not the moment of its effect on the economy. A policy maker knows  $v_{z,\tau}$ ,  $\tau \leq t$  when he decides on  $f_t$ . This notation turns out to be convenient. We stack  $z_t$  with  $d_t$ .  $d_t$  is a possibly non-stationary variable, of which the evolution is determined by an accounting equation:

$$d_{t+1} = a_{0d} + a'_d z_t + d_t. \tag{2}$$

In adding an accounting equation to the VAR we deviate from the optimal control literature. Stacking equations (1) and (2) yields:

$$\begin{aligned} x_{t+1} &= a_0 + A x_t + a f_t + v_{t+1}, \\ x_t \equiv \begin{bmatrix} z_t \\ d_t \end{bmatrix}, \quad a_0 &\equiv \begin{bmatrix} a_{0z} \\ a_{0d} \end{bmatrix}, \quad a \equiv \begin{bmatrix} a_z \\ 0 \end{bmatrix}, \quad A \equiv \begin{bmatrix} A_z & 0 \\ a'_d & 1 \end{bmatrix}, \quad v_{t+1} \equiv \begin{bmatrix} v_{z,t+1} \\ 0 \end{bmatrix}. \end{aligned} \tag{3}$$

The second building block is the intertemporal welfare function. The pay-off in a period is defined as the weighted sum of a non-stationary variable  $y_t$  and a quadratic function of  $x_t$ . Welfare is equal to the net discounted value of the expected pay-offs in every year:

$$\begin{aligned} W_t &= \mathbb{E}_t \left[ \sum_{s=t}^{\infty} (1 + \beta)^{t-s-1} \left( y_s + \theta' x_s + \frac{1}{2} x_s' \Theta x_s \right) \right] \Rightarrow \\ \beta W_t &= y_t + \theta' x_t + \frac{1}{2} x_t' \Theta x_t + \mathbb{E}_t[W_{t+1}] - W_t, \end{aligned} \quad (4)$$

where  $\beta > 0$  is the rate of time preference. The parameters  $\theta$  and  $\Theta$  are the weights of  $x_t$  and  $x_t^2$  in the welfare function. The relation between  $y_t$  and  $x_t$  satisfies:

$$\Delta y_t \equiv \beta \left( h_0 + h' x_t + \frac{1}{2} x_t' H x_t \right). \quad (5)$$

The difference between the  $x_t$  terms directly in the welfare function and those determining  $\Delta y_t$  is that those in the welfare function impact welfare directly, whereas those in  $\Delta y_t$  impact welfare via their impact on  $y_t$ . We assume  $d_t$  impacts welfare quadratically negative.

Equations (3) and (4) imply that the expected future evolution of the economy is fully summarized by  $x_t$  and  $y_t$  and future values of the policy variable  $f_t$ . Since policy makers do not have information on future realisations of  $v_t$ ,  $f_t$  can depend only on past and current realisations of  $x_t$  and  $y_t$ . It will turn out to be convenient to consider the case where  $f_t$  is a linear function of  $x_t$ :

$$f_t = b_0 + b' x_t, \quad (6)$$

$$x_{t+1} = c_0 + C x_t + v_{t+1}, \quad (7)$$

$$\text{with } c_0 \equiv a_0 + a b_0, \quad C \equiv A + a b'.$$

We shall refer to this class of rules as linear policy rules. This relation can be used to eliminate future policy from expected welfare and from the reduced-form model of the economy, equation (3). Hence, expected welfare under a linear policy rule depends solely on the state variables  $x_t$  and  $y_t$ . Let  $W(x_t, y_t) \equiv W_t$  be the expected welfare conditional on the information available at time  $t$ . Proposition 1 characterizes the function  $W(x_t, y_t)$  for any linear policy rule  $f_t$ .

**Proposition 1** (Welfare under a linear policy rule). *For any linear policy rule, welfare  $W(x_t, y_t)$  is a linear function in  $y_t$  and a quadratic function in  $x_t$ :*

$$W(x_t, y_t) = w_0 + w_y y_t + w' x_t + \frac{1}{2} x_t' W x_t, \quad (8)$$

with  $w_0$ ,  $w_y$ ,  $w$  and  $W$  functions of  $a_0$ ,  $A$ ,  $a$ ,  $b_0$ ,  $b$  and the parameters  $\beta$ ,  $h_0$ ,  $h$ ,  $H$ ,  $\theta$  and  $\Theta$ .

The proofs of all propositions and explicit expressions are in Appendix A. If a steady-state solution for  $\bar{x}$  for  $x_t$  exists, it reads:

$$\bar{x} = (I - C)^{-1} c_0. \quad (9)$$

Not all linear policy rules yield a steady-state solution for  $\bar{x}$ , and not all steady-state solutions are stable. When variables with possible unit root behaviour are included in a VAR, the other variables should respond to lags of them in order to prevent non-stationarity (Favero and Giavazzi, 2007). In our set-up,  $z_{t+1}$  does not depend on  $d_t$  directly, which makes  $I - A$  not invertible. This means that discretionary policy  $f_t$  should respond to  $d_t$  to obtain a steady-state solution. Even then,  $d_t$  might explode. This happens when one of the eigenvalues of  $C$  is outside the unit circle. To avoid this, we impose the no-ponzi game condition that all eigenvalues are within the unit circle. A sustainable policy is a policy that satisfies this condition. In contemplating particular policy rules, we always check whether this condition is satisfied.

**Proposition 2** (Natural rate). *The steady state solution  $\bar{z}$  of any linear policy rule does not depend on the coefficients  $b_0$  and  $b$  of the policy rule.  $\bar{d}$  is a linear function of  $b_0$ .*

This is a remarkable result. The parameters  $b_0$  and  $b$  enter the expression for a steady state of  $\bar{x}$  (see equation (9)) in a complex non-linear way. It is not easy to see why these parameters would drop out of this expression. Technically, this is due to the functional form of equation (3) and the linearity of the policy rule. Economically, this implies that the policy rule has an effect on the short-run variations of  $z_t$  around its steady state, but not on the steady state itself. Whatever the policy, as long as it is sustainable, the  $z_t$  will always move back to the same long-run equilibrium  $\bar{z}$ .

**Proposition 3** (Optimal policy). *A policy rule that maximizes welfare over the entire state space  $\{x_t, y_t\}$  exists. This policy rule  $f_t^* = b_0^* + b^* x_t$  is indeed linear.  $b^*$  can be determined independently of  $b_0^*$  and depends on the preferences for stabilization and the effect of policy on  $x_t$ .  $b_0^*$  determines  $\bar{d}$  and weighs the welfare gains of short-run discretionary policy against its long-run losses.*

Superscript \* denotes the value of an object under the optimal policy. In the Appendix we explain how to obtain the optimal policy rule. We consider two extensions of the optimal policy rule. These extensions turn out to be directly related. First, suppose that instead of the VAR model in equation (3) we need a VAR model with more than one lag. What would be the implications of that extension for the previous analysis? The answer is straightforward: none. The reason is that a system of second-order difference equations of dimension  $N$  can be rewritten as a system of first-order difference equations of dimension  $2N$ . Since none of the steps in the proofs of Propositions 1 to 3 depend on the dimension of the matrix  $A$ , these propositions apply mutatis mutandis to the case where the VAR model has two lags.

The second extension considers the case where the policy maker must condition its fiscal policy on information of  $x_t$  of two periods ago instead of one. Hence,



the policy  $f_t$  that conditions on information on  $x_t$  does not affect  $x_{t+1}$  but only  $x_{t+2}$ . This is a more realistic appraisal of the effectiveness of economic policy. This assumption implies that equation (3) should be rewritten as

$$x_{t+1} = a_0 + Ax_t + af_{t-1} + \nu_{t+1}, \quad (10)$$

and the linear policy rule in equation (6) as

$$f_t = b_0 + b'x_t + b_f f_{t-1}. \quad (11)$$

Let symbol  $\tilde{\phantom{x}}$  on top of a variable refer to the extension of that variable for time  $t$  with a row (and column, in case of a matrix) for  $f_{t-1}$ .

$$\begin{aligned} \tilde{x}_{t+1} &= \tilde{c}_0 + \tilde{C}\tilde{x}_t + \tilde{v}_{t+1}, \\ \tilde{x}_t &\equiv \begin{bmatrix} x_t \\ f_{t-1} \end{bmatrix}, \quad \tilde{c}_0 \equiv \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}, \quad \tilde{C} \equiv \begin{bmatrix} A & a \\ b' & b_f \end{bmatrix}, \quad \tilde{v}_t \equiv \begin{bmatrix} \nu_t \\ 0 \end{bmatrix}. \end{aligned} \quad (12)$$

We shall refer to the class of linear policy rules governed by equations (10) and (11) as delayed linear policy rules. Equation (12) is analogous to equation (7); hence, Proposition 1 applies. Furthermore, the steady states are subject to the same natural rate and optimal delayed policy rules can be derived.

**Proposition 4** (Natural rate with lagged information). *The steady-state solution  $\bar{z}$  of any delayed linear policy rule does not depend on the coefficients  $b_0$ ,  $b$  and  $b_f$  of the policy rule and is equal to the steady-state solution from Proposition (2).*

**Proposition 5** (Optimal policy based on lagged information). *Policy conditional on delayed information that maximizes welfare over the entire state space  $\{x_t, y_t\}$  exists, and its policy rule  $f_t^+ = b_0^+ + b^+x_t + b_f^+ f_{t-1}$  is linear (superscript + denotes the value of these objects under the optimal delayed policy).*

Welfare can also be determined for alternative policy rules as long as they are sustainable. The following proposition is useful for comparing the effect of these alternative rules on welfare.

**Proposition 6** (Welfare in the steady state). *For any linear policy with a steady state, welfare in the steady state depends only on current  $y_t$ , the steady-state value  $\bar{d}$  and the matrix of second-order derivatives  $G = W + H$  via its interactions with the covariance matrix  $V$ ,  $\frac{1}{2}\iota'V * G\iota$  (here  $*$  denotes element by element multiplication).*

Welfare losses can be determined by calculating welfare under different policies in the same state. Since  $b$  can be determined independently from  $b_0$ , and  $b_0$  effectively determines the steady-state level of  $d_t$ , we set this parameter such that it has the same steady state across policies. Since  $\bar{d}$  is the same across policies, the welfare comparison is fully determined by the interaction with the covariance matrix which represents its success in reducing volatility.

Two remarks are in order here. Not all simple policies are sustainable, and the steady state that maximizes steady-state welfare is not equal to the steady state of the optimal policy.

**Proposition 7** (Stabilizing a single variable is unsustainable). *Any linear policy rule that aims to stabilize only one element of  $z_t$  is not sustainable.*

The rationale is that such a policy disregards the effects that shocks on the other variables of  $z_t$  have on  $d_t$ . Welfare in the steady state is a function of  $\bar{d}$ ; hence, the level  $\bar{d}^{\text{ss}}$  that maximizes welfare in the steady state does so irrespective of the policy rule and solves  $dE_t [W(x_{t+1}, y_{t+1})] / d\bar{d}^{\text{ss}}$ , whereas optimal policy solves  $dE_t [W(x_{t+1}, y_{t+1})] / df^*$ .

### 3 Application to optimal fiscal policy

We apply our framework to fiscal policy, keeping our model as simple as possible while capturing the most important features of business cycle fluctuations. Our analysis includes two second-order effects. The first is the capacity constraint on labour: unemployment cannot be negative. Expansionary fiscal policy has a bigger effect on unemployment and GDP when there is spare capacity; that is, when the unemployment rate is high. In the limiting case of a zero unemployment rate, expansionary policy will have no impact at all on unemployment. We approximate this non-linearity by using the square root of unemployment.<sup>5</sup> The second non-linearity is the impact of government debt on GDP growth. Reinhart and Rogoff (2010) put this on the agenda by claiming that this effect is small for low values of the debt-to-GDP ratio, but jumps up when this ratio passes a threshold of 90% of GDP. At this value, government debt is reported to depress the growth of GDP by one percentage point. Our interpretation is that this effect captures debt overhang (i.e. investments decline because of expected future tax increases). Recent studies by Baum *et al.* (2013), Checherita-Westphal and Rother (2012) and Cecchetti *et al.* (2011) find evidence for non-linearities of similar magnitude, but different shape. We incorporate a non-linearity in the relation between GDP growth and the debt-to-GDP ratio by a second-order polynomial following Checherita-Westphal and Rother (2012).

The VAR model includes three endogenous variables: the potential growth rate of GDP  $g_t$ , the square root of the unemployment rate  $u_t$ , and the primary surplus of the government as a share of GDP  $s_t$ . It includes discretionary fiscal policy as the policy variable  $f_t$ . The core of this VAR, growth of GDP and unemployment, is the same as in Blanchard and Quah (1989). Having the growth rate instead of the level of GDP in the VAR has important implications for the welfare analysis: transitory shocks to the growth rate weigh heavily in welfare as they have permanent effects on the level of GDP. Equation (5) for the actual growth rate  $\Delta y_t$  reads:

$$\Delta y_t \equiv g_t - \frac{1}{2}\chi\Delta u_t^2 - \frac{1}{2}\psi d_t^2. \quad (13)$$

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<sup>5</sup>A logarithmic transformation would be the most adequate transformation to account for the non-negativity constraint. We nevertheless apply a square root, as its square -the unemployment rate- shows up as one of the arguments in the welfare function.

The first term is the potential growth rate, the second term is the business cycle effect or Okun's law, and the third term is the quadratic effect of the debt-to-GDP ratio on the growth rate. The parameters  $\chi$  and  $\psi$  can be estimated, interpreting the potential growth  $g_t$  as the residual of the equation. This equation is a cornerstone of our approach, as it allows us to use potential rather than actual growth in the VAR. This allows us to keep the VAR linear by "exporting" the non-linear effects of the capacity constraint on labour and the debt overhang effect to the welfare function, while at the same allowing for a simultaneous non-linear effect of  $f_t$  on unemployment  $u_{t+1}^2$  and  $\Delta y_{t+1}$ . Similar to Auerbach and Gorodnichenko (2012), the fiscal multipliers in our model vary with the state of the business cycle:

$$\begin{aligned}\frac{du_{t+1}^2}{df_t} &= \frac{du_{t+1}^2}{du_{t+1}} a_2 = 2u_{t+1} a_2, \\ \frac{d\Delta y_{t+1}}{df_t} &= a_1 - \frac{1}{2} \chi \frac{du_{t+1}^2}{du_{t+1}} a_2 = a_1 - \chi u_{t+1} a_2,\end{aligned}\tag{14}$$

where  $a_i$  is the  $i$ -th coefficient of the vector  $a$  in equation (3). Hence, this multiplier is higher for higher levels unemployment - that is, when the economy is in a recession. Note that we have an unexploited degree of freedom by our transformation from unemployment to square root of unemployment; we may subtract a constant level of unemployment to influence the curvature of  $d\Delta y_{t+1}/df_t$ . This transformation has no impact on equation (13).

$d_t$  is the public debt-to-GDP ratio at the beginning of the year. We linearize the accounting equation for the debt-to-GDP ratio:

$$d_{t+1} = a_{41} g_t - s_t + d_t,\tag{15}$$

and estimate the coefficient  $a_{41}$ .<sup>6</sup> Following equation (3), we stack the estimated VAR with the accounting equation (15) of debt. From the discussion below Proposition 1 follows that debt sustainability requires discretionary action by the policy maker.

Welfare in equation (4) reads:

$$\beta W_t = y_t - \frac{1}{2} \gamma u_t^2 + E_t[W_{t+1}] - W_t.\tag{16}$$

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<sup>6</sup> The accounting equation for the debt ratio reads:

$$d_{t+1} = \frac{1+r_t}{1+g_t} d_t - s_t \cong (1+r_t-g_t) d_t - s_t,$$

where  $r_t$  is the real interest rate. Suppose the interest rate is positively related to growth,  $r_t = \alpha_0 + \alpha g_t + v_t^r$ . Then,

$$\begin{aligned}(r_t - g_t) d_t &= [\alpha_0 - (1 - \alpha) d_t] g_t + \alpha d_t v_t^r \cong [\alpha_0 - (1 - \alpha) \bar{d}] g_t + \alpha \bar{d} v_t^r \Rightarrow \\ a_{41} &= \alpha_0 - (1 - \alpha) \bar{d}.\end{aligned}$$

We ignore the first-order effect of variations in  $d_t$ , since the difference between the real interest rate and the growth rate,  $r_t - g_t$ , is small relative to unity.

The parameter  $\frac{1}{2}\gamma$  measures the cost of unemployment in terms of log GDP.<sup>7</sup> For  $\gamma = 0$ , log GDP is the only relevant factor. This is the case of a clearing labour market where all unemployment is voluntary, so that the only cost of a recession is the loss in output. Unemployment per se does not yield additional cost. Evidence shows the contrary: people attach a high negative value to unemployment. Winkelmann and Winkelmann (1998) use panel data to show that people’s well-being is strongly affected by switching back and forth from employment to unemployment. Di Tella *et al.* (2001, 2003) use cross-section data to show that an increase in unemployment of 1% yields a decline in well-being of half a standard deviation of well-being among the total population. This evidence suggests  $\frac{1}{2}\gamma$  to be in the order of magnitude of 2.2.<sup>8</sup> Using log GDP as the argument in the welfare function implies that we account for risk aversion with a coefficient of relative risk aversion equal to unity, which is consistent with the coefficient derived from surveys on well-being (Gándelman and Hernández-Murillo, 2011). It is the lower bound of risk aversion usually reported.<sup>9</sup> However, the current value is the only value for which the model has an analytical solution. Since the welfare function allows for risk aversion, the optimal policy rule accounts for precaution. The discount rate  $\beta$  is assumed to be equal to 6.4%.<sup>10</sup>

The quadratic effect of the debt-to-GDP ratio on growth  $\frac{1}{2}\psi d_t^2$  is crucial. Without this term, the optimal policy rule would stabilize unemployment. Such a policy is unsustainable by Proposition 7. Fiscal policy is then used only to eliminate the effect of  $x_t$  on  $E_t[u_{t+1}]$ . Hence, the policy rule does not allow for a feedback mechanism to keep the debt-to-GDP ratio bounded. The only way to include an effect of the debt-to-GDP ratio in the policy rule is to make the marginal cost of debt increasing, which is what we do by introducing a quadratic effect. The quadratic effect of unemployment on welfare  $\frac{1}{2}\gamma u_t^2$  works more subtly. Without this effect, the policy maker will merely aim to maximize growth, which means stabilizing the debt-to-GDP ratio at a lower level.

<sup>7</sup>Thus  $h_0 = 0$ ,  $h = \beta^{-1}e_1$  and  $H = \beta^{-1}(\chi e_2 e'_2 - \chi e_6 e'_6 + \psi e_4 e'_4)$  by equation (13) and  $\theta = 0$  and  $\Theta = \gamma e_2 e'_2$  by equation (16).

<sup>8</sup>Di Tella *et al.* (2003) pp. 818-819: By column 1 of table 10 the well-being cost of 1% reduction of GDP per capita is  $0.01 * 1.41 * 0.7809 = 0.011$  and the well-being cost of 1 % unemployment increase is  $0.01 * (0.5 + 1.91) = 0.024$ . The cost of unemployment adds up the effect of a 1% unemployment on general well being, 0.5, and the personal effect of becoming unemployed for 1% of the population, 1.91.

<sup>9</sup>Empirical research indicates wide bounds (see Lanot *et al.* (2006) for a review): macro studies report coefficients up to 50, labour supply studies suggest a value of one, micro studies show coefficients between 1 and 2, while experimental research yields coefficients between 0.6 and 7.

<sup>10</sup>Frederick *et al.* (2002) show in their review that estimates of time discount rates permit negative as well as three-digit positive rates, and are highly dependent on the problem at hand, the time period and the elicitation method. We rely on Cameron and Gerdes (2005), who determined the discount rates for non-tradable durable goods, or public goods, from a large survey. They found significant heterogeneity, discount rates between 2% and 18% were accepted, with clustering around a mean of 6.4%.

The effect of a change in fiscal policy  $f_t$  on welfare in the steady state can be decomposed into four components:

$$\Delta W_t = \begin{pmatrix} \frac{1}{\beta} e'_1 & -\frac{\gamma}{1+\beta} \bar{u} e'_2 & -\frac{\chi}{\beta} \bar{u} (e'_2 - e'_6) & -\frac{\psi}{\beta} \bar{d} e'_4 \\ \text{growth} & \text{unemployment} & \text{Okun's law} & \text{debt} \end{pmatrix} Da\Delta f_t. \quad (17)$$

See equation (35) in the Appendix for the derivation.  $D \equiv \sum_{i=0}^{\infty} (1 + \beta)^{-i} C^i$ , so  $Da\Delta f_t$  is the net discounted value of today's shift in  $f_t$  on future values of  $x_t$ . The first component is the effect of a change in fiscal policy on potential growth. The second is the direct effect of unemployment on welfare. The third is the indirect effect of unemployment, via Okun's law: lower unemployment raises output. The final component is the indirect effect of a stimulus on GDP via debt. At the steady debt level of optimal policy, the marginal welfare costs of a change in fiscal policy  $\Delta f_t$  should equal marginal welfare benefits. Since the natural rate of  $\bar{z}$  is independent of the policy rule by Proposition 2, solving  $\Delta W_t = 0$  yields an expression for the steady-state debt level  $\bar{d}^*$  under optimal policy.

Since the welfare cost of unemployment is equal to  $\frac{1}{2}\gamma E[u_t^2] = \frac{1}{2}\gamma (\bar{u}^2 + \text{Var}[u_t])$ <sup>11</sup>, the welfare cost is increasing in the variance of the square root of unemployment. Similarly, the cost of debt to growth is  $\frac{1}{2}\psi/\beta (\bar{d}^2 + \text{Var}[d_t])$ . Since the natural rate of unemployment does not depend on the policy rule, and the steady-state debt-to-GDP ratio is fixed by the intercept of the policy rule  $b_0$ , the policy maker sets the parameters  $b$  of the policy rule to minimize the variance of the square root of unemployment and the debt level. Hence, the optimal policy faces a trade-off between stabilizing the square root of unemployment and stabilizing the debt-to-GDP ratio.

## 4 Can the model be estimated?

Correct estimation relies on identification of discretionary fiscal policy as the intentional plan of the policy maker as opposed to automatic changes in the budget from changing economic conditions. We identify discretionary fiscal policy using the narrative approach<sup>12</sup>, which employs additional information to determine causality. This allows us to distinguish the effects of fiscal policy on macroeconomic variables from the effects of macroeconomic variables on fiscal policy. Furthermore, in contrast to the real-time data<sup>13</sup> and restriction approach<sup>14</sup> fiscal policy can be identified

<sup>11</sup>The contribution of Okun's law to welfare is zero, since  $E[u_t^2 - u_{t-1}^2] = 0$ .

<sup>12</sup>Ramey and Shapiro (1998) pioneered the narrative approach, Romer and Romer (2010) and Ramey (2011) are major contributions.

<sup>13</sup>The real time data approach uses the difference between the information available on the state of the economy and the actual state of the economy measured ex post, to obtain exogenous shocks in fiscal variables. Auerbach and Gorodnichenko (2012) employ this method.

<sup>14</sup>The restriction approach imposes restrictions on the causal ordering of variables to obtain exogenous shocks in fiscal variables. Contributions by Giavazzi and Pagano (1990), Blanchard and Perotti (2002) and Perotti (1999) characterize this field, and Alesina and Ardagna (n.d.), Ilizetki *et al.* (2010) and Corsetti *et al.* (2012) are recent contributions using these methods.

ex ante with the narrative approach, which allows the policy maker to actively set policy.

Our approach is vulnerable to the Sims and the Lucas critique. Sims (1980) argues that statistical inference is hampered when the policy plan is correlated to the other (un)observed variables in the model. Policy makers respond to the state of the economy when deciding on the appropriate economic policy. The observed policy outcome is therefore not independent of the expected state of the economy, violating the assumption of the independence of  $f_t$  and  $v_t$  in equation (1). The feedback of the expected state of the business cycle on discretionary policy is not accounted for. In the extreme, if we assume that the policy maker already follows optimal policy, economic policy will be a linear function of the state variables  $x_t$ . Then, we will not be able to establish empirically the effect of deviations from the optimal policy rule, which means that, we will not be able to calculate the optimal policy rule itself. Hence, our ability to estimate the effect of deviations from the optimal policy rule depends on the assumption that historically the policy maker hasn't followed this rule in the first place. This cannot be due to informational constraints<sup>15</sup>, but should be because the policy maker does not know the optimal policy rule, faces constraints that force him to deviate from the optimal or makes occasional mistakes in implementation of the policy. Fortunately, the data suggest that  $f_t$  depended historically on  $x_t$  in a different fashion than optimal (compare the optimal response in Table 1 with the empirical response in Table 14).

The second line of critique by Lucas (1976) states that the structure of the economy itself depends on the policy rule that is applied. Within-policy-regime variations cannot be used for the identification shifts in the policy rule itself because occasional deviations from the policy rule leave the structure of the economy unaffected, while systematic changes do not. In terms of our notation: the vectors  $a_0$  and  $a$ , and the matrix  $A$  depend on the policy rule as captured by the coefficients  $b_0$  and  $b$ . Hence, variation in the policy  $f_t$  itself is to no avail for the estimation of the effect of a policy rule on the economy.<sup>16</sup> Only the variation in policy rule itself allows an assessment of the cost of sub-optimal rules. Since there is little variation in the policy rule, this is a serious constraint. The only solution here is to ignore this problem and to assume that the dependence of  $a_0$ ,  $a$ , and  $A$  on the actual policy rule is limited.

Whether or not this assumption is justified depends on the economic model underlying the relations embedded in  $a_0$ ,  $a$ , and  $A$ . The Lucas critique is particularly forceful when applied to the traditional Phillips curve. In this setting, the monetary policy maker took advantage of the money illusion to shift the natural rate of unemployment. Lucas argued that it is unlikely that agents will not adjust their inflationary expectations, especially since the policy maker does not act in their direct interests. Our model does not have this feature, as our variables converge to a

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<sup>15</sup>If the policy maker does not know  $v_t$ , the lagged policy rule is applicable and the same issue emerges for lagged policy.

<sup>16</sup>Fudenberg and Levine (2009) argue that ignoring this criticism may establish the actual policy rule that is applied (i.e. self-fulfilling prophecy) or it may point towards an equilibrium that is not there (i.e. Lucas critique on the Philips curve).

natural rate independent of the behaviour of the policy maker. Instead, the policy maker aims to impact agents' expectations of the second moments and does so in a way that aligns with the agents' interests.

Our approach departs from the traditional response of the literature to the Sims and Lucas critique, which includes a theory for the policy makers' rule setting (countering Sims) and the behavioural response of the agents (countering Lucas). Using a structural model with these features allows comparison of the theoretical impact of policy rules; it does not allow testing the empirical validity of these predictions. Recent critiques on the applicability of prevalent DSGE models for policy analysis stress this point (Chari *et al.*, 2009; Caballero, 2010).

## 5 Panel VAR estimation results

The theoretical framework developed in the previous sections is applied now to a sample of 17 OECD countries.<sup>17</sup> We use annual data from the the Ameco database of the European Commission from 1970 to 2009 on the real growth rate  $\Delta y_t$ , the square root of unemployment minus the country-specific minimum<sup>18</sup>  $u_{it} = \sqrt{\text{unemp}_{it} - \min(\text{unemp}_i)}$ , the primary surplus  $s_t$ ,<sup>19</sup> and the debt-to-GDP ratio  $d_t$ . The fiscal policy measure  $f_t$  is obtained from a recently published dataset of the IMF (Devries *et al.*, 2011) containing information on the size of discretionary fiscal consolidation as a percentage of GDP from from 1979 until 2009. As this measure describes only consolidation policies, and not fiscal expansions, it is equal to zero for years in which no fiscal consolidation takes place. For years in which fiscal consolidation does take place it is negative. Summary statistics of the data are displayed in Appendix B, as well as the results for the Im Pesaran Shin panel unit root test: we may reject the null hypothesis of a unit root for all variables, except for the debt-to-GDP ratio. Since we do not include the debt-to-GDP ratio in our estimation of equation (1), the stationarity of this variable is not relevant for the validity of the estimation results.

The estimation results for Okun's law and the effect of the debt-to-GDP ratio on potential growth, equation (13), read:

$$\Delta y_t = \begin{matrix} .0300 \\ [.0009] \end{matrix} - \begin{matrix} -1.405 \\ [.1715] \end{matrix} \Delta u_t^2 - \begin{matrix} .0104 \\ [.0017] \end{matrix} d_t^2 + v_t^g$$

with  $\sigma_g = .0078$ . The effect of Okun's law is 1.4, which squares well with the literature (see Ball *et al.* (2013), Lee (2000), Freeman (2001) and Balakrishnan *et al.*

<sup>17</sup>These are the countries for which Devries *et al.* (2011) provide data: Belgium, Germany, Finland, France, Ireland, Netherlands, Spain, Portugal, Italy, Austria, Sweden, Denmark, Canada, United Kingdom, United States, Australia and Japan.

<sup>18</sup>This specification determines the size of the state dependency of the unemployment and growth response of fiscal policy. Note that this is the maximum we can subtract from unemployment without having to deal with imaginary numbers.

<sup>19</sup>For the Netherlands some corrections are made for accounting problems, in particular, the privatization of public housing in 1995.

(2010)). A debt-to-GDP ratio of 100% reduces growth by 1.0%, which is also consistent with the literature (see Baum *et al.* (2013), Cecchetti *et al.* (2011), Reinhart and Rogoff (2010), Checherita-Westphal and Rother (2012)). Hence,  $\chi = 2.81$  and  $\psi = .021$ . We use this regression to construct series for potential growth  $g_t = 0.0300 + v_t^g$  that will be used for the estimation of the VAR.

We estimate the real interest rate  $r_t$  as a function of the potential growth rate:

$$r_t = \begin{matrix} .0368 \\ [.0016] \end{matrix} + \begin{matrix} .2323 \\ [.0475] \end{matrix} g_t + v_t^r,$$

with  $\sigma_r = .0118$ . Following footnote 6, this implies  $a_{41} = -.45$  with  $\bar{d} = 63\%$ . Direct estimation of equation (15) yields:

$$d_t = \begin{matrix} .0623 \\ [.0136] \end{matrix} - \begin{matrix} .7239 \\ [.1855] \end{matrix} g_{t-1} - \begin{matrix} .8661 \\ [.0955] \end{matrix} s_{t-1} + \begin{matrix} .9756 \\ [.0189] \end{matrix} d_{t-1} + v_t^d,$$

with  $\sigma_d = .0196$ , which does not differ significantly from the a priori specified theoretical relation (15) with  $a_{41} = -.45$  from footnote 6. We use the latter.

We estimate a panel VAR model as developed by Holtz-Eakin *et al.* (1988) and implemented in the Stata routine written by Love and Zicunno (2006) using structural GMM. Our variables are forward mean-differenced following Arellano and Bover (1995), because introducing regular country fixed effects biases our results. Mean differencing eliminates cross country differences in the structural growth rate of GDP and the natural rate of unemployment. The AIC criterion indicates we should use two lags, while the SIC criterion indicates one (see Table 6 in Appendix C). The differences are small, however. We use two lags.

The estimation results are shown in Table 7 in Appendix C. The policy variable  $f_t$  only has a significant direct impact on unemployment. Announcing fiscal consolidation leads to higher unemployment. Hence, consolidation has no immediate effect on the primary surplus. There are two potential explanations for this finding. First, planned spending cuts might not be fully implemented in the same year. Apparently the timing is such that the effects of stimulus on potential growth and the budget balance manifest only later through their effects on unemployment. Second, spending cuts reduce growth and hence tax revenues. Part of the effect of the cut therefore leaks away due to lower economic activity.

Since the steady state of  $z_t$  is independent of the policy rule (see Proposition 2), it can be calculated irrespective of the policy regime:  $\bar{g} = 2.4\%$ ,  $\bar{u}^2 + \min(\bar{\text{unemp}}) = 8.1\%$ , and  $\bar{s} = -0.4\%$ . These values fit our expectations. Application of equation (14) shows that the fiscal multiplier of GDP varies between 0.50 when the unemployment rate is 2% above its natural rate and 0.29 when the unemployment rate is 2% below its natural rate. These numbers are somewhat lower than those reported by Auerbach and Gorodnichenko (2012), which might be a result of our reliance on announced policy plans, as opposed to the realizations used by Auerbach and Gorodnichenko (2012). Announced consolidations are likely to be higher than their realizations, leading to an underestimation of the fiscal multiplier based on policy announcements. More importantly, the variation in fiscal multipliers between



good and bad states of the economy is lower than that reported in Auerbach and Gorodnichenko (2012). This suggests that the non-linearity introduced by using the square root is insufficient to capture the actual degree of non-linearity in the fiscal multiplier.

We ran a number of specification tests, which are reported in Appendix D. Separate VAR models are estimated for each country. Then, most coefficients become insignificant. However, most coefficients are within two standard deviations of the pooled VAR. A detailed comparison of the coefficients for individual countries shows the largest variation to be in the effect of discretionary fiscal policy on the surplus. We assess the effect of using the square root of unemployment rather than unemployment itself by estimating both versions of the model. There is no conclusive evidence in favour of the one or the other. The IMF data on fiscal policy register only episodes of fiscal consolidation. Hence, data on expansionary years are missing. The data can be interpreted as a censored version of the underlying fiscal policy variable. We use several approaches to assess the sensitivity of our estimation results to this censoring. The first approach is to limit the model to fiscal consolidation years. This leaves us with only 70 observations, reducing the significance of the coefficients, and changing some as well. A second approach is to include a dummy for the censored observations. This dummy would be a proxy for the average amount of expansion. This changes the results in two ways. The dummy seems to pick up periods where a stimulus has a positive impact on the budget balance, whereas our censored variable now picks up periods where a stimulus has a negative impact on the budget balance. A final approach is to apply a Tobit model. In this case, the effect of discretionary policy on the primary surplus does not change. The IMF data contain separate information on consolidation by tax increases and by cuts in spending. Distinguishing between these policies yields no new insights; the coefficients are approximately the same. One would expect the coefficients on fiscal policy for small open economies to be smaller than for large (and hence less open) economies, since part of the effect of fiscal policy leaks away to other countries. We use trade openness as a proxy for that. We reestimate the model including the openness variable and find no significant differences. Our results are robust for removing the financial crisis from the sample (2008, 2009).

## 6 Policy rules

The estimation results from Section 5 together with the parameter values of the welfare function from Section 3 are used to derive the policy rules. As a point of reference, we also estimate the policy rule that is actually applied in the data by regressing  $f_t$  on  $x_t$ . To obtain confidence intervals for the policy rules we bootstrap by resampling the observations. We estimate the Panel VAR and derive optimal policy rules for every iteration.<sup>20</sup> Table 15 gives the median value of  $\gamma$ ,  $\psi$ ,  $a_{41}$

<sup>20</sup>Our results are based on 10,000 successful iterations. Unsuccessful iterations, those with inconsistent policy rules, are discarded.

and the natural rate of growth, unemployment and primary surplus within their bootstrapped confidence interval. Table 16 in Appendix E shows the bootstrapped estimation results of equation (1). The results in both tables are close to those from Section 5.

Table 1: Policy rules  $b$ ,  $b_f$  and  $b_0$  and the steady-state debt level  $\bar{d}$  under several policy regimes.

	Optimal	.., delayed	Max growth	.., delayed	Empirical
g	0.23 [-0.04, 0.62]	-0.05 [-0.47, 0.42]	0.68 [0.29, 1.28]	0.41 [-0.07, 1.06]	0.09 [0.02, 0.16]
u	1.56 <sup>†</sup> [1.20, 2.20]	1.74 <sup>†</sup> [1.28, 2.58]	1.52 <sup>†</sup> [1.17, 2.13]	1.76 <sup>†</sup> [1.29, 2.58]	-0.14 [-0.18, -0.10]
s	0.29 [0.05, 0.61]	0.50 [0.18, 0.92]	0.63 <sup>†</sup> [0.31, 1.07]	0.84 <sup>†</sup> [0.47, 1.38]	-0.01 [-0.09, 0.08]
d	-0.13 <sup>†</sup> [-0.20, -0.08]	-0.13 <sup>†</sup> [-0.20, -0.08]	-0.24 <sup>†</sup> [-0.36, -0.16]	-0.24 <sup>†</sup> [-0.36, -0.16]	-0.01 <sup>†</sup> [-0.02, -0.01]
L.g	0.22 [0.07, 0.44]	0.20 [0.00, 0.47]	0.33 [0.15, 0.61]	0.31 [0.09, 0.64]	-0.02 [-0.09, 0.06]
L.u	-0.59 <sup>†</sup> [-0.88, -0.42]	-0.92 <sup>†</sup> [-1.45, -0.62]	-0.54 <sup>†</sup> [-0.83, -0.36]	-0.89 <sup>†</sup> [-1.42, -0.60]	0.09 <sup>†</sup> [0.05, 0.12]
L.s	0.33 [0.16, 0.51]	0.51 [0.27, 0.78]	0.29 [0.11, 0.49]	0.49 [0.24, 0.77]	0.23 <sup>†</sup> [0.16, 0.31]
L.f		-1.50 <sup>†</sup> [-1.66, -1.37]		-1.53 <sup>†</sup> [-1.69, -1.40]	
Const.	-0.24 <sup>†</sup> [-0.36, -0.15]	-0.21 <sup>†</sup> [-0.33, -0.11]	-0.42 <sup>†</sup> [-0.60, -0.28]	-0.39 <sup>†</sup> [-0.58, -0.26]	0.02 <sup>†</sup> [0.02, 0.02]
$\bar{d}$	-27% [-70%, 53%]	-27% [-70%, 53%]	-79% [-118%, -39%]	-79% [-118%, -39%]	119% <sup>†</sup> [101%, 146%]

The median bootstrap result is shown in normal font; below this, in smaller font, a 1 standard deviation confidence interval is shown between brackets. † denotes significance at the 5% level.

Table 1 presents the policy rules. The optimal policy rule depends positively on the potential growth rate, the square root of unemployment, and the primary surplus, and negatively on the debt-to-GDP ratio. The positive effect of potential growth might be somewhat surprising. However, Blanchard and Quah (1989)'s decomposition of shocks into transitory and permanent components provides an explanation. A shock is classified as transitory when its accumulated effect on GDP is equal to zero ( $\sum_{t=0}^{\infty} \Delta y_t = 0$ ), so that there is no long-run effect on GDP. All other shocks are classified as permanent. By this classification, transitory shocks are characterized by a strong negative correlation between growth and unemployment. A negative transitory shock justifies a fiscal stimulus. Permanent shocks (e.g. TFP shocks) have hardly any effect on unemployment. A positive permanent shock justifies a fiscal stimulus, simply because a country becomes richer. Hence, a pos-

itive shock to growth with hardly any effect on unemployment is an indicator of a permanent shock, which justifies a fiscal stimulus.

The effect of the debt-to-GDP ratio on the policy is large. This can be seen by realizing that a 1%-point increase in the debt-to-GDP ratio justifies 0.13%-point fiscal contraction. The effect of unemployment on the optimal policy is also large. Other things being equal, and starting from an average level of unemployment, a 1% increase in unemployment should lead to a 2.7% ( $= 1.56/2\bar{u}$ ) increase in discretionary policy. Due to the square root transformation, this effect is non-linear. Expansionary fiscal policy is therefore most effective for high levels of unemployment. All coefficients are much larger than for the empirical policy rule, which is reported in the last column of the table. This suggests that the actual fiscal policy is much less activist than optimal. When the policy maker can only use information on the shocks  $v_t$  of the year before when setting the fiscal policy of the next year (Optimal policy, delayed), the rule becomes somewhat less activist. A rule that aims to stabilize debt is unsustainable.<sup>21</sup> A rule that ignores the direct effect of unemployment on welfare,  $-\frac{1}{2}\gamma u_t^2$ , implies a stronger response to growth, primary surplus and debt relative to unemployment. Note that even though this policy rule stems from a welfare function without unemployment as an argument, the response to unemployment remains large, positive and significant.

At the steady-state debt level, welfare is unchanged for a marginal change in policy. Both the absence of a direct positive response of potential growth and a reduction in potential growth later on lead to a negative first-order effect of fiscal stimulus on welfare under the optimal policy. This is compensated by two positive second-order effects, which run via unemployment; a direct effect via the welfare contribution of reduced unemployment, and an indirect effect via the welfare contribution of increased output, as actual growth increases through Okun's law when unemployment decreases. The second-order contribution of stimulus on growth via an increase in the debt level is negative and dependent on the steady-state debt level. We can solve for the steady-state debt level by setting equation (17) to 0 and solving for  $\bar{d}$ :

$$0 = \begin{matrix} -0.90 \\ [-1.33, -0.52] \end{matrix} + \begin{matrix} 0.43 \\ [0.25, 0.69] \end{matrix} + \begin{matrix} 0.27 \\ [0.15, 0.44] \end{matrix} + \begin{matrix} -0.76 \\ [-1.06, -0.49] \end{matrix} \times \bar{d} .$$

Table 1 presents the steady-state level of the debt-to-GDP ratio  $\bar{d}$  for various policy rules. The optimal steady state debt level is not well determined by our analysis. Its median value at  $-27\%$  is not significantly different from zero.<sup>22</sup> The empirical steady-state debt level at  $119\%$ , however, is significantly different.

<sup>21</sup>At first this may seem counter-intuitive, however, a policy that completely stabilizes the debt level has to react with ever increasing magnitude to a small initial shock to off-set its own effect on debt in the next period.

<sup>22</sup>A slight change in the accounting equation for debt, setting  $a_{44} = 1 + \beta$  in equation (3), sets the steady-state debt level at  $0\%$  without altering the policy rules or welfare implications significantly. Unfortunately, for  $a_{44} \neq 1$  Propositions 2 and 4 no longer hold, even though the resulting difference in the steady state is limited.

Figures 5-8 show impulse responses of actual growth, unemployment, debt and discretionary policy after shocks to 'growth' and 'unemployment' under our five policy rules (optimal, optimal delayed, max growth, max growth delayed and empirical). We use generalized impulse response functions (Pesaran and Shin, 1998), which show the impulse responses to a shock in one of the reduced form VAR variables.<sup>23</sup> As no structural decomposition is imposed, these shocks cannot be identified. Nevertheless we will refer to them colloquially as 'growth' and 'unemployment' shocks as the diagonal elements in the covariance matrix dominate the non-diagonal elements. We find that a rise in unemployment should be countered by expansionary policy, and a fall in potential GDP by fiscal consolidation.

An average positive potential growth shock leads to an immediate increase in actual GDP of 1.2% and an increase in unemployment of 0.1%-points. All policy rules respond procyclically to growth shocks; the outcomes under different policies in terms of gdp, unemployment and government debt are qualitatively similar. The response under optimal policy is to implement a one-year fiscal stimulus of 1% of GDP, and to reverse 70% of it the year after. This reduces unemployment below its natural rate by 0.3%-points for a long time and leads to a fall in the debt ratio of 6.0%-points. In the years that follow, this leads to additional spending. The response of delayed policy is somewhat smaller in size and has a one-period delay, the response to a growth-maximizing policy is larger in size. The response under the empirical policy rule is similar to the optimal response, except for the initial stimulus of 1% in year 1 and its retraction in in year 2. This initial stimulus reduces the volatility in GDP-growth and unemployment.

An average positive unemployment shock leads to an immediate decrease in actual GDP of 1.5% and an increase in unemployment of 1.2%-points. The optimal and growth-maximizing response to unemployment is to implement a one-year fiscal stimulus of 4.2% of GDP, of which 1.8% is reversed the year after. The delayed policy response to unemployment shocks is qualitatively the same, but larger in size and delayed one period. The empirical policy rule, however, is pro-cyclical and small, and lasts several periods. Both optimal and optimal delayed policy eliminate the initial impact on unemployment of an unemployment shock in two and three years time, while this would take eight years in the empirical policy rule. This reduces volatility comes at a cost: under optimal policy the debt level increases by 1.8%-points, whereas it decreases under empirical policy by 7.3%.

In Table 2 we report welfare losses under alternative policy rules. As the optimal rule maximizes welfare for any point in the state space  $x_t$ , welfare associated with optimal policy should exceed the welfare of any other policy, provided that both policies are evaluated at the same point in the state space and are sustainable. We evaluate the welfare losses for the steady-state level of various policies as a percentage of log output. For this purpose,  $W_t$  is divided by  $\beta$ . The welfare costs of applying the empirical rather than the optimal policy rule are substantial. Across several points

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<sup>23</sup>The  $j$ -th shocktype is defined as  $E[v_t|v_{tj}] = \mathbf{v}_j^{-1/2}V\mathbf{e}_j$ , where  $V$  is the covariance matrix,  $\mathbf{v}_j$  the  $j$ -th diagonal element of  $V$  and  $\mathbf{e}_j$  the  $j$ -th column of the identity matrix.

Table 2: Welfare losses compared to optimal policy of applying an alternative policy rule, in percent of log output as a function of steady-state debt.

	-27% ( $\bar{d}^*$ )	0%	60%	119% ( $\bar{d}^e$ )
Optimal, delayed	0.5 [0.4, 0.7]	0.6 [0.5, 0.9]	0.9 [0.6, 1.3]	3.1 [1.3, 2.2]
Max growth	3.1 [1.3, 8.9]	3.9 [2.4, 6.4]	6.7 [4.2, 10.1]	10.3 [6.6, 15.9]
..., delayed	3.5 [1.8, 8.9]	4.4 [3.0, 6.8]	7.2 [5.2, 10.2]	11.1 [7.9, 16.0]
Empirical	16.6 [13.1, 43.8]	17.9 [9.7, 29.7]	17.2 [9.0, 30.0]	18.4 [8.5, 33.6]

The median bootstrap result is shown in normal font; below that, in a smaller font a one standard deviation confidence interval is shown between brackets.

in the state space, the median welfare differential ranges from 16.6% to 18.4% of GDP. This either indicates that the empirical policy rule optimizes another welfare function (i.e. it follows a political business cycle) or that there are substantial inefficiencies in the current policy making process (such as the procyclicality of the empirical policy rule with respect to unemployment shocks), which lead to these suboptimal outcomes. Papers that examine optimal monetary policy from a VAR using a linear-quadratic optimal control problem (Sack, 2000; Martin and Salmon, 1999; Polito and Wickens, 2012) also show optimal policy rules that differ markedly from the empirical ones. The cost of having to use the information on the state of the economy  $x_t$  of last year instead of the current year when setting the fiscal policy for next year is smaller, only about 0.5%-1.3% of GDP. Here we reproduce the standard argument against activist policy: policy responses are too slow to have a positive impact. The welfare cost of ignoring unemployment (maximizing growth) is bigger, and depend heavily on the steady-state debt level at which it is evaluated, 3.1%-10.3% of GDP.

Our specification is particularly sensitive to the choice of the discount rate  $\beta$ . We therefore run two robustness checks on the parameters, one with  $\beta^h = 1.5\beta = 9.6\%$  and one with  $\beta^l = 0.5\beta = 3.2\%$ . Tables 17 - 20 in the Appendix show the policy rules and the welfare losses under these assumptions. The debt level in the optimal steady state depends on the discount rate: for higher discount rates the steady-state debt level is higher. If the discount rate increases, the direct contribution of unemployment to the welfare function increases, just as the contribution via Okun's law; furthermore, the long-run effect of debt naturally decreases. This also implies that with increasing discount rates the welfare losses of delayed rules reduce, and those which maximize growth and ignore unemployment increase. Furthermore, with increasing discount rates the welfare losses of applying the empirical policy rule reduce as well, suggesting that policy makers may have higher discount rates than the economy in which they work.

## 7 Evaluation

Motivated by the question of what may be the optimal path of public finances towards equilibrium, we have derived a simple prescription for optimal fiscal policy. Our prescription shows how discretionary fiscal policy should respond to growth, unemployment and the primary surplus shocks. We represent the structure of the economy by a simple VAR model. The non-linear effects in the economy are captured in the aggregate welfare function. This aggregate welfare function is positive in log GDP and negative in unemployment. Furthermore, debt has an increasingly negative effect on GDP growth. We use the square root of unemployment to denote the capacity constraint of the economy together with Okun's law to ensure that fiscal multipliers depend on the state of the economy.

We find that there is a natural growth and unemployment rate, irrespective of the policy rule applied. This means that the effect of discretionary fiscal policy on growth and unemployment is temporary: the most that sustainable policy can aspire to is stabilizing these variables. In the long run both will converge toward their natural rate. Policy has, however, an effect on the debt level and via the debt level an effect on actual growth rates. Our model contains a trade-off regarding the steady-state debt level: If debt is below this level, the short-term gains of stimulating the economy outweigh the long-term losses of lower growth. If debt is above this level, it works the other way around.

Alternative policy prescriptions have lower aggregate welfare in the same state. The current empirical policy rule has 16.6% to 18.4% less lifetime welfare. The most pronounced difference is in the response to unemployment shocks. The empirical policy responds pro-cyclically, whereas the optimal policy responds anti-cyclically. If policy makers are only able to implement policy next year based on last year's information, the cumulative welfare loss is 0.5% to 1.3%. This motivates against multi-year budget plans without some space for discretionary fiscal policy. For a policy that focusses solely on maximizing growth the welfare losses depend on the steady-state debt level, ranging from 3.1% to 10.3%. Debt stabilization rules are unsustainable. Welfare losses are a function of the discount rate applied. The higher the discount rate, the lower the welfare loss under empirical policy and the higher the welfare loss of ignoring unemployment. This indicates that policy makers may have a higher discount rate than the economy they serve.

For explanatory reasons chose to use the simplest model specification that could capture business cycles and model the effects of fiscal policy explicitly. This means our conclusions are necessarily limited to cases where these apply. As nominal shocks and both monetary policy and financial conditions are absent (or assumed to be exogenous), our model could find an application in setting the path to sustainability of public finances in the EMU. At the same time, our model could be extended to include nominal shocks, monetary policy and financial conditions.

In the current crisis, our model argues for an unconventional policy approach. In 2008-2009 the European economies experienced substantial output losses, while unemployment remained unaffected at first. Our results suggest that policy should

have responded pro-cyclical to this 'permanent' loss in output, cutting budgets substantially immediately, and reversing this policy later on. From 2010 onward, growth went nearly flat, while unemployment started to increase. The optimal response here would have been a significant stimulus in 2011 and probably also in 2012.

Several issues remain unsolved. Our fiscal policy variable only contains information on fiscal consolidations, which could bias our results, even though we ran several robustness checks. Our VAR does not explicitly take into account the degree of openness of the economy. Beetsma and Giuliadori (2011) show that fiscal multipliers depend on the degree of openness with exogenous monetary policy. Including openness does not significantly alter our results. As described in Section 4 we are vulnerable to the Sims and the Lucas critiques. Regarding the Sims critique we simply assume that the policy maker does not currently follow optimal policy, which allows us to determine optimal policy. This assumption seems warranted by the data. Regarding the Lucas critique we assume that it is more difficult for agents to game a policy maker who aims to impact higher (instead of the first) moments of the distribution of economic outcomes, and that agents are less willing to game a policy maker who acts in their interest.

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## A Proofs

### Proposition 1: Welfare under a linear policy rule

Substitution of  $W_t = W(x_t, y_t)$  in equation (4) yields:

$$\beta W(x_t, y_t) = y_t + \theta' x_t + \frac{1}{2} x_t' \Theta x_t + E_t [W(x_{t+1}, y_{t+1})] - W(x_t, y_t), \quad (18)$$

where  $e_i$  is a  $i$ -th unit vector. We conjecture that  $W(x_t, y_t)$  is a second-order polynomial in the state variables  $x_t$  and a first-order polynomial in  $y_t$  as given by equation (8) with  $W$  a symmetric matrix. If this conjecture applies, equation (5) implies that  $E_t [W(x_{t+1}, y_{t+1})]$  satisfies:

$$\begin{aligned} E_t [W(x_{t+1}, y_{t+1})] &= W(x_t, y_t) \\ &\quad + E_t \left[ w_y \Delta y_{t+1} + w' \Delta x_{t+1} + x_t' W \Delta x_{t+1} + \frac{1}{2} \Delta x_{t+1}' W \Delta x_{t+1} \right] \\ &= W(x_t, y_t) + w_y \beta h_0 + (w_y \beta h' + w') E_t [x_{t+1}] \\ &\quad + \frac{1}{2} E_t [x_{t+1}' (W + w_y \beta H) x_{t+1}] - w' x_t - \frac{1}{2} x_t' W x_t. \end{aligned} \quad (19)$$

We substitute equation (19) in equation (18) and use  $E_t [x_{t+1}] = c_0 + C x_t$  (see equation (7)),  $E[v_{t+1}' W v_{t+1}] = \iota' V * W \iota$  (the operator  $*$  denotes the element wise multiplication  $K = L * M \Rightarrow \{k_{ij}\} = \{l_{ij}\} \{m_{ij}\}$ ) and  $\iota' = [1, 1..1]$ . This yields:

$$\begin{aligned} &\beta \left( w_0 + w_y y_t + w' x_t + \frac{1}{2} x_t' W x_t \right) \\ &= y_t + w_y \beta h_0 + \left( w_y \beta h' + w' + \frac{1}{2} c_0' (W + w_y \beta H) \right) c_0 + \frac{1}{2} \iota' V * (W + w_y \beta H) \iota \\ &\quad + \theta' x_t + w_y \beta h' C x_t + w' (C - I) x_t + c_0' (W + w_y \beta H) C x_t \\ &\quad + \frac{1}{2} x_t' \Theta x_t + \frac{1}{2} x_t' C' (W + w_y \beta H) C x_t - \frac{1}{2} x_t' W x_t. \end{aligned}$$

This relation should apply identically for all  $x_t$  and  $y_t$ . This yields an identical solution for the parameters of the polynomial (8), which confirms the conjecture. ■

For the parameters of (8) we find the following expressions:

$$w_y = \beta^{-1}, \quad (20)$$

$$\beta w_0 = h_0 + \left[ h' D + \theta' D + c_0' G \left( D - \frac{1}{2} I \right) \right] c_0 + \frac{1}{2} \iota' V * G \iota, \quad (21)$$

$$w = (D' - I) (h + G c_0 + \theta), \quad (22)$$

$$(1 + \beta) W = \Theta + C' G C, \quad (23)$$

with  $D \equiv \left[ I - \frac{C}{1+\beta} \right]^{-1} = \sum_{i=0}^{\infty} \left( \frac{C}{1+\beta} \right)^i$  and  $G \equiv W + H$ , respectively. For any sustainable policy,  $G$  contains the policy effects of the quadratic contributions to welfare. This can be seen by repeatedly substituting  $W$  from equation (23).

$$G = \sum_{i=0}^{\infty} \left( \frac{C'}{1+\beta} \right)^i \left( \frac{1}{1+\beta} \Theta + H \right) C^i. \quad (24)$$

**Proposition 2: Natural rate**

Define the matrices  $\bar{A}$ ,  $\bar{A}_z$ ,  $\bar{a}_d$ ,  $b_z$  and  $b_d$  as follows:

$$\bar{A} \equiv I - A \equiv \begin{bmatrix} \bar{A}_z & 0 \\ \bar{a}'_d & 0 \end{bmatrix}, \quad b \equiv \begin{bmatrix} b_z \\ b_d \end{bmatrix}.$$

$b_z$  denotes the policy response to  $z_t$  and  $b_d$  to  $d_t$ . Then define the inverse, with  $F$  having equal dimensions to  $\bar{A}_z$  as

$$(I - C)^{-1} \equiv \begin{bmatrix} F & g \\ h' & p \end{bmatrix}.$$

$I - C$  times  $(I - C)^{-1}$  yields the identity matrix:

$$\begin{bmatrix} (\bar{A}_z - a_z b'_z) F + a_z b'_d h' & (\bar{A}_z - a_z b'_z) g + a_z b'_d p \\ \bar{a}'_d F & \bar{a}'_d g \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \quad (25)$$

We shall show that  $F$  and  $g$  do not depend on  $b_z$ . Suppose  $b_z = \bar{a}_d$ . Since  $\bar{a}'_d F = 0$  and  $\bar{A}_z$  is invertible, the upper left equation of (25) simplifies to

$$F = \bar{A}_z^{-1} (I - a_z b'_d h').$$

Premultiplying this by  $\bar{a}'_d$  and some further algebra yields

$$b'_d h'_d = \alpha \bar{a}'_d \bar{A}_z^{-1}, \quad F = \bar{A}_z^{-1} (I - \alpha a_z \bar{a}'_d \bar{A}_z^{-1}), \quad (26)$$

where the suffix  $d$  for  $h$  denotes the solution for the specific case  $b_z = \bar{a}_d$  and  $\alpha \equiv (\bar{a}'_d \bar{A}_z^{-1} a_z)^{-1}$ . Premultiplying the upper right equation of (25) with  $\bar{a}'_d \bar{A}_z^{-1}$  and some algebra yields:

$$b'_d p_d = 1 - \alpha, \quad g = \alpha \bar{A}_z^{-1} a_z, \quad (27)$$

where the suffix  $d$  for  $p$  denotes the solution for the specific case  $b_z = \bar{a}_d$ .

Next, we derive the inverse for the general case. We conjecture that indeed  $F$  and  $g$  do not depend on  $b_z$ . Define:

$$b_{\Delta z} \equiv b_z - \bar{a}_d, \quad h_{\Delta} \equiv h - h_d \quad \text{and} \quad p_{\Delta} \equiv p - p_d.$$

Equation (25) applies for  $b_z = \bar{a}_d$ . Our conjecture implies that we only have to consider the marginal effect of  $b_z$  on  $h$  and  $p$ . Hence, if the conjecture applies, it must be true that

$$b'_d h'_{\Delta} = b'_{\Delta z} F \quad \text{and} \quad b'_d p_{\Delta} = b'_{\Delta z} g, \quad (28)$$

where  $F$  and  $g$  are given by equations (26) and (26). The equations show that a solution for  $h_{\Delta}$  and  $p_{\Delta}$  is available, proving the conjecture.

Next, we prove that  $\bar{z}$  does not depend on  $b$  and  $b_0$ . Expanding equation (9) yields:

$$\begin{pmatrix} \bar{z} \\ \bar{d} \end{pmatrix} = \begin{pmatrix} F & g \\ h' & p \end{pmatrix} (a_0 + ab_0).$$

The rows corresponding to  $\bar{z}$  of  $(I - C)^{-1}$  do not depend on  $b$  and  $b_0$ , so this proposition is proven if the elements of  $(I - C)^{-1}a$  corresponding to  $\bar{z}$  equal 0. Since  $a = \begin{bmatrix} a_z \\ 0 \end{bmatrix}$ , this requires  $Fa_z = 0$ . This follows immediately from equation (26).

Finally, from the expansion of equation (9) it follows that  $\bar{d}$  is a linear function of  $b_0$ . ■

### Proposition 3: Optimal policy

We conjecture that the optimal policy is linear in  $x_t$  and does not depend on  $y_t$ . In that case, Proposition 1 applies. The optimal policy  $f_t^*$  maximizes  $W(x_t, y_t)$  given by equation (18). Here, only expected future welfare  $E_t[W(x_{t+1}, y_{t+1})]$  is affected by policy, and since  $f_t$  enters  $E_t[W(x_{t+1}, y_{t+1})]$  via  $E_t[x_{t+1}]$  alone, the first-order condition reads:

$$\frac{dE_t[W(x_{t+1}, y_{t+1})]}{E_t[x_{t+1}]} \frac{E_t[x_{t+1}]}{df_t^*} = (h' + w^{*'} + E_t[x_{t+1}'] G^*) a = 0. \quad (29)$$

The first derivative stems from equation (19). Substituting  $E_t[x_{t+1}] = a_0 + Ax_t + af_t$  shows that the optimal policy  $f_t^*$  is a linear function of the state variables in  $x_t$ , which confirms the conjecture. The second-order condition requires:

$$a'G^*a < 0.$$

Substituting  $E_t[x_{t+1}] = c_0^* + C^*x_t$  in equation (29) disentangles the equation that determines  $b_0^*$  from the one that determines  $b^*$ :

$$(h' + w^{*'} + (a_0' + b_0^{*'} a') G^*) a = 0, \quad x_t' (A' + b^{*'} a') G^* a = 0, \quad (30)$$

with  $c_0^* = a_0 + ab_0^*$  and  $C^* = A + ab^{*'}$ .  $b^*$  is determined by the second part of equation (30), which holds for any  $x_t$ . Hence  $b^*$  is a function of the matrix  $G^*$ , which contains the interaction of policy with the quadratic welfare contributions. As neither the second part of equation (30) nor equation (24) contains any reference to  $b_0^*$ ,  $b^*$  can be determined independently of  $b_0^*$ .

When  $b^*$  is obtained, we get  $b_0^*$  by calculating optimal policy for some  $E_t[x_{t+1}]$ . We calculate optimal policy for the steady state. By Proposition 2  $b_0$  is a linear function of  $\bar{d}$ , so  $\bar{d}^*$  determines  $b_0^*$ . In equation (29) we substitute  $E_t[x_{t+1}] = \bar{x}^*$  and  $w^*$  from equation (22) and use  $c_0^* = (I - C^*)\bar{x}^*$  and the second part of equation (30) to obtain:

$$[h'D^* + \theta'(D^* - I) + \bar{x}^*(I - C^{*'})G^*D^*] a = 0 \quad (31)$$

Note that  $\bar{x}^{*'} = \begin{pmatrix} \bar{z}' & \bar{d}^{*'} \end{pmatrix}$ . As  $\bar{z}$  does not depend on the policy rule, a linear expression for  $\bar{d}^*$  can be derived. Under the optimal policy the level of  $d_t$  is adjusted until the short-run gains of discretionary policy balance achieved via temporary deviations of  $z_t$  from  $\bar{z}$  equal the long-run losses of a higher  $d_t$ . ■

The coefficients  $b_0^*$  and  $b^*$  are obtained by working out equation (30):

$$b_0^* = -(a'G^*a)^{-1} a' (D^{*'}h + D^{*'}G^*a_0 + (D^{*'} - I)\theta), \quad (32)$$

$$b^* = -(a'G^*a)^{-1} A'G^*a, \quad (33)$$

Substitution of  $C^* = A + ab^{*'}$  and equation (33) in equation (23) yields the Ricatti equations:

$$(1 + \beta)W^* = \Theta + A'G^*A - (a'G^*a)^{-1} A'G^*aa'G^*A. \quad (34)$$

Using  $G^* = W^* + H$ , the matrix  $W^*$  can be solved iteratively. We pick  $W_1$  and calculate  $W_{i+1}$  given  $W_i$  from equation (34) and continue until  $W_i$  converges. Having solved for  $W^*$ ,  $b^*$  follows from equation (33) and  $w_0^*$ ,  $w^*$  and  $b_0^*$  can be solved from equations (21), (22) and (32).

The Ricatti equations (34) allow for multiple solutions of  $b^*$ . The second-order condition and the sustainability requirement of all eigenvalues of  $C^*$  being smaller than unity may rule out some of these solutions. If multiple solutions remain, the correct rule can be selected by picking the one yielding the highest welfare  $W(x_t, y_t)$ .

To calculate steady-state debt levels, equation (31) can be simplified. We use:

$$\begin{aligned} (I - C^{*'})G^*D^*a &= (I - C^{*'}) [JD + C^{*'}G^*C^*/(1 + \beta)D^*] a \\ &= (I - C^{*'}) [JD + C^{*'}G^*(D^* - I)] a \\ &= (I - C^{*'}) [JD + C^{*'}G^*D^*] a \\ &= \dots = (I - C^{*'}) \sum_{i=1}^{\infty} C^{*i} JDa = JDa \end{aligned}$$

with  $J = \left(\frac{1}{1+\beta}\Theta + H\right)$ , where we substituted  $x_t C^{*'} G^* a = 0$  for all  $x_t$  from equation (30). This yields:

$$[h'D^* + \theta'(D^* - I) + \bar{x}^* J D^*] a = 0. \quad (35)$$

#### Proposition 4: Natural rate with lagged information

The structure of this proof is identical to that of Proposition 2. Following the definitions in equation (12) we define:

$$\begin{aligned} \bar{\bar{C}} = I - \tilde{C} &= \begin{pmatrix} \bar{A}_z & \mathbf{0} & a_z \\ \bar{a}'_d & \mathbf{0} & 0 \\ b'_z & \beta_d & \bar{b}_f \end{pmatrix}, \\ \bar{\bar{C}}^{-1} &= \begin{pmatrix} F & g & i \\ h' & \phi & \delta \\ k' & \psi & \gamma \end{pmatrix}, \end{aligned}$$

where  $\bar{A}_z$ ,  $\bar{a}_d$ ,  $a_z$ ,  $b_z$  and  $\beta_d$  have been defined as in Proposition 2. This yields the following nine equations:

$$\begin{aligned}
\bar{A}_z F + a_z k' &= I, \\
\bar{A}_z g + a_z \psi &= 0, \\
\bar{A}_z i + a_z \gamma &= 0, \\
\bar{a}'_d F &= 0, \\
\bar{a}'_d g &= 1, \\
\bar{a}'_d i &= 0, \\
b'_z F + \beta_d h' + \bar{b}_f k' &= 0, \\
b'_z g + \beta_d \phi + \bar{b}_f \psi &= 0, \\
b'_z i + \beta_d \delta + \bar{b}_f \gamma &= 1.
\end{aligned}$$

Combining the first, fourth and seventh equation, the second, fifth and eighth equation and the third, sixth and ninth equation yields:

$$\begin{aligned}
F &= \bar{A}_z^{-1} \left( I - \alpha a_z \bar{a}'_d \bar{A}_z^{-1} \right), \\
g &= \alpha \bar{A}_z^{-1} a_z, \\
i &= 0.
\end{aligned}$$

With  $\tilde{c}'_0 = (a'_0, b'_0)$ , this proves the proposition. ■

### Proposition 5: Optimal policy based on lagged information

We conjecture the optimal policy  $f_t^+$  to be a linear function of  $x_t$  and  $f_{t-1}^+$  as in equation (11). In that case Proposition 1 applies and welfare is given by:

$$\tilde{W}(\tilde{x}_t, y_t) = w_0 + w_y y_t + \tilde{w}' \tilde{x}_t + \frac{1}{2} \tilde{x}'_t \tilde{W} \tilde{x}_t.$$

$E_t [W(\tilde{x}_{t+1}, y_{t+1})]$  satisfies:

$$\begin{aligned}
E_t [\tilde{W}(\tilde{x}_{t+1}, y_{t+1})] &= \tilde{W}(\tilde{x}_t, y_t) + w_y \beta h_0 + (w_y \beta \tilde{h}' + \tilde{w}') E_t [\tilde{x}_{t+1}] + \\
&\quad \frac{1}{2} E_t [\tilde{x}'_{t+1} (\tilde{W} + w_y \beta \tilde{H}) \tilde{x}_{t+1}] - \tilde{w}' \tilde{x}_t - \frac{1}{2} \tilde{x}'_t \tilde{W} \tilde{x}_t. \quad (36)
\end{aligned}$$

Expressions for  $w_0$ ,  $w_y$ ,  $\tilde{w}$  and  $\tilde{W}$  are given by equations (20) - (23). They are obtained by substituting equation (36) in equation (18), using  $E_t [\tilde{x}_{t+1}] = \tilde{c}_0 + \tilde{C} \tilde{x}_t$  and  $E_t [\Delta y_{t+1}]$  from equation (5). Since  $dE_t [\tilde{x}_{t+1}] / df_t = e_f$  ( $e_f$  denoting the unit vector relating to  $f_{t-1}$  in  $\tilde{x}_t$ ), the first-order condition for the optimal policy  $f_t^+$  reads:

$$0 = e'_5 (\tilde{w}^+ + \tilde{h}) + w_5^{+'} (a_0 + A x_t + a f_{t-1}) + w_{55}^+ (b_0^+ + b^+ x_t + b_f^+ f_{t-1}).$$

where  $w_{ff}^+ = e'_f (\widetilde{W}^+ + H) e_f$  and  $w_f^+ = \widetilde{w}_f^+$  omitting the element  $w_{ff}^+$ . The policy rule is linear and the second-order conditions satisfy  $w_{ff}^+ < 0$ . ■

The coefficients of equation (11) are:

$$\begin{aligned} b_0^+ &= -w_{ff}^{+-1} \left( e'_f (\widetilde{w}^+ + \widetilde{h}) + w_f^{+'} a_0 \right), \\ b^{+'} &= -w_{ff}^{+-1} w_f^{+'} A, \\ b_f^+ &= -w_{ff}^{+-1} w_f^{+'} a, \end{aligned} \tag{37}$$

$\widetilde{W}^+$ ,  $b^+$ , and  $b_f^+$  can be solved iteratively from equation (23), the definition of  $\widetilde{C}^+$  in equation (12), and equation 37). Having solved for  $\widetilde{W}^+$ ,  $b^+$ , and  $b_f^+$ ,  $\widetilde{w}^+$ ,  $w_0^+$ ,  $b_0^+$  and  $\widetilde{c}_0^+$  can be solved from the equations (12) and (37) and equation (20).

### Proposition 6: Welfare in the steady state

For any linear policy rule that responds to  $d_t$  Propositions 1 and 2 apply. Substitution of equation (9) in equations (19) and (18) yields an expression for steady-state welfare:

$$\beta W(\bar{x}, y_t) = y_t + h_0 + h' \bar{x} + \frac{1}{2} \bar{x}' H \bar{x} + \theta' \bar{x} + \frac{1}{2} \bar{x}' \Theta \bar{x} + \frac{1}{2} l' V * G l.$$

$W(\bar{x}, y_t)$  does not depend on  $w_0$  and  $w$ , but only on the matrix of second-order derivatives  $W$ . ■

### Proposition 7: Stabilizing a single variable is unsustainable

A policy that aims to stabilize the  $i$ -th element of  $z_t$  follows the rule:  $b_i = -\frac{1}{a_i} \bar{a}_i$ , where  $a_i$  is the  $i$ -th element of  $a$  and  $\bar{a}_i$  denotes the vector of the  $i$ -th row of  $A$ . Then matrix  $C$  satisfies  $C = A - \frac{1}{a_i} a \bar{a}_i'$ :

$$C = \begin{bmatrix} \bar{a}_1' - \frac{a_1}{a_i} \bar{a}_i' & 0 & \dots \\ \dots & \dots & \dots \\ 0 & 0 & \dots \\ \dots & \dots & \dots \\ \bar{a}_j' - \frac{a_j}{a_i} \bar{a}_i' & 0 & \dots \\ \dots & \dots & \dots \\ a'_d & 1 & \dots \end{bmatrix}.$$

Now the steady state  $\bar{x} = (I - C)^{-1} c_0$  is undefined, as the last column of  $I - C$  is zero; and hence,  $(I - C)^{-1}$  does not exist. ■



## B Summary statistics

Table 3: Summary statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
dy	510	.0238	.0238	-.0802	.1146
g	485	.0300	.0179	-.0517	.1221
u	510	.1733	.0795	.0000	.3982
s	474	.0144	.0319	-.1222	.1162
d	501	.6310	.3037	.0910	2.139
f	510	-.0033	.0072	-.0474	.0075

Table 4: Correlation coefficients

	g	u	s	d	f
g	1				
u	.1228	1			
s	.1501	-.1207	1		
d	.1015	.0889	-.0684	1	
f	-.0641	-.2951	.136	-.2226	1

Table 5: T-statistics of unit root tests

	g	u	s	d	f
IPS AIC, lag selection	-6.13***	-3.99***	-2.66***	-0.50	-8.16***
IPS BIC, lag selection	-6.93***	-4.68***	-2.48***	-0.52	-9.62***
IPS HQIC, lag selection	-6.13***	-3.99***	-2.66***	-0.30	-8.26***

\*\*\*  $p < 0.01$ .

## C PVAR estimation results

Table 6: Lag selection using information criteria

f	no	no	no	yes	yes	yes
lags	1	2	3	1	2	3
AIC	-10.25228	-10.362401	-10.3178	-10.4485	-10.51*	-10.4527
SIC	-10.184365	-10.2225331	-10.1015	-10.45*	-10.3429699	-10.2057

Table 7: Regression outcomes excluding (1) and including (2) discretionary budget measures.

VARIABLES	(1)			(2)		
	g	u	s	g	u	s
L.g	0.55*** [0.114]	-0.437*** [0.151]	0.415*** [0.102]	0.577*** [0.119]	-0.411*** [0.153]	0.361*** [0.099]
L.u	0.072* [0.038]	1.304*** [0.069]	-0.085** [0.034]	0.082** [0.034]	1.248*** [0.065]	-0.091*** [0.029]
L.s	-0.178** [0.084]	-0.412*** [0.146]	0.793*** [0.112]	-0.169** [0.08]	-0.225* [0.136]	0.746*** [0.101]
L2.g	0.214*** [0.082]	0.198 [0.132]	0.159 [0.098]	0.2** [0.079]	-0.011 [0.116]	0.229** [0.089]
L2.u	-0.029 [0.036]	-0.373*** [0.068]	0.131*** [0.034]	-0.046 [0.035]	-0.38*** [0.059]	0.144*** [0.03]
L2.s	0.137** [0.061]	0.466*** [0.121]	-0.091 [0.08]	0.123** [0.06]	0.298*** [0.112]	-0.022 [0.072]
f				-0.005 [0.123]	-0.735*** [0.215]	-0.178 [0.117]
std	0.016	0.028	0.016	0.015	0.026	0.015
n		417			402	

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## D PVAR Robustness

The variance weighted coefficient estimates and standard deviations from the individual country VAR models are shown columns (3) in Table 8.<sup>24</sup> Most coefficients are within two standard deviations of the Panel VAR regression coefficients of Table 7; the exceptions are the first-order autoregression coefficient of growth, which is significantly lower, and the effect of discretionary fiscal policy on primary surplus, which is significantly higher. The higher value of this last coefficient is more in line with our prior expectations. Unfortunately, almost all individual country coefficients for the responses to growth, unemployment and primary surplus to discretionary fiscal policy are indistinguishable from zero. Hence, using a panel data approach is necessary. Figures 2 - 4 plotted the value of the coefficients from each country's VAR with its standard deviation and the coefficient estimate from the panel VAR. The standard errors from the Panel VAR are smaller by a factor two to four than the estimation results of the individual country VARs. As can be seen from Figures 2 - 4, Spain is an outlier. Column (4) in Table 8 shows that the Panel VAR results of Table 7 excluding Spain are virtually the same.

Table 9 shows the estimation results for equation (3) using unemployment instead of square root unemployment. The coefficients of unemployment can be compared to those in Table 7 by multiplication/division by a factor  $2\bar{u} \approx 0.40$ . None of the coefficients differs substantially and a test on heteroskedasticity of  $\text{abs}(e_u)$  indicates that it is not significant in either the square root version or the level version of the model. This suggests that the data has no clear preference for either the square root or the level version. Figure 1 provides a scatterplot of the absolute value of the residuals for the square root version and the level version.

Following Alesina and Ardagna (n.d.), we distinguish between cutting expenditure and raising taxes in Table 10. Our results indicate, however, that none of the coefficients changes significantly. In columns (14) of Table 11 we included trade openness as an additional explanatory variable, and in column (15) we report the product of openness and discretionary fiscal policy. The effects of openness alone and combined with a cross term with fiscal policy are all insignificant. In column (16) in Table 12 the years 2008 and 2009 are removed from the sample. The coefficients are robust. Column (17) presents the coefficients of a VAR(1) model.

Next we examine the consequences of censoring in the discretionary fiscal policy variable. We assume  $\text{Cov}[x, f] \neq 0$  and let  $f$  be determined by

$$f = \gamma_0 + x'\gamma + u,$$

where  $u \sim N(0, \psi^2)$ . First, estimate a Tobit model:

$$f = \text{Tobit}(\gamma_0 + x'\gamma)$$

---

<sup>24</sup>To calculate the weighted means we used:

$$\bar{x} = \frac{\sum_i \sigma_i^{-2} x_i}{\sum_i \sigma_i^{-2}}$$

Table 8: The variance weighted mean and variance from the individual country VAR models (3). Results without Spain (4).

VARIABLES	(3)			(4)		
	g	u	s	g	u	s
L.g	0.146*** [0.051]	-0.375*** [0.07]	0.258*** [0.047]	0.577*** [0.119]	-0.411*** [0.153]	0.361*** [0.099]
L.u	0.114*** [0.026]	1.182*** [0.049]	-0.028 [0.026]	0.082** [0.034]	1.248*** [0.065]	-0.091*** [0.029]
L.s	0.003 [0.049]	-0.235*** [0.072]	0.732*** [0.05]	-0.169** [0.08]	-0.225* [0.136]	0.746*** [0.101]
L2.g	0.069 [0.049]	0.061 [0.066]	0.1** [0.047]	0.2** [0.079]	-0.011 [0.116]	0.229** [0.089]
L2.u	-0.052** [0.026]	-0.387*** [0.047]	0.082*** [0.026]	-0.046 [0.035]	-0.38*** [0.059]	0.144*** [0.03]
L2.s	0.03 [0.047]	0.089 [0.069]	-0.007 [0.049]	0.123** [0.06]	0.298*** [0.112]	-0.022 [0.072]
f	-0.006 [0.102]	-0.303* [0.168]	-0.441*** [0.086]	-0.005 [0.123]	-0.735*** [0.215]	-0.178 [0.117]
std				0.015	0.026	0.015
n		402			390	

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

This model yields estimates of  $\gamma_0, \gamma$  and  $\psi^2$ . Next, we calculate  $E[f^{\text{imp}}|f, x]$ :

$$E[f^{\text{imp}}|f, x] = \begin{cases} 0 & \text{if } f < 0 \\ \gamma_0 + x'\gamma + \psi\phi\left[\frac{\gamma_0 + x'\gamma}{\psi}\right] / \Phi\left[\frac{\gamma_0 + x'\gamma}{\psi}\right] & \text{if } f = 0 \end{cases}, \quad (38)$$

where  $\phi$  is the probability distribution function of the standard normal distribution and  $\Phi$  its cumulative distribution function. Finally, estimate:

$$y = a_o + x'a + \beta_1 f + \beta_2 E[f^{\text{imp}}|f, x] + \varepsilon^{\text{imp}}. \quad (39)$$

The Tobit regression outcomes are shown in Table 14.

Table 9: Estimation of equation (1) with unemployment instead of square root unemployment.

VARIABLES	(5)		
	g	$u^2$	s
L.g	0.568*** [0.119]	-0.149*** [0.047]	0.407*** [0.1]
L.u	0.228* [0.132]	1.37*** [0.069]	-0.521*** [0.092]
L.s	-0.186** [0.082]	-0.034 [0.038]	0.712*** [0.101]
L2.g	0.179** [0.078]	-0.033 [0.03]	0.203** [0.084]
L2.u	-0.084 [0.124]	-0.504*** [0.067]	0.616*** [0.084]
L2.s	0.139** [0.061]	0.086*** [0.031]	-0.005 [0.069]
f	0.035 [0.124]	-0.271*** [0.063]	-0.271** [0.111]
std	0.015	0.007	0.014
n	402		

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Figure 1: Left: Scatterplot of the absolute value of residuals of square root unemployment from Table 7 equation against unemployment. Right: Scatterplot of the absolute value of residuals of unemployment equation from Table 9 against unemployment.

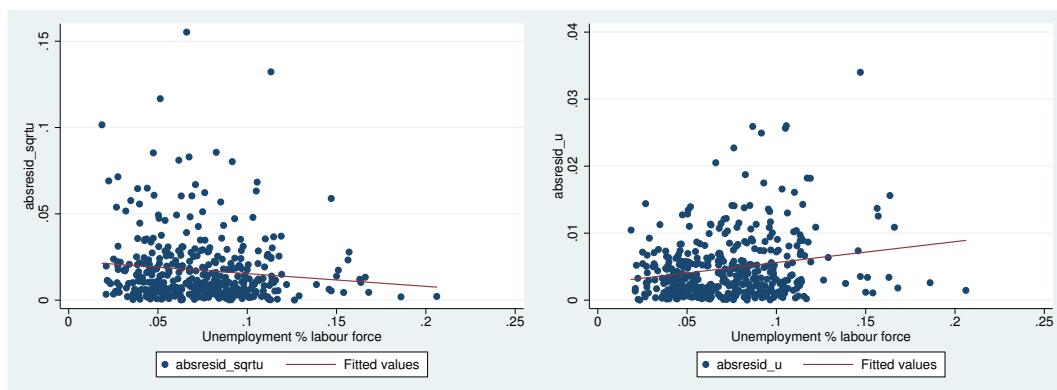


Table 10: Regression outcomes using discretionary spending (11), discretionary tax revenue (12) and both of them (13) as alternative measures for discretionary fiscal policy.

VARIABLES	(11)			(12)			(13)		
	g	u	s	g	u	s	g	u	s
L.g	0.581*** [0.118]	-0.382*** [0.147]	0.364*** [0.098]	0.573*** [0.118]	-0.416*** [0.153]	0.364*** [0.1]	0.574*** [0.118]	-0.417*** [0.153]	0.363*** [0.099]
L.u	0.082** [0.036]	1.286*** [0.064]	-0.082*** [0.029]	0.075** [0.034]	1.228*** [0.067]	-0.089*** [0.029]	0.072** [0.035]	1.23*** [0.067]	-0.085*** [0.03]
L.s	-0.172** [0.08]	-0.24* [0.134]	0.746*** [0.1]	-0.166** [0.08]	-0.23* [0.133]	0.743*** [0.101]	-0.17** [0.08]	-0.228* [0.134]	0.747*** [0.101]
L2.g	0.201** [0.078]	0.03 [0.114]	0.238*** [0.087]	0.193** [0.079]	-0.031 [0.116]	0.231** [0.091]	0.19** [0.079]	-0.03 [0.116]	0.235*** [0.09]
L2.u	-0.044 [0.036]	-0.391*** [0.06]	0.139*** [0.03]	-0.044 [0.034]	-0.366*** [0.06]	0.145*** [0.03]	-0.04 [0.035]	-0.368*** [0.06]	0.14*** [0.03]
L2.s	0.122** [0.06]	0.282** [0.112]	-0.025 [0.072]	0.126** [0.059]	0.304*** [0.11]	-0.024 [0.072]	0.126** [0.059]	0.304*** [0.111]	-0.024 [0.072]
fspend	0.219 [0.219]	-0.633 [0.413]	-0.389* [0.219]				0.31 [0.232]	-0.158 [0.369]	-0.367 [0.242]
ftax				-0.135 [0.182]	-1.159*** [0.302]	-0.145 [0.186]	-0.214 [0.192]	-1.119*** [0.296]	-0.052 [0.199]
std	0.015	0.026	0.015	0.015	0.026	0.015	0.015	0.026	0.015
n		402			402			402	

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table 11: Regression results check using openness as an additional explanatory variable (14) and the product of openness times discretionary spending (15).

VARIABLES	(14)			(15)		
	g	u	s	g	u	s
L.g	0.52*** [0.121]	-0.364** [0.144]	0.316*** [0.095]	0.56*** [0.119]	-0.39*** [0.149]	0.353*** [0.097]
L.u	0.11** [0.044]	1.223*** [0.063]	-0.07** [0.035]	0.099** [0.039]	1.23*** [0.063]	-0.08*** [0.031]
L.s	-0.14 [0.093]	-0.255* [0.133]	0.765*** [0.102]	-0.157* [0.087]	-0.245* [0.134]	0.75*** [0.101]
L2.g	0.21** [0.084]	-0.015 [0.12]	0.238*** [0.091]	0.225*** [0.086]	-0.025 [0.123]	0.252*** [0.093]
L2.u	-0.073* [0.043]	-0.356*** [0.056]	0.124*** [0.034]	-0.063* [0.038]	-0.363*** [0.056]	0.134*** [0.031]
L2.s	0.121* [0.065]	0.301*** [0.112]	-0.024 [0.07]	0.126** [0.062]	0.297*** [0.111]	-0.019 [0.07]
f	-0.314 [0.316]	-0.46 [0.358]	-0.41 [0.253]	0.156 [0.186]	-0.764*** [0.292]	0.033 [0.215]
open	0.045 [0.039]	-0.039 [0.043]	0.034 [0.031]	0.033 [0.026]	-0.032 [0.032]	0.023 [0.022]
f*open				-0.66 [0.473]	0.428 [0.635]	-0.622 [0.435]
std	0.016	0.026	0.015	0.015	0.026	0.015
n		402			402	

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table 12: Regression results using just the time period (1979-2007) (16) and a VAR(1) model (17).

VARIABLES	(16)			(17)		
	g	u	s	g	u	s
L.g	0.48*** [0.11]	-0.52*** [0.157]	0.402*** [0.105]	0.591*** [0.117]	-0.281* [0.162]	0.388*** [0.097]
L.u	0.087** [0.035]	1.29*** [0.064]	-0.071** [0.031]	0.044** [0.018]	0.918*** [0.029]	0.046*** [0.017]
L.s	-0.188** [0.076]	-0.193 [0.134]	0.695*** [0.1]	-0.043 [0.039]	-0.188** [0.084]	0.833*** [0.043]
L2.g	0.318*** [0.077]	-0.019 [0.117]	0.293*** [0.092]			
L2.u	-0.02 [0.035]	-0.402*** [0.057]	0.127*** [0.03]			
L2.s	0.196*** [0.057]	0.297*** [0.112]	0.024 [0.071]			
f	-0.066 [0.126]	-0.784*** [0.214]	-0.107 [0.122]	-0.009 [0.118]	-0.551** [0.239]	-0.281** [0.124]
std	0.014	0.025	0.015	0.015	0.029	0.015
n		387			419	

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .



Table 13: Regression results omitting observations for which  $f = 0$  (6) and regression results including  $D$  for the omitted truncated variables (7).

VARIABLES	(6)			(7)		
	g	u	s	g	u	s
L.g	-0.036 [0.082]	-0.135 [0.208]	0.176 [0.169]	0.56*** [0.119]	-0.376*** [0.145]	0.344*** [0.092]
L.u	0.239*** [0.067]	1.536*** [0.144]	-0.161 [0.104]	0.081** [0.034]	1.25*** [0.065]	-0.091*** [0.029]
L.s	0.224** [0.09]	-0.003 [0.151]	0.895*** [0.179]	-0.157** [0.08]	-0.249* [0.129]	0.757*** [0.094]
L2.g	0.161** [0.07]	0.091 [0.136]	0.228** [0.11]	0.192** [0.076]	0.006 [0.111]	0.221*** [0.084]
L2.u	-0.186*** [0.062]	-0.603*** [0.139]	0.27*** [0.089]	-0.05 [0.034]	-0.373*** [0.058]	0.141*** [0.029]
L2.s	-0.05 [0.1]	0.253 [0.18]	-0.063 [0.123]	0.102 [0.062]	0.342*** [0.11]	-0.044 [0.066]
f	-0.271 [0.2]	-0.374 [0.308]	-0.167 [0.182]	-0.109 [0.141]	-0.522** [0.23]	-0.282** [0.137]
Df/100				0.293 [0.273]	-0.595 [0.438]	0.293 [0.284]
std	0.009	0.018	0.013	0.015	0.026	0.015
n		70			402	

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table 14: Robustness check using extrapolation for the omitted truncated variables. The first column shows the Tobit regression outcomes, the later columns the panel VAR regression outcomes with the extrapolated variables.  $f^{\text{imp}}$  is defined by equation (38)  $f' = f^{\text{imp}} + f$ .

VARIABLES	$f^-$	g	(8) u	s	g	(9) u	s
L.g	0.089 [0.066]	0.605*** [0.122]	-0.53*** [0.176]	0.413*** [0.126]	0.585*** [0.12]	-0.471*** [0.163]	0.37*** [0.106]
L.u	-0.146*** [0.036]	0.107** [0.053]	1.142*** [0.084]	-0.044 [0.052]	0.089** [0.037]	1.193*** [0.068]	-0.081** [0.032]
L.s	-0.001 [0.069]	-0.167** [0.082]	-0.233 [0.142]	0.749*** [0.107]	-0.171** [0.081]	-0.22 [0.141]	0.74*** [0.104]
L2.g	-0.012*** [0.003]	0.241** [0.113]	-0.186 [0.178]	0.306** [0.143]	0.21** [0.086]	-0.095 [0.132]	0.239** [0.103]
L2.u	-0.031 [0.065]	-0.053 [0.037]	-0.353*** [0.061]	0.132*** [0.034]	-0.048 [0.035]	-0.367*** [0.059]	0.143*** [0.029]
L2.s	0.093*** [0.035]	0.098 [0.072]	0.407*** [0.112]	-0.07 [0.074]	0.117* [0.062]	0.348*** [0.111]	-0.027 [0.069]
L.d	0.232*** [0.068]						
Constant	0.021*** [0.004]						
f		-0.185 [0.276]	0.046 [0.45]	-0.522* [0.292]			
f <sup>imp</sup>		0.383 [0.555]	-1.653* [0.878]	0.73 [0.595]			
f'					0.052 [0.113]	-0.663*** [0.187]	0 [0.116]
$\sigma/\text{std}$	0.015***	0.015	0.027	0.016	0.015	0.026	0.015
n	421		402			402	

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Figure 2: VAR regression outcomes of the coefficients (red dots), their confidence intervals with two standard deviations on both sides (black vertical lines) and the value of the panel VAR regression outcome (red horizontal line)

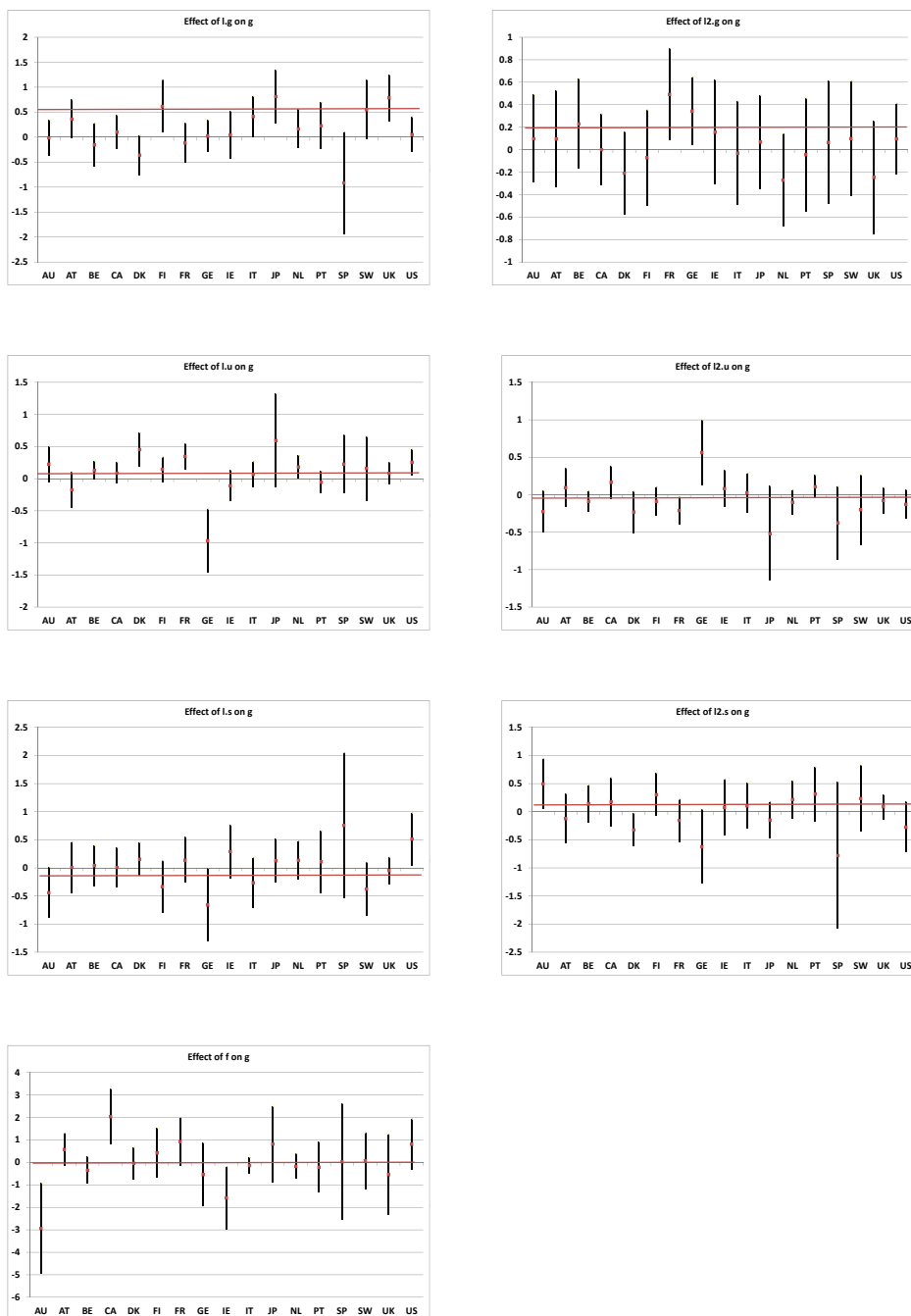


Figure 3: Continuation of the figure from previous page: VAR regression outcomes of the coefficients (red dots), their confidence intervals with two standard deviations on both sides (black vertical lines) and the value of the panel VAR regression outcome (red horizontal line)

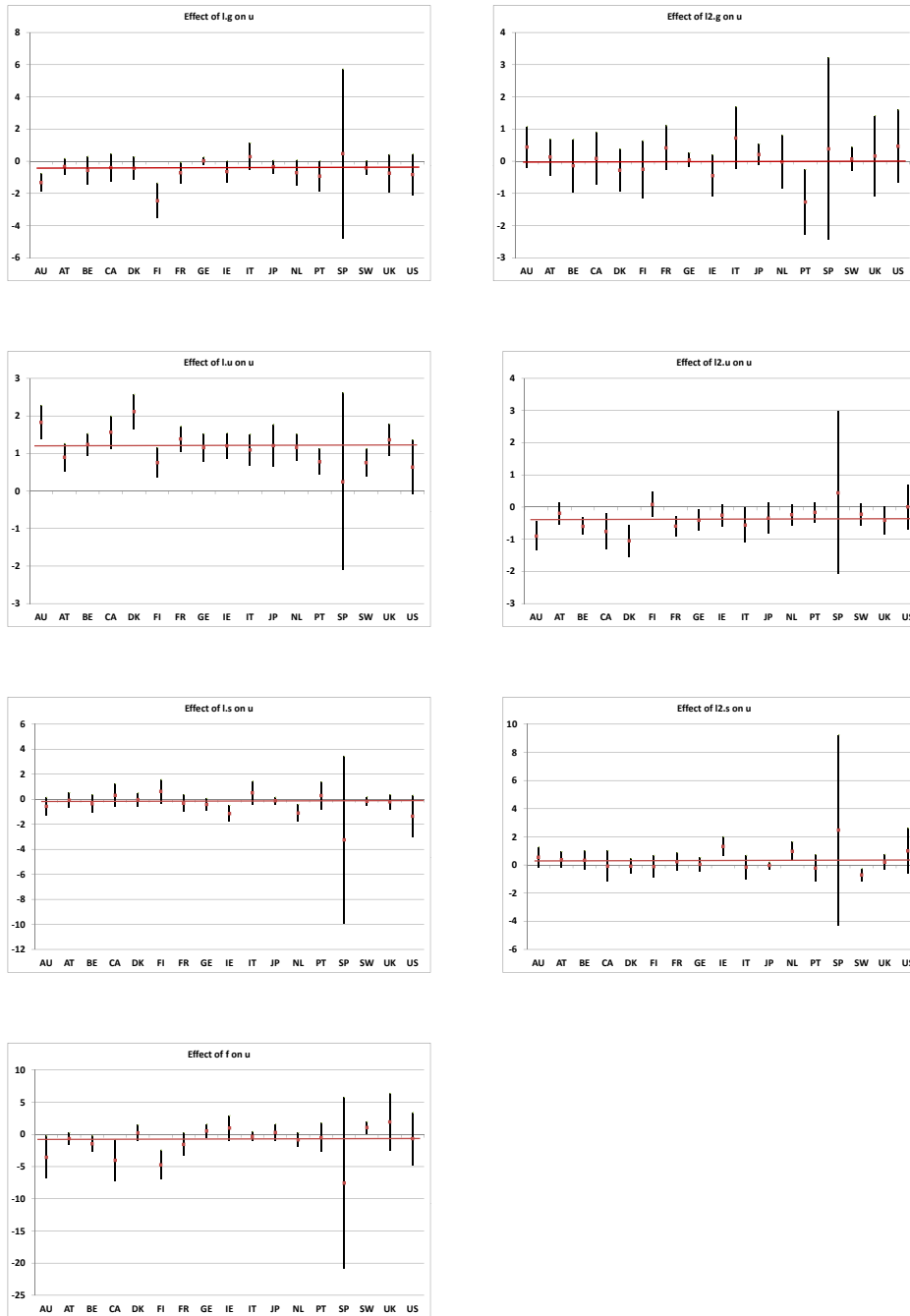
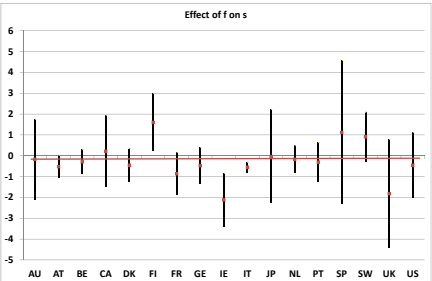
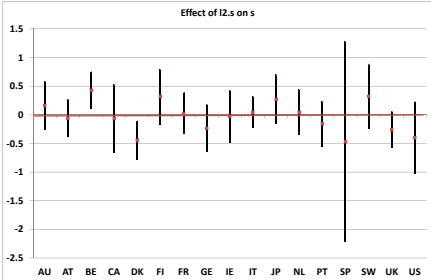
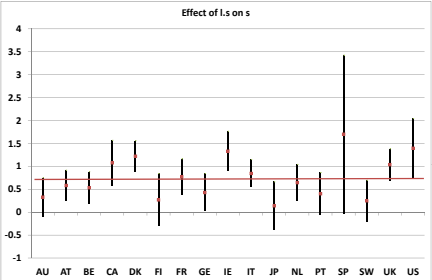
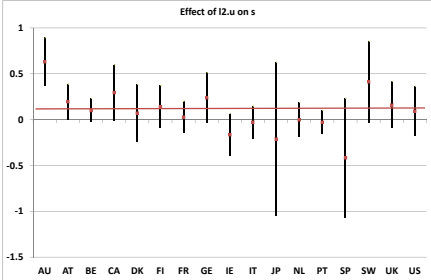
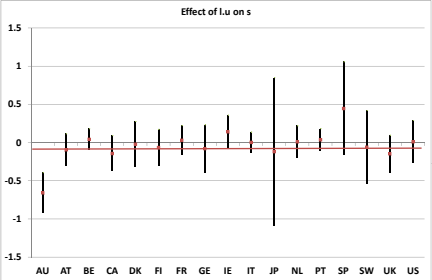
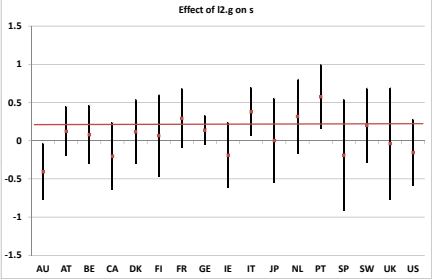
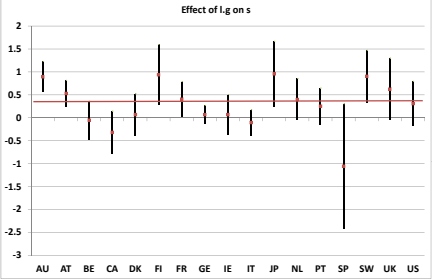


Figure 4: Continuation of the figure from previous page: VAR regression outcomes of the coefficients (red dots), their confidence intervals with two standard deviations on both sides (black vertical lines) and the value of the panel VAR regression outcome (red horizontal line)



## E Bootstrap results

Table 15: Bootstrap results for  $\gamma$ ,  $\psi$ ,  $a_{41}$  and the natural rate of  $g_t$ ,  $u_t$  and  $s_t$ .

$\gamma$	$\psi$	$a_{41}$
2.768 <sup>†</sup>	0.021 <sup>†</sup>	-0.422 <sup>†</sup>
[2.586, 2.962]	[0.016, 0.025]	[-0.450, -0.394]
$\bar{g}$	$\bar{u}$	$\bar{s}$
0.025 <sup>†</sup>	0.199 <sup>†</sup>	-0.005 <sup>†</sup>
[0.023, 0.027]	[0.187, 0.218]	[-0.007, -0.003]

The median bootstrap result is shown in normal font; below that, in a smaller font a one standard deviation confidence interval is shown between brackets. † denotes significance at the 5% level.

Table 16: Bootstrap estimation of equation (1).

VARIABLES	(18)		
	g	u	s
L.g	0.472***	-0.673***	0.418***
	[0.105]	[0.224]	[0.105]
L.u	0.108**	1.582***	-0.149**
	[0.047]	[0.173]	[0.058]
L.s	-0.077	0.015	0.769***
	[0.086]	[0.251]	[0.107]
L2.g	0.175**	0.142	0.136
	[0.088]	[0.15]	[0.095]
L2.u	-0.071	-0.671***	0.177***
	[0.045]	[0.169]	[0.057]
L2.s	0.084	0.254	-0.078
	[0.062]	[0.189]	[0.077]
f	-0.034	-0.974***	-0.097
	[0.105]	[0.277]	[0.125]
std	0.017	0.044	0.019
n	402		

Standard errors in brackets \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table 17: Policy rules  $b$ ,  $b_f$  and  $b_0$  and the steady-state debt level  $\bar{d}$  under several policy regimes for  $\beta^h = 9.6\%$ .

	Optimal	..., delayed	Max growth	..., delayed	Empirical
g	-0.03 [-0.23, 0.26]	-0.31 [-0.72, 0.08]	0.29 [0.01, 0.72]	0.02 [-0.40, 0.53]	0.08 [0.02, 0.16]
u	1.57 <sup>†</sup> [1.19, 2.22]	1.73 <sup>†</sup> [1.26, 2.52]	1.56 <sup>†</sup> [1.18, 2.15]	1.75 <sup>†</sup> [1.27, 2.53]	-0.14 [-0.18, -0.10]
s	0.11 [-0.09, 0.37]	0.31 [0.02, 0.69]	0.37 [0.10, 0.71]	0.56 [0.23, 1.00]	-0.01 [-0.09, 0.08]
d	-0.07 <sup>†</sup> [-0.12, -0.04]	-0.07 <sup>†</sup> [-0.12, -0.04]	-0.15 <sup>†</sup> [-0.24, -0.09]	-0.15 <sup>†</sup> [-0.24, -0.09]	-0.01 <sup>†</sup> [-0.02, -0.01]
L.g	0.16 [0.01, 0.36]	0.12 [-0.08, 0.37]	0.23 [0.08, 0.47]	0.21 [0.00, 0.49]	-0.02 [-0.09, 0.06]
L.u	-0.61 <sup>†</sup> [-0.91, -0.42]	-0.93 <sup>†</sup> [-1.47, -0.61]	-0.57 <sup>†</sup> [-0.86, -0.40]	-0.91 <sup>†</sup> [-1.44, -0.60]	0.09 <sup>†</sup> [0.05, 0.12]
L.s	0.35 [0.19, 0.51]	0.51 [0.28, 0.79]	0.32 [0.16, 0.49]	0.49 [0.27, 0.76]	0.23 <sup>†</sup> [0.16, 0.31]
L.f		-1.48 <sup>†</sup> [-1.63, -1.34]		-1.50 <sup>†</sup> [-1.66, -1.37]	
Const.	-0.17 <sup>†</sup> [-0.29, -0.10]	-0.14 [-0.25, -0.06]	-0.36 <sup>†</sup> [-0.57, -0.23]	-0.33 <sup>†</sup> [-0.53, -0.20]	0.02 <sup>†</sup> [0.02, 0.02]
$\bar{d}$	26% [-58%, 192%]	26% [-58%, 192%]	-101% [-165%, -30%]	-101% [-165%, -30%]	118% <sup>†</sup> [100%, 142%]

The median bootstrap result is shown in normal font; below this, in smaller font, a 1 standard deviation confidence interval is shown between brackets. † denotes significance at the 5% level.

Table 18: Welfare losses compared to optimal policy of applying an alternative policy rule, in percent of log output as a function of steady-state debt with  $\beta^h = 9.6\%$ .

	26% ( $\bar{d}^*$ )	0%	60%	118% ( $\bar{d}^e$ )
Optimal, delayed	0.5 [0.4, 0.6]	0.6 [0.4, 1.0]	0.7 [0.5, 1.0]	0.8 [0.5, 1.4]
Max growth	8.3 [3.8, 18.1]	6.2 [3.9, 9.1]	9.2 [6.1, 12.6]	12.6 [8.9, 17.5]
..., delayed	8.2 [3.9, 17.1]	6.4 [4.3, 8.9]	9.1 [6.5, 12.2]	12.6 [9.2, 16.8]
Empirical	9.4 [4.4, 24.4]	8.8 [4.8, 17.1]	7.6 [4.3, 15.0]	7.6 [4.0, 15.4]

The median bootstrap result is shown in normal font; below that, in a smaller font a one standard deviation confidence interval is shown between brackets.

Table 19: Policy rules  $b$ ,  $b_f$  and  $b_0$  and the steady-state debt level  $\bar{d}$  under several policy regimes for  $\beta^l = 3.2\%$ .

	Optimal	..., delayed	Max growth	..., delayed	Empirical
g	0.77 <sup>†</sup> [0.38, 1.39]	0.47 [0.03, 1.13]	1.40 <sup>†</sup> [0.84, 2.25]	1.14 <sup>†</sup> [0.56, 2.07]	0.08 [0.02, 0.16]
u	1.50 <sup>†</sup> [1.15, 2.14]	1.74 <sup>†</sup> [1.28, 2.58]	1.46 <sup>†</sup> [1.10, 2.05]	1.77 <sup>†</sup> [1.30, 2.59]	-0.14 [-0.18, -0.10]
s	0.65 <sup>†</sup> [0.31, 1.08]	0.84 <sup>†</sup> [0.46, 1.36]	1.15 <sup>†</sup> [0.70, 1.73]	1.35 <sup>†</sup> [0.86, 2.05]	-0.01 [-0.09, 0.08]
d	-0.25 <sup>†</sup> [-0.35, -0.18]	-0.25 <sup>†</sup> [-0.35, -0.18]	-0.41 <sup>†</sup> [-0.58, -0.30]	-0.41 <sup>†</sup> [-0.58, -0.30]	-0.01 <sup>†</sup> [-0.02, -0.01]
L.g	0.35 <sup>†</sup> [0.17, 0.62]	0.32 [0.12, 0.62]	0.50 <sup>†</sup> [0.27, 0.87]	0.49 <sup>†</sup> [0.24, 0.90]	-0.02 [-0.09, 0.06]
L.u	-0.53 <sup>†</sup> [-0.80, -0.36]	-0.88 <sup>†</sup> [-1.42, -0.59]	-0.46 <sup>†</sup> [-0.75, -0.26]	-0.85 <sup>†</sup> [-1.36, -0.54]	0.09 <sup>†</sup> [0.05, 0.12]
L.s	0.30 [0.11, 0.50]	0.49 [0.25, 0.79]	0.26 [0.03, 0.49]	0.45 [0.19, 0.77]	0.23 <sup>†</sup> [0.16, 0.31]
L.f		-1.53 <sup>†</sup> [-1.67, -1.39]		-1.57 <sup>†</sup> [-1.73, -1.44]	
Const.	-0.30 <sup>†</sup> [-0.43, -0.22]	-0.28 <sup>†</sup> [-0.40, -0.19]	-0.43 <sup>†</sup> [-0.61, -0.32]	-0.42 <sup>†</sup> [-0.59, -0.30]	0.02 <sup>†</sup> [0.02, 0.02]
$\bar{d}$	-31% [-50%, -10%]	-31% [-50%, -10%]	-44% [-64%, -28%]	-44% [-64%, -28%]	117% <sup>†</sup> [100%, 145%]

The median bootstrap result is shown in normal font; below this, in smaller font, a 1 standard deviation confidence interval is shown between brackets. † denotes significance at the 5% level.

Table 20: Welfare losses compared to optimal policy of applying an alternative policy rule, in percent of log output as a function of steady-state debt with  $\beta^l = 3.2\%$ .

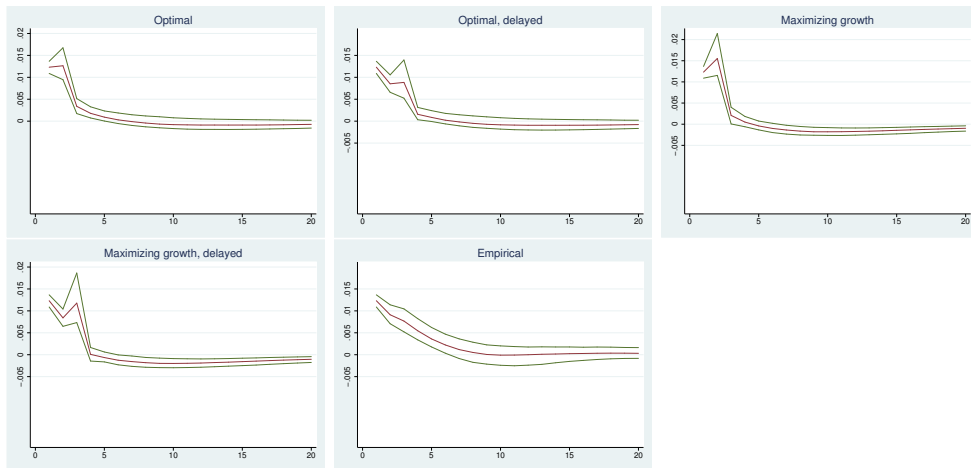
	-31% ( $\bar{d}^*$ )	0%	60%	117% ( $\bar{d}^e$ )
Optimal, delayed	0.6 [0.4, 0.8]	0.7 [0.5, 1.0]	1.2 [0.8, 1.5]	1.2 [0.8, 1.5]
Max growth	0.7 [0.4, 1.9]	1.1 [0.7, 2.4]	2.4 [1.6, 4.6]	4.7 [2.9, 8.8]
..., delayed	1.4 [0.9, 2.5]	1.9 [1.3, 3.0]	3.7 [2.8, 5.5]	6.8 [4.9, 10.8]
Empirical	40.5 [25.4, 66.5]	43.2 [27.3, 66.1]	47.8 [31.2, 69.8]	54.4 [35.2, 82.3]

The median bootstrap result is shown in normal font; below that, in a smaller font a one standard deviation confidence interval is shown between brackets.



Figure 5: Generalized impulse response functions after a growth shock under five policy rules. The graphs show the median and  $\pm\sigma$  response.

### Growth shock, response of growth



### Growth shock, response of unemployment

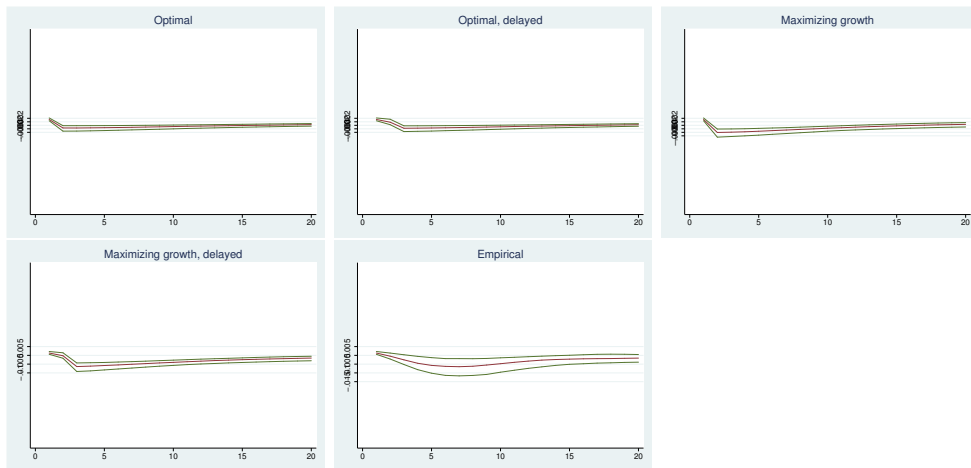
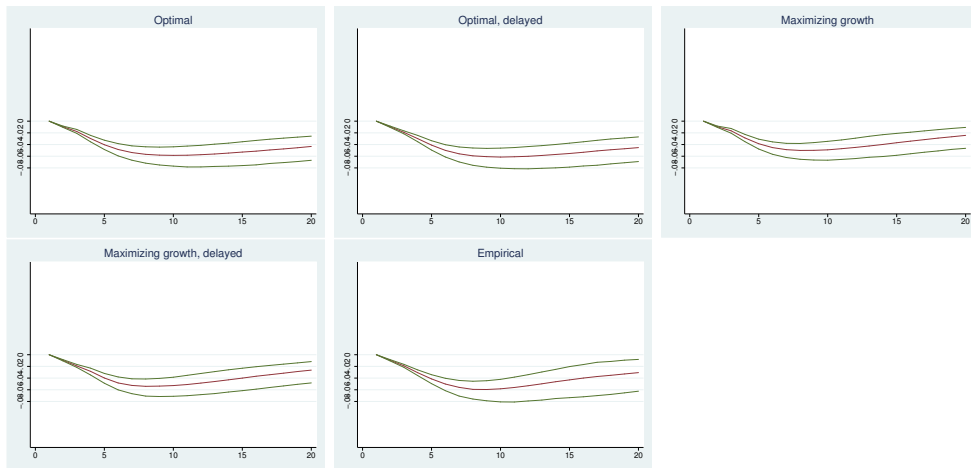


Figure 6: Continuation: Generalized impulse response functions after a growth shock under five policy rules. The graphs show the median and  $\pm\sigma$  response.

Growth shock, response of debt



Growth shock, response of discretionary policy

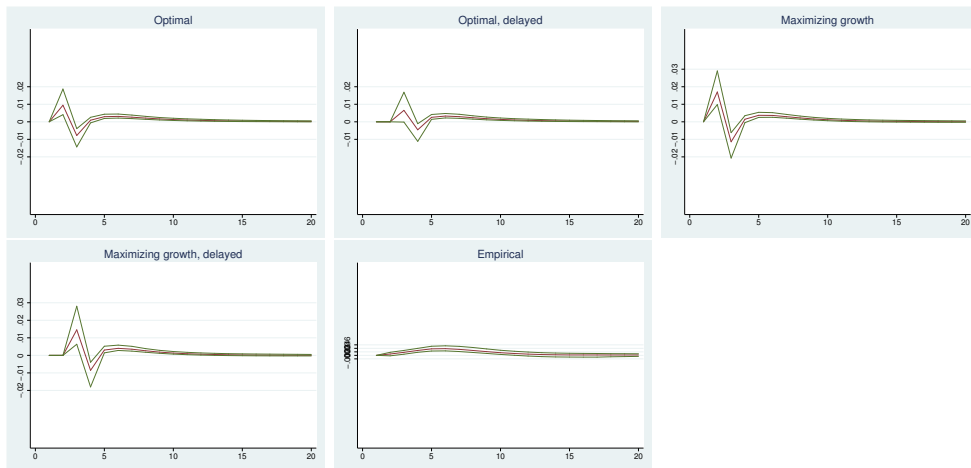
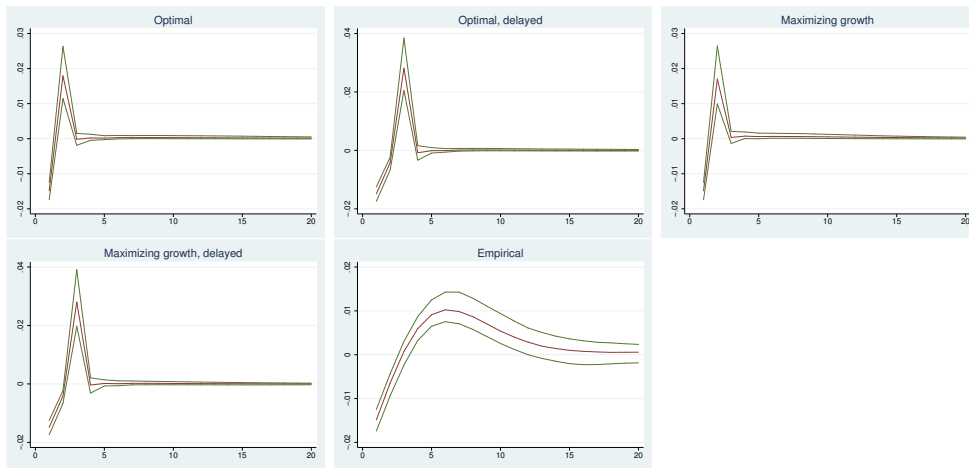


Figure 7: Generalized impulse response functions after an unemployment shock under five policy rules. The graphs show the median and  $\pm\sigma$  response.

### Unemployment shock, response of growth



### Unemployment shock, response of unemployment

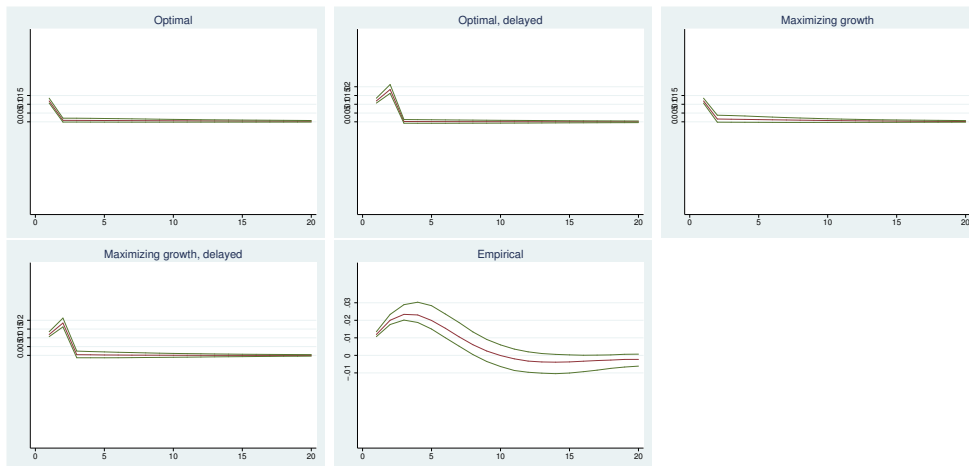
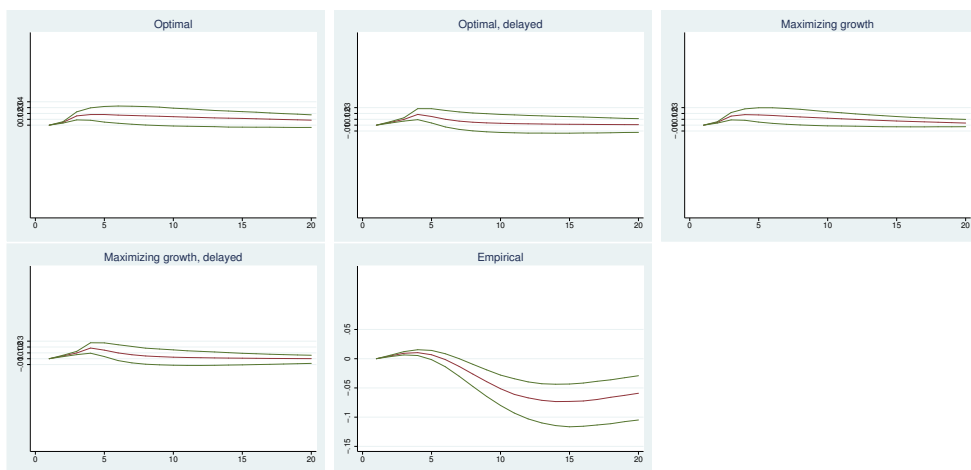


Figure 8: Continuation: Generalized impulse response functions after an unemployment shock under five policy rules. The graph shows the median and  $\pm\sigma$  response.

Unemployment shock, response of debt



Unemployment shock, response of discretionary policy

