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Breakthrough Renewables and the Green Paradox

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Abstract

We show how a monopolistic owner of oil reserves responds to a carbon-free substitute becoming available at some uncertain point in the future if demand is isoelastic and variable extraction costs are zero but upfront exploration investment costs have to be made. Not the arrival of this substitute matters for efficiency, but the uncertainty about the timing of this substitute coming on stream. Before the carbon-free substitute comes on stream, oil reserves are depleted too rapidly; as soon as the substitute has arrived, the oil depletion rate drops and the oil price jumps up by a discrete amount. Subsidizing green R&D to speed up the introduction of breakthrough renewables leads to more rapid oil extraction before the breakthrough, but more oil is left in situ as exploration investment will be lower. The latter offsets the Green Paradox.

JEL-Code: D810, H200, Q310, Q380.

Keywords: hotelling principle, exhaustible resources, carbon-free substitute, regime switch, oil stock uncertainty, hold-up problem, green R&D, Green Paradox.

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1. Introduction

The idea that well-intended climate policy may have undesirable unintended consequences has received a lot of attention in recent years (e.g., Sinn, 2008; Gerlagh, 2011; Grafton et al., 2010; Hoel, 2010). By levying a steeply rising carbon tax or subsidizing the use of renewables, oil well owners are encouraged to extract and sell their oil more quickly, thereby exacerbating carbon emissions and global warming. This counterintuitive result has been coined the Green Paradox (and is the intertemporal variant of the spatial notion of import leakage). However, if oil extraction becomes more costly as fewer reserves are left, the total amount of oil extracted from the earth is endogenous and not all oil reserves are necessarily fully exhausted. Over time, oil will become less attractive relative to the carbon-free backstop. Hence, a rising schedule for the carbon tax or a renewables subsidy makes it more attractive to keep more oil reserves in the crust of the earth. This offsets and can overturn the Green Paradox, both in terms of green welfare and total welfare (van der Ploeg and Withagen, 2011).

Our objective is to provide an alternative rationale for the Green Paradox not to hold. To make our case as stark as possible, we abstract from stock-dependent extraction costs and allow instead for initial outlays on exploration investment (cf., Gaudet and Laserre, 1988; Daubanes and Laserre, 2012). This also gives two margins: how quickly to extract oil and how much oil in total to extract from the earth. We then argue that the prospect of some breakthrough in the invention and bringing to the market of a carbon-free substitute induces oil to be pumped up more rapidly. As a result, carbon is more quickly emitted into the atmosphere and thus global warming is exacerbated. These effects are less strong if the carbon-free backstop is a worse substitute for oil (cf., Grafton et al., 2012). At the moment the carbon-free substitute becomes available, oil use jumps down by a discrete amount and the oil price jumps up by a discrete amount unless the cost reduction of renewables and the degree of substitutability is large enough in which case the oil price jumps down. From then on, the rate of decline in the rate of oil depletion and the rate of increase in the oil price follow Hotelling paths, albeit starting from a lower level of oil reserves than if there would have been no hazard of a cheaper substitute coming to the market. This inefficiency is stronger if the risk of discovery and drop in the price of the substitute are higher. Once the cheap carbon-free substitute is on the market, oil is depleted in Hotelling manner. Uncertainty about timing of the breakthrough causes inefficiencies, not the breakthrough itself.

However, the prospect of cost-effective renewables becoming available at some random moment in the future implies also that exploration investment is curbed and thus that the total stock of available oil reserves diminishes. This inefficiency in exploration investment is akin to the hold-up problem (e.g., Rogerson, 1992; Holmström and Roberts, 1998). It reduces the total of carbon emitted into the atmosphere and thus alleviates the problem of global warming. Subsidizing green R&D to speed up the introduction of breakthrough renewables leads to more rapid oil extraction before the breakthrough, but more oil is left in situ as exploration investment will be lower. The latter offsets and can even reverse the Green Paradox.

To highlight the inefficiencies from the eventual arrival of breakthrough carbon-free substitutes in the most striking manner, we suppose isoelastic demand and zero variable resource extraction costs. This is a useful benchmark, since the monopolistic resource extraction problem is efficient under these two assumptions (cf., Stiglitz, 1976).

The idea of a discrete change in demand resulting from a breakthrough technology occurring at some unknown date in the future goes back a long time (e.g., Dasgupta and Heal, 1974; Dasgupta and Stiglitz, 1981) and has recently been used to argue that innovation unsupported by carbon pricing can lead to runaway global warming worse if the carbon cycle contains strong positive feedback effects (Winter, 2012). Our contribution is to give a tractable analysis of the effects of a breakthrough in renewables technology for the path of oil extraction and exploration investment and investigate the robustness of the Green Paradox.

Our model is closely related to the ones found in the literatures on potential machine failure (Kamien and Schwartz, 1971), nationalization, expropriation and confiscation risk (Long, 1975; Bohn and Deacon, 2000; Laurent-Luchetti and Santaguni, 2012; van der Ploeg, 2012), collapses of the resource stock and changes in system dynamics (regime switches) in pollution control (e.g., Cropper, 1976; Heal, 1984; Clarke and Reed, 1994; Tsur and Zemel, 1996; Naevdal, 2006; Polasky et al., 2011; de Zeeuw and Zemel, 2012), the effects on the speed of resource extraction of uncertainty about the time at which a resource cartel is broken up (Benchekroun et al., 2006), and the interplay between political risk and foreign investment (e.g., Cherian and Perotti, 2001).

Section 2 presents a tractable model of oil extraction and exploration investment by a monopolistic owner of oil reserves faced with the possible arrival of breakthrough renewables. It faces a constant hazard of a breakthrough at some unknown future date. Section 3 derives the optimal oil depletion and price paths before and after the breakthrough. Section 4 characterizes the solution and gives illustrative simulations. Section 5 shows that a higher chance of a renewables breakthrough depresses exploration investment. Section 6 discusses climate policy and the Green Paradox. Section 7 summarizes results and offers suggestions for further research.

2. The model

We suppose that the economy needs two types of energy, viz. fossil fuel or oil for short, O, and renewables, R. Oil has zero extraction cost, but needs investment outlays I which lead to proven initial oil reserves S_0. The price of oil is endogenous and denoted by p. The breakthrough occurs at time $T > 0$ and calendar time is denoted by t. Before the breakthrough ($t < T$), renewables are infinitely elastically supplied at cost $\tilde{b}(t) = b$. After the breakthrough ($t \geq T$), they are supplied at cost $\tilde{b}(t) = b - \Delta$ where $0 < \Delta \leq b$.

The owner of the oil reserves chooses its level of exploration investment and extraction path to maximize the present value of its profits,

$$(1) \qquad \underset{O,I}{\text{Max}}\ \mathrm{E}\left[\int_0^\infty p(t)O(t)\mathrm{e}^{-rt}\,dt\right] - qI$$

subject to the oil depletion equations,

$$(2) \qquad \dot{S}(t) = -O(t),\ \forall t \geq 0, \quad S(0) = S_0 > 0, \quad \int_0^\infty O(t)dt \leq S_0,$$

the oil exploration investment schedule,

$$(3) \qquad S_0 = \Theta(I), \quad \Theta' > 0, \Theta'' < 0,$$

the oil demand schedule,

$$(4) \qquad O(t) = \Upsilon p(t)^{-\varepsilon} b^{\sigma},\ 0 \leq t < T, \quad O(t) = \Upsilon p(t)^{-\varepsilon}(b - \Delta)^{\sigma},\ \forall t \geq T,$$

the probability that the breakthrough technology occurs in the interval ending at time t,

$$(5) \qquad \Pr(T \leq t) = 1 - \exp(-ht),\ \forall t \geq 0, \quad h \geq 0,$$

where S, q, I and r denote the stock of oil reserves, the price of oil exploration investment, the volume of oil exploration investment and the market interest rate, respectively. The price of oil exploration investment (q) and the market rate of interest (r) are exogenously determined on world markets and constant over time. The concavity of $\Theta(.)$ ensures decreasing returns to exploration investment. The own price elasticity of oil demand (ε) exceeds unity, so that marginal oil revenue is positive.[1] With the demand function (4), marginal revenue is always finite and oil reserves are fully exhausted asymptotically. Oil and renewables are supposed to be gross substitutes, so that the constant cross price elasticity of oil demand (σ) is positive. The inverse demand function for oil is $p = (\Upsilon \tilde{b}^{\sigma} / O)^{1/\varepsilon} \equiv p(O, \tilde{b})$.

The probability that the breakthrough technology has not taken place before time t is $\Pr(T > t) = \exp(-ht)$. The expected time for the breakthrough to occur is $\mathrm{E}[T] = 1/h$. The exponential distribution has a constant hazard rate h, so the conditional probability that the breakthrough does not take place for another t years given that the breakthrough has not already taken place in the first s years is the same as the initial probability that the breakthrough does not take place for another t years: $\Pr(T > s + t \,|\, T > s) = \Pr(T > t),\ \forall s, t \geq 0.$

3. Optimal oil depletion paths before and after the renewables breakthrough

Using the principle of dynamic programming, we work backward in time and first solve the problem from unknown time T onwards when the cheap carbon-free substitute is on the market, then solve the more interesting problem of oil extraction before the substitute has arrived on the market, and finally solve for the optimal level of exploration investment. We denote the problems of oil extraction *after* and *before* the breakthrough technology with superscripts A and B, respectively, and solve them for given S_0 in the rest of this section and characterize the outcomes in section 4. Section 5 then solves for the optimal level of exploration investment I.

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[1] Aggregate oil demand is relatively inelastic, but the relevant elasticity for an individual oil-producing firm is much higher as it cannot easily manipulate the price without losing market share.

After the breakthrough

Marginal oil revenue must equal the scarcity rent, λ, which according to the Hotelling rule must rise at a rate equal to the market interest rate, r:

(6) $$(1-1/\varepsilon)p^A = \lambda, \quad \dot{\lambda}/\lambda = r.$$

It follows from (6) and the isoelastic demand schedule (4) with zero extraction costs that the oil price and depletion paths are efficient despite the oil owner being a monopolist:

(7) $$\dot{p}^A / p^A = r > 0, \quad \dot{O}^A / O^A = -\varepsilon r < 0.$$

Using (7) in (2), we solve for the optimal paths of oil depletion, oil reserves and (using (7)) the oil price after the breakthrough:

(8) $$\begin{aligned} & O^A(t) = \varepsilon r S(t) = \varepsilon r e^{-\varepsilon r(t-T)} S(T), \quad S^A(t) = e^{-\varepsilon r(t-T)} S(T) \le S(T) < S_0, \\ & p^A(t) = e^{r(t-T)} \left[\frac{(b-\Delta)^\sigma \Upsilon}{\varepsilon r S(T)} \right]^{1/\varepsilon}, \quad \forall t > T. \end{aligned}$$

A lower cost of the backstop ($\Delta > 0$) thus pushes down the oil price, especially if the backstop is a good substitute for oil (high σ), but does not affect the path of oil depletion rates except for depressing the final stock of oil (see outcomes before the breakthrough). Substituting (8) into (1), we get the present value of profits of the oil firm after the breakthrough technology comes to market (i.e., the value function after the regime switch):

(9) $$V^A(S(t), b-\Delta) = \left[\frac{(b-\Delta)^\sigma \Upsilon}{\varepsilon r} \right]^{1/\varepsilon} S(T)^{1-1/\varepsilon}, \quad \forall t \ge T.$$

A future breakthrough ($\Delta > 0$) reduces the cost of the substitute and thus curbs the future price of oil. As a result, the present value of oil profits is lower. Oil profits after the breakthrough are high if remaining reserves at the time of the breakthrough are high.

Before the breakthrough

The Hamilton-Jacobi-Bellman equation for the optimization problem before the breakthrough is:

$$(10) \qquad \underset{O^B}{\text{Max}}\left[p(O^B,b)O^B - V_S^B(S,b,\Delta,h)O^B \right] - h\left[V^B(S,b,\Delta,h) - V^A(S,b-\Delta) \right] = rV^B(S,b,\Delta,h),$$

where $V^B(S,b,\Delta,h)$ denotes the value function (i.e., the present value of profits to go excluding the cost of the initial outlay on exploration investment) before the breakthrough (see appendix for a mathematical derivation). Equation (10) states that maximum oil rents *minus* the expected loss in value terms of the carbon-free substitute coming to market must equal the return from investing proceeds at the market rate of interest. The maximization of oil rents in (10) requires marginal oil revenue to be set to the marginal value of in situ oil reserves:

$$(11) \qquad (1-1/\varepsilon)\,p^B(t) = V_S^B(S(t),b,\Delta,h), \quad 0 \le t < T.$$

Using (4) and (11), we obtain the optimal oil depletion rate before the regime switch:

$$(12) \qquad O^B(t) = \Upsilon b^\sigma \left(\frac{V_S^B(S(t),b,\Delta,h)}{1-1/\varepsilon} \right)^{-\varepsilon}, \quad 0 \le t < T.$$

Upon substitution of (11) and (12) into (10), we obtain:

$$(13) \qquad \frac{1}{\varepsilon}\left(\frac{V_S^B(S,b,\Delta,h)}{1-1/\varepsilon} \right)^{1-\varepsilon} \Upsilon b^\sigma - h\left[V^B(S,b,\Delta,h) - V^A(S,b-\Delta) \right] = rV^B(S,b,\Delta,h).$$

To solve (13), we guess the value function $V^B(S,b,\Delta,h)=KS^{1-1/\varepsilon}$, substitute it with (9) into (13), and use the method of undetermined coefficients to solve for *K*. It then turns out that $K=K(b-\Delta,h)$ must satisfy the nonlinear equation:

$$\text{(14)}\qquad \frac{1}{\varepsilon}\Upsilon b^{\sigma}K^{1-\varepsilon}+h\left[\frac{(b-\Delta)^{\sigma}\Upsilon}{\varepsilon r}\right]^{1/\varepsilon}=(r+h)K.$$

Using the resulting value function in (11) and using the oil demand function (4), we get:

$$\text{(15)}\qquad p^B(t)=K(b-\Delta,h)S(t)^{-1/\varepsilon},\quad O^B(t)=L(b,\Delta,h)S(t),\quad 0\le t<T.$$

where $L(b,\Delta,h)\equiv K(b-\Delta,h)^{-\varepsilon}\Upsilon b^{\sigma}$. Solving for the time paths from (15) and (2), we obtain:

$$\text{(16)}\qquad p^B(t)=e^{Lt/\varepsilon}KS_0^{-1/\varepsilon},\quad O^B(t)=Le^{-Lt}S_0,\quad S^B(t)=e^{-Lt}S_0,\quad 0\le t<T.$$

4. Characterization of the solution: aggressive oil depletion

To understand the solution more fully, we characterize the function $K=K(b-\Delta,h)$. The benchmark corresponds to a zero hazard rate. If *h* = 0, (14) gives $K(b-\Delta,0)=(\Upsilon b^{\sigma}/\varepsilon r)^{1/\varepsilon}$ and thus $O^A(t)=O^B(t)=\varepsilon rS(t),\,\forall t\ge 0.$ If $h\to\infty$, (14) indicates that $K\to\left[\frac{\Upsilon(b-\Delta)^{\sigma}}{\varepsilon r}\right]^{1/\varepsilon}$ and

$L\to\varepsilon r\left(\frac{b}{b-\Delta}\right)^{\sigma}>\varepsilon r.$ The rate of oil depletion is thus faster if the breakthrough is imminent rather than if the expected time of arrival is infinite. If the breakthrough leads to an infinitely elastic supply of the carbon-free substitute (*b* = Δ), (14) gives $K(0,h)=\left[\Upsilon b^{\sigma}/\varepsilon(r+h)\right]^{1/\varepsilon}$ $<K(b-\Delta,0),\,\forall h>0.$ and thus *L*(*b*, *b*, *h*) = ε (*r* + *h*). The possibility of oil being made completely obsolete thus depresses expected profits to go for any stock of oil reserves. It also leads to more aggressive depletion of oil reserves. Total differentiation of equation (14) yields:

$$(14') \qquad dK = \frac{-\dfrac{K}{\varepsilon h}(L-\varepsilon r)dh + \dfrac{h\sigma}{\varepsilon(b-\Delta)}\left[\dfrac{(b-\Delta)^{\sigma}\Upsilon}{\varepsilon r}\right]^{1/\varepsilon} d(b-\Delta)}{r+h+(1-1/\varepsilon)\Upsilon b^{\sigma}K^{-\varepsilon}}.$$

Since $L = \varepsilon r$ if $h = 0$ and $L > \varepsilon r$ if $h \to \infty$, (14′) implies $K_h(b-\Delta,h)<0, L_h(b,\Delta,h)>0, \forall h>0.$

This reflects that a higher probability of a renewables technology breakthrough reduces the expected profit to go for the oil well owner, lifts up the path for the oil depletion rate, and depresses the oil price path before the switch. It follows that $\varepsilon r < L < \varepsilon(r+h)$ for any $0< h < \infty$.

Also, if $h > 0$, $K_{b-\Delta}(b-\Delta,h)>0, \forall \Delta \in (0,1)$ from (14′). A bigger size of the climate calamity thus curbs profits to and makes oil depletion more aggressive.

Suppose that the breakthrough occurs at date T. We know from (16) that just before we have $O^B(T-) = Le^{-LT}S_0$ and $p^B(T-) = e^{LT/\varepsilon}(LS_0)^{-1/\varepsilon}$. Using $S^A(T) = S^B(T) = e^{-LT}S_0$ in (8), we get:

$$(17) \qquad \begin{aligned} &O^A(T+) = \varepsilon r e^{-LT}S_0 < O^B(T-) = Le^{-LT}S_0, \\ &p^A(T+) = \left[\frac{(b-\Delta)^{\sigma}\Upsilon}{\varepsilon r S_0}\right]^{1/\varepsilon} e^{LT/\varepsilon} > p^B(T-) = \left(\frac{b^{\sigma}\Upsilon}{LS_0}\right)^{1/\varepsilon} e^{LT/\varepsilon} \text{ iff } \left(\frac{b}{b-\Delta}\right)^{\sigma} > \frac{L}{\varepsilon r}. \end{aligned}$$

We thus arrive at the following proposition.

Proposition 1: *After the breakthrough the oil depletion rate and oil reserves decline at the rate εr and the oil price rises at the rate r with the corresponding time paths given by (8). Before the breakthrough the oil depletion rate and oil reserves decline too rapidly at the rate* $L \equiv K(b-\Delta,h)^{-\varepsilon}\Upsilon b^{\sigma} > \varepsilon r$ *and the oil price rises too rapidly at the rate L/ε > r with the time paths given by (16) where* $K = K(b-\Delta,h), K_{b-\Delta} > 0, K_h < 0$ *solves (14). At the time of the* *breakthrough, there is a discrete increase fall in oil extraction. If renewables enjoy a big enough cost reduction and are a good enough substitute, the oil price falls by a discrete amount.*

Anticipation of a future breakthrough in renewables technology boosts the initial oil depletion rate and depresses the initial oil price, especially if the chance of a breakthrough occurring and the expected cost reduction are high. Whilst the breakthrough technology is not on the market, the oil depletion rate falls and the oil price rises too rapid and may even cross their efficient paths. Once the breakthrough technology is on the market, the oil depletion rate jumps down and the oil price jumps down by a discrete amount if the breakthrough yields a big enough cost reduction and renewables are a good enough substitute, else the oil price jumps up by a discrete amount (see (17)). From then on oil depletion and oil prices follow Hotelling paths, but starting out from an inefficiently low level of oil reserves.

4.1. Benchmark: certainty-equivalent outcome

As benchmark we also calculate outcomes if the breakthrough in renewables technology is introduced *with certainty* at time $T = 1/h$ (i.e., the expected date of the breakthrough). After the breakthrough equations (8) hold. Since there cannot be a discontinuity in the price path at time $1/h$ and prices follow a Hotelling path, the initial oil price is $p(0) = e^{-r/h}\left[\frac{(b-\Delta)^{\sigma}\Upsilon}{\varepsilon r S(1/h)}\right]^{1/\varepsilon}$. Equation (4) gives $O(0) = \varepsilon r e^{\varepsilon r/h}\left(\frac{b}{b-\Delta}\right)^{\sigma} S(1/h)$ and so $O(t) = \varepsilon r e^{-\varepsilon r(t-1/h)}\left(\frac{b}{b-\Delta}\right)^{\sigma} S(1/h),\ \forall t \in [0,1/h].$

Putting this in (2) and integrating, we get the stock of oil at the expected time of breakthrough:

(19) $$S(1/h) = \frac{S_0}{1+(e^{\varepsilon r/h}-1)\left(\frac{b}{b-\Delta}\right)^{\sigma}} \le e^{-\varepsilon r/h} S_0 \le S_0.$$

Given (19), the certainty-equivalent oil price, depletion and reserves paths before and after the breakthrough can be calculated. In particular, we have:

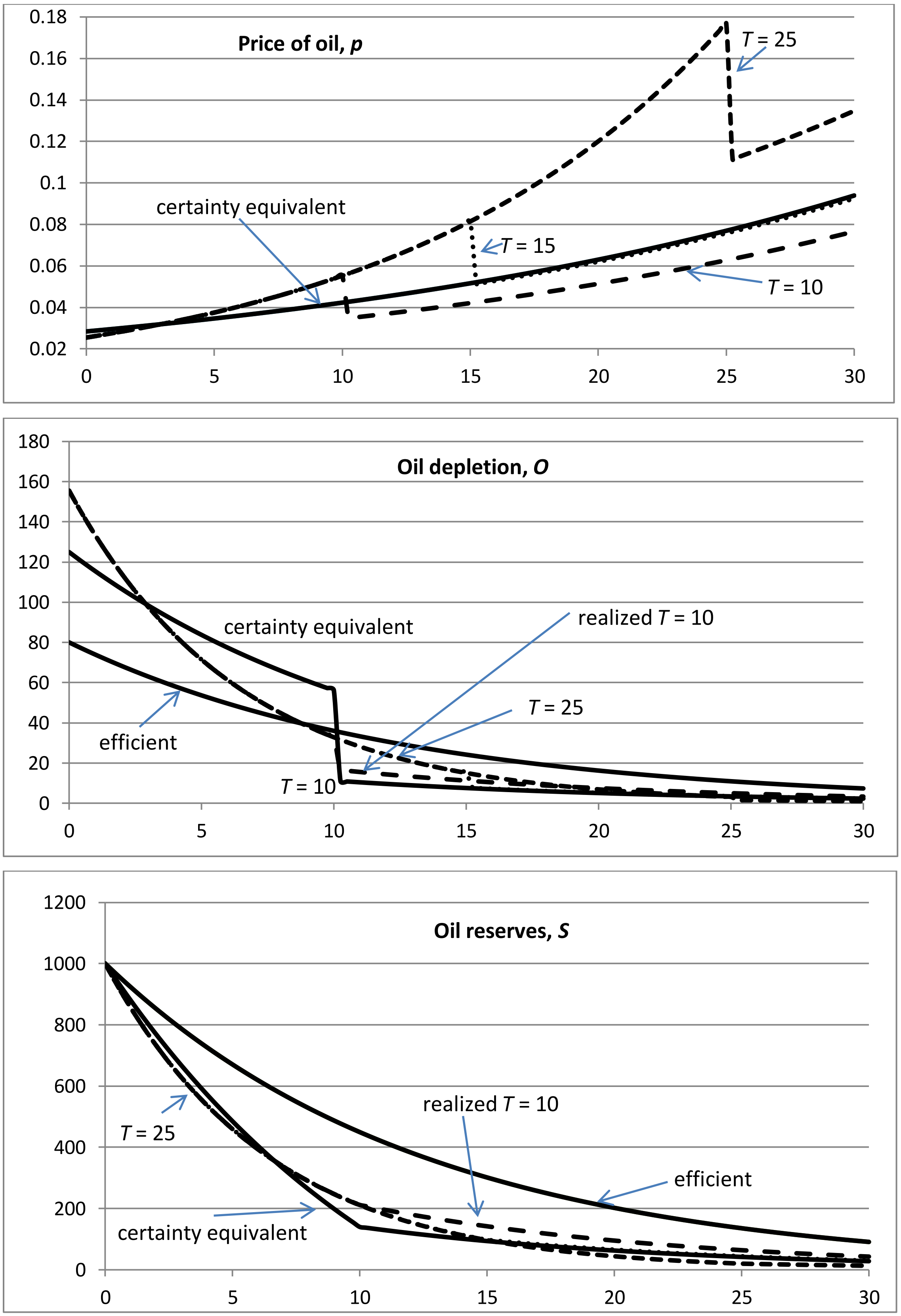
$$(19') \quad S(t) = \frac{1+\left[e^{\varepsilon r(1/h-t)}-1\right]\left(\frac{b}{b-\Delta}\right)^{\sigma}}{1+(e^{\varepsilon r/h}-1)\left(\frac{b}{b-\Delta}\right)^{\sigma}} S_0,\ t\in[0,1/h], \quad S(t) = \frac{e^{\varepsilon r(1/h-t)}S_0}{1+(e^{\varepsilon r/h}-1)\left(\frac{b}{b-\Delta}\right)^{\sigma}},\ t\geq 1/h.$$

Equation (19) indicates that the path of oil depletion rates is unaffected by the cost of renewables if there is no breakthrough. However, a renewables breakthrough ($\Delta > 0$) at the known time $1/h$ induces a lower stock of oil reserves at time $1/h$ and thus before the breakthrough oil depletion occurs at a faster rate than after the breakthrough, especially if renewables are a good substitute for oil (high σ). The path for oil prices satisfies the Hotelling arbitrage principle and is efficient given that the breakthrough is certain to occur at time $1/h$. The efficient paths for oil depletion rates and oil reserves corresponding to a constant cost of renewables of either b or $b-\Delta$ from time zero onwards are identical; these paths are unaffected by the breakthrough. However, the efficient oil price paths corresponding to cost b and to cost $b-\Delta$ rise at the market rate of interest r and are, respectively, above and below the certainty-equivalent path of oil prices.

4.2. Simulation of the impact of expected breakthrough in renewables technology

To illustrate proposition 1, fig. 1 offers some illustrative simulations of our model. We set the own price elasticity of oil to $\varepsilon = 2$, the cross price elasticity of oil to $\sigma = 1$ and autonomous oil demand to $\Upsilon = 1$. We set the interest rate to $r = 0.04$. The hazard rate for the breakthrough is set to $h = 0.1$, so the expected time it takes for the breakthrough is 10 years. Hence, $0.08 < L < 0.28$. The cost of renewables is set to 100 before the breakthrough and to 20 after the breakthrough, so $b = 100$ and $\Delta = 80$. Finally, the initial stock of oil reserves is set to $S_0 = 1000$.

Figure 1: Impact of threat of breakthrough renewables on oil extraction and prices



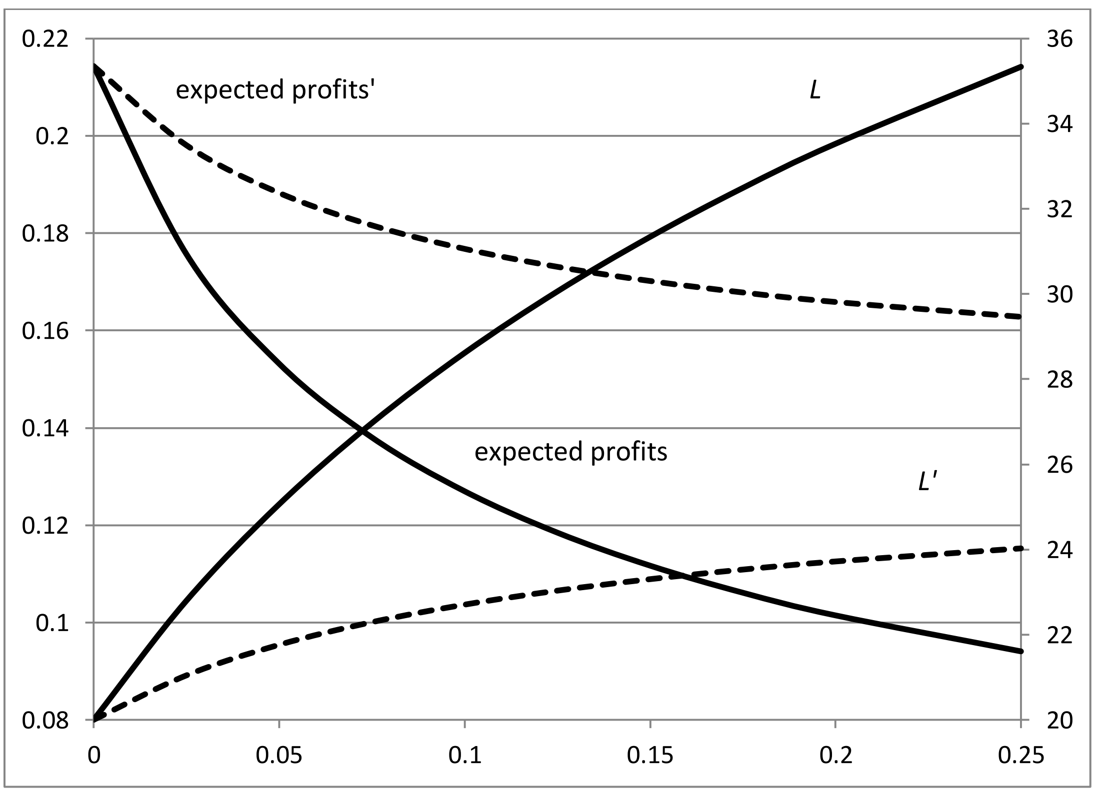
With the parameters set to these values, we find that the solution to (14) is $K = 0.802$ and thus that $L = 0.155$. The speed of oil depletion, 0.155, is thus almost twice as high as the speed after the breakthrough, $\varepsilon r = 0.08$. Fig. 1 shows simulations with realized times of the breakthrough technology occurring at times 10, 15 and 25 by long dashes, dots and short dashes, respectively. We compares these with the certainty-equivalent paths and the efficient paths when cost of renewables are constant from time zero onwards. Although not shown in fig. 1, the initial oil price if there is never a breakthrough is 0.0354 and if there is an immediate breakthrough in renewables technology it is 0.0158. From then on oil prices follow a Hotelling path in each of these two cases. The paths for oil depletion rates and reserves do not depend on whether there is never or an immediate breakthrough. The certainty-equivalent path starts off with an oil price in between, 0.0283, and then also follows a Hotelling path. Oil depletion is affected by the certainty of a breakthrough at some future date: until the breakthrough reserves are depleted at a rapid rate and therefore at a lower rate after the breakthrough.

Not knowing the date of the breakthrough also speeds up the rate of oil extraction before the breakthrough compared with the certainty-equivalent (and a fortiori the efficient) path. This means that initially oil depletion is higher and oil prices lower than in the certainty-equivalent path, but after some time as a consequence of the faster rate of oil depletion oil depletion is lower and oil prices higher than in the certainty-equivalent outcome. At the moment the breakthrough comes to market, both the rate of oil depletion and oil prices jump down and thereafter continue along their Hotelling paths, albeit from an inefficient base. If the cost reduction would have not been so substantial or the renewables would not have been such a good substitute, the oil price would have jumped up by a discrete moment of the breakthrough. A sufficient condition for this not to occur is from (17) that $b/(b-\Delta)^{\sigma} > (r+h)/h = 3.5$.

4.3. Sensitivity of outcomes

Fig. 2 plots the expected present value of oil profits at time zero, $V^B(S_0) = K(b - \Delta, h)S_0^{1-1/\varepsilon}$, and the speed at which oil is extracted, L, against the hazard rate h, both for a potential cost reduction in renewables Δ of 80 and 40. The highest feasible level of expected oil profits is 35.36, which occurs if there is no chance of a breakthrough. The expected present value of oil profits is lower for higher hazard rates and for larger potential cost reductions in renewables. Effectively, a more serious threat of being put out of business by a revolution in breakthrough technology damages prospects for oil producers. As a result, when the threat of a breakthrough and the size of the breakthrough become more substantial, oil producers start extracting oil and more rapid rates before renewables come to market and oil prices fall.

Fig. 2: Effects of chance of breakthrough on oil profits and speed of extraction



Key: Hazard rate h on horizontal axis, speed of oil extraction L on left vertical axis and present value of expected profits on right vertical axis. Dashed indicate $\Delta = 40$ instead of 80.

5. Exploration investment and the hold-up problem

The final stage of solving the problem stated in section 2 is to solve for the optimal I. Using (3) and the expression for the value function at time zero, $V^B(\Theta(I)) = K(b-\Delta,h)\Theta(I)^{1-1/\varepsilon}$, we find that this requires setting the marginal return on marginal exploration investment to its cost:

(20) $$(1-1/\varepsilon)K(b-\Delta,h)\Theta(I)^{-1/\varepsilon}\Theta'(I) = q.$$

Total differentiation of (20) gives $q\ [\Theta'(I)/\varepsilon\Theta(I)] - \Theta''(I)/\Theta'(I)\ dI = q(K_b d(b-\Delta) + K_h dh)/K$ $-dq,$ so optimal exploration investment declines with its cost q, the breakthrough hazard h and the size of the reduction in the cost of renewables after the breakthrough Δ:

(21) $$I = I(b-\Delta,h,q), \quad I_{b-\Delta} > 0, I_h, I_q < 0.$$

If exploration investment is subsidized at the rate θ, the optimality condition (20) becomes $(1-1/\varepsilon)K(b-\Delta,h)\Theta(I)^{-1/\varepsilon}\Theta'(I) = q-\theta.$ In the efficient outcome with never a breakthrough or a breakthrough from the outset, one has $(1-1/\varepsilon)K(b-\Delta,0)\Theta(I)^{-1/\varepsilon}\Theta'(I) = q$ with $K(b-\Delta,0) = (\varepsilon r)^{-1/\varepsilon}$. It thus follows that the optimal exploration investment subsidy increases with the breakthrough hazard and the potential cost advantage of breakthrough renewables:

(22) $$\theta = \left[(\varepsilon r)^{-1/\varepsilon} - K(b-\Delta,h)\right](1-1/\varepsilon)\Theta(I)^{-1/\varepsilon}\Theta'(I) \equiv \theta(b-\Delta,h) > 0, \quad \theta_{b-\Delta} < 0, \theta_h > 0.$$

Equations (21) and (22) give rise to the following proposition.

Proposition 2: *The inefficiencies induced by the uncertain timing of a breakthrough in renewables are exacerbated by a drop in exploration investment, especially if the threat of a better carbon-free substitute and the potential cost reduction are higher. These inefficiencies can be eliminated by subsidizing exploration investment at the rate (22).*

This proposition is an illustration of the hold-up problem (e.g., Rogerson, 1992; Holmström and Roberts, 1998). One way to overcome this is vertical integration, which may be feasible if the government can nationalize the oil firm. There may also be contractual solutions. Here an appropriate exploration investment subsidy gets rid of the inefficiency. As oil producers are typically in different jurisdictions to oil users, such a subsidy is unlikely to be implemented.

6. Climate policy and the Green Paradox

It is easy to see that with isoelastic demand and zero oil extraction costs a *constant* ad valorem carbon tax is equivalent to a fall in oil demand (lower Υ) or a *constant* subsidy to renewables use (lower b throughout) do not affect the paths of oil extraction and oil reserves. A rising path of ad valorem carbon taxes or a constant specific carbon tax will affect the rate of oil depletion, but we will abstract from these for it is difficult to muster political support for these policies. Policy makers find the carrot easier than the stick, so they focus at subsidizing green R&D instead of taxing carbon emissions. Subsidizing green R&D is meant to bring forward the introduction of carbon-free substitutes for fossil fuel, so we specify $h = H(\theta), H' > 0, H'' < 0,$ where θ is the subsidy for green R&D. The subsidy thus increases the hazard rate h and cuts the expected time of the breakthrough, $1/h$. We see from (21) and proposition 2 (abstracting from exploration investment subsidies) that subsidizing green R&D makes oil more obsolete and thus depresses exploration investment I and thus curbs the total amount of recoverable oil reserves, S_0, and the total amount of carbon that can be emitted into the atmosphere. Of course, we also know from proposition 1 that a higher hazard rate slows down the speed of oil extraction and thus the speed at which carbon is emitted into the atmosphere (see also fig. 2). Subsidizing green R&D to speed up the development of carbon-free substitutes thus leads to a Green Paradox in the short run.

However, as the total amount of carbon that can be emitted into the atmosphere is curbed, the Green Paradox is likely to be reversed in the long run.

To see this more precisely, let the present value of global warming damages be defined by:

$$(23) \qquad G \equiv \int_0^\infty D\left[E_0 + \Theta\left[I\left(b-\Delta, H(\theta)\right)\right] - S(t)\right] e^{-\rho t} dt, \quad D'>0, D''>0,$$

where $E_0 > 0$ denotes the initial stock of carbon in the atmosphere and $\rho > 0$ the social rate of discount. This formulation supposes, for simplicity, that all carbon that is emitted into the atmosphere stays there forever. Total carbon in the biosphere is thus $E_0 + S_0$ and $E \equiv E_0 + S_0 - S$ thereof is in the atmosphere. Global warming damages depend on total carbon emissions (so oil is measured in GtC) and are described by a convex function. We then have:

$$(24) \qquad \frac{\partial G}{\partial \theta} = \underbrace{-\int_0^\infty \frac{\partial S(t)}{\partial h} H'(\theta) D'\left[E(t)\right] e^{-\rho t} dt}_{+} + \underbrace{\Theta' I_h H' \tau}_{-},$$

where $\tau \equiv \int_0^\infty D'\left[E(t)\right] e^{-\rho t} dt > 0$ defines the social cost of carbon (present value of marginal damages). The first term on the right-hand side of (24) indicates that a green R&D subsidy speeds up oil extraction and thus exacerbates damages (the Green Paradox) and the second term shows that the subsidy discourages exploration of oil fields and thus limits the total amount of carbon emitted into the atmosphere. If the latter effect dominates, the Green Paradox is reversed.

7. Conclusion

We have used a tractable model of a resource-owning monopolist with isoelastic demand and zero variable oil extraction costs to gain more insights into the Green Paradox. The anticipation of the arrival of a carbon-free substitute at an uncertain moment of time in the future induces oil well owners to pump oil more quickly and to push down the oil price. Since this leads to more rapid emissions of the *given* amount carbon in the soil, global warming is exacerbated (the Green Paradox). As soon as the carbon-free substitute has arrived, the oil depletion rate jumps down. If the new carbon-free fuel is not a very good substitute and the cost reduction is not too substantial, the oil price jumps up by a discrete amount at that moment. If the breakthrough is substantial enough and a good enough substitute for oil, the oil price jumps down. From then on the oil depletion rate declines at the Hotelling rate, albeit starting out from a lower level of reserves than would be the case if there was no anticipation of renewables being introduced, and the oil price rises at the market rate of interest. Interestingly, if the carbon-free substitute was introduced from the outset with certainty, oil extraction would be unaffected.

An uncertain introduction date for the carbon-free substitute depresses oil exploration investment and thus more oil is left in the crust of the earth. Hence, the total amount of carbon emitted into the atmosphere is reduced, albeit that what is emitted is emitted more rapidly. This can easily overturn the Green Paradox. The exploration investment inefficiency can be corrected for with an appropriate subsidy, which increases in the chance of the breakthrough occurring.

The consequences of cheaper renewables are thus not as bleak as suggested by proponents of the Green Paradox. Future work will benefit from a better grasp of regime switches, whether they relate to arrival of carbon-free substitutes or potential tipping points and climate disasters.[2] It is also of interest to put the breakthrough approach in a strategic setting.[3]

[2] If positive feedback effects in the carbon cycle are strong enough, runaway global warming may result (Winter, 2012). However, this will lead to an irreversible doomsday scenario.

[3] Hoel (1978) and Gerlagh and Liski (2011) analyze the strategic pricing policies of an oil producer faced with a substitute coming to market. Jaakkola (2012) analyzes in more detail the strategic dynamic interactions with oil importers developing substitutes.

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Appendix: Derivation of the HJB equation (10)

Since the probability of a regime shift in an infinitesimally small time period Δt is $h\Delta t$, the Principle of Optimality from the perspective of time zero can be written as follows:

(A1) $$e^{-rt}V^B\ \ S(t)\ = \underset{O^B}{\mathrm{Max}}\Big[\int_t^{t+\Delta t} e^{-rs} p\ \ O^B(s)\ \ O^B(s)ds + (1-h\Delta t)e^{-r(t+\Delta t)}V^B\ \ S(t+\Delta t) + h\Delta t e^{-r(t+\Delta t)}V^A\ \ S(t+\Delta t)\ \Big]$$

(suppressing the arguments b, Δ and h in the value function $V^B(.)$). Multiplying both sides by e^{rt}, rearranging and dividing by Δt, we rewrite (A1) as:

(A2) $$\underset{O^B}{\mathrm{Max}}\left[\frac{\int_t^{t+\Delta t} e^{-r(s-t)} p\ \ O^B(s)\ \ O^B(s)ds}{\Delta t} - he^{-r\Delta t}V^B\ \ S(t+\Delta t)\ + he^{-r\Delta t}V^A\ \ S(t+\Delta t) \frac{e^{-r\Delta t}-1\ V^B\ \ S(t+\Delta t)}{\Delta t} + \frac{V^B\ \ S(t+\Delta t)\ - V^B\ \ S(t)}{\Delta t}\right] = 0.$$

Evaluating the integral in (A2) for infinitesimally small Δt and taking the limit as $\Delta t \to 0$ whilst using l'Hôpital's Rule for $\lim_{\Delta t\to 0}\frac{\exp(-r\Delta t)-1}{\Delta t} = -r,$ we get:

(A3) $$\underset{O^B}{\mathrm{Max}}\Big[p\ \ O^B(t)\ \ O^B(t) - \dot{V}^B\ \ S(t)\ - hV^B\ \ S(t)\ + hV^A\ \ S(t)\ - rV^B\ \ S(t)\ \Big] = 0.$$

Substituting $\dot{V}^B = V_S^B\dot{S}$ and using (2), rearranging and dropping the time index, we get (10).