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CESIFO WORKING PAPER NO. 3953
CATEGORY 12: EMPIRICAL AND THEORETICAL METHODS
OCTOBER 2012

CESifo *Working Paper No. 3953*

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Abstract

High income growth in many countries in East Asia and the Middle East has been accompanied by increasing income inequality and widening gaps between rich and poor, and urban and rural. It is therefore it is important to examine the interrelationships between inequality and economic growth. This paper develops a simple model to establish that the change in income growth rate is a non-linear function of the income growth if policy makers try to influence economic growth. As a result, inequality and growth bear a non-linear relationship: for low values of inequality, economic growth rate is an inverted U-shaped \function of inequality. This function becomes U-shaped for values of inequality beyond a critical value of inequality. As a result, the relationship between growth and inequality can take the form of a wave. This simple theoretical model is a sufficient case to explain why previous empirical studies might have failed to reach a consensus between economic growth and inequality. The paper estimates the model empirically by using a set of panel data for ten Middle Eastern countries. The empirical analysis finds statistical support for a possible wave like relationship between growth and inequality, which can bear ominous messages for using equitable growth in fighting poverty. Rapid growing developing economies need to adopt appropriate policies for achieving an optimal mix of inequality and growth. Promoting inclusive growth together with good governance is crucial to ensure more equity and social stability.

JEL-Code: H210, O110, O150, O410.

Keywords: growth, inequality, poverty, East Asia and the Middle East.

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Can there be a Wave-Like Association between Economic Growth and Inequality? Theory and Lessons for East Asia from the Middle East

By

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1. Introduction

During the last five decades two critical changes of paramount significance have shaped the global economy. First and foremost, there arose a continual transfer of the production capacity of the global economy from the West to the East Asian nations - namely Japan, Taiwan, South Korea and finally to China[1]. Secondly, due to geological serendipity, the Middle East has much of the world's oil reserves and productive capacity of oil while oil exporters from the region accumulated rich returns from their dollar deposits from the Eurodollar markets. Both East Asia and West Asia, as a direct consequence, have experienced unprecedented economic opulence. Due to asymmetric distribution of oil endowments, the benefits of economic growth in the Middle East has been uneven as economic growth benefited a few exporters of oil more than others, which has possibly led to the widespread protests in recent years for democracy, freedom and economic

[1] This region has grown into the economic powerhouse of the global economy. In 1955 China, Japan, South Korea, and Taiwan encompassed over one quarter of the global population but generated only 9 percent of the gross domestic product (GDP of the globe. Within a span of five decades East Asia's population, measured against the world's total, had fallen to 23.24 percent while its share of the global GDP had shot up nearly three-fold to 25 percent. During the five decades since 1955 these East Asian economies grew from among the poorest to among the richest in the world.

equity in the Middle East. On the other hand, during 1955-2000 East Asia became a showcase of progressive development as rapid economic growth in the region was accompanied with sustained declines in inequality (see Fei, Ranis and Kuo, 1979 and United Nations, 1999)[2]. With the advent of China as an engine of growth in East Asia we now increasingly witness spectacular economic growth and rising income inequality within East Asia. It is evident that China not only leads the region's economic growth but also leads the region's growth in inequality[3]. We also note a general trend of increasing inequality in the entire region. In Japan, Taiwan and South Korea, by the early twenty-first century - a half century after the initiation of the growth with equity pattern, levels of income inequality in these societies mostly exceeded what they had had about a half-century earlier[4]. Over the long haul the differences in income and wealth, relatively small to begin with, have moved apart to become wider and wider and have evolved into a durable new social stratification of the region in terms income inequality within and between nations. The level of inequality in East Asia today is comparable to what have

[2] The regional economic progress not only increased their total GDP it also benefitted most, if not all, of the population through increasing income levels and improving standards of living. The economic growth seen in Japan, South Korea, and Taiwan was accompanied by a relatively equitable distribution of income throughout their populations.

[3] The most significant case of rising inequality in East Asia is China. Income inequality measured by the Gini index rose from around 0.30 in the early 1980s to over 0.45 by the turn of the century. In 2009 the Gini index stood at .48. Such a change marked China as having the fastest income-inequality increase of any large country over the last three decades and, now, one of the countries with the highest income inequality in the world.

[4] There are various local and global factors responsible for increasing inequality in Japan, South Korea and Taiwan: among global factors, one often highlights the collapse of communism by the 1990s, which paved the way for the global capitalist expansion driven by multinaitonalisation and propelled by the strengthening of the free market ideology and a gradual withdrawal of the state from the domain of economic management. Among local factors we note a gradual transition of these economies from their initial focus upon large-scale manufacturing to their current concentration on service sectors. It is widely held that large-scale manufacturing is more equitable than service sectors that naturally tend to create economic heterogeneity in a society. At the same time due to changes in demographic patterns and shifts n cultural and value systems, the region experienced a gradual weakening of the traditional family-based social networks and kinships. The weakening of these traditional collectivist norms also led to the rising inequality in the region.

been the inequality levels in the Middle East during the oil boom till the Iraq war. Both in East Asia today and in the Middle East in the past, policy makers have placed an emphasis on *inclusive growth*, which traditionally implies a desirable mix of economic growth and income inequality. Yet the Middle East seems to have failed to strike a balance between growth and inequity during the oil boom. Our central concern is to explain why nations may fail to achieve inclusive growth, which will form the core of lessons for East Asia from the experience of the Middle East in terms of growth and inequity. In order to understand the process of inclusive growth, our focus is on the interrelationship between changes in economic growth and inequality.

The interrelationships between inequality and economic growth of a nation have been extensively studied in economics while an apparent inconclusiveness of the literature has become one of the classic examples of the most enduring economic debates in macroeconomics (see Barro, 2000; Dollar and Kraay, 2002; Easterly, 1999; Forbes, 2005; Kraay, 2005; 2006; Lopez, 2004; 2005; Ravallian1997; 2004). In an important contribution Banerjee and Duflo (2003) questioned the tenability of the assumed *linear* relationship between growth and inequality in the existing literature by establishing an inverted U-curve between growth and inequality[5]. The main goal of our paper is two-fold: first, we develop a simple model of policy-induced growth in order to establish a wave-like relationship between growth and inequality. Secondly, we provide empirical support to our model to establish that the intention to use economic growth and inequality

[5] Banerjee and Duflo (2003) marshalled evidence and offered a political economy model to explain why there is little theoretical salience to the assumed linear, or even monotonic relationship between growth and inequality. From the cross-country data they established that changes in inequality and growth rate bear an inverted U-shaped relationship, which may either be caused by measurement errors or by their model. The inverse U-curve can explain the divergence of estimates of the previous studies on the impact of inequality on growth.

as policy instruments to shape economic development can backfire. The plan of the paper is as follows: in Section 2 we provide a brief literature review. Section 2 also introduces the baseline model, economic data and the modeling framework. Section 3 provides the estimation procedure and basic results. Section 4 discusses the main findings. Section 5 concludes.

2. Modeling Growth and Inequality: Related Literature

There are several interesting and important issues are at stake in the context of growth and inequality: first and foremost, an extensive literature exists on the policy framework and institutional details that promote equitable growth (see Kanbur, 2005 for a review). Secondly, some attempts have been made to understand the dynamics of choice of a society of those specific policies and institutions that are responsible for creating, fueling and driving equitable growth. The rational choice models of political economy provide some insights into the success, or failures, of a society in choosing appropriate institutional structures and relevant policies for promoting equitable growth[6]. There are obvious difficulties in isolating precise links between economic policies and their impacts on economic growth, as highlighted by Easterley (2001). Thirdly, the role of equitable growth is adequately reflected in the United Nations' strategy to reduce the incidence of global poverty by half, under the Millennium Development Goals (MDGs), by creating equitable growth by the year 2015. There is a convergence of views, or opinions, on two related themes: first, increasing economic growth holding inequality unchanged is good for a society. Admittedly, there is little discussion on the impacts of economic growth on

[6] See Besley and Case (2003), Besley and Coate (2003), Case (2001), Drazen (2000), Persson and Tabellini (2000, 2003).

environment. Secondly, inequality holding the rate of economic growth unchanged is bad for a society. However, once inequality and growth both vary, the statistical results are inconclusive about their interrelationship. Though, economists tend to still get influenced by the "Kuznets curve", in an early work, Anand and Kanbur (1993) showed that the cross-country data cannot establish any precise relationship. Our work will try to establish the *raison-d'etra* for this finding, which was confirmed by others in subsequent work (e.g. Deininger and Squire, 1996 and Li, Squire and Zou, 1998 among others).

2.1.1. Growth, Inequality and their Interrelationships

An extensive literature has already explored how distribution of income affects the GDP growth (see early work by Persson and Tabellini 1994, Alesina and Rodrik, 1994). Note that the direction of causality is postulated to run opposite to the much-celebrated Kuznets' Hypothesis that argues that income inequality first rises and then falls during the course of economic development, or economic growth (Kuznets, 1955). Alesina and Rodrik (1994) find a negative relationship between inequality and growth in a political-economy-model of endogenous growth, if government spending is devoted entirely to production. Persson and Tabellini (1994) confirm the result as Alesina and Rodrik (1994) in a two-period overlapping-generations model. On the other hand, Li and Zou (1998) came to the opposite conclusion by examining the relationship between income inequality and economic growth in an endogenous growth model with distributive conflicts among agents. They find that when the household utility function is logarithmic

in public consumption and exhibits a higher-than-unity degree of risk aversion in private consumption, a more equal distribution of income causes a higher rate of capital taxation in a majority voting mechanism. An increase in the rate of capital taxation lowers economic growth, which shows that income inequality can foster faster economic growth. Empirical results based on the cross-country evidence, undertaken by Li and Zu, 1998, Clarke (1995), Benabou (1996), Deininger and Squire (1996, 1998), Li and Zou (1998), Li, Squire and Zou (1998), Barro (2000), Savvides and Stengos (2000), Forbes (2000), Li, Xu and Zou (2000), Banerjee and Duflo (2000), Li, Xie and Zou (2000), Chen (2002), among many others, are somewhat inconclusive.

2.1.2. The 95% Theory of Kuznets' Inverted-U Hypothesis: Just a Glorified Speculation?

Growth and inequality and their mutual feedbacks on each other can hardly escape the tyranny of the oft-repeated "iron law of empirical regularity" popularly known as the inverted-U hypothesis of Kuznets. The hypothesis posits that economic growth is initially accompanied with an increasing inequality till a point, which is the hilltop of the inverted-U curve, and then they bear an inverse relationship. The causality is believed to run from growth to inequality. There is no gainsaying to the fact that Kuznets' inverted-U hypothesis has played an important role in the continuing debate on the interrelationship between inequality and growth since his classis work published in 1955. In his own opinion, yet, Kuznets underscored the inverted-U as a 95% speculation and 5% "empirical verification". Moreover, his "empirical verification" was centred on three advanced nations Germany, England and the US. The inverted-U hypothesis proposes two mutually exclusive phenomena: first, at lower levels of economic development,

increasing economic growth promotes rising inequality. The rising inequality is caused by economic growth since economic growth results in an important transition of an economy, at a lower level of economic development, from predominantly agrarian to an industrial society. The fundamental assumption is that the industrial sector is richer and also more "unequal" than the agrarian sector. The rising weights and importance of the industrial sector thus cause the inequality to rise until a critical point. Secondly, economic growth beyond this critical point lowers inequality due to another important transition in the society – namely the organization of industrial workers into powerful lobbies and unions to advance their self-interests. Kuznets (1955) was cautious in labelling his own hypothesis as 'speculation' since such transitions are neither guaranteed nor sacrosanct. If there are forces within the society that thwart, or cause multiple recurrences of, these transitions the Kuznets-inverted U will never materialise. In what follows we show the possibility of a wave function, instead of an inverted-U-shape, between economic growth and inequality with significant implications.

2.1.3. The Exalted Status of the Interrelationships between Growth and Inequality: The Immortal Triangle of Growth-Inequality-Poverty

In their important initial work Kakwani et el. (2004), Ravallion and Chen (2003), and Ravallion (2004), and subsequent finessing, they have provided the foundation for the important goal of maximizing the *reduction of poverty* via finetuning economic growth and equity. For the reduction of poverty, they have tended to agree that both faster economic growth and greater equity should be the policy priorities of national

governments and international agencies[7]. The essence of the argument of the pro-poor growth (PPG) of Ravallian and Chen (2003) requires that as an inequality index, say the Gini coefficient, increases, the rate of PPG will decline relative to the actual rate of growth. Similarly, if the index falls, the rate of PPG will rise relative to the actual rate of growth. The definition of Kakwani *et el*. (2004) is known as the poverty-equivalent growth (PEG) that is the product of the actual growth rate and the poverty elasticities with respect to income growth and income inequality. If the PEG exceeds the actual growth rate then growth is pro-poor, otherwise not[8]. Both these definitions are based on the effects of growth and inequality in reducing poverty. In simple terms both theories seek to maximise the 'Total Poverty Elasticity' (with respect to both the growth of income and changes in inequality), by assuming a *complementarity* between economic growth and income equality in reducing poverty. However, the problem is that the cross-country regressions have not provided empirical support to the complementarity between growth and equity.

[7] It is well-known in the literature that Kakwani et el. (2004) and Ravallian (2004) had different definitions of 'Pro-Poor Growth'. Kakwani et el. unequivocally noted the importance of identifying a *relative* improvement in the condition of the poor, which convinced them to argue that "the incomes of the poor grow faster than those of the non-poor". On the other hand, Ravallion's original position recognised that more rapid growth is 'pro-poor' if it is more poverty-reducing in terms of headcount ratios.

[8] The PEG is given by the percentage change in the poverty headcount relative to the percentage change in income per capita. The 'Total Poverty Elasticity' (TPE) combines both the 'Poverty Elasticity of Growth' and the 'Poverty Elasticity of Inequality' (PEI). The PEI is the percentage change in the poverty headcount relative to the percentage change in the Gini Coefficient. Hence, if the 'Total Poverty Elasticity' exceeds the 'Poverty Elasticity of Growth', then the reduction in inequality is reducing poverty and, by definition, the Poverty Equivalent Growth Rate exceeds the actual growth rate.

2.2 Our Modeling Framework

X represents economic growth while x is the change in economic growth over time. In a similar vein Y is economic inequality and y is the change in inequality over time. We assume that the policy maker receives a positive return R that is predicated on economic growth and given by[9]:

$$[R(X)/X]=a-bX,\ a>0,\ b>0,\ \text{and}\ X>(b/a) \qquad (1a)$$

We assume that economic inequality imposes a cost on the policy maker[10] and the policy cost, C, depends both on X and Y and given as

$$C(X, Y)=c(Y/X)^2/2,\ c>0 \qquad (1b)$$

Note $\partial C/\partial X<0$ and $\partial C/\partial Y>0$. The higher the growth the lower is the cost of inequality. The policy cost increases with increased inequality, *ceteris paribus*. Some of the policy costs may be purely pecuniary such as social security payment, unemployment benefits while others may be purely social like conflicts, jealousy, social deprivation etc.

We further posit that inequality and growth will have impacts on the time profile of change in growth x and we express the relationship as:

$$x=F(y,X) \qquad (1c)$$

We assume that increase in inequality induces growth and hence $\partial F/\partial y>0$[11]. We also assume $\partial F/\partial X<0$.The higher is the initial growth X, the lower is the change in growth rate x. We express (1C) as a simple linear function[12]:

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[9] One can argue governments seek economic growth since growth reduces poverty. Kraay (2005) showed that 70% short-run changes in poverty are propelled by growth in average incomes of nations.

[10] There are various ways one can rationalise the cost of inequality on policy makers and one possibility is due to Ravallian (1997; 2004) who established that the effectiveness of growth in reducing poverty depends on the initial level of inequality. His 2004 estimates show that 1% increase in average income will result in a decline of 4.3% of poverty for very low inequality nations, or as little as .6% for high inequality nations.

[11] Following the unanimity of the empirical literature, we posit that growth does not impact on inequality (see Dollar and Kraay, 2002; Easterly, 1999).

[12] First, it is widely recognised and empirically verified that increases in inequality promote economic

$$x=hy-mX \text{ with } h>0 \text{ and } m>0 \qquad (1d)$$

It is imperative that we carefully explain equation (1d) and our model of agent behaviour here before making any further progress: we postulate that the policy maker and all economic agents have "learned to believe" the economist's model that there is a linear and positive relationship between inequality and growth. It is important to note that the so-called "threshold effects" offer a theoretical justification in terms of political economy models for higher inequality at a point in time to slower future economic growth. Banerjee and Duflo (2003) examine some of these threshold-effect models and develop an overarching model to capture various causal links running from inequality to growth[13]. Banerjee and Duflo (2003) suggested the possibility of an inverted U-curve as an empirical association between economic inequality and economic growth. The problem is that there are various causal links by which inequality impacts on growth and empirical verification of each is a serious problem (see Kanbur, 2005). This problematic issue is pithily outlined by Kanbur (2005) as:

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growth (see Banerjee and Duflo, 2003; Li and Zou, 1998; Arellano and Bond, 1991). In contrast, Barro (2000) and Lopez (2004) did not find *strong* dependence of growth on inequality. Lopez (2006) and Lopez and Serven (2006) reversed their earlier findings. Secondly, impacts of X on x represent an implicit condition for convergence of growth paths.

[13] These models postulate that there are threshold effects in the return to human capital in the sense that substantial returns are generated only after a critical threshold of human capital is reached by decision-makers. If capital market is imperfect then these decision-makers will have to self-finance their building of human capital. In such a scenario, under a set of conditions, increase inequality will cause the accumulation of human capital to decline, which will thereby lower labour productivity and thereby reduce future economic growth.

"The jury is still out, and the literature swings between combinations of papers that claim to show causality from high inequality to low growth, to those that claim to show no causality - or even that more inequality leads to higher growth". (pp. 226).

It is instructive to note that the choice of (1d) is robust, which can easily incorporate the "threshold effect" by altering the signs of the coefficients to h (<0) and m (<0), which will not change. These changes in signs will have no effect on the subsequent equilibria X_i* and their stability properties. Our model is thus capable of generating wave-like functions even when h<0 and m<0, which are likely to be the case for threshold-effect models. What is also important is that we postulate that the linear relationship is not only the "shared mental model" but also the correct model. However, the problem starts the very moment the policy maker tries to exploit this linear relationship to achieve a desirable mix of inequality and growth. What we will show is that the attempt to influence changes in growth by changing inequality by the policy maker will create the wave-like relationship between growth and inequality. Let us now get back to the basics of the model.

The policy-induced growth model is represented by a policy maker who solves the following present value problem:

$$\text{Maximise } V(x)=\int_0^T e^{-rt}[R(X)-C(Y, X)]dt$$

Subject to

$$[R(X)/X]=a-bX,\ a>0,\ b>0 \text{ and } X>(b/a) \quad (1a)$$

$$C(X, Y)=c(Y/X)^2/2,\ c>0 \quad (1b)$$

$$x=hy-mX \quad (1d)$$

$$X(0)=a \quad (1e)$$

The Hamiltonian-Jacobi-Bellman (HJB) equation is given by

$$rV(x)=Max[R(X)-C(X,Y)+V'(x)x] \quad (2a)$$

Proposition 1: If X* represents the steady state economic growth, the Hamiltonian-Jacobi-Bellman equation is reduced to:

$$X^*[h^2aX^*/(cr)- h^2bX^{*2}/(cr)-m]=X^*M(X^*)=0 \quad (2b)$$

M is a quadratic function of X*. Thus there are three possible steady state equilibria:

$$X_1^*=0 \quad (2c)$$

$$X_2^*=a+SQRT[a^2-4bmcr)/2b] \quad (2d)$$

$$X_3^*=a-SQRT[a^2-4bmcr)/2b] \quad (2e)$$

Proof: By definition X* is given by

$$x=hy-mX=0 \quad (3a)$$

From the HJB equation we have

$$V(X^*)=[R(X^*)-cy^2/(2X^*)^2]/r \quad (3b)$$

$$V(X^*)=[R(X^*)-cm^2/(2h^2)]/r \quad (3c)$$

Hence $V'(X^*)=R'(X^*)/r$ (3d)

The Left Hand Side (LHS) of the HJB is:

$$LHS=rV(X^*)=R(X^*)-(cm^2)/(2h^2) \quad (3e)$$

The Right Hand Side (RHS) of the HJB is:

$$RHS=\max_{\{y\}}[R(X^*)-(cm^2)/(2h^2)+ xR'(X^*)/r] \quad (3f)$$

The first order condition requires:

$$\frac{\partial[(R(X^*) - cm^2/(2h^2) + (hy - mX^*)R'(X^*)/r]}{\partial y} = 0 \qquad (4a)$$

Note that (4a) yields:

$$y=h(a-bX^*)/(c^*r) \qquad (4b)$$

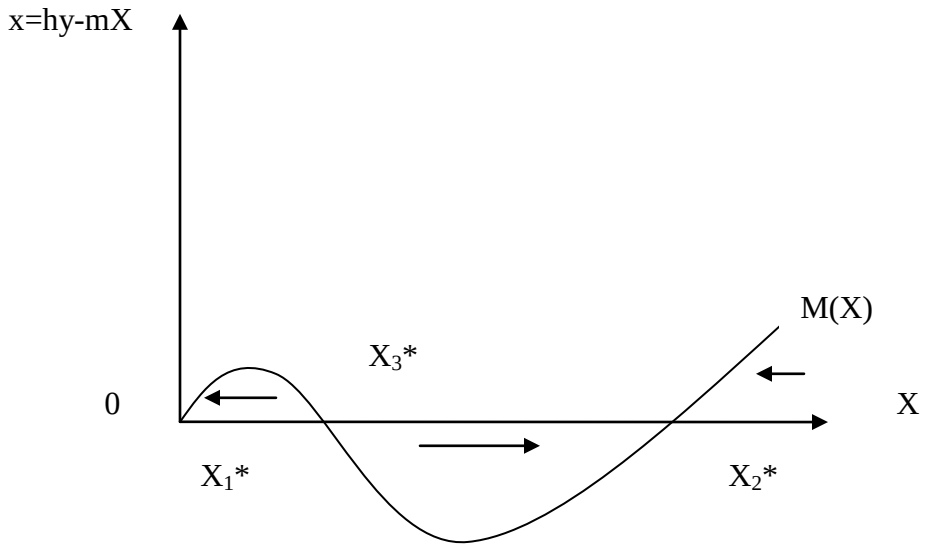
Substituting (4b) into (3a) yields:

$$X^*[(h^2a)/(c^*r)X^* - (h^2b)/(c^*r)\ X^{*2} - m]=0 \qquad (4c)$$

Equation (4c) has three roots as given by equations (2c), (2d) and (2e) that are the three steady states. QED.

The above equilibria can be depicted in a diagram as follows:

DIAGRAM 1: MULTIPLE GROWTH EQUILIBRIA



2.3 Discussion of the Theoretical Findings

In Diagram 1, we plot economic growth along the horizontal axis and the change in growth along the vertical axis and equation (2b) is drawn as M(X) that intersects the horizontal axis at X_1*, X_2* and X_3* that are the three equilibrium growth rates and their stability is described arrows: X_3* is the unstable equilibrium that separates the other two stable equilibrium. We note that X_1*, X_2* and X_3*can be Pareto-ranked from the standpoint of growth. X_1* is the Pareto-worst, X_2* is the Pareto-best and are the extremal equilibria (Milgrom and Roberts, 1990 and Vives, 2005) and X_3*acts as a separatrix between the extremal equilibria. If the initial rate of growth $X<X_3$*, the system monotonically converges to the Pareto-worst equilibrium X_1*. If the initial economic growth exceeds X_3*, $X> X_3$*, then the system monotonically converges to the Pareto-dominant equilibrium X_2*. It is also important to note that the dynamics of growth will bring the growth rates X* within (X_1*<X*<X_2*) as the mixed strategy outcomes, correlated equilibria and rationalization equilibria will lie in the zone (X_1*<X*<X_2*). Any kind of adaptive dynamics will take the system monotonically to either of the extremal equilibria (see Vives, 1990). One can also impose an explicit dynamics to generate cyclical fluctuations within the extremal equilibria (see Vives, 2005; pp. 430). Furthermore, properly mixed equilibria can also be shown to be unstable with respect to a general adaptive dynamics (see Echenique and Edlin., 2004).

2.4 Measurement of Variables in the Model

We will estimate equation (5a) by using a set of panel data including observations for ten Middle Eastern countries covering the period 1963–1999. Unfortunately, there are limited

freely available data on Arab countries. As a consequence, we choose the following seven Arab countries in this study: Algeria, Egypt, Jordan, Kuwait, Morocco, Syria, and Tunisia. The three non-Arab countries are Iran, Israel, and Turkey. We consider the following variables for each country:

- RGROWTH is the growth rate of the real GDP at constant 1995 US$ (Variable RGROWTH),
- INQ is the estimated income inequality measured by the Gini coefficient (Variable INQ),
- FDI is foreign direct investment as a percentage of GDP, constant 1995 US$, (Variable Z_1)
- INF is annual inflation as measured by the year-to-year change in the consumer price index (Variable Z_2),
- IMN is the immigrant population to the US as a proportion of the population in the country of origin, (Variable Z_3)
- ME is military expenditure as a percentage of GDP, constant 1995 US$, (Variable Z_4),
- MILPER is the number of military personnel, (Variable Z_5)
- POP is the total population, (Variable Z_6)
- PP is GDP per capita (constant 1995 US$), (Variable Z_7)
- WRG is the workers' remittance as a percentage of GDP, (Variable Z_8)

2.4.1 Inequality and Growth Data

The real income growth data are from the GDP figures reported in the Penn World Table 6.1. The inequality data is drawn from the Estimated Household Income Inequality Data Set (EHII) — a global dataset derived from the econometric relationship between UTIP-UNIDO, other conditioning variables, and the World Bank's Deininger and Squire data set (see http://utip.gov.utexas.edu/about.html). The UTIP-UNIDO data set source computes inequality measures for nearly 3200 country/year observations, covering over 150 countries during the period 1963 to 1999. Inequality is linked to a number of mathematical concepts such as skewness, variance, and dispersion. Consequently, there are several methods to compute inequality, for example the McLoone Index, the coefficient of variation, range, range ratios, the Gini Coefficient, and Theil's T statistic. The main justification for choosing Theil's T statistic is that it offers a more flexible structure that often makes it more suitable than other measures[14]. If we had permanent access to all necessary individual-level data for the population of interest, measures like the Gini coefficient or the coefficient of variation would be generally satisfactory for describing inequality. Yet, in the real world, individual data is hardly ever reachable, and researchers make do with aggregated data.

[14] Pedro Conceição and Pedro Ferreira provide a much more detailed analysis of these issues in their UTIP working paper 'The Young Person's Guide to the Theil Index: Suggesting Intuitive Interpretations and Exploring Analytical Applications.'

3 Estimation and Empirical Results

To model potential nonlinear effects of inequality (INQ) on the real income growth (RGROWTH), we use a cubic polynomial in inequality in our econometric models. Therefore, our benchmark regression model for country i is as follows:

$$(RGROWTH)=\alpha+\Sigma_k \beta_k*Z_k+\gamma_1*INQ+ \gamma_2*INQ^2 + \gamma_3*INQ^3 + error \quad (5a)$$

Table 1: Empirical Association between Growth-Inequality

Dependent Variable: RGROWTH
Method: Panel Least Squares
Date: 05/14/11 Time: 19:38
Sample (adjusted): 1970 1996
Periods included: 27
Cross-sections included: 10
Total panel (unbalanced) observations: 186
(RGROWTH)= α + β_1*FDI+ γ_1*INQ+ γ_2*INQ*INQ+ γ_4*NQ*INQ*INQ+ β_2*INF + β_3*IMN+ β_4*ME+ β_5*MILPER+ β_6*POP+ β_7*PP+ β_8*WRG

	Coefficient	Std. Error	t-Statistic	Prob.
α	-599.2532	350.4205	-1.710097	0.0890
β_1	0.729246	0.520750	1.400377	0.1632
γ_1	50.99392	22.69368	2.247054	0.0259
γ_2	-1.145777	0.511015	-2.242158	0.0262
γ_3	0.008500	0.003807	2.232649	0.0268
β_2	0.003969	0.014454	0.274588	0.7840
β_3	15.32921	19.74486	0.776365	0.4386
β_4	0.066952	0.066320	1.009536	0.3141
β_5	-0.655965	0.707594	-0.927036	0.3552
β_6	-0.075280	0.077845	-0.967055	0.3349
β_7	-5.70E-05	0.000137	-0.417858	0.6766
β_8	10.68395	8.666678	1.232762	0.2193

R-squared	0.171233	Mean dependent var	1.183911
Adjusted R-squared	0.052517	S.D. dependent var	6.826291
S.E. of regression	6.783433	Akaike info criterion	6.729185
Sum squared resid	8006.603	Schwarz criterion	6.937297
Log likelihood	-613.8142	Hannan-Quinn criter.	6.813520
F-statistic	1.213190	Durbin-Watson stat	1.846341
Prob(F-statistic)	0.281308		

$$RGROWTH=C + 50*INQ - 1.14*INQ^2 + .008*INQ^3 \quad (5a')$$

Assuming C=Y-599.23>0, we get

$$\frac{d(RGROWTH)}{d(INQ)} = 50\text{-}2.28INQ+.024INQ^2 \quad (5b)$$

$$\frac{d^2(RGROWTH)}{d(INQ)^2} = -2.28+.048*INQ \quad (5c)$$

(5b) Implies:

$$\frac{d(RGROWTH)}{d(INQ)} = 0 \text{ for } X_{Min}=46.87,\ X_{Max}=48.61 \quad (5d)$$

(5c) implies:

$$\frac{d^2(RGROWTH)}{d(INQ)^2} = 0 \text{ for } X^{CR}=47.5 \quad (5e)$$

Combining (5b), (5c) and (5e) one will get the wave function (reversed S-function) in the relationship between growth and inequality, the inverse association arising for the Gini coefficient within the range of {46.87, 48.61}. Outside this range, there exists a positive association between RGROWTH and INQ. It is instructive to note that the relationship between growth and inequality will be inverted-U shaped for values of INQ such that 0<INQ<46.87. In other words, if INQ<46.87, the Kuznets curve depicts the true relationship between growth and inequality in our model. On the other hand, if INQ lies in the region {46.87, ∞}, our results show that the relationship between growth and inequality will be U shaped. In other words, an inverted Kuznets curve will depict the relationship between growth and inequality for those values of INQ such that 46.87<INQ< ∞. In the appendix we present alternative econometric formulations that all support the cubic relationship. However, it is important to bear in mind that the adjusted

R^2 is quite low – yet the nonlinear relationship between RGROWTH and INQ is supported.

DIAGRAM 2: GROWTH AND INEQUALITY IN THE MIDDLE EAST

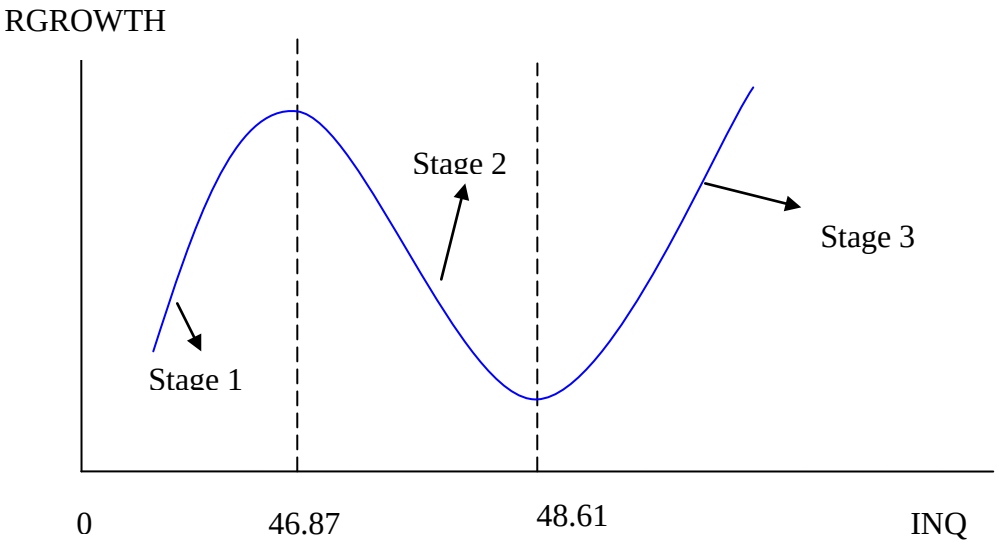


Diagram 2 illustrates the changes in real growth rate with respect to changes in inequality In Stage 1, the change in the real growth rate is positively related to inequality. As the Gini coefficient reaches a critical value of 46.87 Stage 1 is replaced by Stage 2. In Stage 2 inequality has a dampening effect on the (real) rate of growth, which gives rise to an inverse relationship between growth and inequality. When the Gini coefficient exceeds the value of 48.61, the real growth bears a positive relationship with inequality and we have Stage 3 in which inequality seems to promote economic growth.

Note that for most of the European nations, especially European Union (EU) nations, the Gini coefficient is less than 30, which, according to our results, suggests that

the relationship between growth and inequality will be inverted U-shaped. On the other hand, Gini coefficient for the US is much higher: Table 2 shows the values in recent years, with lowest in 2000 (46.2) and highest in 2006 (47). In other words, the graphical representation of the values will be inverted U-shaped when INQ<46.87, and U-shaped when 46.87<INQ. Thus, an S-shaped function relating RGROWTH to INQ arises for reasonable values of the Gini coefficient.

Table 2: The Gini Coefficient for the US Economy (2000-2009)

Year	Gini
2000	46.2
2005	46.9
2006	47
2007	46.3
2008	46.69
2009	46.8

Source: The US Census Bureau 2001-2010

4. Discussion

In the existing literature limited attempts have been made to generate a dynamic theory of income and wealth distribution integrating microeconomic models of accumulation and macroeconomic theories of factors' remuneration (see Stiglitz, 1969). In this framework it is established that the distribution of income and wealth tends asymptotically toward equality if and only if saving functions are either linear or concave. It is Stiglitz who clearly indicated that the distribution of income and wealth can have two attractors, or long-term equilibria, if the saving functions are convex. In Stiglitz's words, the convexity

of saving function will generate a “two-class equilibrium”. Our paper shows the possibility of multiple equilibria in a dynamic setting for the first time, to our best understanding, without exploiting the non-concavity of saving functions. It is also important to note that the cross-sectional studies point to the possibility that the marginal propensity to save increases with income and/or wealth and this empirical fact is behind the commonly held view that income equality might conflict with growth and aggregate welfare. Our findings are independent of whether saving functions are convex or concave.

In an immensely interesting work, Bourguignon (1981) showed that locally stable unegalitarian equilibria, or “stationary distributions” will exist along with the egalitarian one if the saving function is convex. Bourguignon (1981) also observed important welfare implications of the multiplicity of equilibrium as he showed that the non-egalitarian equilibria are Pareto-superior to the egalitarian equilibrium. Economic inequality in the dynamic neo-classical framework causes not only the generation of higher aggregate income and consumption per capita as could have been expected, but also higher income and consumption for all individuals. This result holds only to equilibria where all individuals have a positive wealth. Our results confirm the main finding of Bourguignon (1981) that higher inequality (unegalitariacan equilibrium) can sustain a Pareto efficient growth equilibrium (X_2*) characterized by higher inequality. Our result also confirms that the egalitarian equilibrium (X_1*=0) is inefficient and characterized by zero inequality. These two equilibria are separated by an unstable equilibrium (X_3*) that creates a threshold effect. In contrast to the earlier papers, our model establishes that there is no monotonic relationship between inequality and growth

if policy makers seek to influence economic growth and inequality. From the empirical study we confirm the theoretical findings. Since growth and inequality have U shaped and inverted-U shaped relationships, policy makers cannot utilize the interrelationship between growth and equity to achieve a desirable mix of growth and inequality.

5. Conclusion

The main goal of this paper is to establish that the desire of a policy maker to *choose* an optimal mix of inequality and growth, given a correctly expected *linear* model of growth and inequality, can lift the lid off the Pandora's box: the linear relationship between growth and inequality will break down to give way to a wave-like relationship, multiple equilibria and resultant complexities will emerge and the pertinence of the linear model to investigate the relationship between growth and inequality will disappear. From the empirical work, we find a statistical support for the wave like relationship between growth and inequality, which casts a serious doubt on the possibility of using appropriate policies to achieve a desirable mix of growth and equity. In other words, the feasibility of using appropriate institutional structures to stimulate equitable growth via suitable economic policies can become untenable. As a result, the millennium goals of eradicating poverty through equitable growth can never be achieved, even if all the underlying growth models are correct and correctly predicted by policy makers. As our theoretical model shows, which is supported by the empirical study, that growth and inequality can have an inverted-S-shaped relationship if policy makers try to achieve a desirable mix of growth and equity. In other words, the attempt to influence growth and inequality can give rise to a non-uniform association between growth and equity: there is a critical value

of inequality below which the Kuznets curve relationship will hold. We also find another critical value of inequality beyond which the inverse Kuznets curve relationship becomes operational. Our empirical finding is that these critical values of inequality are reasonable values, which can therefore create enormous problems for policy makers to use growth and inequality in an instrumental fashion to reduce poverty. In view of recent popular uprising and political turmoil in the Middle East and North Africa, particularly Egypt, Tunisia, Syria and Jordan, policy makers of developing economies should find a proper strategy of promoting inclusive and pro-poor growth together with good governance to ensure more equity and social stability which is in turn crucial for sustainable growth and poverty reduction.

APPENDIX: ALTERNATIVE ECONOMETRIC FORMULATIONS FOR THE CUBIC FUNCTION

Experiment 1: Cross-section fixed effect, non-period effect

Dependent Variable: RGROWTH
Method: Pooled EGLS (Cross-section weights)
Date: 10/18/11 Time: 21:52
Sample (adjusted): 1970 1996
Included observations: 27 after adjustments
Cross-sections included: 10
Total pool (unbalanced) observations: 186
Linear estimation after one-step weighting matrix

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-359.0356	274.9525	-1.305810	0.1934
FDI	-0.338406	0.353435	-0.957476	0.3397
INQ	25.45809	18.84426	1.350973	0.1786
INQ^2	-0.583152	0.427324	-1.364660	0.1742
INQ^3	0.004415	0.003208	1.376491	0.1705
INF	-2.79E-05	0.007519	-0.003708	0.9970
IMN	-35.09547	48.11870	-0.729352	0.4668
ME	0.197368	0.076214	2.589665	0.0105
MILPER	-2.705694	1.164469	-2.323543	0.0214
CON	-3.689322	1.570595	-2.348997	0.0200
PP	-0.000438	0.000427	-1.025862	0.3065
WRG	-11.72375	15.97734	-0.733774	0.4641
Fixed Effects (Cross)				
_ALG—C	-4.605544			
_EGY—C	-2.020845			
_IRN—C	-5.388607			
_ISR—C	12.26246			
_JOR—C	5.959153			
_KWT—C	-2.346617			
_MOR—C	-3.035500			
_SYR—C	-3.498547			
_TUN—C	-2.708144			
_TUR—C	5.345341			

Effects Specification

Cross-section fixed (dummy variables)

Weighted Statistics

R-squared	0.160786	Mean dependent var	2.347041
Adjusted R-squared	0.059063	S.D. dependent var	6.827805
S.E. of regression	6.288390	Sum squared resid	6524.736
F-statistic	1.580623	Durbin-Watson stat	2.152398
Prob(F-statistic)	0.062726		

Unweighted Statistics

R-squared	0.129143	Mean dependent var	1.183911
Sum squared resid	7507.374	Durbin-Watson stat	1.978210

Experiment 2: Cross-section fixed effect with period fixed effect

Dependent Variable: RGROWT
Method: Pooled Least Squares
Date: 10/18/11 Time: 21:54
Sample (adjusted): 1970 1996
Included observations: 27 after adjustments
Cross-sections included: 10
Total pool (unbalanced) observations: 186

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-590.9603	378.6986	-1.560503	0.1209
FDI	-0.447628	0.671596	-0.666515	0.5062
INQ	42.46050	25.92794	1.637635	0.1038
INQ^2	-1.000514	0.586215	-1.706737	0.0901
INQ^3	0.007795	0.004381	1.779174	0.0774
INF	0.013531	0.015407	0.878223	0.3813
IMN	51.59896	67.70179	0.762151	0.4473
ME	0.330850	0.097585	3.390376	0.0009
MILPER	-5.117111	1.429883	-3.578693	0.0005
CON	-4.874948	1.874902	-2.600108	0.0103
PP	0.000253	0.000339	0.747062	0.4563
WRG	-9.704127	20.59597	-0.471166	0.6383
Fixed Effects (Cross)				
_ALG--C	-2.508207			
_EGY--C	1.637906			
_IRN--C	-5.387867			
_ISR--C	7.552149			
_JOR--C	2.369877			
_KWT--C	-12.86854			
_MOR--C	0.697549			
_SYR--C	0.190162			
_TUN--C	-1.198165			
_TUR--C	9.515135			
Fixed Effects (Period)				
1970--C	5.540887			
1971--C	-15.88055			
1972--C	22.94172			
1973--C	1.309283			
1974--C	5.896635			
1975--C	1.353757			
1976--C	7.193140			
1977--C	1.005089			
1978--C	4.422324			
1979--C	3.501785			
1980--C	-1.150507			
1981--C	-2.671960			
1982--C	-0.260470			
1983--C	1.602546			
1984--C	0.462071			
1985--C	-1.621555			
1986--C	-3.063513			
1987--C	0.601368			
1988--C	-4.126955			
1989--C	2.138849			
1990--C	0.517952			
1991--C	-4.180749			

1992--C	0.940415
1993--C	1.306342
1994--C	-2.769046
1995--C	0.687053
1996--C	-1.418766

Effects Specification

Cross-section fixed (dummy variables)
Period fixed (dummy variables)

R-squared	0.404170	Mean dependent var	1.183911
Adjusted R-squared	0.206989	S.D. dependent var	6.826291
S.E. of regression	6.078894	Akaike info criterion	6.661626
Sum squared resid	5136.460	Schwarz criterion	7.476734
Log likelihood	-572.5312	Hannan-Quinn criter.	6.991939
F-statistic	2.049738	Durbin-Watson stat	1.968951
Prob(F-statistic)	0.000748		

Experiment 3: Random Effect

Random-effects GLS regression — Number of obs = 186
Group variable: country — Number of groups = 10

R-sq: within = 0.0587 — Obs per group: min = 7
between = 0.5374 — avg = 18.6
overall = 0.0847 — max = 27

Random effects u_i ~ Gaussian — Wald chi2(11) = 16.10
corr(u_i, X) = 0 (assumed) — Prob > chi2 = 0.1373

Rgrowth	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
FDI	.5437817	.5259602	1.03	0.301	-.4870813	1.574645
INQ	43.18826	22.54565	1.92	0.055	-1.000412	87.37693
INQ^2	-.9724456	.5073517	-1.92	0.055	-1.966837	.0219454
INQ^3	.0072205	.0037774	1.91	0.056	-.000183	.014624
INF	.0088888	.0146437	0.61	0.544	-.0198122	.0375899
IMN	14.63556	19.60128	0.75	0.455	-23.78225	53.05336
ME	.0773689	.0661534	1.17	0.242	-.0522894	.2070271
MILPE	-.8845165	.7161434	-1.24	0.217	-2.288132	.5190988
CON	-3.042013	1.623557	-1.87	0.061	-6.224127	.1401014
PP	-.0000505	.0001356	-0.37	0.710	-.0003162	.0002152
WRG	11.91361	8.60843	1.38	0.166	-4.958598	28.78583
C	-631.3121	331.0998	-1.91	0.057	-1280.256	17.63161

sigma_u	0
sigma_e	6.5644988
rho	0 (fraction of variance due to u_i)

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