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Abstract

The paper presents an adjusted Faustmann Rule for optimal harvest of a forest when there is a social cost of carbon emissions. The theoretical framework takes account of the dynamics and interactions of forests' multiple carbon pools and assumes an infinite time horizon. Our paper provides a theoretical foundation for numerical model studies that have found that a social cost of carbon implies longer optimal rotation periods and that if the social cost of carbon exceeds a certain threshold value the forest should not be harvested. At the same time we show that it could be a net social benefit from harvesting even if the commercial profit from harvest is negative. If that is the case, the optimal harvest age is decreasing in the social cost of carbon.

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1 Introduction

There has been extensive research on the question of what is optimal forest management when there is a social cost of carbon emissions to the atmosphere. A broadly accepted conclusion from this literature is that a social cost of carbon emissions should lead to longer rotation periods and that if the social cost of carbon exceeds a certain level, the considered stand should not be harvested, see for example Asante and Armstrong (2012), Asante et al. (2011), Daigneault et al. (2010), Gutrich and Howarth (2007), Kötke and Dieter (2010), Karpainen et al. (2004), Price and Willis (2011), Pukkala (2011), Raymer et al. (2011), and van Kooten et al. (1995).

While most contributions to this strand of the literature have been based on numerical simulation models, our main contribution is to analyze the issue theoretically with less restrictive assumptions than earlier theoretical studies. In addition, we illustrate the theoretical results with numerical examples. We will show that our less restrictive assumptions turn out to be important for the conclusions.

With regard to theoretical studies of the question of how a social cost of carbon should influence forest management, van Kooten et al. (1995) represent to our knowledge the most thorough study of the issue. They applied a multi-rotation infinite time horizon model and provided an adjusted Faustmann rule for determination of the length of the rotation period when there is a social cost of carbon emissions. However, the theoretical framework of van Kooten et al. (1995) did not incorporate the dynamics of important carbon pools as roots, stumps, tops and branches, harvest residues and naturally dead organic matter.

Asante and Armstrong (2012) is another theoretical contribution. In contrast to van Kooten et al. (1995) they included the forests' multiple carbon pools in their model. At the same time they considered a single rotation model only and their time horizon was limited to the length of the single rotation. As van Kooten et al. (2012), Asante and Armstrong (2012) found that a social cost of carbon emissions increases optimal harvest age. However, their numerical analysis indicated that incorporating the pools of dead organic matter and wood products in their model have the effect of reducing rotation age. And finally, they found that the higher are the initial stocks of carbon in dead organic matter or wood products the shorter is the optimal harvest age.

Holtsmark et al. (2013) discussed the results of Asante and Armstrong (2012) and Asante et al. (2011) and found that the surprising result that the higher are the initial stocks of carbon in dead organic matter or wood products the shorter is the optimal harvest age, was an artefact of their limited time horizon. Holtsmark et al. (2013) found that from a theoretical point of view the initial stocks of carbon in dead organic matter or wood products should not influence the harvest age. Moreover, the numerical analyses in Holtsmark et al. (2013) indictated that accounting for dead organic matter has the effect of increasing the rotation age, also in contrast to the results of Asante and Armstrong (2012) and Asante et al. (2011).

Although Holtsmark et al. (2013) applied an infinite time horizon, it presented a single rotation analysis only and presented few theoretical results. This underlines the need for a theoretical, multi-period infinite horizon analysis of the issue, which includes the dynamics of the forests' main carbon pools. Therefore, this paper presents a comprehensive theoretical analysis of the question of how a social cost of carbon should influence the length of rotation and the harvest level.

The present paper combines the multi-rotation infinite time horizon model of van Kooten et al. (1995) with the single-rotation, multiple carbon pools approach of Asante and Armstrong (2012) and Holtsmark et al. (2013). Compared to the many numerical model studies of the issue, our theoretical analysis is superior in its potential to reveal the drivers behind the obtained results. While it is generally more difficult to disentangle the important assumptions in a numerical model, our theoretical framework allows us to discuss these more thoroughly.

Our starting point is Faustmann (1849), who has been attributed a formula for determination of the length of the rotation period when a forest owner's goal is to maximize the discounted yield, see also Clark (2010), Samuelson (1976) and Scorgie and Kennedy (1996). We develop an adjusted

Faustmann Rule when there is a social cost of carbon emissions, while taking into account the dynamics and interactions of the forest's multiple carbon pools. From this rule it follows if there is a positive commercial profit from harvesting and the socially optimal harvest age is finite, then the optimal harvest age is increasing in the social cost of carbon. If there is a negative commercial profit from harvesting, one cannot on theoretical basis rule out that the socially optimal rotation length is finite. If the socially optimal rotation length is finite in the case with negative commercial profit from harvesting, then the rotation length is decreasing in the size of the social cost of carbon. However, our numerical model indicates that with reasonable levels of the discount rate and other parameters, negative commercial profit means that the optimal rotation length is infinite. The numerical model also indicate that if there is a positive commercial profit from harvesting, and the social cost of carbon exceeds a certain threshold, then the forest should not be harvested. The numerical examples showed that even at quite moderate levels of the social cost of carbon, social welfare is maximized by never harvesting the forest. This last result was also found in the single rotation analysis of Holtsmark et al. (2013). However, a single rotation analysis of the type reported in Holtsmark et al. (2013) will to some extent provide somewhat too high estimates of the effect on the rotation length of a social cost of carbon. The reason is that a single rotation analysis does not take into account the regrowth of the considered stand in later rotations.

To our knowledge, no one has undertaken a full theoretical analysis of optimal forest management in the presence of a social cost of carbon that includes all the following five realistic features, which are all included in our model:

1. Only about half of the carbon in the forests' living biomass is contained in the tree trunks. Tops, branches, roots and stumps constitute the remaining half of the carbon stored in living biomass.

2. Harvest residues will gradually decompose and release carbon to the atmosphere. Moreover, natural deadwood constitutes an important part of the carbon stock of a forest. The dynamics of these carbon pools are included in the analysis.

3. We allow an exogenous fraction of tops, branches, roots and stumps to be harvested and used for energy purposes, and study the consequences of changing this fraction.

4. Tree trunks that are harvested may either be used in a way that immediately releases carbon to the atmosphere (e.g. for energy purposes) or as materials for buildings and furniture. The size of the fraction of the harvest used for such purposes and the lifetime of this carbon stock could be varied. We study different assumptions with regard to these parameters.

5. We apply an infinite time perspective, not only with a single harvest perspective.

Before we embark on the analysis, we should also mention Hartman (1976), who provided an adjusted rule for optimal rotation length. However, he considered a case where a forest provides valuable services in addition to the values provided by timber harvesting and did not focus on a social cost of carbon.

The next four sections present our theoretical model and our main theoretical results. Section 6 presents numerical examples, and Section 7 concludes. An appendix contains proofs of our main results, a discussion of how our results would change if some parameters were changing over time, as well as a background discussion of whether the social cost of carbon is rising over time.

2 A model for calculation of optimal rotation length

We consider a forest stand where the stock of living biomass, measured in units of its carbon content, develops according to the function $B(t)$, where t is the time since last harvest, and $B(0) = 0$.[1] In accordance with what is common in the literature we assume that the stock of living biomass increases with age t up to some maximum value $\bar{B} = B(\bar{t})$. In order to simplify the

[1]We assume throughout the paper that the land occupied by the forest has such low value in alternative uses that these are irrelevant.

analysis we assume that when $t \geq \bar{t}$, the stock of living biomass is constant, i.e. $B(t) = \bar{B}$ for any $t \geq \bar{t}$. We did not analyze the case where $B(t)$ is decreasing when t exceeds a certain threshold level.

It is assumed that the trunks $R(t)$ constitute a share $\alpha \in (0,1)$ of the total stock of living biomass $B(t)$. Obviously, this assumption is a simplification. In reality the ratio between stems and total biomass is increasing over time, see e.g. Asante and Armstrong (2012). However, as argued in the Appendix, our assumption of $R(t)/B(t)$ being constant is not important for our results as long as this ratio does not increase rapidly in t for values close to the optimal rotation time.

The forest owner is assumed to harvest a share $\sigma \in [0,1]$ of the residues in addition to the trunks $R(t)$. Hence, in total a share $\alpha + \sigma (1-\alpha) \in [\alpha, 1]$ of the total living biomass $B(T)$ is harvested, where T is the length of the rotation period. In the formal analysis σ is assumed constant. In reality, marginal harvesting costs of the residues are likely to be increasing in σ, making σ endogenously determined and depending on the price of energy. We return to this issue in section 4.

We assume that a share $\beta \in [0,1]$ of the trunks harvested is used as building materials and furniture. The remaining share of the trunks is used for energy purposes. Note also that we assume that all of the harvested residues are used for energy purposes. The assumption of β being exogenous and independent of T is a simplification. The choice of using trunks for building materials and furniture versus for energy, will to some extent depend on the size and quality of the trunks. It seems reasonable to believe that more will be used for building materials and furniture the larger is T (Grutich and Howarth, 2007, Pukkala 2011). In the Appendix we show that our main results are not changed if β is increasing in T instead of constant.

The relative price between the two uses of trunks may also influence the ratio β: The higher is the price of energy relative to the price of building materials, the lower is β likely to be. This is discussed further in section 3.

A further simplification is that net profit per unit harvest (the net price) is assumed independent of T. It is probably more realistic to assume that the net price is increasing in T, at least up to a certain threshold value of T.

However, in the Appendix we show that if the net price is increasing in T, it strengthens our main result.

Before we proceed, we list the following stock and flow variables that all are important in the subsequent analysis:

B	the total stock of biomass
$R = \alpha B$	the stock of trunks
$(1-\alpha)B$	residues generated by harvesting
$\sigma(1-\alpha)B$	residues harvested
$(1-\sigma)(1-\alpha)B$	residues left on the stand
$(1-\beta)\alpha B + \sigma(1-\alpha)B$	energy
$\beta\alpha B = \beta R$	building materials

Other relevant stocks of carbon are natural deadwood, as well as the stock of carbon stored in wood-based building materials and furniture with their origin in the considered stand. Below, the dynamics of all these stocks of carbon are modeled.

The present value of the commercial profits from the next harvest is

$$V_P(p, T, \sigma) = e^{-\delta T} p \left(\alpha + \sigma\left(1-\alpha\right)\right) B(T), \tag{1}$$

where $\delta \in (0,1)$ is the discount rate and p is commercial profit per unit of harvest.[2] Clearly, p will depend on both prices and costs of the two uses of the harvested stand. Changes in e.g. the price of energy are likely to affect p; this is discussed in section 4. We assume throughout most of the paper that $p > 0$, but briefly discuss the case of a commercially unprofitable forest ($p \leq 0$) in Section 5.

[2]The social value of the harvest is the same as the commercial profits, provided fossil fuel use that is affected by the harvest is taxed according to the social cost of carbon.

We assume that the social cost of carbon emissions is $s(t)$, with the property that the present value $e^{-\delta t}s(t)$ is declining over time. To simplify the formal analysis, we assume that $s(t)$ is constant and equal to s. However, as argued in the Appendix, it is the assumption that the present value of the carbon price is declining over time that is important, not the simplification of $s(t)$ being constant.

With a constant carbon price, the present value social cost of immediate combustion of the harvest that is used for energy is

$$V_F(T,s,\beta,\sigma) = e^{-\delta T}s\left(\alpha\left(1-\beta\right)+\sigma\left(1-\alpha\right)\right)B\left(T\right). \tag{2}$$

At the time of harvest, a stock of building materials and furniture, $M\left(T\right)$, is generated; from our assumptions above we have

$$M\left(T\right) = \beta\alpha B(T). \tag{3}$$

Within each time period a share $\kappa \in (0,1)$ of the stock of building materials and furniture is scrapped and combusted. Hence, at time t the remaining stock of building materials/furniture from the first harvest is equal to $e^{-\kappa(t-T)}M(T)$, while emissions at time t due to combustion of this wood are $\kappa e^{-\kappa(t-T)}M(T)$.

Correspondingly, the amount of harvest residues left in on forest floor after a single harvest event is

$$D(T) = \left(1-\sigma\right)\left(1-\alpha\right)B(T). \tag{4}$$

Within each period, a share $\omega \in (0,1)$ of the stock of residues left in the forest decomposes. Hence, at time t the remaining stock of residues from the first harvest is equal to $e^{-\omega(t-T)}D(T)$, while emissions at time t due to decomposition of these residues are $\omega e^{-\omega(t-T)}D(T)$. It follows that the present value social cost of these emissions from combustion of building materials and furniture, $V_M(T)$, and from decomposition of residues, $V_D(T)$, are:

$$V_M(T, s, \beta) = \int_T^\infty e^{-\delta x} s\kappa e^{-\kappa(x-T)} \beta\alpha B(T) dx, \tag{5}$$

$$V_D(T, s, \sigma) = \int_T^\infty e^{-\delta x} s\omega e^{-\omega(x-T)} (1-\sigma)(1-\alpha) B(T) dx. \tag{6}$$

These expressions are simplified to:

$$V_M(T, s, \beta) = e^{-\delta T} s \frac{\kappa}{\delta + \kappa} \beta B(T), \tag{7}$$

$$V_D(T, s, \sigma) = e^{-\delta T} s \frac{\omega}{\delta + \omega} (1-\sigma)(1-\alpha) B(T). \tag{8}$$

As the stand grows, it will capture and store carbon. The social present value of carbon capture in living biomass over the first rotation is:

$$V_{CC}(T, s) = s \int_0^T e^{-\delta x} B'(x) dx. \tag{9}$$

Finally, we have to take into consideration that the stand contains a stock of naturally dead biomass, denoted by $N(t)$, and with $N(0) = 0$. We can here ignore any remaining natural deadwood that might have been generated in earlier rotation periods, see Holtsmark et al. (2013). We assume that the inflow of the stock of natural deadwood is a constant fraction $\gamma \in (0, 1)$ of the living biomass, while the stock decomposes at the same rate as harvest residues. Hence, the accumulation of natural deadwood is:

$$N'(t) = \gamma B(t) - \omega N(t) \text{ for } t \in (0, T). \tag{10}$$

Solving the differential equation gives:

$$N(t) = \gamma e^{-\omega t} \int_0^t e^{\omega x} B(x)\, dx, t < T, \tag{11}$$

resulting in:

$$N'(t) = \gamma \left(B(t) - \omega e^{-\omega t} \int_0^t e^{\omega y} B(y)\, dy \right), \tag{12}$$

$$N(T) = \gamma e^{-\omega T} \int_0^T e^{\omega x} B(x) dx. \tag{13}$$

At time T, when the stand is harvested, accumulation of a new stock of natural deadwood begins. At the same time, the stock of natural deadwood from the first rotation enters a phase of decomposition (see comment on this below), and we assume that natural deadwood decomposes with the same rate ω as harvest residues.

It follows from (12) that:

$$\lim_{t\to\infty} N'(t) = \gamma \left(\lim_{t\to\infty} B(t) - \omega \lim_{t\to\infty} \frac{e^{\omega t} B(t)}{e^{\omega t}} \right) = 0.$$

Hence, the stock of natural deadwood will approach steady state if the forest is never harvested.

The net accumulation of natural deadwood gives rise to a positive welfare effect through additional carbon capture in the forest. The present social value of carbon capture due to accumulation of natural deadwood during the first rotation period is:

$$V_{NCC}(T, s) = s \int_0^T e^{-\delta x} N'(x)\, dx. \tag{14}$$

In the Appendix we show that this may be written as

$$V_{NCC}\left(\cdot\right) = s\gamma\left(\frac{\delta}{\delta+\omega}\int_{0}^{T} e^{-\delta x}B\left(x\right)dx + \frac{\omega}{\delta+\omega}e^{-(\delta+\omega)T}\int_{0}^{T} e^{\omega x}B\left(x\right)dx\right). \tag{15}$$

Furthermore, the discounted social cost of emissions from decomposition of natural deadwood that was accumulated during the first rotation cycle is:

$$V_{ND}(T,s) = e^{-\delta T}s\int_{0}^{\infty}\omega e^{-(\delta+\omega)x}N(T)dx.$$

By using (13) we may rewrite this as:

$$V_{ND}(T,s) = s\gamma\frac{\omega}{\delta+\omega}e^{-(\delta+\omega)T}\int_{0}^{T}e^{\omega x}B(x)dx. \tag{16}$$

Note that the second term on the right hand side of (15) is identical to the right hand side of (16). We may then define the present time social value of net accumulation of natural deadwood:

$$V_N(\cdot) := V_{NCC}(\cdot) - V_{ND}(\cdot), \tag{17}$$

or

$$V_N(\cdot) = s\gamma\frac{\delta}{\delta+\omega}\int_{0}^{T}e^{-\delta x}B\left(x\right)dx. \tag{18}$$

Summing up, all terms in the net social welfare generated by the first harvest cycle, $V(p,T,s,\beta,\sigma)$, is then:

$$\begin{aligned} V(p,T,s,\beta,\sigma) := {} & V_P(\cdot) + V_{CC}(\cdot) - V_F(\cdot) - V_M(\cdot) \\ & - V_D(\cdot) + V_N(\cdot), \end{aligned} \tag{19}$$

where all terms on the right hand side are defined above. Next, define:

$$\Psi(T) := \left(1 + \frac{\gamma}{\delta + \omega}\right) \left(1 - e^{-\delta T} - \delta \frac{\int_0^T e^{-\delta x} B(x) dx}{B(T)} \right) \tag{20}$$

(21)

$$\Omega := p\left(\alpha + \sigma\left(1 - \alpha\right)\right) + sh \tag{22}$$

where

$$h := \left(1 - \alpha\right)\left(1 - \sigma\right)\left(1 - \frac{\omega}{\delta + \omega}\right) + \alpha\beta\left(1 - \frac{\kappa}{\delta + \kappa}\right) \in (0, 1) \tag{23}$$

From the defintions above it follows that we may write:

$$V(\cdot) = \left[e^{-\delta T}\Omega + s\left(\left(1 - e^{-\delta T}\right)\left(1 + \frac{\gamma}{\delta + \omega}\right) - \Psi(T)\right)\right] B(T) \tag{24}$$

Next, define a welfare function including the sum of the discounted welfare of all future rotation cycles:

$$W(p, T, s, \beta, \sigma) := V(\cdot) + e^{-\delta T} V(\cdot) + e^{-\delta 2T} V(\cdot) + ...,$$

which is simplified to:

$$W(\cdot) = \frac{1}{1 - e^{-\delta T}} V(\cdot). \tag{25}$$

In preparation for our first result, note that if the rotation period T is increased by one time unit, the first harvest takes place one time unit later, the second harvest two time units later, and so forth. A rule of harvesting simply saying that the growth rate of the stock of stems should drop to the level of the discount rate does not account for this. The contribution of the German forester Martin Faustmann (1849) was to take into account the complete added delay of profits from harvesting when the rotation period is prolonged.

When a social cost on carbon emissions is introduced, similar and additional effects come into play. When increasing the rotation period, the amount of carbon stored on the stand at time of harvesting increases, and emissions from immediate combustion, and from combustion of building materials and furniture, in addition to decomposition of harvest residues, are postponed. And these delays apply to future rotations as well. However, the beginning of the process of carbon capture *after* each harvest is also delayed. Furthermore, the process of accumulation of natural deadwood is affected by increasing the rotation period. In a period of time after harvest there will be net release of C from natural deadwood, as the generation of natural deadwood is small in a young stand. Postponing harvest means an additional period with positive net accumulation of natural deadwood. The trade off between carbon storage now or in the future, as well as between profits now or in the future, determines the optimal length of the rotation period.

For later use, we recall from (22) that $\Omega > 0$ for $p > 0$. Moreover, we show in the Appendix that $\Psi(T)$ is positive and increasing in T for $T < \bar{t}$, and equal to $\Psi(\bar{t})$ for $T \geq \bar{t}$.

3 The social optimum

To find the social optimum, we differetiate W given by (24) and (25) with respect to T. This is done in the Appendix, where we derive the Lemma below. Our main theoretical result will follow from this Lemma; an adjusted Faustmann formula taking the social costs of carbon emissions into account:

Lemma 1. *If social welfare $W(p, T, s, \beta, \sigma)$ is maximized for a finite value of T, this value satisfies:*

$$\frac{B'(T)}{B(T)} = \frac{\delta}{1 - e^{-\delta T}} \left(1 - \frac{s}{\Omega}\Psi(T)\right). \tag{26}$$

If

$$\lim_{T \to \infty} W(p, T, s, \beta, \sigma) > W(p, T, s, \beta, \sigma) \text{ for all finite } T, \tag{27}$$

then social welfare $W(p,T,s,\beta,\sigma)$ *is maximized by never harvesting the stand. A necessary condition for (27) to hold is that*

$$\Psi(\bar{t}) > \frac{\Omega}{s} \ . \tag{28}$$

All functions and parameters in (26) – (28) *are defined above.*

Proof. See Appendix. ■

Condition (28) is simply the condition for the derivative $W_T(p,T,s,\beta,\sigma)$ to be positive for large T. This condition is also sufficient for (27) unless the function $W(p,T,s,\beta,\sigma)$ has a local maximum for $T = T^*$ and a local minimum for $T = T^{**} > T^*$, which seems implausible for reasonable specifications of $B(T)$. In the proceeding discussion we therefore assume that it is optimal to never harvest the stand if and only if the inequality (28) holds.

The l.h.s. of (28) is positive, and can be lower or higher than 1. The fraction Ω/s is monotonically declining in s (for $p > 0$), with a lower bound of $h \in (0,1)$. Depending on the parameters, it may be the case that a finite value of T is optimal no matter how large s is. It may also be the case that there is a threshold value, which we label $\bar{s}$, such that if $s > \bar{s}$, then (28) holds and the stand should not be harvested.

It follows from Lemma 1 (more precisely from equation (26)) that if $s = 0$, then the rotation period that maximizes social welfare is defined by:

$$\frac{B'(T)}{B(T)} = \frac{\delta}{1 - e^{-\delta T}}, \tag{29}$$

which is the classical formula attributed to Faustmann (1849) for maximization of the forest owner's profit. Furthermore, if $s = 0$ and the discount rate δ approaches zero, then (26) reduces to

$$\frac{B'(T)}{B(T)} = \frac{1}{T}. \tag{30}$$

If T satisfies (30), then the the rotation length gives the maximum sustained yield.

Our next section discusses how the optimal length of the rotation period depends on the size of the social cost of carbon, s.

4 The optimal rotation period and the social cost of carbon

From Lemma 1 it is easily verified that the optimal T depends on s/Ω, and hence on s/p. For a given ratio of s/p, the optimal T is unaffected by s. We mentioned in the previous section that p might depend on the price of energy, since p is average profit per unit harvest, some of which is used for energy purposes. It is beyond the scope of the present paper to give a full analysis of how prices of energy and other uses of forest harvests may depend on the social cost of carbon. It may nevertheless be useful to illustrate the issue with a very simple example. Let $p = wp_1 + (1-w)p_2$ where w is the share of the harvest used for building materials and furniture, assumed for now exogenous.[3] Profits per unit harvest used for building materials and furniture are exogenous and equal to p_1, assumed positive. Profits per unit harvest used for energy are given by $p_2 = q+s-c$, where c is the average cost of harvest for energy purposes, and $q+s$ is is the energy price. An obvious interpretation is that bioenergy and fossil energy are perfect substitutes, fossil energy is competitively supplied at the unit cost q, and s is a carbon tax on fossil energy only.

With the notation and assumptions above we have

$$\frac{s}{p} = \frac{s}{wp_1 + (1-w)(q+s-c)}$$

This relative price will be increasing in s if $wp_1 + (1-w)(q-c) > 0$. A sufficient condition for this to hold is that $q - c > 0$, i.e. that there are positive profits from producing bioenergy even in the absence of any carbon tax. It is not obvious that this holds. In the rest of the paper we shall nevertheless assume that s/p increases when s increases. The results below

[3]It follows from the assumptions in section 2 that $w = \frac{\alpha\beta}{\alpha+(1-\alpha)\sigma}$.

are changed in obvious ways if the opposite were true.

In section 2 we argued that σ and β might depend on s. We return to this below, but first consider the case of a change in s for given values of σ and β.

Our main result concerns the effect on the optimal length of the rotation period of an increase in the social cost of carbon, s:

Proposition 1 *If $p > 0$ and the optimal T is finite, the length of the rotation period that maximizes social welfare is strictly increasing in the social cost of carbon, s.*

Proof. See Appendix. ■

This result provides a theoretical foundation for a number of numerical studies that pointed in the same direction. Proposition 1 is also in agreement with the main results of the theoretical models of van Kooten et al. (1995) and Asante and Armstrong (2012) although their models were less general.

The main driver of the result in Proposition 1 is the decreasing present value of the social cost of carbon emissions. If emissions in the future are preferred over emissions today, a higher cost of emissions implies longer optimal rotation periods, since delaying harvest also delays emissions. We show in the appendix that the optimal length of the rotation period is independent of the social cost of carbon if the present value of this cost is constant over time.

We argued previously that σ, the share of residues that is harvested, might depend on the social cost of carbon, s. Independently of whether or not this is the case, it is of interest to see how an increase in σ affects the optimal length of the rotation period.

Proposition 2 *If and only if the social cost of carbon is sufficiently low relative to the per unit commercial profits from harvest, an increase in the share of the living biomass that is harvested in addition to trunks, σ, will strictly decrease the optimal length of the rotation period.*

Proof. In the appendix, we show that the optimal rotation period, T, is strictly decreasing in σ if and only if

$$\frac{s}{p} < \frac{\delta + \omega}{\delta}. \tag{31}$$

■

If the inequality in (31) does not hold, the optimal length of the rotation period will either be increased or unaffected by an increase in σ. An increase in σ means that more biomass is harvested and used for energy purposes, and less harvest residues are left in the forest. The result is that both commercial profits and emissions immediately after harvest are increased. If the per unit profit is large enough, this decreases the optimal length of the rotation period. However, if the social cost of carbon emissions is large compared to the per unit profit, the optimal length of the rotation period is increased.

Assume that due to increased profitability of bioenergy, σ is an increasing function of s. From Proposition 2 we know that if s is sufficiently high, an increase in σ will make T go up (or stay unchanged). In this case σ increasing with s thus strengthens our conclusion that T increases with s. However, for lower values of s we get the opposite: an increase in σ will make T go down. If σ increases with s the total effect of an increase in s hence has a theoretically ambiguous effect on T. The direct effect is to increase T (Proposition 1), while the indirect effect via a higher σ tends to reduce T (Proposition 2). In our numerical illustration in section 4 we find that for reasonable assumptions about how much σ is affected by a change in s, the direct effect dominates. Hence, for this case Proposition 1 remains valid.

We argued previously that β, the share of trunks used for building materials and furniture, might depend on the social cost of s. Independently of whether or not this is the case, it is of interest to see how an increase in β affects the optimal length of the rotation period.

Proposition 3 *If the optimal T is finite, an increase in β, the share of trunks used for building materials and furniture, will strictly reduce the optimal length of the rotation period, T.*

Proof. See Appendix. ■

When a larger share of harvested biomass is used for building materials and furniture, emissions immediately following harvest are reduced. This implies a smaller social gain from postponing harvest, and hence a shorter optimal rotation period.

Assume that due to increased profitability of bioenergy, β is a decreasing function of s. From Proposition 3 we know that a reduction in β will make T go up. It follows that β decreasing with s strengthens our conclusion from Proposition 1 that T increases with s.

We conclude this section by considering the limiting case of no residuals ($\alpha = 1$), and all the harvested stems are stored in a safe place forever ($\beta = 1$ and $\kappa = 0$). In this case there is no release of carbon after harvesting, so we might expect that that the optimal T is finite for all values of s in this case. It is straightforward to see that $\Omega = p + s$ for this case, implying $\Omega/s = (p + s)/s$, which has a lower bound equal to 1. If $\Psi(\bar{t}) \leq 1$, the inequality in (28) can therefore not hold no matter how high s is, implying that the optimal T is finite for all values of s. However, due to the growth of deadwood ($\gamma > 0$),we cannot theoretically rule out the possibility of $\Psi(\bar{t}) > 1$. If this inequality holds and s is sufficiently large, it will be optimal to never harvest the forest. The interpretation of this is that with a sufficiently large value of s, the importance of deadwood growth for social welfare will be so high that the forest should never be harvested.

5 A commercially unprofitable forest

If $p \leq 0$, there will be no profit from harvesting an existing forest (disregarding alternative uses of the land, see footnote 1). Leaving the forest unharvested is also socially optimal as long as $s = 0$. However, we shall see below that this may no longer be true if s is positive.

Consider first the case of s positive but so small that $\Omega \leq 0$. In this case (28) must hold, implying that it is optimal to never harvest the stand.

Consider next the case of $p \leq 0$ and s so large that $\Omega > 0$. From (22) it is clear that Ω/s is monotonically increasing in s in this case, with an upper bound equal to $h \in (0, 1)$. (If $p = 0$, $\Omega/s = h$ for all values of s.). For a sufficiently high value of s, the inequality (28) may therefore no longer hold, and the optimal T may hence be finite.

To interpret the possibility of a finite T being socially optimal for a commercially unprofitable forest, it is useful to return to the limiting case discussed in the end of Section 4: With no residuals ($\alpha = 1$) and all the harvested stems stored in a safe place forever ($\beta = 1$ and $\kappa = 0$) there is no release of carbon after harvesting. Harvesting and replanting in this case acts as carbon sequestration device and may be optimal if s is sufficiently large. Formally, $\Omega/s = (p + s)/s \in (0, 1)$. It therefore follows from Lemma 1 that a finite T is socially optimal if s is sufficiently high and $\Psi(\bar{t}) < 1$.

Proposition 1 showed us how T depends on s for the case of $p > 0$. For the case of $p \leq 0$ we have the following Proposition:

Proposition 4 *If $p \leq 0$ and the optimal T is finite, the length of the rotation period that maximizes social welfare is strictly decreasing in the social cost of carbon, s, for $p < 0$, and independent of s for $p = 0$.*

Proof. See Appendix. ■

For the optimal T to be finite in the case when $p < 0$, the discount rate has to be relatively low. According to simulations with the numerical model applied in the next section, and assuming that $p < 0$, $\alpha = 0.48$, $\beta = 1$, and $\kappa = 0$, then, for any discount rate equal to or larger than 0.011, the optimal T is infinite for any $s > 0$. Note that this applies also when the stems harvested are stored on a safe place forever ($\beta = 1$, and $\kappa = 0$). If we instead, more realistically, assumed that $\beta = 0.25$, and $\kappa = 0.014$, then, for any discount rate equal to or larger than 0.0001, the optimal T is infinite for any $s > 0$. Hence, with discount rate levels that are usually applied, the forest should not be harvested if there is a negative commercial profit from harvesting.

6 Numerical illustrations

In order to provide further intuition to the theoretical results in section 2 and 3, this section provides numerical simulations of the consequences of implementation of a social cost of carbon for optimal harvest from a forest stand. We will in this section only consider cases where the social cost of carbon is constant over time.

6.1 Model and parameter values

Figure 1 provides an overview of the dynamic development of the considered forest stand with 150 years long rotations. Below follows a detailed description of the model.

(Figure 1 here).

After harvest at time $t = 0$ the stock of stems is assumed to develop along the function

$$R(t) = v_1(1 - e^{-v_2 t})^{v_3}.$$

We have followed Asante et al. (2011) in choice of parameter values, which are as follows: $v_1 = 100.08$, $v_2 = 0.027$, $v_3 = 4.003$. (Note that as Asante et al. (2011) applied m^3/ha as their unit of measurement, $v_1 = 500.4$ in their set up.) The chosen numerical representation gives maximum sustained yield at 88 year old stands. Hence, it is representative for a Scandinavian forest where the dominating spruce and pine forests typically are mature after 80 – 110 years. With regard to development of the stock of other living biomass, it is assumed that the trunks constitute 48 percent of total biomass in the forest stand, i.e. $\alpha = 0.48$ (NCPA, 2010).

With regard to the stock of natural deadwood, it is assumed that $\gamma = 0.001$, see equation (10) for definition. This parameter value gives an accumulation of natural deadwood corresponding to what is found in Asante et al. (2011). The decomposition rate for deadwood, ω, is set to 0.04 (Holtsmark 2012).

With regard to the share β of the harvested stems that are used for building materials and furniture, based on NCPA (2011) it is assumed that $\beta = 0.25$ in the base case. However, simulations are provided where other values of this parameter is applied. We have assumed that building materials and furniture are durable goods in the sense that only a share $\kappa = 0.014$ of this stock of wood is scrapped and combusted annually.

The amount of residues harvested is determined by the share σ, which is set to 0.2 in the base case. However, additional simulations are carried out considering higher and lower assumptions with regard to the value of σ. Figure 1 provides a description of how the different components of the considered stand's carbon stock develops if the rotation length is 150 years.

In the simulations presented in the next subsection it is assumed that the forest owner's net profit is 15 USD/m^3 wood harvested. As one cubic meter of wood contains approximately 0.2 tonnes carbon, this corresponds to 75 USD/tC, for short labeled the (net) price of wood. Note that only the relative price of the social cost of carbon, s/p, matters.

The discount rate is set to 0.05 in all simulations.

6.2 Simulation results

Figure 2 shows the results of simulations carried out in a case where 20 percent of residues are harvested ($\sigma = 0.2$). The solid single-lined curve shows the case where $\beta = 0$, i.e. the share of the harvested stems that are used for building materials and furniture is zero. The dashed curve shows the case where $\beta = 0.25$, while the dotted curve shows the case where $\beta = 0.5$. In addition, the double-lined curve shows for illustrative purposes the less realistic case where all harvested roundwood is stored forever.

The curves in Figure 2 confirm the result of Proposition 2, that increasing the social cost of carbon s should lead to longer rotation periods. This applies also in the case where a reasonable share of the harvested stems in some way or another are converted to a durable carbon storage, i.e. when $\beta > 0$. In addition, Figure 2 illustrates that increasing β, i.e. the share of the harvested stems that are used for building materials and furniture, has a significant effect and draws in the direction of shorter rotation. The double-lined curve shows illustrates that the theoretical results of Lemma 1 and Proposition 1 applies also when $\beta = 1$ and $\kappa = 0$.

(Figure 2 - 4 and Table 1 here)

Table 1 presents results of a number of model simulations given different levels of the share of residues that is harvested as well as different levels of the social cost of carbon. In these simulations it was assumed that the share β of the harvested trunks that are used as building materials and furniture is fixed at 0.25, as this is likely to be close to a realistic level (NCPA 2011). Table 1 shows that the optimal length of the rotation period is influenced by the share of the residues that are harvested. However, changes in the social cost of carbon have a significantly stronger effect on the optimal rotation length than the size of the share of residues harvested. One should at this point also have in mind that we ignored that the amount of residues harvested is likely to influence the carbon balance of the soil. Intensive removal of residues from the forest floor might lead to release of soil carbon to the atmosphere. The carbon stock of the soil constitutes a significant share of the carbon stock of boreal and temperate forests (Kasischke 2000). Hence, this effect might be significant (Nakane and Lee 1995, Palosuo et al. 2001, Nilsen et al. 2008, Repo et al. 2010). Moreover, as mentioned in section 2, we assumed that the unit costs related to harvesting of residues are constant to scale and that the commercial profit from harvesting residues is as high as the commercial profit from harvesting stems (per m^3). These simplifications have a common bias and draw in the direction of too high estimates of to what extent increasing the share of residues harvested should reduce the rotation period.

Both Table 1 and Figure 2 illustrate that the social carbon cost has a certain threshold value above which the stand should not be harvested. The higher is the share of the harvest stored in furniture and buildings, the higher is the mentioned threshold value.

It is here appropriate to recall that only the size of the social cost of carbon *relative* to the price of wood (s/p) matters. Hence, if we for example are considering a marginal forest in the sense that the commercial profit from harvesting is low, then the threshold value of the social cost of carbon, above which the forest should not be harvested, is lower than found in the presented simulation. And correspondingly, if we consider a forest with high commercial profit from harvesting, the threshold value is higher than found here.

In this paper we have emphasized the importance of taking account of the forests' different carbon pools, not only the trunks. Figure 3 shows the importance of this. The solid curve in Figure 3 shows the estimates of optimal rotation period in the case where all carbon pools other than the trunks are ignored. The dotted curve shows the estimates when only the trunks and the pool of wooden products are included. Finally, the dashed curve shows the result when all carbon pools are taken account of. The figure shows that these choices influence the estimates of the optimal rotation period significantly. The inclusion of the wood product pool means shorter rotation and a higher threshold value above which the forest should not be harvested. Inclusion of harvest pools as other living biomass than the stems, harvest residues and NDOM draws in the direction of significantly longer rotation periods and a significantly lower threshold value above which the forest should not be harvested.

As mentioned in the introduction, our results with regard to the effects of inclusion of dead organic matter in the analysis contrast the main finding in Asante and Armstrong (2012) and Asante et al. (2011). They found that incorporating dead organic matter has the effect of reducing the rotation period. In addition, they found that high initial stocks of dead organic matter and wood products have the effect of reducing the rotation period. With regard to the latter result, Holtsmark et al. (2013) demonstrated that it follows from the consideration of a single rotation period only and the fact that Asante and Armstrong (2012) and Asante et al. (2011) ignored the release of carbon from decomposition of dead organic matter after the time of the first harvest T. With that simplification it is obvious that a large initial stock of dead organic matter draws in the direction of earlier harvest. Holtsmark et al. (2013) demonstrated that if it had been taken into account that the time profile of the decomposition of the initial carbon pools over the infinite time horizon $t \epsilon (0, \infty)$ is not influenced by the harvest age, the size of the initial carbon pools has no effect on the optimal harvest age. The first mentioned result in Asante and Armstrong (2012) and Asante et al. (2011) with regard to the effects of incorporating multiple carbon pools in the analysis should also be considered in the light of their fail to see the importance of the release of carbon from dead organic matter after time T.

An interesting question is how the choice of a single rotation vs. a multiple rotation analysis influence the relationship between the social cost of carbon and the optimal length of the rotation period. Direct comparison of the results reported by Holtsmark et al. (2013) with the results reported here is not fruitful because Holtsmark et al. (2013) included a fixed harvest costs that for simplicity has not been included in this paper's analysis. However, Figure 4 makes a comparison of a multiple harvest case and a single harvest case, with all other things being equal. It shows that the single rotation analysis to some extent will exaggerate the effect of the social cost of carbon with regard to the optimal harvest age. The intuition behind this result is that the single harvest analysis does not take into account the regrowth in the forest in future rotation periods.

As underlined by van Kooten et al. (1995), longer rotation periods do not necessarily reduce the supply of timber in the long term. Figure 5 illustrates this. When the social cost increases from zero, the long term supply of timber is firstly increasing before a maximum is reached. If the social cost of carbon is further increased, the long term supply is reduced and becomes zero if the social cost of carbon settles above the mentioned threshold value. Figure 4 also illustrates the importance of taking the forests' multiple carbon pools into account.

7 Discussion and conclusion

The increasing use of subsidies in order to encourage the use of biofuels, including wood fuels from forests, calls for a theoretical clarification of how a social cost of carbon should influence forest management. Searchinger et al. (2009) claimed that current regulation regimes might lead to overharvesting of the world's forests. In order to increase insight, this paper provides a theoretical model of the relationship between forest management and the interaction and dynamics of the forest's multiple carbon pools. A theoretical study that includes the dynamics of the forest's main carbon pools in a multiple rotation infinite horizon model is to our knowledge new. The theoretical analysis leads to an adjusted Faustmann Rule for optimal harvest when there is a social cost of carbon emissions.

Let us first consider the case when there is a positive net commercial profit from harvesting ($p > 0$). In that case, and if the rotation period that maximizes social welfare is finite, the adjusted rule implies that the optimal T is strictly increasing in the social cost of carbon, s. Depending on the parameters, it may be the case that a finite value of T is optimal no matter how large s is. It may also be the case that there is a threshold value, which we labeled $\bar{s}$, such that if $s > \bar{s}$, then the stand should not be harvested. It could here be mentioned that the numerical simulations show that if the disount rate is not lower than 0.01, any realistic set of parameter values of our numerical model gives the conclusion that such a threshold value exists above which the forest should not be harvested.

Next, consider the case when there is negative commercial profit from harvesting ($p < 0$). If s positive but below a certain threshold level (such that $\Omega \leq 0$), then it is optimal to never harvest the stand. If s is above the mentioned threshold level, (such that $\Omega > 0$), depending on the parameters, it could be optimal to harvest, i.e. the rotation period that maximizes social welfare might be finite. If the optimal T is finite when $p < 0$, then the adjusted Faustmann rule implies that the optimal T is strictly *decreasing* in the social cost of carbon, s. A finite optimal T when $p < 0$ is not a very likely case, however. Numerical simulations showed that if the disount rate is 0.01 or above, and $p < 0$, any realistic set of parameter values of the applied numerical model gives the conclusion that the stand should never be harvested.

The main driver of these results is the assumption that the present value of the climate damage caused by emissions is decreasing over time - emissions in the future are preferred over emissions today. A single harvest leads to an increase in the stock of carbon in the atmosphere in the short run, and the damage resulting from this increase would have been postponed with a longer rotation period. We discuss the development over time of the present value of the damage resulting from a marginal increase in the atmospheric carbon stock in the appendix. This present value will be determined by the time profile of the stock of carbon in the atmosphere, the profile of the marginal damage as the stock increases, and the discount rate. To be able to characterize the present value of the damage from emissions precisely, a specification of the climate cost function is necessary. As is also shown in the appendix, when assuming that the present value of the social cost of carbon is constant over time, the result is no longer that a social cost of carbon implies longer rotation periods. Therefore, in order to focus on the main driver of the results, we have chosen to model the social cost of carbon, $s(t)$, as constant over time, giving a declining present value of the damage from emissions. Compared to using a general social cost function $s(t)$, this simplifies the calculations, while still allowing the timing of emissions to affect the optimal rotation period. Intuitively, if the decline in the present value of the social cost of carbon is slower, the effect of this cost on the optimal rotation period is weaker.

Compared to other theoretical studies, our contribution is to investigate this issue in a considerably less restrictive theoretical framework. We take into account that less than half of the carbon in the forests' biomass is contained in the tree trunks. Tops, branches, roots and stumps constitute approximately half of the carbon stored in living biomass, and to the extent that these components are not harvested together with the trunks, they will gradually decompose and release carbon to the atmosphere. The dynamics of these carbon pools as well as the stock of natural deadwood is included in both the theoretical and numerical analyses. In addition, we allow an exogenous fraction of tops, branches, roots and stumps to be harvested and used for energy purposes. And finally, the dynamics of a stock of carbon stored in building materials and furniture is also taken into account.

With our less restrictive approach, including both multiple rotation periods and multiple carbon pools in the analysis, the threshold value of the social cost of carbon above which harvest should not take place, is significantly lower than found in studies with a more restrictive approach. The multiple carbon pool approach also means that the effect of a social cost of carbon on the length of the rotation period is significantly stronger than found in previous studies. Our model allows us to investigate the effect of changes in the composition and dynamics of forests. In order to fully understand the mechanisms underlying the effect on the rotation period of a social cost of carbon, a model that is not too restrictive is useful. We have found that increasing the share of residues harvested and/or the share of stems used for durable storage in buildings and furniture reduces the effect of a social cost of carbon on the optimal rotation period. Conclusions regarding the effect on the optimal rotation periods of changes in harvesting procedures or use of harvested material might potentially have important policy implications.

Finally, it should be noted that all conclusions in the paper are based on the implicit assumption that there is a tax or similar instrument related to combustion of fossil fuels, that corresponds to the social cost of carbon. A general equilibrium approach is needed in order to evaluate optimal second-best policy if this is not the case.

Appendix

Proofs

Properties of the function $\Psi(T)$

Appliying l'Hospital's rule to (20) we find that

$$\lim_{T\to 0}\left(1-e^{-\delta T}-\frac{\delta}{B(T)}\int_0^T e^{-\delta x}B(x)dx\right)=-\lim_{T\to 0}\frac{\delta e^{-\delta T}B(T)}{B'(T)}=0. \quad \text{(A.1)}$$

Hence, as T approaches 0, also $\Psi(T)$ approaches zero. Moreover, we have:

$$\Psi'(T)=\left(1+\frac{\gamma}{\delta+\omega}\right)\frac{B'(T)}{(B(T))^2}\int_0^T e^{-\delta x}B(x)dx. \quad \text{(A.2)}$$

Since $B'(T)>0$ for $T<\bar{t}$ and $B'(T)=0$ for $T\geq\bar{t}$, it follows that $\Psi(T)$ is positive and increasing in T for $T<\bar{t}$, and equal to $\Psi(\bar{t})$ for $T\geq\bar{t}$.

Proof of Lemma 1. We want to find the T that maximizes $W(p,T,s,\beta,\sigma)$. From (24) and (25) we have:

$$W(\cdot)=\frac{1}{1-e^{-\delta T}}\left[e^{-\delta T}\Omega+s\left(\left(1-e^{-\delta T}\right)\left(1+\frac{\gamma}{\delta+\omega}\right)-\Psi(T)\right)\right]B(T) \quad \text{(A.3)}$$

Define:

$$\Delta_1:=\Omega B'(T)+\frac{\delta}{1-e^{-\delta T}}\left(s\Psi(T)-\Omega\right)B(T). \quad \text{(A.4)}$$

Then we could write the first order condition:

$$\frac{\partial W(p,T,s,\beta,\sigma)}{\partial T}=\frac{1}{e^{\delta T}-1}\Delta_1=0, \quad \text{(A.5)}$$

which gives (26). Furthermore, the inequality in (28) is equivalent to $\Delta_1>0$ for $T\geq\bar{t}$, and hence a necessary condition for

$$\frac{\partial W(p,T,s,\beta,\sigma)}{\partial T}>0 \quad \text{(A.6)}$$

for all $T > 0$. If this inequality applies for all $T > 0$, then the first order condition (26) does not hold for any $T > 0$, and social welfare is maximized by never harvesting.□

Carbon capture due to accumulation of natural deadwood

Equation (14) may be rewritten as

$$V_{NCC}(\cdot) = s\gamma \left(\int_0^T e^{-\delta x} B(x)\, dx - \omega \underbrace{\int_0^T \overbrace{e^{-(\delta+\omega)x}}^{g'(x)} \overbrace{\int_0^x e^{\omega y} B(y)\, dy}^{f(x)}\, dx}_{K} \right) \quad \text{(A.7)}$$

where we implicitly have defined K, $g(x)$ and $f(x)$, and we have:

$$g(x) = -\frac{1}{\delta+\omega} e^{-(\delta+\omega)x}, \quad \text{(A.8)}$$

$$f'(x) = e^{\omega x} B(x). \quad \text{(A.9)}$$

Then apply the formula for integration by parts:

$$\int_0^T g'(x) f(x) dx = |_0^T g(x) f(x) - \int_0^T g(x) f'(x) dx. \quad \text{(A.10)}$$

Hence, K may be written:

$$K = \frac{1}{\delta+\omega} \int_0^T e^{-\delta x} B(x)\, dx - \frac{1}{\delta+\omega} e^{-(\delta+\omega)T} \int_0^T e^{\omega x} B(x)\, dx, \quad \text{(A.11)}$$

and it follows that $V_{NCC}(\cdot)$ can be simplified to (15).

Proof of proposition 1. From $(A.5)$ it follows that the second order condition for the maximization problem can be written as:

$$\frac{\partial^2 W(p,T,s,\beta,\sigma)}{\partial T^2} = \frac{\partial}{\partial T}\left(\frac{1}{e^{\delta T}-1}\right) \cdot \Delta_1 + \frac{1}{e^{\delta T}-1} \cdot \frac{\partial \Delta_1}{\partial T} \leq 0. \quad \text{(A.12)}$$

It follows from the first order condition $(A.5)$ that $\Delta_1 = 0$. Hence, the second order condition is reduced to $\partial\Delta_1/\partial T \leq 0$. Define:

$$\Delta_2 := \frac{\partial \Delta_1}{\partial T}.$$

By use of $(A.5)$ we have that:

$$\Delta_2 = \left(\frac{\delta}{e^{\delta T}-1}B'(T) - \frac{(B'(T))^2}{B(T)} + B''(T)\right)\Omega + \frac{\delta}{1-e^{-\delta T}}s\Psi'(T)B(T).$$

Furthermore, when taking the derivative of (26) with respect to s, we find that:

$$\frac{\partial T}{\partial s} = \frac{1}{\Delta_2}\frac{\delta}{1-e^{-\delta T}}\left(\frac{s}{\Omega}\frac{\partial \Omega}{\partial s} - 1\right)\Psi(T)B(T). \tag{A.13}$$

We want to show under what conditions $\partial T/\partial s > 0$. From the second order condition (A.12) we have that $\Delta_2 < 0$. Moreover, we know that have that $\Psi(T)B(T) > 0$. It follows that

$$sign\left(-\frac{\partial T}{\partial s}\right) = sign(\frac{s}{\Omega}\frac{\partial \Omega}{\partial s} - 1)$$

From (22) it is immediately clear that (for $\Omega > 0$, which must hold for the optimal T to be finite)

$$\frac{s}{\Omega}\frac{\partial \Omega}{\partial s} - 1 > 0 \text{ for } p < 0,$$
$$\frac{s}{\Omega}\frac{\partial \Omega}{\partial s} - 1 = 0 \text{ for } p = 0,$$
$$\frac{s}{\Omega}\frac{\partial \Omega}{\partial s} - 1 < 0 \text{ for } p > 0.$$

It follows that

$$\frac{\partial T}{\partial s} < 0 \text{ for } p < 0,$$
$$\frac{\partial T}{\partial s} = 0 \text{ for } p = 0,$$
$$\frac{\partial T}{\partial s} > 0 \text{ for } p > 0.\square$$

Proof of proposition 2. In line with the proof of proposition 1, taking the derivative of (26) with respect to σ and rearranging yields:

$$\frac{\partial T}{\partial \sigma} = \frac{\delta}{1 - e^{-\delta T}} \frac{1}{\Delta_2} \frac{s\Psi(T)}{\Omega^2} \frac{\partial \Omega}{\partial \sigma}. \tag{A.14}$$

We have that:

$$\frac{\partial \Omega}{\partial \sigma} = (1 - \alpha) \left(p - s \left(1 - \frac{\omega}{\delta + \omega} \right) \right) \begin{cases} > 0 \text{ if } s/p < \frac{\delta+\omega}{\delta} \\ \leq 0 \text{ if } s/p \geq \frac{\delta+\omega}{\delta} \end{cases}, \tag{A.15}$$

and it follows that

$$\frac{\partial T}{\partial \sigma} \begin{cases} < 0 \text{ if } s < \frac{\delta+\omega}{\delta} \\ \geq 0 \text{ if } s \geq \frac{\delta+\omega}{\delta} \end{cases}, \tag{A.16}$$

which is equivalent to the statement in Proposition 2.□

Proof of proposition 3. In line with the proof of Proposition 1 and 2, taking the derivative of (26) with respect to β and rearranging yields:

$$\frac{\partial T}{\partial \beta} = \frac{\delta}{1 - e^{-\delta T}} \frac{1}{\Delta_2} \frac{s\Psi(T)}{\Omega^2} \frac{\partial \Omega}{\partial \beta}. \tag{A.17}$$

We have that:

$$\frac{\partial \Omega}{\partial \beta} = s\alpha \left[1 - \frac{\kappa}{\delta + \kappa} \right] > 0 \tag{A.18}$$

Since $\Delta_2 < 0$ and $\Psi(T) > 0$, it follows that $\frac{\partial T}{\partial \beta} < 0$, which is equivalent to the statement in Proposition 3.□

Time-dependent prices and parameters

In the Introduction we argued that p, α and β might be increasing functions of the rotation period T. We wish to investigate what implications such extensions may have for our main result given in Proposition 1, i.e. that the optimal rotation time increases with an increased carbon price.

Let p, α and β be replaced with increasing functions $p(T)$, $\alpha(T)$ and $\beta(T)$. The welfare function that is maximized is now instead of (15) given by

$$\Gamma(T,s) \equiv W(T,p(T),s,\alpha(T),\beta(T),\sigma) = \frac{1}{1-e^{-\delta T}}V(T,p(T),s,\alpha(T),\beta(T),\sigma) \tag{A.19}$$

The optimal choice of T (assuming it exists) is given by

$$\Gamma_T(T,s) \equiv W_T + [W_p p'(T) + W_\alpha \alpha'(T) + W_\beta \beta'(T)] = 0$$

Differentiating gives

$$\frac{dT}{ds} = \frac{\Gamma_{Ts}}{-\Gamma_{TT}}$$

From the second-order conditions for an optimum we have $\Gamma_{TT} < 0$, implying that

$$sign\left(\frac{dT}{ds}\right) = sign\left(\Gamma_{Ts}\right)$$

Moreover,

$$\Gamma_{Ts} = W_{Ts} + [W_{ps} p'(T) + W_{\alpha s} \alpha'(T) + W_{\beta s} \beta'(T)] \tag{A.20}$$

We showed in Proposition 1 that the optimal T was an increasing function of s when p, α and β were independent of T, i.e. that $W_{Ts} > 0$. We now turn to the three terms in square brackets in (A.20)

W_{ps} has the same sign as V_{ps}; by examining each term in the expression for V (given by (**??**)) we find that $V_{ps} = 0$. Hence, the fact that p may be increasing in T does not affect our conclusion that T is increasing in s.

$W_{\alpha s}$ has the same sign as V_{as}; by examining each term in the expression for V (given by (**??**)) we find that $V_{\alpha s}$ consists of two negative terms (associated with $+V_{CC}$ and $+V_{NCC}$) and three positive terms (associated with $-V_F$,

$-V_D$ and $-V_N$). More specifically, we have that

$$V_{as} = \frac{1}{\alpha^2}\left[e^{-\delta T}\frac{\omega+\sigma\delta}{\delta+\omega}B(T) - \int\limits_0^T e^{-\delta x}B'(x)dx + \gamma\int\limits_0^T \left(e^{-\delta T}\frac{\omega}{\delta+\omega} - e^{-\delta x}\right)B(x)dx\right]. \tag{A.21}$$

With regard to the terms in the square bracket above, the first term is less than the second term, while the third term is less than the fourth term. Hence, $V_{\alpha s} < 0$, implying that we cannot rule out the possibility that $\alpha'(T) > 0$ may reverse the conclusion that T is increasing in s. However, this can only occur if $\alpha'(T)$ is sufficiently large.

$W_{\beta s}$ has the same sign as $V_{\beta s}$; from (**??**) and the expressions for each of the terms in V we find

$$V_{\beta s} = e^{-\delta T}R(T)\left[1 - \frac{\kappa}{\delta+\kappa}\right] > 0$$

Together with $\beta'(T) > 0$ this strengthens our conclusion that T is increasing in s.

The social cost of carbon

The social cost of carbon is the present value of all future climate costs caused by one unit of current emissions. In formal notation this is often written as

$$s(t) = \int_t^\infty e^{-(\delta+\rho)(\tau-t)}C'(A(\tau))d\tau \tag{A.22}$$

where δ is the discount rate, ρ is the depreciation rate for carbon in the atmosphere, $A(\tau)$ is the stock of carbon at date τ (above natural or preindustrial level) and C is a measure of climate costs, assumed at any time to depend on the stock of carbon in the atmosphere at that time.

The size of the appropriate discount rate has been discussed extensively in the literature, and we have nothing to add to this discussion. The formula above is based on the assumption that an amount $\rho A(\tau)$ of the carbon in the atmosphere at date τ is transferred from the atmosphere to other carbon sinks (in particular to the ocean). Although used frequently in economic models, it is well-known that this assumption is a very inaccurate description of the true carbon cycle. In particular, the assumption means that if emissions drop to zero, the amount of carbon in the atmosphere will eventually drop down to its preindustrial level. The assumption also implies that if emissions are constant and equal to $\rho A(\tau)$ from τ onwards, carbon in the atmosphere will remain constant from τ onwards.

It is true that a rapid increase of carbon in the atmosphere will gradually decline over time, as it is transferred to other sinks. However, a significant portion (about 25% according to e.g. Archer, 2005) remains in the atmosphere for ever (or at least for thousands of years). Even if emissions are constant the carbon in the atmosphere will eventually grow; the only possibility for the amount of carbon in the atmosphere to be constant for a long period is to have zero emissions. Moreover, for a given amount of fossil fuels extracted, there is a corresponding long-run increase in the amount of carbon in the atmosphere.

From the discussion above it is clear that $\rho = 0$ in many ways gives a better representation of some important features than $\rho > 0$. Some analyses explicitly take into account the fact that some but not all carbon emissions remain in the atmosphere, see e.g. Farzin and Tahvonen (1996). In our subsequent discussion we simply assume $\rho = 0$, so that (A.22) implies

$$\dot{s}(t) = \delta s(t) - C'(A(t)) \tag{A.23}$$

An immediate conclusion from this is that the present value of $s(t)$ declines over time provided $C' > 0$. To be able characterize the path of $s(t)$ any further we must first discuss the properties of the climate cost function $C(A)$.

The function $C(A)$ is typically assumed increasing and convex−often strictly convex. The background for this is that the global temperature increase above preindustrial average is rising in A, and that climate costs−and probably marginal climate costs−are increasing in the temperature increase. Even if climate costs are an increasing and strictly convex function of the temperature increase, it is not obvious that $C''(A) > 0$. The reason for this is that there is a complex and non-linear relationship between A and temperature increase. In particular, radiative forcing, which is the prime cause of the temperature increase, is a logarithmic function of A. If climate costs were approximately proportional to temperature increase, this suggests $C''(A) < 0$. Although it hence is not obvious that $C''(A) \geq 0$, we shall stick to this assumption as it is frequently used elsewhere in the literature, and will hold if marginal climate costs rise sufficiently with increased temperature.

For the limiting case of $C'' = 0$ it follows from (A.22) that $s(t)$ is constant (equal to C'/δ for $\rho = 0$). For the more general case of $C''(A) > 0$, it follows from (A.22) that $s(t)$ must be rising as long as emissions are positive and hence $A(t)$ is increasing (for $\rho = 0$). However, the growth rate of $s(t)$ will be below δ as long as $C' > 0$.

It is sometimes assumed that there is a climate goal of a maximum permitted temperature increase, and that one is not concerned about the temperature increase as long as this limit is not violated. This corresponds to a maximal limit on A, and $C(A) = 0$ below this limit. For this case $C'(A) = 0$ as long as A is below its maximal limit, implying that $\dot{s}(t) = \delta s(t)$ as long as A is below its maximal limit. While the case of a constant present value of the social cost of carbon is of some interest as a limiting case, this case is not particularly relevant in practice: Even if one has a goal of a maximal permitted temperature increase, one would usually also have some concern of temperature increases below this level. If so, $C' > 0$ and $\dot{s}(t) = \delta s(t)$ also when A is below its maximal limit.

To conclude: The reasoning above suggests that $s(t)$ is rising over time, while the present value of $s(t)$ is declining over time. Our analysis considers the two limiting cases of $s(t)$ constant and $\dot{s}(t) = \delta s(t)$.

A rising social cost of carbon

We now turn to the other limiting case, i.e. the case of $s(t)$ rising at the rate of interest δ, i.e. $s(t) = s_0 e^{\delta t}$. Total welfare for one rotation period is given by

$$V_P(\cdot) + s_0 \Sigma(\cdot), \tag{A.24}$$

where $V_P(\cdot)$ is defined by (1) and where

$$\Sigma(\cdot) := -V_F^*(\cdot) - V_M^*(\cdot) - V_D^*(\cdot) + V_{CC}^*(\cdot) + V_N^*(\cdot), \tag{A.25}$$

where all $V_i^*(\cdot)$ are defined as $V_i(\cdot)$ in the previous subsection except that we replace s with $e^{\delta t}$. This gives

$$\begin{aligned}
V_F^*(\cdot) &= (\alpha\,(1-\beta) + \sigma\,(1-\alpha))\, B\,(T)\,,\\
V_M^*(\cdot) &= \beta\alpha B\,(T)\,,\\
V_D^*(\cdot) &= (1-\sigma)\,(1-\alpha)\, B(T),\\
V_{CC}^*(\cdot) &= B(T),\\
V_N^*(\cdot) &= 0.
\end{aligned}$$

It follows that $\Sigma(\cdot) = 0$. This means that the social welfare for a infinite horizon constant rotation case is given by

$$W(p,T,\sigma) = \frac{1}{1-e^{-\delta T}} V_P(p,T,\sigma), \tag{A.26}$$

and the value of T that maximizes this is simply the standard Faustmann rule given in (29), independent of the size of s_0.

This result is not surprising. Consider again the one period rotation model: We start out with zero carbon tied up in biomass. As time passes, carbon in biomass increases. Once the forest is harvested, all of the carbon is released to the atmosphere (some immediately and some only gradually). As long as the present value of the social cost of carbon is constant, the initial increase of carbon in biomass has exactly the same social value as the later reduction. Hence, the one rotation period social welfare is independent of the level of the social cost of carbon. It immediately follows that the same must be true of the present social value for the infinite horizon constant rotation period case.

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Table 1. Optimal length of the rotation period (T) with regard to different values of the social cost of carbon (s), as well as different values of the share of residues harvested (σ).*

Social cost of carbon		The share of residues harvested (σ)		
s/p	USD/tC	0	0.25**	0.50***
0	0	39	39	39
0.49	36.67	75	66	61
0.73	55.00	125	96	83
1.00	75.00	∞	∞	176
1.22	91.70	∞	∞	∞

* The share of the harvested trunks that are used for durable storage in buildings and furniture (β) is set to 0.25 in all simulations presented in this table.
** σ=0.25 means that all tops and branches are harvested.
** σ=0.5 means that a share of stumps and roots is harvested in addition to tops and branches.

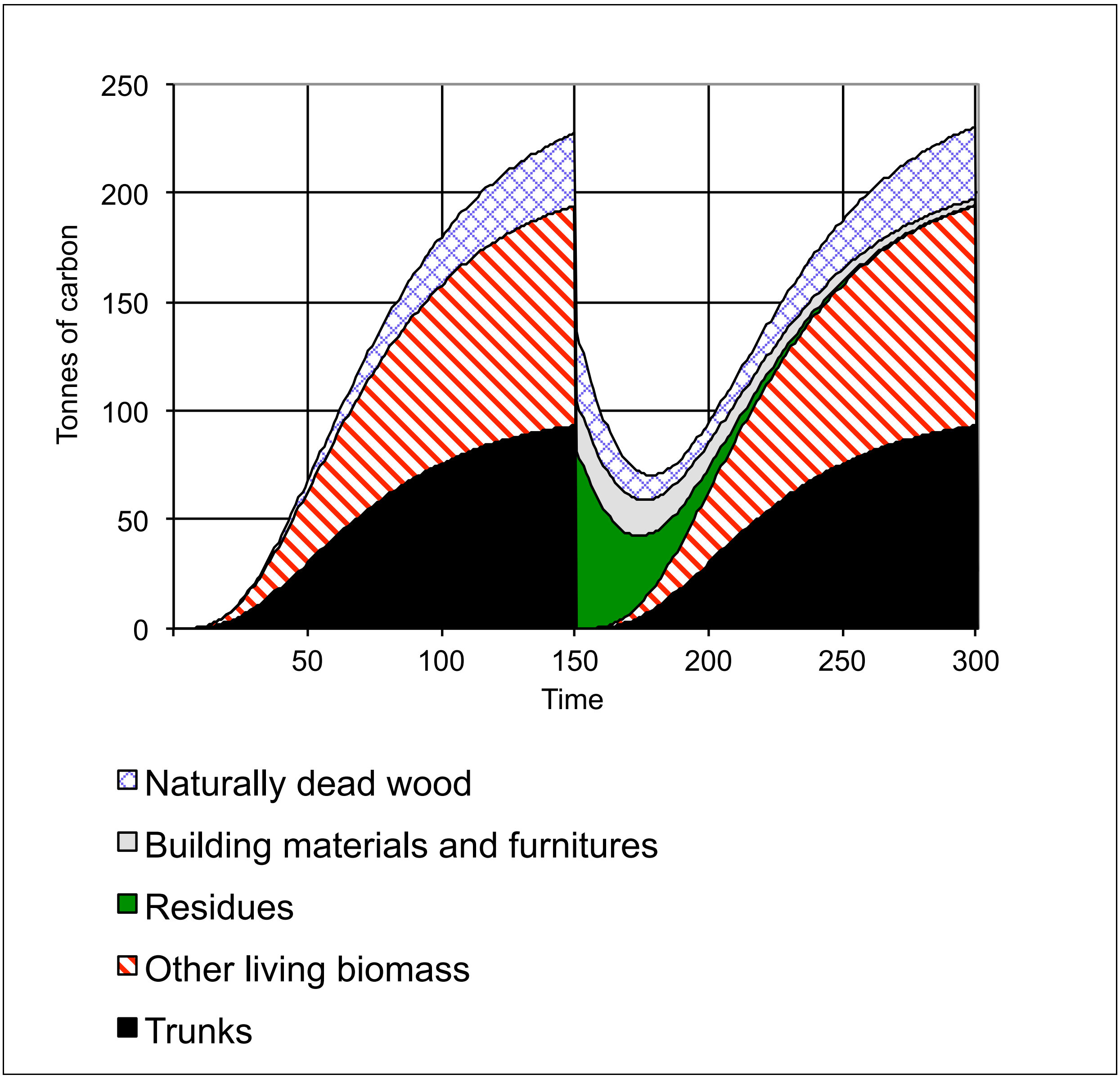

Figure 1. The development of the components of the stock of carbon in the forest and in building materials/furniture with a rotation length of 150 years.

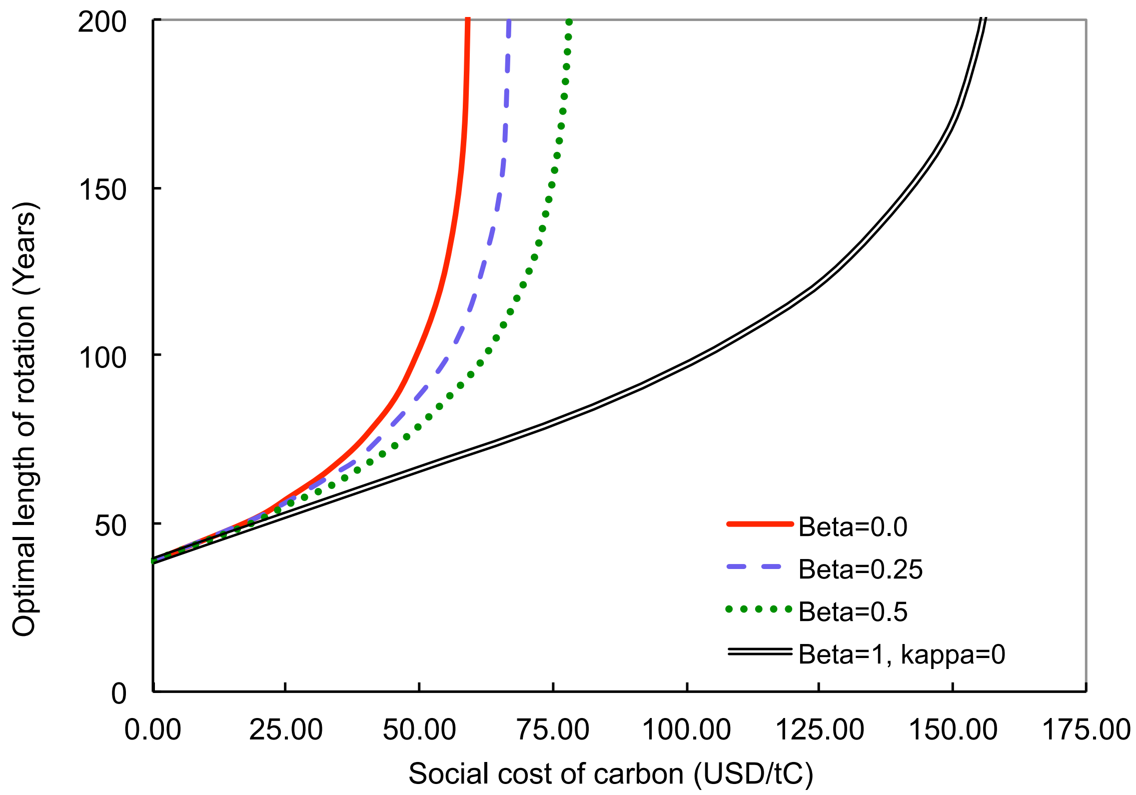

Figure 2. The optimal length of the rotation period given different shares of the harvest that are used for durable storage in buildings and furniture (β). The net commercial profit to the forest owner is 15 USD/m^3 wood, which corresponds to 75 USD/tC. Hence, $s/p = 1$ if the social cost of carbon is 75 USD/tC. In the cases where β is 0.0, 0.25, and 0.5, then κ is 0.04. In the case where β is 1.0, then κ = 0.0.

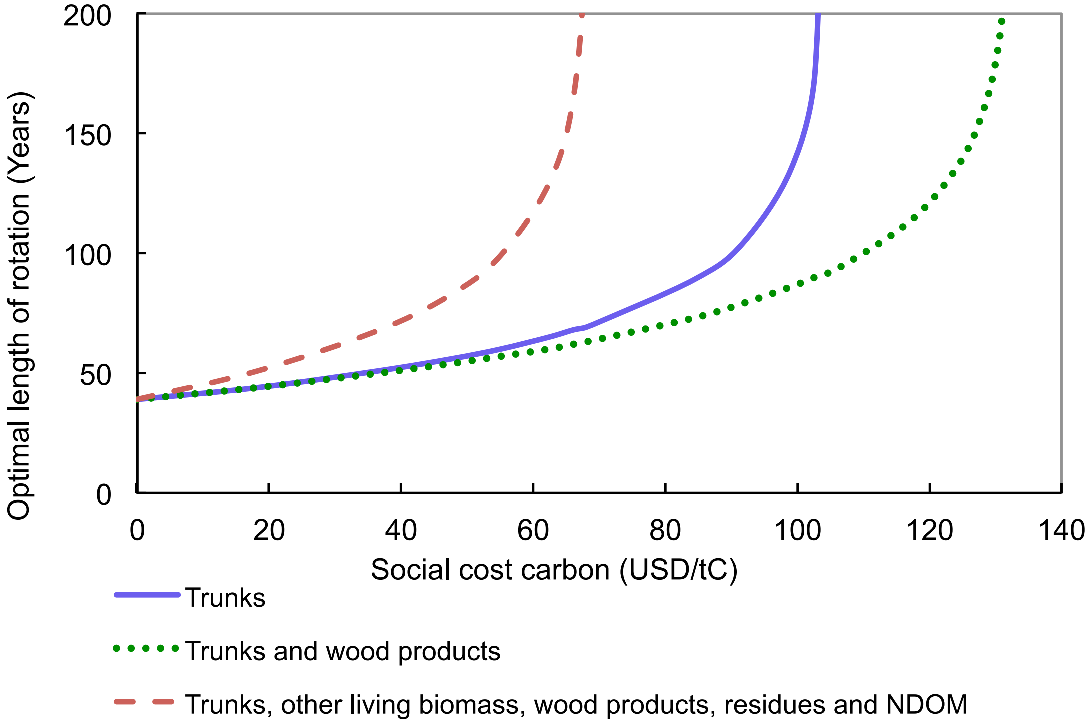

Figure 3. The optimal length of the rotation period in the main multiple carbon pool case (the double lined curve) and cases where one or more carbon pools are not included in the analysis.

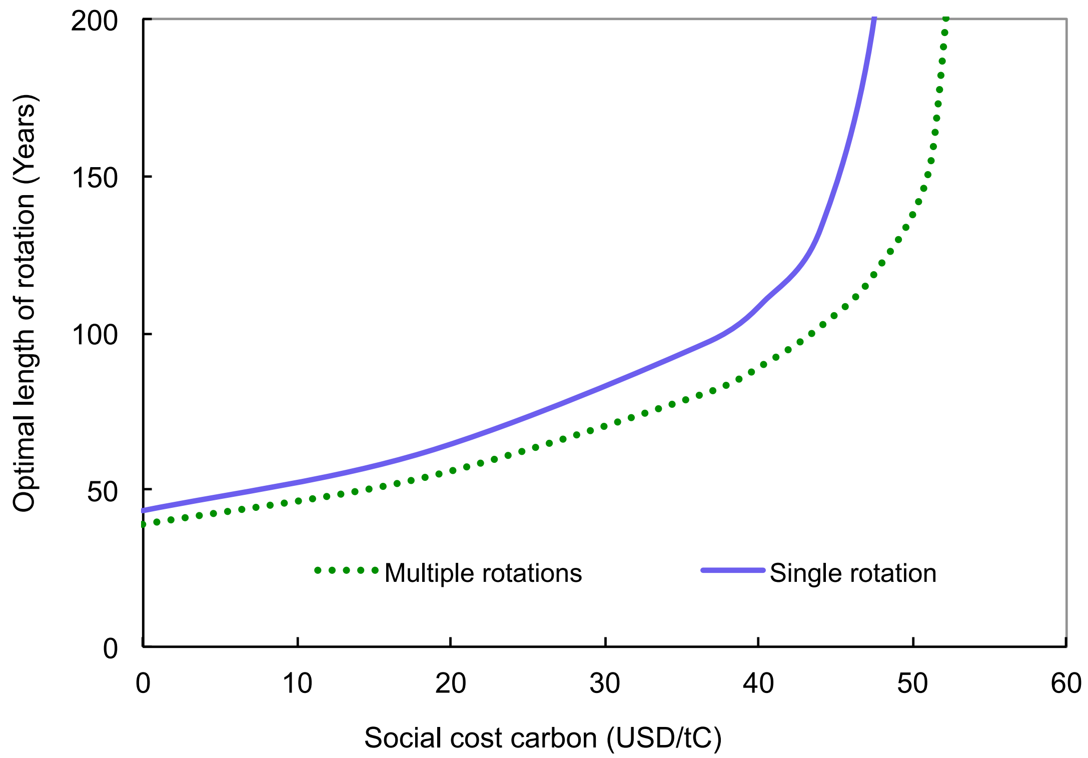

Figure 4. The optimal long run average annual supply of wood per hectar given different social costs of carbon in a single harvest analysis and when multiple rotations are considered.

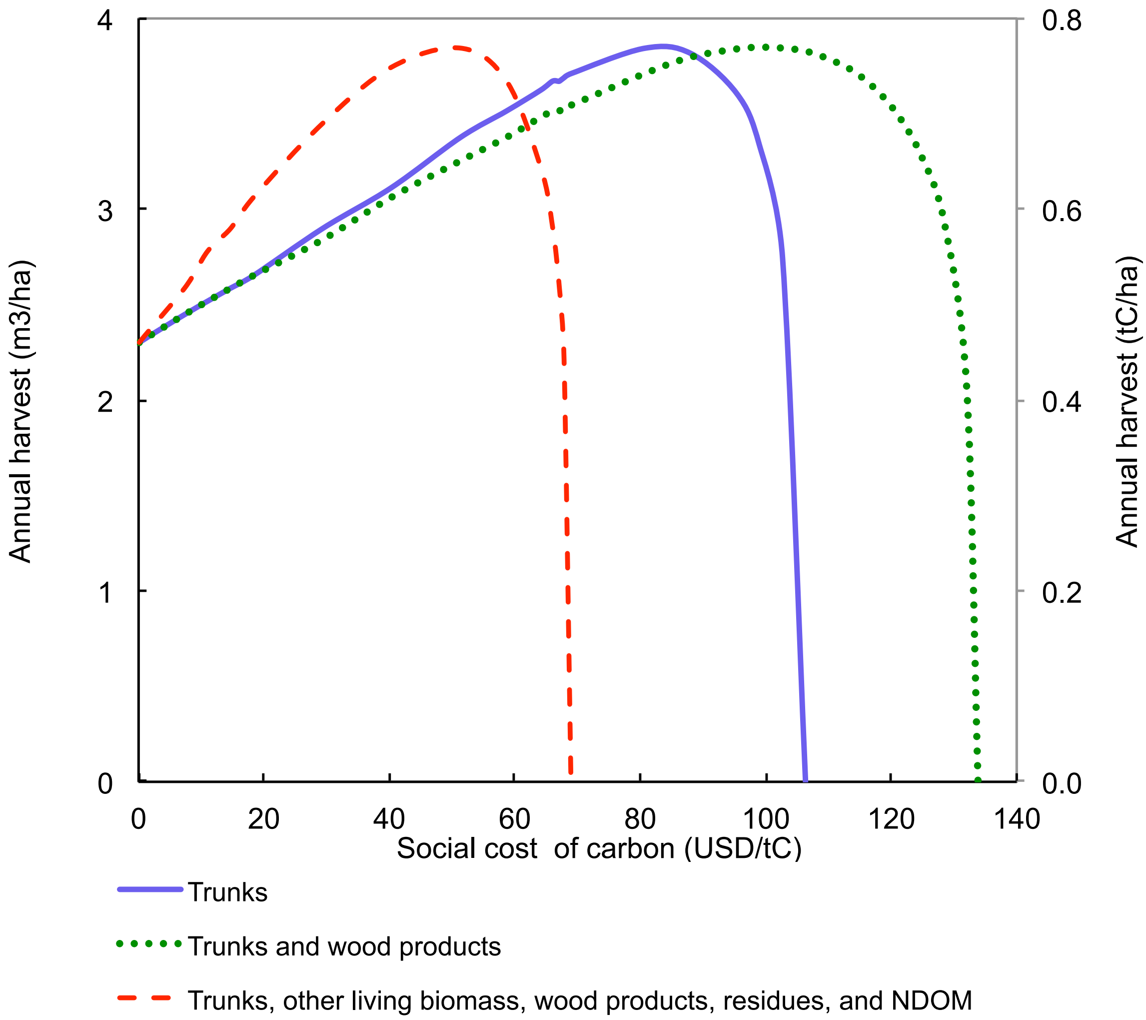

Figure 5. The optimal long run average annual supply of wood per hectar given different social costs of carbon when different carbon pools are included in the model.