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Lexicographic Voting

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Abstract

This paper reconsiders the division of the literature on electoral competition into models with forward-looking voters and those with backward-looking voters by combining ideas from both strands of the literature. As long as there is no uncertainty about voters' policy preferences and parties can commit in advance to a policy platform but not to a maximal level of rent extraction, voters can limit rents to the same extent as in a purely backward-looking model. At the same time, the policy preferred by the median voter is implemented as in a standard forward-looking model of political competition on an ideological policy dimension. Voters achieve this outcome by following a simple lexicographic voting strategy. They cast their vote in favor of their preferred policy position, but make their vote dependent on the incumbent parties' performance in office whenever they are indifferent. When uncertainty about the bliss point of the median voter is introduced into the model, voters have to accept higher rent payments, but they still retain some control over rent extraction.

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1. Introduction

Do voters reward incumbents for past success and honesty or do they disregard the past and only consider the future when they vote? This is one of the most fundamental questions for a positive theory of electoral competition. However, theoretical models of elections usually assume either backward-looking (also called retrospective) or forward-looking (also called prospective) voting. The voters' motivation at the ballot box is an assumption of the model rather than an outcome of the equilibrium analysis. In models of pre-election politics, candidates commit to their post-election actions before elections take place. In contrast, in models of postelection politics, politicians are free to decide about their policies when they are in office. However, in the successive elections, the voters can condition their vote on the performance of the incumbent party.[1,2]

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[1]Retrospective and prospective voting seem to be self-explanatory terms. Either voters consider past performance or expectations about future performance when they make their voting decision. However, as soon as we use game theoretic models of elections the distinction turns out to be far from trivial. By the very definition of Nash-equilibrium every voting strategy that is part of an equilibrium must be prospective in the sense that is maximizes the (expected) utility of the voter who plays it. Nonetheless, it seems reasonable to call strategies that can be completely described by past experience of (dis-)utility as retrospective and strategies that depend only on variables that influence a voter's utility only in the future as prospective. A formal definition along these lines is provided by Duggan (2000).

[2]Models of preelection politics are especially popular for modeling spatial policy choices in the tradition of Downs (1957), where voters decide between announced policy positions, while models of postelection politics are often, but not exclusively, applied to accountability

In this paper, I combine a simple prospective model of Downsian spatial electoral competition on an ideological policy dimension and a simple retrospective model of electoral accountability with rent extraction. Specifically, parties can commit to a policy position before elections take place, as in Downs (1957), but decide on the level of rent extraction once they are in office, as in Barro (1973) and the simplified model of political accountability discussed in Persson and Tabellini (2000).

In the basic model in Section 2, I show that, as long as there is certainty about the position of the median voter, having voters with divergent policy preferences does not at all restrict the possibility of holding politicians accountable. The possible equilibrium rent levels are the same as they would be in a model without the additional ideological policy dimension. The voters achieve this by following a straightforward and intuitive lexicographic voting strategy. Specifically, if the parties commit to policy positions that differ in attractiveness to a voter, the voter casts her vote in favor of the party which minimizes her disutilty on the policy dimension. Only when a voter is indifferent with respect to the parties' policy platforms does she condition her vote on the rent extraction of the incumbent party. She supports the incumbent party only if the rents have not exceeded a maximum acceptable level.

issues. Politicians are induced to put in more effort (Ferejohn, 1986) or to limit rent extraction due to the possibility of losing the successive elections and office if they do not comply (Barro, 1973). Essentially, these accountability models apply a principal-agent framework to elections with the politicians as agents and the voters as their principals. For an overview of both types of model, see Persson and Tabellini (2000). For an overview especially of models of accountability, see Besley (2006).

I call this voting strategy "lexicographic" because voters cast their votes as if they had lexicographic preferences over policy and rents. My model is the first one to show that lexicographic voting can achieve a reconciliation of backward-looking and forward-looking voting.[3] The lowest possible rent level that is sustainable in an equilibrium is positive but smaller than the maximum rent the incumbent party could take. Moreover, the existence of the ideological policy dimension does not influence the bound on rents.

The lexicographic voting strategy forces the parties to converge on the policy dimension, but also allows for control of the incumbent's party rent extraction. In addition, it is intuitive that a voter who is indifferent will take past actions of the parties into account, whereas it is impossible for a rational forward-looking voter to consider the past when she is not indifferent with respect to the future.

Generally, the equilibria in backward looking models hinge on the fact that voters are indifferent between the incumbent party and the opposition and can therefore reward or punish past actions while playing undominated strategies. The fact that a simple strategy can solve the accountability problem in a model combining rent extraction with Downsian competition can be explained by the fact that competition on the ideological policy dimension forces both parties to choose the same platform so that voters are indeed

[3] The term lexicographic voting has been used before to describe similar voting strategies, for example in Dutter (1981) and Soberman and Sadoulet (2007). However, in these papers, lexicographic voting follows directly from lexicographic preferences. In my model, lexicographic voting is part of an equilibrium of the voting game, although the voters' preferences are not lexicographic.

indifferent between the parties in equilibrium. Full convergence of policy is a result of the lack of uncertainty over the preferences of the median voter in the basic model.

Section 3 shows that when uncertainty over voters' preferences is introduced into the model, the minimum equilibrium rent extraction by the incumbent party increases. Because the parties do not know the position of the median voter's bliss point with certainty, the opposition party now has a chance of winning office by offering a different policy position to that of the incumbent party. Nonetheless, the incumbent party has an incentive to accept somewhat reduced rent payments in return for being re-elected whenever the voters are indifferent because in this way, it can ensure that it will be re-elected with positive probability.

In Section 4, I show that if parties are also motivated by policy and not only by rents, the ideological aspect of political competition can even increase the accountability of politicians compared to a pure accountability model. Ideological parties give voters the additional option of threatening the incumbent party to allow the opposition party to win with policies that make the incumbent party worse off than the bliss point of the median voter. However, this requires more coordination among voters than the simple and straightforward lexicographic voting strategy given in Section 2. Therefore, the lexicographic voting strategy from the main model which continues to constitute an equilibrium in the case with ideological parties seems the most plausible outcome even in the case of ideological parties.

The paper proceeds as follows. Section 2 develops the main model with certainty about the position of the median voter and discusses its equilibrium.

Section 3 shows that uncertainty over the positions of the median voter leads to less electoral control. Section 4 presents an extension to policy oriented parties. An Appendix contains the proofs of the results in Section 3 and the examples of Section 4.

2. The model

I consider a polity with two parties interested in winning office only for rent-seeking purposes, and an odd number N of voters $i = 1, 2, ..., n$ interested in policy as well as rent reduction. The ideological policy space is the interval $[0, 1]$. Party $j \in \{x, y\}$ maximizes:

$$U_p^j = E_0 \sum_{t=0}^{\infty} \beta^t r_t^j, \tag{1}$$

where rents in future periods are discounted by the factor $\beta \in (0, 1)$. r_t^j is the rent extracted by party j in period t. The party in government (also called the incumbent party) in period t is denoted by $I_t \in \{x, y\}$. The opposition party in period t is denoted by $O_t \in \{x, y\}$, $O_t \neq I_t$. Parties decide how much rent $r_t \in [0, R]$ they extract in a period in which they are in office. R is the total amount of available public funds and constitutes the maximum per period rent. Parties out of office cannot acquire any rents. Hence $r_t^{O_t} = 0$ in all periods t.

Voters $i = 1, 2, ..., n$ maximize:

$$U_v^i = E_0 \sum_{t=0}^{\infty} \beta^t (-(p_t - b^i)^2 + (R - r_t)), \tag{2}$$

where b^i is the policy bliss point for voter i, r_t the rent extraction and p_t the policy implemented in period t. Because the policy platform announced by the incumbent is always implemented and $r_t^{O_t} = 0$ in all periods t we have $p_t = p_t^{I_t}$ and $r_t = r_t^{I_t}$. I abstract from any details on how rents are extracted and assume that rent payments reduce a given amount of public funds, which reduces every voter's utility in the same way. Hence, $R - r_t^{I_t}$ gives the amount of public funds that are used in the voters' interest. For simplicity, I assume that the utility from public good spending is uncorrelated with the ideological policy position. The variable p_t denotes the policy in period t and the vector $B = (b^1, b^2, ..., b^N)$ the policy bliss points of the voters. $b_m = median(B)$ is the bliss point of the median voter. For the moment, this is assumed to be constant over time. In Section 3, the more general case of uncertainty about the median voter's position is discussed. Disutility in policy is quadratic in the distance from the bliss point. This standard functional form assumption is made for convenience of notation. All the following results only depend on increasing disutility in distance of policy from a voter's bliss point.

2.1. The order of moves

The order of moves is the following: In any period t, the policy position p_t^I of the incumbent party $I_t \in \{x, y\}$ is implemented, then rents $r_t^{I_t}$ and a new policy position p_{t+1}^I are chosen by the incumbent party. An alternative policy position $p_{t+1}^{O_t}$ is chosen by the opposition after observing the policy position of the incumbent party and the rent r_t. Then, elections take place and every voter i casts her vote $v_t^i \in \{x, y\}$. Abstentions are not possible.

Let $V_t = (v_t^1, v_t^2, .., v_t^N)$ be the vector containing the votes of all voters. After the elections have taken place, the new period $t+1$ begins and the party with the majority of votes in period t becomes the incumbent party:

$$I_{t+1} = \text{mod}(V_t).$$

In Period 0, the identity and the policy positions of the incumbent party and the opposition are exogenously given.

The incumbent party is thus assumed to choose its position first, instead of the more standard assumption that policy positions are chosen simultaneously.[4] For the basic model, this is of no great importance (however, the best reply of the opposition is no longer unique), but it plays some role when I introduce uncertainty in Section 3, where it is essential for the existence of equilibria in pure strategies. The timing assumption is made to keep the analysis there as simple as possible.

2.2. Strategies

To denote the entire history of a variable z_t up to period t, I use a superscript t such that $z^t = \{z_0, z_1, z_2, ..., z_t\}$. Then $h_t = \{p^{y,t}, p^{x,t}, I^t, V^{t-1}, r^{t-1}\}$ denotes the complete history of the game up to the beginning of period t because it contains all past values of all variables. A strategy for a party j is the decision about a policy platform $p^j_{t+1}(h_t) \in [0,1]$ for all possible histories with $j = I_t$ and $p^j_{t+1}(h_t, p^{I_t}_{t+1}, r_t) \in [0,1]$ for all possible histories with $j = O_t$. In addition, the strategy contains the rent payment $r^j_t(h_t)$ for all possible histories with $j = I_t$. Because the opposition can observe the policy position of the incumbent party, the party that is out of office can take both the policy position and the rent payment to the incumbent party into account when announcing its policy position, while the incumbent party cannot. A strategy for a voter i is a vote $v^i_t(h_t, p^y_{t+1}, p^x_{t+1}, r_t) \in \{y, x\}$ for every period t

[4]This assumption is less common than simultaneous policy announcements, but has been made in many papers. For an early example see Wittman (1973).

and every possible history up to the time of her voting decision. However,in all equilibria discussed in the paper votes depend only on $p^y_{t+1}, p^x_{t+1}, r_t$ and I_t. At the time of the voting decision only p^y_{t+1}, p^x_{t+1} are directly relevant for the future utility of the voters.

2.3. An equilibrium with lexicographic voting

The strategies formulated in Proposition 1 below constitute an interesting equilibrium which has all the essential features of a backward-looking model in the tradition of Barro (1973) and Ferejohn (1986) as well as those of a forward-looking model in the tradition of Downs (1957). Parties converge on the ideological dimension and the rents are at the lowest level sustainable in the purely backward-looking model without policy dimension. This is the result of an intuitive lexicographic voting strategy. A voter casts her ballot in favor of her preferred policy position. Only when she is indifferent in this respect does she decide according to past rent extraction by the incumbent party. It is clear that with such a strategy, she encounters no credibility or time-inconsistency problem.

Proposition 1. *An equilibrium of the game is constituted by the following strategies:*

The parties play:

$$\begin{aligned} p^j_{t+1} &= b_m \text{ for } j = y, x \text{ in all } t, \qquad (3) \\ r^{I_t}_t &= \bar{r} \text{ in all } t, \end{aligned}$$

where $\bar{r} = (1-\beta)R$.

The voters play:

$$v_t^i = \begin{cases} y \text{ if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 < 0 \\ x \text{ if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 > 0 \\ I_t \text{ if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 = 0 \text{ and } r_t \leq \bar{r} \\ O_t \text{ if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 = 0 \text{ and } r_t > \bar{r} \end{cases} \text{ in all } t. \quad (4)$$

Given the strategies, it follows that:

$$\begin{aligned} I_t &= I_0 \text{ in all } t, \\ p_t &= b_m \text{ in all } t \geq 1, \\ r_t &= \bar{r} \text{ in all } t. \end{aligned} \quad (5)$$

Proof. Given the voters' strategy, the median voter is decisive: If $v_t^m = j$, it follows that $(p_{t+1}^j - b^m)^2 - (p_{t+1}^{\sim j} - b^m)^2 \leq 0$. This implies that $(p_{t+1}^j - b^i)^2 - (p_{t+1}^{\sim j} - b^i)^2 \leq 0$ for all $b_i \leq b_m$ or all $b_i \geq b_m$ and therefore for a majority of voters. Thus, the majority of voters cast their votes for the same candidate as the median voter and the party with the support of the median voter wins. Given the equilibrium strategies of the parties, $(p_{t+1}^j - b^i)^2 = (p_{t+1}^{\sim j} - b^i)^2$ in all periods. Because $r_t = \bar{r}$ in all periods, all voters vote for the incumbent party, which remains in office and implements $p_{t+1}^I = b_m$.

Given the strategies of the parties, a voter in period t neither influences future rent payments nor future policy platforms (that is any p_s^j with $s > t+1$) with her vote. Therefore, a voter has no utility-increasing deviation from voting for the party that offers the policy closest to her bliss point in period $t+1$. In the case that a voter is indifferent between the candidates' policy platforms in period $t+1$, there is no utility-increasing deviation from voting according to the past performance of the incumbent because, again, it does not influence future policy or rent payments.

The fact that the opposition party cannot be better off by deviating follows from the fact that given the position and rent extraction of the incumbent party and the strategy of the voters, it either wins with certainty or has no possibility of winning office and, moreover, it cannot influence any election results or rent payments in the future with its choice of policy position. For the incumbent party, any policy position different from $p_{t+1}^I = b_m$ leads to a loss of office (and therefore rent payments) forever because given the reply of the opposition, the latter is preferred by the median voter. The same is true for the combination of any policy position p_{t+1}^I with any rent $r_t > \bar{r}$. Therefore, re-election is only possible with $r \leq \bar{r}$. Hence, there is no possibility for the incumbent party of increasing its utility by deviating with a strategy that leads to its re-election. If it accepts defeat by deviating in an arbitrary period s, the incumbent party can, at most, achieve a rent of R in the period in which it deviates and then lose office and rents forever. This gives the same utility level that the incumbent party achieves by not deviating and receiving a rent of $r_t = \bar{r} = (1 - \beta)R$ forever, because the present discounted value of future rent payments in period s is the same:

$$\sum_{t=0}^{\infty} \beta^t \bar{r} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \sum_{t=s}^{\infty} \beta^t \bar{r} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \sum_{t=s}^{\infty} \beta^s \frac{\bar{r}}{1-\beta} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \beta^s R.$$

Therefore, no deviation from the given strategy increases the utility of the incumbent party. ■

Which party is the incumbent party in period 0 is exogenously given. This party remains in office forever, as in the standard case of backward-looking models without uncertainty. However, this will no longer be the case when I introduce some uncertainty in Section 3.

Corollary 2. *There is no equilibrium with a present discounted value of future rent payments in any period s of the game that is lower than the maximum per-period rent extraction R.*

Proof. Suppose that there is an equilibrium with $\sum_{t=s}^{\infty} \beta^{t+s} r_t < R$ in any period s. Then, the incumbent party in period s is better off by deviating and taking a rent of $r_s = R$. This is a contradiction. ■

Therefore, the equilibrium rent level in Proposition 1 gives a lower bound for rents in equilibrium.[5] The rent level is identical to the lower bound on rent extraction in a model without a policy dimension.[6]

As is also common in models of political accountability, the given equilibrium is not unique and other equilibria with larger rent payments exist. However, the existence of this equilibrium is sufficient to establish that retrospective and prospective motives in voting are not inconsistent with each other. Voters have just one instrument, namely their single vote, but this is sufficient to control policy as well as to hold politicians accountable to a

[5] There are equilibria with a lower rent payment $r_t < \bar{r}$ in period t that are sustainable because the incumbent party expects higher rent payments in the future. However, from Corollary 2, we know that the present discounted value of rent extraction cannot be smaller than R. Equilibria with increasing rent payments over time seem rather implausible. The opposition party could in this case try to convince the voters that it would only demand a constant rent payment of $\bar{r}$ once in office.

[6] This can easily be established following the same line of reasoning as in the proof of Corollary 2.

certain degree.

A strategy is stationary if it depends only on state variables, not on past play or the period of the game. A voter votes sincerely if she votes for the party whose victory maximizes her intertemporal utility function.[7]

Corollary 4 below shows that convergence on the policy dimension is the rule rather than the exception, but first I derive a useful Lemma:

Lemma 3. *If parties play symmetric stationary strategies and voters vote sincerely, then: a) A voter votes for a party that offers the bliss point minimizing her disutility from policy in the next period. b) A party's present discounted utility only depends on being the incumbent party in the next period and its rent in the current period.*

Proof. Stationarity together with symmetry of the parties strategies imply that, from period $t+1$ onwards,the set of available policy positions and the level of rent extraction are independent of past periods. The only state variable is incumbency, but voters are indifferent to which party is in office and which party offers which policy position as long as the available policy positione are the same. From this, the lemma directly follows. ■

[7] Sincere voting is not necessary for the equilibrium of a voting game (even if it is one-shot) because a vote does not necessarily change the identity of the party that wins the election. In addition, it is unusual to apply the concept of sincere voting to a repeated game. The reason is simply that because of the effects on future play and payoffs that a vote can potentially have the concept is not necessarily well-defined in a repeated setup . However, because in this section we only consider equilibria in which parties are restricted to symmetric stationary strategies sincere voting is well-defined because a voters' vote can affect her utility only by determining the policy implemented in the next period.

Corollary 4. *There is no equilibrium with symmetric stationary strategies by the parties, voters who vote sincerely, rent payments $r_t < R$ and policy $p_{t+1} \neq b^m$ in any period t.*

Proof. From Lemma 3, it follows that in any equilibrium with stationary symmetric strategies played by the parties, a party's policy position influences its utility only in so far as it determines the winner of the elections and the rent extraction. Suppose that $r_t < R$. This can only be part of an equilibrium if the incumbent party is re-elected with positive probability; if not it would play $r_t = R$, because a lower rent r_t could not improve its situation once in opposition. If both parties play symmetric history-independent strategies, the incumbent party can only be re-elected with positive probability if it plays $p^I_{t+1} = b_m$, because all other positions would be beaten by $p^O_{t+1} = b_m$. To see this, consider the problem of a voter who votes sincerely: By definition of b_m, a majority of voters must prefer b_m to any $b \neq b_m$, and in equilibrium the opposition would have to choose a position that wins the election to maximize its utility. Therefore, if $r_t < R$ the incumbent party offers $p_{t+1} = b_m$ and, in equilibrium, a party offering b_m wins. ■

However, there exist equilibria with $r_t = R$ and $p_{t+1} \neq b_m$. This is due to the unusual timing assumption that the opposition party chooses its policy position after the incumbent party. There are, for example, history-independent equilibria where the incumbent party always takes R and is never re-elected. In such equilibria, the incumbent party has no incentive to take the median position. However, if the incumbent party does not take the median position, the opposition party does not have to take it either to win, because any policy position that is different from b_m can be beaten by another policy position that is different from b_m, but slightly closer to the bliss point of the median voter. With the standard timing assumption of simultaneous announcement of policy positions, this is not possible. However, a similar equilibrium in which policy does not converge to the median position is possible in a purely Downsian framework with the incumbent party choosing its position first, and the result should therefore not be attributed to the combination of prospective and retrospective voting motives. On the contrary, only in combination with the outcome of $r_t = R$ in all periods can it be sustained in the combined model.

2.4. Discussion of the different treatment of rents and policy

A crucial assumption is that commitments to electoral platforms are credible in the policy dimension but lack credibility in the rent dimension. A first justification is that these are widely accepted standard assumptions for both types of models and that it is worth exploring whether combining them leads to results that cannot be found by looking at the models separately. Moreover, in the basic model as well as in the extension with uncertainty over the position of the median voter (Sections 2 and 3), parties have no reason to break their electoral promises with regard to policy because it does not enter their utility function. A further justification is that if parties announce policy motivated candidates who run for office, they can indeed credibly commit to policies, but not to limits of rent extraction. Osborne and Slivinski (1996) and Besley and Coate (1997) introduced citizen-candidates into the voting literature. In these models, not parties, but citizens with policy preferences run for election. Commitment to a policy position does not constitute a problem because voters vote for ideological candidates whom they know to implement their favorite policy. As long as there is a candidate with a certain ideology, voters can vote for that candidate. The principal-agent problem of the voters is solved by delegation to an agent with the right preferences. However, empirically, citizen-candidates who run independently of any party appear to be the exception rather than the rule. The basic idea that a certain type of candidate will implement a certain kind of policy can be incorporated into models with parties if the parties have the chance to decide before the elections who the candidate is and achieve office in case of victory, and if the choice of potential candidates is sufficiently large. I do not explicitly model such a candidate choice stage, but the fact that parties usually run with candidates who have their own ideology is a good justification for the assumption that parties can commit to a policy. However, as long as there are no candidates with purely altruistic motives without interest in rent payments available, parties cannot credibly commit to refrain from rent seeking.

3. Uncertainty about the median bliss point

So far, I have assumed that the identity of the median voter is known when parties decide on their policy platforms. How robust are the results to relaxing this assumption? This section shows that voters retain some control over rent extraction in a straightforward and plausible equilibrium where they follow the same lexicographic voting strategy as in Section 2.

The assumptions and the order of moves are the same as in Section 2. The only difference is that the favorite position of the median voter is now uncertain at the point when parties announce their policy positions. Voters keep some control over rent extraction, but the control is limited because sometimes the incumbent party loses office even when it does not deviate and therefore can demand higher rents in equilibrium.

For simplicity, I assume from now on that there is only one voter. She can be thought off as representing the decisive median voter.[8] Her expected utility is given by:

$$U_m = E_0 \sum_{t=0}^{\infty} \beta^t (-(p_t - b_t)^2 + R - r_t), \tag{2'}$$

where b_t is her bliss point in period t. This bliss point is now a random variable that is only determined after the parties have announced their policy positions for period t. The value of b_t is distributed identically and independently of past bliss points. The expected utility function of the parties $j = y, x$ is identical to the expected utility function in Section 2:

$$U_p^j = E_0 \sum_{t=0}^{\infty} \beta^t r_t^j. \tag{1}$$

Let there be K distinct possible policy bliss points b_k of the voter, all within the policy space $[0, 1]$. They are ordered such that $b_k < b_l$ if and only if $k < l$. Let q_k be the probability that the median voter of period t has the bliss point $b_t = b_k$. By assumption, this probability is the same in every period t. Then, $F(b_k) = \sum_{l=1}^{l=k} q_l$ is the cumulative distribution function of b_k. I define:

$$b_m = \min_{k \in K} \ F(b_k) \ s.t. \ F(b_k) \geq 0.5, \tag{6}$$

[8] This simplifies the notation because it rules out the possibility that the identity of the median voter changes between periods without changing the results significantly.

so that b_m is now the median of the possible bliss points of the voter.[9] Moreover, I define for the case $K \geq 2$:

$$b^*(b_k) = \begin{cases} b_2 & \text{for } k = 1 \\ b_{K-1} & \text{for } k = K \\ \left.\begin{matrix} b_{k-1} \text{ if } F(b_{k-1}) \geq 1 - F(b_k) \\ b_{k+1} \text{ if } F(b_{k-1}) < 1 - F(b_k) \end{matrix}\right\} & \text{for } k \in \{2, 3, .., K-1\} \end{cases} \tag{7}$$

$$\pi^* = \begin{cases} F(b_m) \text{ if } b^*(b_m) > b_m \\ 1 - F(b^*(b_{m-1})) \text{ if } b^*(b_m) < b_m \end{cases} \tag{8}$$

$$r^* = \frac{((1 - 2\pi^*)\beta + 1)}{(1 - \pi^*)\beta + 1} R \tag{9}$$

If $K = 1$, then $b^* = b_m = b_1$ and $\pi^* = 1$.

Proposition 5. *An equilibrium of the game entails the following strategies: The parties play:*

$$\left.\begin{aligned} p^I_{t+1} &= b_m, \\ r_t &= r^*, \\ p^O_{t+1} &= \begin{cases} b^*(p^I_{t+1}) \text{ if } r_t \leq r^* \\ p^O_{t+1} = p^I_{t+1} \text{ if } r_t > r^* \end{cases} \end{aligned}\right. \quad \textit{in all } t. \tag{10}$$

[9] Naturally, b_m was also the median of the possible median bliss points in Section 2, where the distribution of the median voter was degenerate. Therefore, there is no need to change the notation.

The voter plays:

$$v_t = \begin{cases} y \text{ if } (p^y_{t+1} - b_{t+1})^2 - (p^x_{t+1} - b_{t+1})^2 < 0 \\ x \text{ if } (p^y_{t+1} - b_{t+1})^2 - (p^x_{t+1} - b_{t+1})^2 > 0 \\ I_t \text{ if } (p^I_{t+1} - b_{t+1})^2 - (p^O_{t+1} - b_{t+1})^2 = 0 \text{ and } r_t \leq r^* \\ O_t \text{ if } (p^I_{t+1} - b_{t+1})^2 - (p^O_{t+1} - b_{t+1})^2 = 0 \text{ and } r_t > r^* \end{cases} \quad \text{in all } t. \tag{11}$$

In every period, the probability that the incumbent party wins is π^*. If the incumbent party wins, b_m is implemented, if the incumbent party loses, $b^*(b_m)$ is implemented. If $K = 1$, the expected utility of the voter is $\frac{R-r^*}{1-\beta}$ because there is no uncertainty and her favorite policy is always implemented. In the case of $K \geq 2$, the expected utility of the voter is:

$$u_{rv} = \begin{cases} \begin{array}{c} \sum_{t=0}^{\infty}(\sum_{k=1}^{m-1} q_k \beta^t(-(b_{m-1} - b_k)^2 + R - r^*) \\ + \sum_{k=m}^{K} q_k \beta^t(-(b_m - b_k)^2 + R - r^*)) \end{array} & \text{if } b^* = b_{m-1} \\ \\ \begin{array}{c} \sum_{t=0}^{\infty}(\sum_{k=1}^{m} q_k \beta^t(-(b_m - b_k)^2 + R - r^*) \\ + \sum_{k=m+1}^{K} q_k \beta^t(-(b_{m+1} - b_k)^2 + R - r^*)) \end{array} & \text{if } b^* = b_{m+1} \end{cases} \tag{12}$$

Proof. See the Appendix ■

The best position any incumbent party can choose is the median of the possible positions of the voter. The intuition is straightforward. The incumbent party must choose its position first. Because the incumbent party will not be re-elected if the voter prefers its opponent even if it constrains itself with respect to rent extraction, the best the incumbent party can do is to choose its position so that the opposition can only achieve less than 50% of the votes. The incumbent party can achieve this by announcing the median bliss point as policy position. The opposition party will then choose a position as close to the median position as possible to ensure the victory whenever the bliss point of the median voter is on the same side of the median position. It chooses the side of the median where this probability is the largest. Therefore, the most useful measure of uncertainty about the election outcome is given by:

$$\pi^* = min(F(b_m), 1 - F(b_{m-1})).$$

It turns out that the larger π^*, the greater is the control of the voter over rent extraction by the parties. In the special case of no uncertainty about the bliss point of the voter, $\pi^* = 1$, an incumbent party that does not extract too high rent payments is re-elected with certainty. The results of Section 2 are confirmed as a special case of the generalized model.

Restricting the strategies of parties to be history-independent and identical, and letting the strategy of the voter only depend on the current policy offers and the last rent payment seems intuitively plausible as the model is completely symmetric. Under these conditions, the equilibrium stated in Proposition 5 is the one with the lowest rent payment that the voter can achieve, as is shown by the following corollary:

Corollary 6. *There is no equilibrium with a rent $r_t < r^*$ if the voter's strategy only depends on rent extraction in the last period and the policy positions of the parties (that is $v_t(h_t, p^y_{t+1}, p^x_{t+1}, r_t) = v_t(r_t, I_t, p_{y,t+1}, p_{x,t+1})$), when both parties play identical history-independent strategies (that is $p^I_{t+1}(h_t) = p_I,\ r_t(h_t) = r$ and $p^O_{t+1}(h_t, r_t, p^I_{t+1}) = p^O_{t+1}(r_t, p^I_{t+1})$.*

Proof. See the Appendix ■

From the voter's perspective, it would potentially enhance expected welfare if the candidates did not choose policy positions the way they actually do. Competition drives parties "almost" to convergence, but this is not necessarily in the voter's interest from an ex ante perspective. The reason is that if she has rather extreme preferences, both parties will offer a policy position that is rather centrist and she will suffer from lack of choice. The expected per-period utility of the voter before her preferences are revealed would increase if only one party chose a centrist position but the other an extreme one.

Bernhardt et al. (2009) show that such a lack of choice in policy provided by parties uncertain about the position of the median bliss point can make voters worse off. This may not be all that surprising in the light of the literature on spatial competition (Hotelling, 1929).

Equilibrium rent extraction r^* is decreasing in π^*. The intuition is straightforward: The larger π^* is, the more likely it is that the incumbent party remains in office if it does not deviate. In addition, the incumbent party is also less likely to regain office once it loses it. Therefore, the rent that has to be paid to make the incumbent party willing to forgo the maximum rent R in favor of re-election decreases.

The voter is essentially playing the same lexicographic strategy as in the model without uncertainty in Section 2. However, she has to accept higher rent payments because there is no longer any guarantee that the incumbent party will be re-elected. Moreover, an incumbent party which loses office can regain office later, which also makes losing power less costly.

3.1. Two interesting cases

There are two interesting cases with intuitive results. First, there is the case of $\pi = 1$, which can only occur if $K = 1$; otherwise there would always be at least a small probability that the incumbent party loses. In this case, we are back to the set-up of Section 2 and it indeed turns out that $r^* = (1-\beta)R = \bar{r}$. The incumbent party once more faces the choice between either remaining in office forever or stealing R once.

The second case is $\pi = 0.5$, which happens if and only if $F(b_m) = 0.5$. Because the probability that $b_t \leq b_m$ is exactly equal to the probability that $b_t > b_m$, incumbents have no possibility of increasing their chances of re-election to more than 50% even when they accept limited rent extraction. This is also what would happen if there were a continuous function of possible positions of the median voter. In this case, $r^* = \frac{1}{0.5\beta+1}R$ or $(1+0.5\beta)r^* = R$. The reason is that when the incumbent party does extract the maximum amount of rent R, it loses $0.5\beta r^*$ in the next period, but from then onwards, it has the same chance of being the incumbent party (50%) that it would have without any deviation from its strategy.

3.2. Discussion of the timing assumption

Without the assumption of the incumbent party moving first, a lexicographic strategy by the voters can only be consistent with an equilibrium if the parties randomize over policy. The reason is that the incumbent party would always like to take the same position as the opposition and win with certainty, and therefore the opposition must randomize over its position. A somewhat similar model has been solved by Aragones and Palfrey (2002). In their set-up, voters are not indifferent because candidates differ in an exogenously given policy attribute, so that the candidate who is preferred in this dimension wins if he can take the same policy position as the other candidate. It should therefore be possible to solve an alternative model without the timing assumption and derive similar results with respect to accountability. However, finding optimal mixed strategies is not the focus of this paper.

4. Parties with policy preferences

In this section, I go back to a world without uncertainty. The model is the same as in Section 2 with the one difference that the expected utility of the parties $j \in \{x, y\}$ is from now on:

$$U_j = E_o \sum_{t=0}^{\infty} \beta^t (r_{t,j} - (p_t - b_j)^2), \qquad (1')$$

with $b_x < b_m < b_y$. In other words, The parties' utility is now influenced by the policy that is implemented and party j is better off whenever policy is close to its bliss point b_j with $j \in \{x, y\}$. It is easy to check that giving parties policy preferences does not change the fact that the strategies given in Proposition 1 continue to constitute an equilibrium because by deviating and committing to a different policy than that preferred by the median voter, a party can never win the elections given the strategies of the other players.

If parties have policy preferences of their own, the question arises how a party is able to commit to a policy in advance, but not to restrictions in rent seeking.[10] As indicated before, a plausible answer is that parties commit to

[10] The fact that partisan parties potentially have a dynamic inconsistency problem with

certain policies by running with certain candidates who are known to have preferences for the policy. If such a party wins an election, its candidate has no incentive to deviate from his preferred policy (although the average party member might still suffer from disutility from a deviation from his or her own policy bliss point).

However, with parties with policy preferences, there are now equilibria with lower rent payments that are not possible if the principle-agent problem and the electoral competition problem are treated separately. The reason is that a party can now be punished by allowing the other party to win with a position different from the bliss point of the median voter. The details can be found in the appendix, here I only provide a brief summary of the results and the intuition behind them. To demonstrate this point three Examples that build on each other that are provided in the Appendix. In all the examples all voters always vote sincerely, which means that they vote for the party whose victory maximizes their intertemporal utility function. Example 1 is a special case of lexicographic voting. It is identical to the equilibrium given in Proposition 1 in Section 2 with the one difference that the incumbent is allowed to take the maximum amount of rents and nonetheless reelected whenever the voters are indifferent with respect to policy. Strategies are identical, just $\bar{r} = R$ instead of $\bar{r} = (1 - \beta)R$. This example constitutes an equilibrium because the voters have no reason to punish the incumbent party in spite of the fact that it extracts the maximum rent level because the opposition party does not behave better once in office.

their policy announcements was first pointed out by Alesina (1988) .

Example 1 is not very interesting in itself, but the threat to revert to it gives parties the possibility to win with a position that is different from the bliss point of the median voter b_m as is shown in Example 2. The idea is that the median voter will accept deviations from the median bliss points if she knows that if she does not the parties will punish her with the high rent equilibrium given in Example 1.

Finally, in Example 3 it is shown that the threat with the equilibrium given in example 2 makes it possible for voters to reach an equilibrium with a per period rent that is smaller than $\bar{r} = (1-\beta)R$. As was shown in Corollary 2, there is no such equilibrium as long as policy does not enter the parties' utility functions. The reason that this is different with ideological parties is that voters can now punish parties that do not comply with policies that they dislike. Therefore, losing office becomes more costly and lower rent payments have to be accepted. In the example, it is assumed that the parameter values are such that parties refrain from any rent seeking in equilibrium.

The examples show that by separating backward-looking and forward-looking motives, some interesting strategic possibilities for voters might be overlooked. Voters are able to decrease rent payments further from $\bar{r}$ without accepting a more ideological policy by threatening not only to vote for the opposition party, but to do so even when it dose not offer the median voter's policy bliss point. This punishment is only credible because the voters end up in an even worse situation if they do not implement it.

Example 3 demands a larger degree of coordination among voters than what seems plausible. Moreover, even if Example 1 constitutes an equilibrium, it is not clear why voters who are as sophisticated as in Example

3 would not manage to switch to the more attractive equilibrium given in Proposition 1 instead once they are in the "bad" equilibrium of Example 1. There is no intuition how they could coordinate and commit to punish themselves for not punishing a party that deviates from the equilibrium given in Example 3. However, the analysis of this Section nonetheless indicates that modeling accountability issues without any consideration of policy in models with partisan parties that derive utility from implemented policy could potentially lead to wrong conclusions.

5. Conclusion

It is surprising that until now, there seem to have been no attempts to combine models of retrospective voting with aspects of Downsian competition. My model shows that forward-looking and backward-looking motives can be reconciled in a single model. This should be considered in future empirical research because so far, the question seems to have been if voters vote retrospectively or prospectively. If there is not necessarily a contradiction, some empirical results might have to be re-evaluated.

As long as there is certainty about the position of the median voter, I find that on the policy dimension where commitment is possible, the usual median voter results apply, while rent extraction by politicians is limited to the same degree as in a standard model without a policy dimension. If there is uncertainty about the position of the median voter, voters cannot limit rent extraction to the same degree as in the certainty case, but accountability is not completely lost either. The reason is that even when the incumbent party complies with the voters demands for limited rent extraction, it will still lose office if the opposition party commits to a policy that is more attractive to the majority of voters. Models of political accountability can explain the often observed incumbency advantage, as is pointed out by Austen-Smith and Banks (1989). Models in the Downsian tradition, on the other hand, provide no explanation for an incumbency advantage. My basic model in Section 2 leads to the implausible result that in equilibrium, the incumbent party is always re-elected. In the extended model with uncertainty about the exact position of the median voter in Section 3, I find that the incumbent party always has a chance exceeding 50% of winning the elections and that its advantage depends on a measure of uncertainty about the preferences of the median voter. This result is consistent with election results in many countries. Incumbent parties win more often than not, but their victory is far from certain.

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Appendix A. Proofs Section 3

Proof. [Proof of Proposition 5] The single deviation principle states that it is sufficient to show that no player can increase his expected utility by a single deviation to prove that the given strategies constitute a subgame perfect Nash Equilibrium. The single deviation principle applies to an infinite game when the overall payoffs are a discounted sum of the per-period payoffs that are uniformly bounded. This applies to the game in Section 3.[11]

First, I show that both the incumbent party and the opposition party maximize their chances of winning the election if they follow their given strategies. In the case of $r_t > r^*$, the opposition party wins with certainty by taking the same policy position as the incumbent party $p^O_{t+1} = p^I_{t+1}$ and wins office with certainty. In the case of $r_t \leq r^*$, if $p^O_{t+1} = p^I_{t+1}$ and therefore $(p^I_{t+1} - b_{t+1})^2 - (p^O_{t+1} - b_{t+1})^2 = 0$, the opposition loses with certainty. If $-(b_k - b_{t+1})^2 + (b_{k-1} - b_{t+1})^2 < 0$, then $(b_k - b_{t+1})^2 - (b_l - b_{t+1})^2 < 0$ for all $l \leq k - 1$. Therefore, if $p^I_{t+1} = b_k$ and $r_t \leq r^*$, the opposition is at least as likely to win with $p^O_{t+1} = b_{k-1}$ as with any $p^O_{t+1} < b_{k-1}$. Similarly, if $-(b_k - b_{t+1})^2 + (b_{k+1} - b_{t+1})^2 < 0$, then $(b_k - b_{t+1})^2 - (b_l - b_{t+1})^2 < 0$ for all $l \geq k + 1$ and therefore the opposition is at least as likely to win with $p^O_{t+1} = b_{k+1}$ as with any $p^O_{t+1} > b_{k+1}$. It follows that either $p^O_{t+1} = b_{k+1}$ or $p^O_{t+1} = b_{k-1}$ maximizes the probability of the opposition winning against $p^I_{t+1} = b_k$. Therefore, from the definition of $b^*(b_k)$, a policy that maximizes the probability of the opposition party winning is given by $p^O_{t+1} =$

[11] See Fudenberg and Tirole (1991) for a formal statement of the single deviation principle.

$b^*(p^I_{t+1})$. It remains to be shown that $p^I_{t+1} = b_m$ maximizes the prospects of the incumbent party given the reply $b^*(p^I_{t+1})$. From its definition and the voter's strategy, π^* gives the probability that the incumbent party wins when $r_t \leq r^*$, $p^I_{t+1} = b_m$ and $p^O_{t+1} = b^*(p^I_{t+1})$. From the definition of b_m, $F(b_{m-1}) < 0.5$ and $1 - F(b_m) \leq 0.5$. Therefore, $\pi^* \geq 0.5$. If $p^I_{t+1} \neq b_m$, the probability of winning for the opposition by choosing b_m itself is at least 0.5 and therefore, the probability that the opposition wins with $p^O_{t+1} = b^*(p^I_{t+1})$ for $p^I_{t+1} \neq b_m$ cannot be smaller than 0.5. Hence, $p^I_{t+1} = b_m$ maximizes the chances of the incumbent party remaining in power given the strategies of the other players, and π^* gives the probability of reelecting the incumbent party in the given equilibrium.

Given the strategies of the other players, the voter will encounter the two policy offers b_m and $b^*(b_m)$ and the rent extraction r^* in all future periods. Therefore, maximizing the current period utility, as she does by voting for the party she prefers if she is not indifferent, is maximizing her expected utility.

Let V denote the value of being in office and W denote the value of being out of office given the strategies. The present expected value of being out of office is determined by the value of being in office and the equilibrium probability of winning the next elections, $1 - \pi^*$:

$$W = (1 - \pi^*)\beta V + \pi^* \beta W \implies W = \frac{\beta(1 - \pi^*)V}{1 - \pi^* \beta}. \tag{A.1}$$

It follows that $W < V$ and being in office is better than being out of office. From this, it directly follows that deviating once from the strategy cannot make the opposition that maximizes its chances of becoming the next incumbent party better off, because a single deviation cannot change the future values of being in and out of office. Therefore, maximizing the probability of being incumbent and achieving V instead of W in the next period is optimal. The value of being the incumbent party depends on the equilibrium rent extraction r^* and the probability of being in and out off office in the next period is:

$$\begin{aligned} V &= r^* + \beta\pi^* V + \beta(1-\pi^*)W = r^* + \beta\pi^* V + \beta(1-\pi^*)\frac{\beta(1-\pi^*)V}{1-\pi^*\beta} \qquad (A.2)\\ &= \frac{((1-2\pi^*)\beta+1)}{(1-\pi^*)\beta+1}R + \beta\pi^* V + \beta(1-\pi^*)\frac{\beta(1-\pi^*)V}{1-\pi^*\beta} \\ &\Longrightarrow \quad V = \frac{\pi^*\beta - 1}{\pi^*\beta + \beta^2 - \pi^*\beta^2 - 1}R. \end{aligned}$$

Given that the future values of being an incumbent party and in opposition cannot be changed by a one-time deviation, it is clear that the incumbent party should maximize the rent payment for a given probability of re-election. Therefore, any rent payment $r_t < r^*$ cannot make the incumbent party better off, because it decreases the rent without increasing the probability of re-election. From the fact that the incumbent party loses the election with certainty if $r_t > r^*$ independently of its chosen policy position, the only deviation that needs to be checked is $r_t = R$ in combination with any arbitrary policy position. The reason is that if the party were to be better off by extracting any rent r such that $r^* < r < R$, it must also be better off extracting R. The expected value of deviating in this way and then being in opposition in the next period is given by the sum of R and the present value in opposition in the next period:

$$
\begin{aligned}
R + \beta W &= R + \beta \frac{\beta(1-\pi^*)V}{1-\pi^*\beta} \\
&= R + \beta \frac{\beta(1-\pi^*)}{1-\pi^*\beta} \frac{\pi^*\beta - 1}{\pi^*\beta + \beta^2 - \pi^*\beta^2 - 1} R \\
&= \frac{\pi^*\beta - 1}{\pi^*\beta + \beta^2 - \pi^*\beta^2 - 1} R = V.
\end{aligned} \tag{A.3}
$$

This gives the party the same utility V as following the strategy given in Proposition 5. Therefore, the incumbent party has no reason to deviate. None of the players is better off with a one time deviation and therefore, the strategies given in Proposition 5 constitute a subgame perfect Nash Equilibrium. ■

Proof. [Proof of Corollary 6] Because $p^I_{t+1}(h_t) = p_I$, $r_t(h_t) = r$ and $p^O_{t+1}(h_t, r_t, p^I_{t+1})$
$= p^O_{t+1}(r_t, p^I_{t+1}) = p^O_{t+1}(r, p_I)$ for all t, the voter's decision can neither change her future policy choice nor future rent extraction. Therefore, in equilibrium, she votes for the party that offers the policy that is closest to her bliss point. Only if both parties offer the same policy position, is voting for either party consistent with an equilibrium. This gives the opposition party the possibility of being elected with a probability of at least $1 - \pi^*$ for any rent payment r_t and the policy position of the incumbent party by offering $p_{O,t+1} = b^*(p_{I,t+1})$. The opposition party maximizes its utility by maximizing the probability of being voted into office since being in office must be better than being out of office. Only in office is any rent extraction possible and the history-independence of the strategies implies that future rents are given by some constant level r. Let $r^{\min}$ be the smallest rent payment that is consistent with an equilibrium. The value

of being in office is given by $V(r^{\min}, \pi) = \frac{(1-\pi\beta)r^{\min}}{(1-\pi\beta)^2-\beta^2(1-\pi)^2}$, where π is the probability of re-election of the incumbent party. V is increasing in π, and the maximum π that is consistent with equilibrium is π^*. Therefore, the maximum V that is consistent with $r^{\min}$ and an equilibrium is given by $V(r_{\min}, \pi^*) = \frac{(1-\pi^*\beta)r^{\min}}{(1-\pi^*\beta)^2-\beta^2(1-\pi^*)^2}$. The second condition that must hold is $R \leq r^{\min} + \beta\pi V(r^{\min}, \pi^*) + \beta(1-\pi)\frac{\beta(1-\pi)r^{\min}V(,\pi^*)}{1-\pi\beta}$, because otherwise the incumbent party would be better off taking R and losing office. This condition can only hold if $r^{\min} \geq r^*$, hence it follows that $r^* = r^{\min}$. ■

Appendix B. Examples illustrating Section 4

Example 1 with maximum rents and median policy constitutes an equilibrium because the voters have no reason to punish the incumbent party in spite of the fact that it extracts the maximum rent level. The reason ist that the opposition does not take less rents when in office.

Example 1 (Subgame perfect equilbrum with median policy and high rents). *The parties play:*

$$
\begin{aligned}
p^I_{t+1} &= b_m \textit{ in all } t, \\
p^O_{t+1} &= \arg\min_{p^O_{t+1}} (p^O_{t+1} - b_{O_t})^2 \textit{ s.t. } (p^O_{t+1} - b_m)^2 \leq (p^I_{t+1} - b_m)^2 \textit{ in all } t, \\
r_t &= R \textit{ in all } t.
\end{aligned}
$$

The voters play:

$$v_t^i = \begin{cases} I_t \text{ if } p_{t+1}^I = b_m \text{ and } (p_{t+1}^I - b^i)^2 - (p_{t+1}^O - b^i)^2 \leq 0 \\ O_t \text{ if } p_{t+1}^I = b_m \text{ and } (p_{t+1}^I - b^i)^2 - (p_{t+1}^O - b^i)^2 > 0 \\ I_t \text{ if } p_{t+1}^I \neq b_m \text{ and } (p_{t+1}^I - b^i)^2 - (p_{t+1}^O - b^i)^2 < 0 \\ O_t \text{ if } p_{t+1}^I \neq b_m \text{ and } (p_{t+1}^I - b^i)^2 - (p_{t+1}^O - b^i)^2 \geq 0 \end{cases} \text{ in all } t.$$

And therefore in equilibrium:

$$\begin{aligned} I_t &= I_0 \text{ in all } t, \\ p_t &= b_m \text{ in all } t \geq 1, \\ r_r &= R \text{ in all } t. \end{aligned}$$

I will show below that voters vote sincerely in the sense of voting always for a party maximizing their intertemporal utility function given equilibrium play in future periods. In an election with only two candidates voting sincerely is always optimal. The tie-breaking rules that apply in case voters are indifferent ensure subgame perfection by giving the opposition party a best response in case the incumbent party deviates from offering the median voter's bliss point as policy. Given the response of the opposition party and the strategy of the voters, the incumbent party loses office with any deviation from the median policy position and then never regains it. Moreover, in this case the opposition party wins with a position that is worse for the incumbent than the median policy position. On the other hand, taking less than the maximum amount R would also make the incumbent party worse off. Therefore, the incumbent party is not better off with any deviation. As long as the incumbent party does not deviate from $p_{t+1}^I = b_m$*, the opposition party loses with any policy. Therefore,* $p_{t+1}^O(b_m) = \arg\min_{p_{t+1}^O} (p_{t+1}^O - b_{O_t})^2$ *s.t.* $(p_{t+1}^O - b_m)^2 \leq 0 = b_m$

is a best response in this case. If the incumbent party deviates on the policy dimension, the opposition party chooses the policy that is closest to its own bliss point but makes the decisive median voter at least indifferent and therefore wins the election. Because in equilibrium future play always leads to $p = b_m$ *and* $r = R$*, maximizing her intertemporal utility function means for voter* i *that she votes for the incumbent party whenever* $(p^I_{t+1} - b^i)^2 < (p^O_{t+1} - b^i)^2$ *and for the opposition party whenever* $(p^I_{t+1} - b^i)^2 > (p^O_{t+1} - b^i)^2$*. Thus, given the equilibrium strategy of the voters, they vote sincerely and the party with the support of the median voter wins the election. Moreover, because voters vote sincerely and the opposition party plays a best response in case the incumbent party deviates, the equilibrium is subgame perfect.*

Building on the fact that there is an equilibrium with high rents, an equilibrium with a party deviating from the median position becomes possible (under further assumptions about the values of the parameters of the model) because voters can be "punished" with high rent payments if they do not accept the deviation.

Example 2 (Deviation from median policy subgame perfect equilibrium). *I Assume that* $(b_m - b_j)^2 \leq R \leq \frac{\beta}{1-\beta}(b_m - b_j)^2$ *for* $j \in \{x, y\}$*. The game begins in Phase* 3.[12]*. If the incumbent party deviates in Phase* 3 *from* $p^I_{t+1} = b_{I_t}$ *and* $r_t = 0$*, Phase* 4 *begins. In this case, the opposition party chooses its position and the voters cast their votes in Phase* 4 *and then Phase* 5 *begins before the new period starts. Phase* 5 *also begins if the incumbent party is not*

[12]Because example 2 is identical with the subgame that begins once Phase 3 is reached in Example 3 this notation facilitates the discussion of Example 3 that follows.

reelected in Phase 3. Once Phase 5 is reached, the game stays in that phase forever.

The parties play:

$$
\begin{aligned}
p_{t+1}^{I} &= b_{I_t} \text{ in Phase } 3, \\
p_{t+1}^{I} &= b_m \text{ in Phase } 5, \\
p_{t+1}^{O} &= b_m \text{ in Phase } 3, \\
p_{t+1}^{O} &= \arg\min_{p_{t+1}^{O}} (p_{t+1}^{O} - b_{O_t})^2 \text{ s.t. } (p_{t+1}^{O} - b_m)^2 \leq (p_{t+1}^{I} - b_m)^2 \text{ in Phase 4 and Phase } 5, \\
r_t &= 0 \text{ in Phase } 3, \\
r_t &= R \text{ in Phase } 5.
\end{aligned}
$$

The voters play:

$$
v_t^i = \begin{cases} I_t \text{ if } (p_{t+1}^{I} - b^i)^2 - (p_{t+1}^{O} - b^i)^2 + \frac{\beta}{1-\beta}\left((b_{I_t} - b^i)^2 - (b_m - b^i)^2\right) \leq \frac{R}{1-\beta} \\ O_t \text{ if } (p_{t+1}^{I} - b^i)^2 - (p_{t+1}^{O} - b^i)^2 + \frac{\beta}{1-\beta}\left((b_{I_t} - b^i)^2 - (b_m - b^i)^2\right) > \frac{R}{1-\beta} \end{cases} \text{ in Phase } 3,
$$

$$
v_t^i = \begin{cases} I_t \text{ if } (p_{t+1}^{I} - b^i)^2 - (p_{t+1}^{O} - b^i)^2 < 0 \\ O_t \text{ if } (p_{t+1}^{I} - b^i)^2 - (p_{t+1}^{O} - b^i)^2 \geq 0 \end{cases} \text{ in Phase } 4,
$$

$$
v_t^i = \begin{cases} I_t \text{ if } p_{t+1}^{I} = b_m \text{ and } (p_{t+1}^{I} - b^i)^2 - (p_{t+1}^{O} - b^i)^2 \leq 0 \\ O_t \text{ if } p_{t+1}^{I} = b_m \text{ and } (p_{t+1}^{I} - b^i)^2 - (p_{t+1}^{O} - b^i)^2 > 0 \\ I_t \text{ if } p_{t+1}^{I} \neq b_m \text{ and } (p_{t+1}^{I} - b^i)^2 - (p_{t+1}^{O} - b^i)^2 < 0 \\ O_t \text{ if } p_{t+1}^{I} \neq b_m \text{ and } (p_{t+1}^{I} - b^i)^2 - (p_{t+1}^{O} - b^i)^2 \geq 0 \end{cases} \text{ in Phase } 5.
$$

And therefore in equilibrium:

$$
\begin{aligned}
I_t &= I_0 \text{ in all } t, \\
p_t &= b_{I_0} \text{ in all } t \geq 1, \\
r_r &= 0 \text{ in all } t.
\end{aligned}
$$

The equilibrium in Example 1 *is identical to the subgame that begins in Phase* 5 *of Example* 2*. The subgame in Phase* 5 *is used to "punish" either the voters for not reelecting the incumbent party in Phase* 3 *or to "punish" the incumbent party in Phase* 3 *if it keeps a positive rent. It is easy to check that given voters and parties strategies the incumbent party is reelected if it does not deviate in Phase* 3 *and loses office if it does deviate and the game enters Phase* 4*. Thus, as long as the players follow their equilibrium strategies, the game stays in Phase* 3 *forever and the subgame that begins in Phase* 5 *is never reached. In Phase* 5*, the incumbent party is reelected as long as it does not deviate. If incumbent party deviates in Phase* 5 *the opposition party wins if the voters do not deviate from their equilibrium strategies. We know from Example* 1 *that the subgame that begins in Phase* 5 *is a subgame perfect equilibrium. It remains to show that none of the players would be better off deviating in Phase* 3 *or* 4*. First, I show that the voters always vote sincerely in the sense of voting for the party whose victory maximizes their intertemporal utility function assuming that there are no deviations from equilibrium play in future periods. Given that there are only two competing parties, sincere voting is utility maximizing. Once the game reaches Phase* 4*, voters know that Phase* 5 *begins before the next elections take place. Therefore, their intertemporal utility depends only on the policies positions the parties* *have chosen for the following period given that there will not be any further deviations from equilibrium play. In Phase* 3*, the elections decide not only the policy in the next period but also future future policies and rents change with the identity of the winning party. The reason is that if the opposition party is elected the game moves to Phase* 5*. Given* p^I_{t+1} *and* p^O_{t+1}*, the difference in intertemporal utility for voter* i *between the incumbent and the opposition party winning the elections in the next period is therefore:*

$$\beta\left(-(p^I_{t+1}-b^i)^2+(p^O_{t+1}-b^i)^2+\frac{\beta}{1-\beta}\left((b_{I_t}-b^i)^2-(b_m-b^i)^2\right)+\frac{R}{1-\beta}\right).$$

Where $\beta(-(p^I_{t+1}-b^i)^2+(p^O_{t+1}-b^i)^2)$ *is the difference in utility from policy in the next period and* $\frac{\beta^2}{1-\beta}\left((b_{I_t}-b^i)^2-(b_m-b^i)^2\right)$ *the difference in utility from policy in all later periods given that there is no further deviation from equilibrium play. If the incumbent party is reelected, it continues to implement its preferred policy* b_{I_t} *forever. If the incumbent party loses, Phase* 5 *begins and* b_m *will be implemented in all future periods. In addition, utility from public goods can only be expected if the game stays in Phase* 3*. Therefore,* $\frac{R}{1-\beta}$ *is the difference in utility from public good provision between reelecting the incumbent party and electing the opposition party. Thus, given their equilibrium voting strategy, the voters vote sincerely. If the game stays in Phase* 3*, we know that the incumbent party has played* b_m*. It follows that the difference between in utility for the median voter between the incumbent party and the opposition party must be at least* $-\frac{1}{1-\beta}(b_{I_t}-b_m)^2+\frac{R}{1-\beta}$*. This term is positive by the assumption that* $(b_m-b_j)^2\leq R$ *for* $j\in\{x,y\}$*. Therefore, either every voter with policy preferences to the left or to the right of the median voter is also better off and thus the majority of voters. From sincere voting follows that, as long as the incumbent party does not deviate from its equilibrium* *policy in Phase 3, it is reelected and the party in opposition can never win office with any deviation. Because* $R \leq \frac{\beta}{1-\beta}(b_m - b_j)^2$ *holds by assumption, the incumbent in Phase 3 is (weakly) better off with no rents at all then with once stealing R but then having to accept median policy instead of his policy bliss point forever.*

Building on Example 2, I can now show that there is also an equilibrium without any rent payments and median policy. This is the case because if the incumbent party deviates by appropriating positive rents, it can be punished with policies that makes it worse off than the median position by allowing the opposition to win with its own bliss point instead of the median position as in Example 2.

Example 3 (Subgame perfect equilbrum with median policy and no rents). *I assume that* $R \leq \frac{\beta}{1-\beta}((b_y - b_x)^2 - (b_j - b_m)^2)$ *and continue to assume that* $(b_m - b_j)^2 \leq R \leq \frac{\beta}{1-\beta}(b_m - b_j)^2$ *for* $j \in \{x, y\}$. *The game begins in Phase 1. As long as the incumbent does not deviate the play remains in Phase 1. If an incumbent in Phase 1 deviates from* $p^I_{t+1} = b_m$ *and* $r_t = 0$ *the play moves to Phase 2. In Phase 2 the opposition party chooses its position and the voters cast their votes. Then Phase 3 begins if the incumbent loses the elections and Phase 5 if the incumbent wins. The game moves from Phase 3 to Phase 4 whenever the incumbent party in Phase 3 deviates from* $p^I_{t+1} = b_{I_t}$ *and* $r_t = 0$. *In Phase 4, the opposition party chooses its position and the voters cast their votes. Then Phase 5 begins. Phase 5 also begins if the incumbent party is not reelected in Phase 3. Once Phase 5 is reached, the game stays in that Phase forever. The following strategies constitute a subgame perfect equilibrium:*

The parties play:

$$
\begin{aligned}
p^I_{t+1} &= b_m \text{ in phases 1 and 5,} \\
p^I_{t+1} &= b_{I_t} \text{ in Phase 3,} \\
p^O_{t+1} &= b_m \text{ in Phase 1 and 3.} \\
p^O_{t+1} &= b_{O_t} \text{ in Phase 2,} \\
p^O_{t+1} &= \arg\min_{p^O_{t+1}} (p^O_{t+1} - b_{O_t})^2 s.t. (p^O_{t+1} - b_m)^2 \leq (p^I_{t+1} - b_m)^2, \text{ in Phase 4 and Phase 5,} \\
r_t &= 0 \text{ in Phases } 1, 2 \text{ and 3,} \\
r_t &= R \text{ in Phase 4 and Phase 5.}
\end{aligned}
$$

The voters play:

$$
v^i_t = \begin{cases} I_t \text{ if } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 \leq 0 \\ O_t \text{ if } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 > 0 \end{cases} \text{ in Phase 1,}
$$

$$
v^i_t = \begin{cases} O_t \text{ if } (p^O_{t+1} - b^i)^2 - (p^I_{t+1} - b^i)^2 + \frac{\beta}{1-\beta}(b_{O_t} - b^i)^2 - (b_m - b^i)^2 \leq \frac{R}{1-\beta} \\ I_t \text{ if } (p^O_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 + \frac{\beta}{1-\beta}(b_{O_t} - b^i)^2 - (b_m - b^i)^2 > \frac{R}{1-\beta} \end{cases} \text{ in Phase 2,}
$$

$$
v^i_t = \begin{cases} I_t \text{ if } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 + \frac{\beta}{1-\beta}(b_{I_t} - b^i)^2 - (b_m - b^i)^2 \leq \frac{R}{1-\beta} \\ O_t \text{ if } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 + \frac{\beta}{1-\beta}(b_{I_t} - b^i)^2 - (b_m - b^i)^2 > \frac{R}{1-\beta} \end{cases} \text{ in Phase 3,}
$$

$$
v^i_t = \begin{cases} I_t \text{ if } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 < 0 \\ O_t \text{ if } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 \geq 0 \end{cases} \text{ in Phase 4,}
$$

$$
v^i_t = \begin{cases} I_t \text{ if } p^I_{t+1} = b_m \text{ and } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 \leq 0 \\ O_t \text{ if } p^I_{t+1} = b_m \text{ and } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 > 0 \\ I_t \text{ if } p^I_{t+1} \neq b_m \text{ and } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 < 0 \\ O_t \text{ if } p^I_{t+1} \neq b_m \text{ and } (p^I_{t+1} - b^i)^2 - (p^O_{t+1} - b^i)^2 \geq 0 \end{cases} \text{ in Phase 5.}
$$

And therefore in equilibrium:

$$
\begin{aligned}
I_t &= I_0 \text{ in all } t, \\
p_t &= b_m \text{ in all } t \geq 1, \\
r_r &= 0 \text{ in all } t.
\end{aligned}
$$

The subgame that begins in Phase 3 is identical to Example 2 that constitutes a subgame perfect equilibrium. It remains to show that none of the players prefers to deviate in Phase 1 or 2 either on or off the equilibrium path. Given voters and parties strategies the incumbent party is reelected if it does not deviate and loses office if it deviates in Phase 1. When they vote in Phase 1, voters know that their vote will not influence policy positions rent levels that the parties choose in future periods in equilibrium. Therefore, the difference in intertemporal utility between the incumbent party and the opposition party winning the elections is given by $\beta(-(b_m - b^i)^2 + (p^O_{t+1} - b^i))$. This is always positive for the median voter and thus the majority of voters. It follows that voters are voting sincerely in Phase 1 and that the incumbent party stays in office if it does not deviate in Phase 1. If the incumbent party does not deviate the opposition party in Phase 1 can neither win the election nor influence future play in any way and thus the given policy is a best response. When they vote in Phase 2, the voters decide not only between policy positions but also between phases of the game. If they vote for the incumbent party equilibrium rents will be the maximum level R in all future periods and future equilibrium policy will be b_m from the next but one period forever. On the other hand, if the opposition wins future equilibrium rents will be 0 and future equilibrium policy b_{O_t}, the policy bliss point of the opposition party, from the next but one period forever. It follows that if they follow their given equilibrium voting rule all voters' again vote for the party whose victory maximizes their intertemporal utility function in equilibrium. We know that if the incumbent party does not deviate in Phase 1, it stays in office and in equilibrium achieve a utility of $-(b_{I_t} - b_m)^2$ in all future periods. If the incumbent party deviates the game moves to Phase 2. In equilibrium, the opposition party chooses the policy position b_{O_t} Given the equilibrium voting by the voters this wins the elections. Moreover, it is a best response because even if other policy positions would also win the election this one maximizes utility in the next period without changing utility in any other period for the opposition party. Thus, none of the players has a reason to deviate from the given strategies in Phase 1 or Phase 2 and Example 3 constitutes a subgame perfect equilibrium.

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