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Seppo Honkapohja
Arja H. Turunen-Red
Alan D. Woodland

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Abstract

We study a many country endogenous growth model in which decisions about innovation and new investment are influenced by growth expectations. Adaptive learning dynamics determine country-specific short run transition paths. Countries differ in basic structural parameters and may impose tariffs on imports of capital goods. Numerical experiments illustrate adjustment dynamics that follow the use of tariffs. We show that countries that limit trade in capital goods can experience dynamic gains both in growth and utility and that such gains persist the longer the larger the structural advantages of the region that applies tariffs. Substantial differences in the levels of innovation, consumption, output, and utility can appear, and asymmetries in economic outcomes that were present before trade restrictions are made more severe.

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Seppo Honkapohja
Bank of Finland
PO Box 160
Finland - 00101 Helsinki
seppo.honkapohja@bof.fi

Arja H. Turunen-Red
University of New Orleans
Department of Economics and Finance
USA - New Orleans, LA 70148
aturunen@uno.edu

Alan D. Woodland
University of New South Wales
Sydney NSW 2052
Australia
a.woodland@unsw.edu.au

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1 Introduction

Whether openness toward international trade contributes toward faster growth is a topic of considerable theoretical and empirical interest. While theory generally obtains a positive connection, empirical work has found it difficult to establish a solid causality between trade and growth. The vast literature on the contribution of trade to economic growth has recently been summarized in Estevadeordal and Taylor (2008). We take a novel viewpoint on this issue by examining the evolution of economies as a sequence of temporary equilibria and show that the impact of openness on growth may differ in the short and long run. In particular, we show that trade restrictions can asymmetrically accelerate growth in locations that impose taxes on imported capital goods. Even if gains from such trade restrictions are a transitory phenomenon, their duration can vary considerably depending on structural asymmetries and the specifics of policy.

We construct a many country endogenous growth model and first briefly describe the long run balanced growth solutions. Beyond comparisons of equilibria in the long run, we solve country-specific adjustment paths that follow the introduction of tariffs and demonstrate that, based on these short run dynamics of temporary equilibria, a motivation for restricting trade in capital goods can exist. The model we employ expands Evans, Honkapohja and Romer (1998) (EHR below) and Honkapohja and Turunen-Red (2002) (HTR) and includes several important features. We include an arbitrary number of heterogenous countries that may differ from each other in trade policy and structural parameters (country size, factor productivity, costs of innovation).[1] Trade occurs both intra-industry in a variety of differentiated capital goods and inter-industry in capital goods and a homogenous aggregate consumption commodity. Capital goods are technological complements and the source of growth is the endogenous invention of new capital varieties. Because productivity and profitability of all capital goods improve as more are developed and traded, there is an incentive for investment to be

[1] These parameters help explain international income differences. For factor productivity, see Caselli (2005), Trefler (1993), and Hsieh and Klenow (2007); for research productivity, Alesina and Giavazzi (2006); for country size effects, Alesina, Spolaore and Wacziarg (2005). We do not model the foundations of parameter asymmetries. For the contribution of geographic agglomeration, see Baldwin, Martin and Ottaviano (2001); for difficult technology transfer, Howitt and Mayer-Foulkes (2004); for social infrastructure as a cause of productivity differences, see Hall and Jones (1999).

synchronous (the best time to innovate and invest is when others do so), and perceptions of profit opportunities rise together with expectations growth.[2]

We present several numerical examples illustrating the role of tariffs on capital goods in creating and sustaining growth cycles that yield asymmetric utility gains.[3] When these tariffs are proposed and implemented, growth expectations shift to reflect the expected reallocation of innovation and investment toward the protected markets. In particular, in countries with tariffs, growth expectations improve and a cycle of optimism and faster growth is set in motion. Shortsighted policy makers who are subject to limited knowledge and emphasize growth as a policy goal (because growth of the local economy largely determines consumption possibilities and aggregate well-being) may well see the cycle of high growth equilibria created by tariffs as a positive reason for imposing them.

Adaptive learning dynamics and resulting dynamics of temporary equilibria are applied to formalize the transition from a long run steady state with free trade to a new steady state after the introduction of tariffs. At any point in time, producers are assumed to apply expectations about the future state of the economy so as to choose investment in innovation and production of new capital goods. Once a temporary equilibrium is attained, discrepancies between previous expectations and the observed economic performance lead to adjustment of expectations for the subsequent time period. In the literature, this adjustment over time is called a learning process.[4] As decision makers learn and adapt over time, a sequence of temporary equilibria converges to a balanced growth state that is a stable outcome in the learning process. During the transition and even assuming structural symmetry, countries do not innovate or grow at the same rate nor do they attain the same levels of technology, consumption, and utility. Countries with tariffs can experience significant gains but in the rest of the world a slowdown is likely, yielding losses that may persist much longer than the gains to countries with tariffs.

The magnitude and duration of gains from tariffs on capital goods depend on many factors, including the asymmetries in structural parameters across countries, the changes in initial expectations, and the speed at which decision

[2] Walz (1997, 1999) has presented a different theoretical treatment of asymmetric trade policy using the Grossman and Helpman (1991) approach.

[3] Estevadeordal and Taylor (2008) empirically observe a significant correlation between tariffs on intermediate capital goods and growth.

[4] For discussions of learning dynamics, see Evans and Honkapohja (2001, 2009).

makers learn. Numerical experiments indicate that gains are more likely when the restrictions are imposed by more than one country and when the rest of the world is at a disadvantage (with smaller markets, lower total factor productivity, or higher innovation costs). Initial asymmetries, if any exist, are widened during the adjustment dynamics.

Overall, results suggest that trade policy and heterogeneity in macroeconomic parameters can be important not only because of their impact on long run equilibria but also because transition dynamics can be much affected. In the present model, balanced growth solutions feature a common rate of growth that depends on patterns of openness toward trade[5] but countries with different structural parameters form separate (endogenous) income (-level) clubs across which the levels of technological development, output, consumption, and long run well-being vary.[6] Numerical experiments demonstrate the wide differences in transition paths that can appear when trade in capital goods is restricted.

Subsequent discussion proceeds as follows. Section 2 describes the model and the long run balanced growth solutions. Section 3 formulates the short run learning dynamics, and Section 4 presents the numerical experiments and our results. Section 5 concludes.

2 Model

We assume that there are N (≥ 3) countries, indexed by $i = 1, ..., N$. The aggregate consumer in each country i maximizes the discounted utility expression

$$U_{it} = \sum_{j=0}^{\infty} \frac{\beta^{t+j} C_{i,t+j}^{1-\sigma}}{1-\sigma}, \qquad 0 < \beta, \sigma < 1, \tag{1}$$

where $C_{i,t+j}$ denotes final consumption in period $(t+j)$. Given a constant interest rate, r_i, each consumer's preferred rate of consumption growth, g_{ic},

[5] For empirics of growth and trade patters, see Quah (1997) and Kali, Mendez and Reyes (2007). For preferential trade agreements and growth, Berthelon (2004).

[6] Waugh (2007) observes (using a static model) that asymmetries in trade costs can have a significant effect on relative incomes across countries. A large literature addresses the lack of convergence in incomes and growth between the poorest and richest countries (e.g., Sala-i-Martin (1996), Quah (1997)). Pritchett (2006) has suggested that the differences can be understood using growth regimes that experience separate balanced growth states (each with its own transition dynamics).

is obtained from the Euler equation

$$\frac{C_{i,t+1}}{C_{it}} \equiv g_{ic} = [\beta(1+r_i)]^{1/\sigma} \,. \tag{2}$$

Final consumption is produced by competitive production sectors that employ immobile resources (labor) and domestic and imported capital goods according to the technology function for country i

$$Y_{it} = \widehat{L}_i^{1-\alpha} \left(\sum_{k=1}^{N} \int_0^{A_{kt}} x_{ikt}(j_k)^\gamma \, dj_k \right)^\phi \,, \quad \widehat{L}_i \equiv \psi_i^{\frac{1}{1-\alpha}} L_i, \quad \phi > 1 \,. \tag{3}$$

In (3), L_i denotes the (fixed) endowment of labor (country size) and ψ_i allows for differences in aggregate productivity (parameters (α, γ, ϕ) are identical in all $i = 1, ..., N$). The $x_{ikt}(j_k)$ give the quantities of capital goods (indexed by j_k) purchased from country k by the competitive production sector in country i.[7] The number of capital goods produced, A_{kt}, defines the technology level of country k at time period t. The parameter $\phi > 1$ imposes technological complementarity between all capital varieties. Linear homogeneity of production requires that $\alpha = \gamma\phi$.

Intra-industry trade in capital inputs takes place even if all countries are symmetric (see HTR (2002)). Inter-industry exchange of aggregate consumption and capital goods appears when structural asymmetries are present. We assume that there are trade barriers, denoted by τ_{ik} (≥ 1), that impact international trade in capital goods (imports to country i from country k); for domestic production the barriers are set to zero ($\tau_{ii} = 1$). We regard the trade barriers as exogenous ad valorem tariffs (usually expressed as $(1+\tau_{ik})$) but other trade costs may partly determine τ_{ik} as well (in the case of tariffs, all tariff revenues are distributed to consumers as lump sum income).[8]

All competitive production sectors observe domestic prices (measured with respect to the world market price of final consumption which is the freely traded *numeraire*) and maximize profit given technology (3). Equating the marginal cost and marginal product of each capital variety, demand

[7] All $x_{ikt}(j_k)$ are treated as service flows from durable capital goods owned by their inventor producers.

[8] For example, τ_{ik} can reflect public infrastructure (transport and communication networks) and legal institutions.

for capital goods in all locations is determined by equations

$$\tau_{ik} R_{ikt}(j_k) = \widehat{L}_i^{1-\alpha} \left(\sum_{l=1}^{N} \int_0^{A_{lt}} x_{ilt}(j_l)^{\gamma}\, dj_l \right)^{\phi-1} \alpha x_{ikt}(j_k)^{\gamma-1}, \tag{4}$$

where $R_{ikt}(j_k)$ denotes the rental price in country i of capital varieties j_k originating in country k.

Intermediate capital goods are supplied by monopolistic competitive inventors (without replicative innovation). A unit of each capital good is produced by converting c_i units of aggregate capital, denoted by Z_i, into a specific capital variety in location i (aggregate capital is not traded). Production is realized at the end of a time period so that, at the end of a period, a producer in country i receives revenue $R_{kit}(j_i)x_{kit}(j_i)$ from sales in each country k. In the beginning of a time period, $c_i x_{kit}(j_i)$ units of Z_i are needed to produce the capital rented out. The rental cost for this aggregate capital is $c_i r_{it} p_{it}^z x_{kit}(j_i)$, where p_{it}^z is the opportunity cost of aggregate capital in country i. Each producer of capital goods maximizes the profit expression

$$\pi_{it}(j_i) = \sum_{k=1}^{N} R_{kit}(j_i)x_{kit}(j_i) - c_i r_{it} p_{it}^z \left[\sum_{k=1}^{N} x_{kit}(j_i) \right], \quad j_i \in [0, A_{it}], \tag{5}$$

where the $x_{kit}(j_i)$ are obtained using (4). The subsequent optimized mark-up rules are

$$R_{kit} = \frac{c_i r_{it} p_{it}^z}{\gamma} \equiv R_{it}(r_{it}, p_{it}^z, c_i), \quad i, k = 1, ..., N, \tag{6}$$

for all varieties of capital $j_i \in [0, A_{it}]$ in location i.

While the rate of technological progress is the same in all countries in the long run, technology levels do not necessarily equate. Country-specific technology proportionality factors, θ_{it}, are defined by setting

$$A_{it} = \theta_{it} A_{1t}, \quad i = 2, ..., N, \text{ and } \theta_{1t} \equiv 1, \ \forall t, \tag{7}$$

where A_{1t} serves as the world technology level index over time. Using (4) and (6)-(7) we obtain the provision of all capital varieties in all markets $(i = 1, ..., N, \ k \neq i)$:

$$x_{iit} = \widehat{L}_i (A_{1t} S_{it})^{\xi} \left(\frac{R_{it}}{\alpha} \right)^{\frac{1}{\alpha-1}}, \ x_{ikt} = x_{iit} \left(\frac{\tau_{ik} R_{kt}}{R_{it}} \right)^{\frac{1}{\gamma-1}}, \ \xi \equiv \frac{\phi-1}{1-\alpha}, \tag{8}$$

$$S_{it}(\hat{\theta}_t, \tau, R_t) \equiv \sum_k \theta_{kt} \left(\frac{\tau_{ik} R_{kt}}{R_{it}} \right)^{\frac{\gamma}{\gamma-1}} = \theta_{it} + \sum_{k \neq i} \theta_{kt} \left(\frac{\tau_{ik} R_{kt}}{R_{it}} \right)^{\frac{\gamma}{\gamma-1}}, \quad (9)$$

where $R_t = (R_{1t}, ..., R_{Nt})^T$, $\hat{\theta}_t = (\theta_{1t}, ..., \theta_{Nt})^T$, and $\tau = (\tau_1, ..., \tau_N)^T$. Supply of aggregate output equals

$$Y_{it} = \widehat{L}_i^{1-\alpha} x_{iit}^{\alpha} (A_{1t} S_{it})^{\phi} = \widehat{L}_i (A_{1t} S_{it})^{1+\xi} \left(\frac{R_{it}}{\alpha} \right)^{\frac{\alpha}{\alpha-1}}. \quad (10)$$

The multiplicative terms S_{it} defined in (9) give an import tariff and relative rental -deflated sum of the technology levels of a country's trade partners and reflect the accessibility of world technology to the aggregate production sector of a country (*openness factor* of a country). These S_{it} determine the impact of a country's trade policy and trade pattern on domestic capital goods production (x_{iit}) and imports of intermediate capital, *ceteris paribus.* The openness factors S_{it} decrease (reducing x_{iit} and x_{ikt}) as trade barriers (τ_{ik}) and the relative rentals (R_k/R_i) rise and this effect is the larger the higher the technology levels of the trade partners (θ_{kt}) and the larger the contribution of capital goods in aggregate production (γ). The aggregate output solution (10) shows that, in addition to institutional and other exogenous factors (ψ_i), a country's total factor productivity depends on its effective technology level ($S_{it} A_{1t}$). As the degree of complementarity among capital varieties (ϕ) increases, the contribution of openness becomes more important.

Analogously to EHR (1998) and HTR (2002), we assume that the development of the (j_i)th capital commodity in country i costs $\upsilon_i p_i^z j_i^{\xi}$ units of aggregate consumption. This specification makes later innovations more costly but, owing to capital complementarity, they are more valuable. Parameters υ_i allow for differences in research productivity (variations in human capital, institutions, and policies toward innovative activities). The extent of innovation per time period is determined by the zero profit condition of the monopolistically competitive producers (equating the discounted monopoly rents from the last capital variety using (5), (6) and (8) to the cost of invention):

$$\upsilon_i p_{it}^z \theta_{it}^{\xi} A_{1t}^{\xi} = \sum_{s=0}^{\infty} (1 + r_{t+s})^{-(s+1)} \pi_{i,t+s}, \quad i = 1, ..., N, \quad (11)$$

$$\pi_{i,t} = (1-\gamma)R_{it}\left[\sum_{j=1}^{N} x_{ji}\right] = k_{it}\Omega A_{1t}^{\xi}(c_i r_{it} p_{it}^{z})^{\frac{\alpha}{\alpha-1}}, \tag{12}$$

where

$$k_i(\hat{\theta}_t; \tau, \widehat{L}, R_t) \equiv \sum_{j=1}^{N} \widehat{L}_j S_{jt}^{\xi} \tau_{ji}^{\frac{1}{\gamma-1}} \left(\frac{R_{it}}{R_{jt}}\right)^{\frac{\gamma\xi}{(1-\gamma)}}, \tag{13}$$

where $\Omega \equiv (1-\gamma)\gamma^{\frac{\alpha}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}$ and $\widehat{L} = (\widehat{L}_1, ..., \widehat{L}_N)^T$. By (12), profitability of invention in a location is affected by the general technology level (A_{1t}) and the cost of producing a unit of a capital commodity there ($c_i r_{it} p_{it}^{z}$). Multipliers $k_{it}(.)$ take into account the negative impact of tariff barriers against a country's exports in all markets (τ_{ji}) but also show the positive effect of productive size ($\widehat{L}_j$), relative rentals (R_i/R_j), and openness (S_{jt}) of markets on profitability.

The opportunity cost of general purpose capital is derived using the production possibility frontier between final consumption and aggregate capital,

$$Y_{it} = C_{it} + Z_{it}\Gamma\left(\frac{Z_{i,t+1} - Z_{it}}{Z_{it}}\right), \quad i = 1, ..., N, \tag{14}$$

where $\Gamma(.)$ is a convex cost function and Z_{it} equals the total investment in aggregate capital in country i in time period t, i.e.,

$$Z_{it} = K_{it} + I_{it} \equiv \sum_{k=1}^{N} \int_{0}^{\theta_{it}A_{1t}} c_i x_{kit}(j_i)\, dj_i + \int_{0}^{\theta_{it}A_{1t}} \upsilon_i j_i^{\xi} dj_i = \tag{15}$$

$$A_{1t}^{1+\xi}\left[c_i\theta_{it}k_{it}\left(\frac{R_{it}}{\alpha}\right)^{\frac{1}{\alpha-1}} + \frac{\upsilon_i\theta_{it}^{1+\xi}}{1+\xi}\right].$$

By (14),

$$p_{it}^{z} = -\frac{dC_{it}}{dZ_{i,t+1}} = \Gamma'\left(\frac{Z_{i,t+1} - Z_{it}}{Z_{it}}\right), \tag{16}$$

where $\Gamma'' > 0$. The common cost function $\Gamma(.)$ in (14) and (16) and the expression for Z_{it} in (15) reflect the sectoral structure of production that is assumed to be the same everywhere: while aggregate technologies for capital goods production and innovation are the same (so that $Z_{it} = K_{it} + I_{it}$ for all i, including possible efficiency differences (c_i, υ_i)), (unspecified) technological differences between the monopolistically competitive sector and the production of aggregate consumption yield the frontier (14).[9]

In a long run balanced growth equilibrium, the interest rate and the opportunity cost of aggregate capital remain constant while technology, output, aggregate capital, and consumption grow at a common constant rate. Assuming that the rate of technology growth is $g_A \equiv A_{1,t+1}/A_{1t}$, then by (15), aggregate capital grows at rate

$$g_z = (g_A)^{1+\xi}, \tag{17}$$

and, due to (10) and (14), aggregate output and consumption grow at this same rate. By (16),

$$p^z = \Gamma'(g_z - 1), \tag{18}$$

and r is determined by the Euler equation

$$g_z = [\beta(1+r)]^{1/\sigma}. \tag{19}$$

Substituting (12) into zero profit conditions (11) and assuming that all countries innovate in equilibrium, we obtain the remaining equilibrium conditions

$$g_z = \left[1 + r - \Omega \widehat{k}_1(.) r^{\frac{\alpha}{\alpha-1}} (p^z)^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}}, \tag{20}$$

$$\widehat{k}_1(\theta; \tau, \widehat{L}, \upsilon, c, R) = \widehat{k}_i(\theta; \tau, \widehat{L}, \upsilon, c, R), \quad i = 2, ..., N, \tag{21}$$

$$\widehat{k}_i(\theta; \tau, \widehat{L}, \upsilon, c, R) \equiv \frac{k_i(\hat{\theta}; \tau, \widehat{L}, c, R)}{c_i^{\frac{\alpha}{1-\alpha}} \upsilon_i \theta_i{}^{\xi}}, \ i = 1, ..., N. \tag{22}$$

Here we introduce the notation $\hat{\theta}^T = (1, \theta^T)$, where $\theta = (\theta_2, ..., \theta_N)^T$, because θ is the relevant state variable (since $\theta_1 \equiv 1$ as indicated in ((7))). Subsequently, $\widehat{k}_i(\theta; \tau, \widehat{L}, \upsilon, c, R)$ for all $i = 1, ..., N$ will play an important role and we refer to them as the countries' *relative profitability parameters.*[10]

[9] If technologies in all three production sectors are the same, then (14) is replaced by the accumulation equation $Y_{it} = C_{it} + (Z_{i,t+1} - Z_{it})$. Then $p^z_{it} \equiv 1$ for all i and t.

[10] By (11)-(12) and at a steady state, $\widehat{k}_i/\widehat{k}_j = (\pi_i/\upsilon_i\theta^{\xi}_{it})/(\pi_j/\upsilon_j\theta^{\xi}_{jt})$ $(= 1)$ reflects relative profitability of innovation (over innovation costs) in countries i and j, $i, j = 1, ..., N$.

Equations (18)-(21) determine the long run growth rate (g_z) and the level variables ($\theta_2, ..., \theta_N; r, p^z$), given the exogenous trade barriers (τ) and other country-specific parameters ($\widehat{L}, \upsilon, c$). Several features of the model contribute to the structure of the long run equilibrium. First, equations (21) determine the allocation of innovation across countries (θ_i) so as to maintain zero profitability of invention everywhere despite country-specific asymmetries and as expansion of technology proceeds at a common rate. Because in a steady state equations (21) are separable from the rest of the equilibrium conditions we can solve technology levels ($\theta_2, ..., \theta_N$) using only information regarding tariffs and ($\widehat{L}, \upsilon, c$).[11] The impact on long run growth of technology levels, trade policies, and other parameters is transmitted by the multiplier $\widehat{k}_1(\theta; \tau, \widehat{L}, \upsilon, c, R)$ in (20).[12] Patterns of trade influence growth because $\widehat{k}_1(.)$ depends on the country by country openness factors ($S_i(.)$). Further, the common aggregate technology between consumption and general capital (the cost function $\Gamma(.)$ in (14)) implies that a shared rate of growth (g_z) at a long run equilibrium converts into a common opportunity cost of aggregate capital in (18). Because preferences are the same everywhere, common growth in output, capital, and consumption yields, by (19), a common interest rate in all locations.

In a steady state, technology level differences partly determine levels of aggregate output since, by (10),

$$y_i \equiv \frac{Y_i}{\widehat{L}_i} \;\Rightarrow\; \frac{y_i}{y_j} = \left(\frac{S_i(\hat{\theta}, \tau, c)}{S_j(\hat{\theta}, \tau, c)}\right)^{1+\xi} \left(\frac{c_i}{c_j}\right)^{\frac{\alpha}{\alpha-1}}, \; i, j = 1, ..., N. \tag{23}$$

Accordingly, a country's relative aggregate output per effective unit of labor (y_i/y_j) is the larger the more open the country's trade policy and the more advanced its trade partners (S_i/S_j). Country by country balances of payments are maintained through inter-industry trade in aggregate consumption and capital goods, i.e., consumption equals ($i = 1, ..., N$)

$$C_{it} = Y_{it} - p_{it}^z Z_{it}(g_z - 1) + A_{1t}\left[\theta_i R_{it}(\sum_{k \neq i} x_{ki}) - \sum_{k \neq i} \theta_k R_{kt} x_{ik}\right]. \tag{24}$$

[11] Separability obtains because i) capital is assumed not to depreciate (with depreciation, equations (14) would include additional terms and θ_i would appear in (18)) and ii) in (21), the common r and p^z terms cancel out at a steady state.

[12] Under full symmetry, by (21), $\theta_i = 1$, $i = 1, ..., N$, and if trade is also free, then $\widehat{k}_1 = \widehat{L} N^{1+\xi} c^{\frac{\alpha}{\alpha-1}} / \upsilon$.

Given that initial consumption levels satisfy (24), trade remains balanced under balanced growth. (Since aggregate capital (Z) is nontraded, the capital account equals zero everywhere.) In subsequent sections, when considering transition dynamics we maintain balanced trade by solving aggregate consumption using the equivalents of the balance of payments condition (24). By (1), aggregate utility along a balanced growth path equals

$$U_i = \frac{C_{i0}^{1-\sigma}}{1-\sigma}\left[\frac{1}{1-\beta {g_z}^{1-\sigma}}\right], \quad i = 1, ..., N, \tag{25}$$

where C_{i0} denotes the initial consumption level in location i and g_z is the equilibrium growth rate.

Figure 1 illustrates the equilibrium conditions (18)-(20). The upward sloping curve CC graphs equation (19) in (r, g_z) space, while curve TT is obtained by substituting into the technology arbitrage condition (20) the opportunity cost of capital ((18)) and the equilibrium technology levels from (21).[13] Steady states are found where curves CC and TT intersect, and multiple equilibria may occur (e.g., $E1$, $E2$ and $E3$ in Fig. 1). Multiplicity arises should equations (21) possess more than one solution for the vector of technology levels or if the slope of curve TT varies as depicted in Fig. 1. Of these, the first possibility can be ruled out under simplifying assumptions (Lemma A1 in the Appendix) and never occurred in numerical simulations; the slope of curve TT, however, is influenced by the complementarity parameter (ϕ) and the sensitivity of the opportunity cost of capital to changes in aggregate investment (Γ'') and can take both positive and negative values (see the Appendix).[14]

FIGURE 1 HERE

At each balanced growth equilibrium, depending on structural heterogeneities, several long run income clubs may exist. Such clubs are defined by the dispersion in innovation (θ_i), output (Y_i), consumption (C_i), and well-being (U_i) in equilibrium.[15] Example 1 offers a numerical illustration; these

[13] Because technology levels θ_i remain fixed along the TT curve, Fig. 1 cannot depict the short term transion dynamics that are considered in Sections 3 and 4.

[14] If $\Gamma'' = 0$, the TT curve is globally upward sloping and, given $(\theta_2, ..., \theta_N)$ that solve (21), there can be two long-run steady states, but only one of them is stable under learning (EHR (1998)). Stability of equilibria under learning is analyzed in the next section.

[15] The term 'income club' refers to country groups that experience similar equilibrium outcomes in level variables.

balanced growth solutions, obtained assuming that all trade is free, serve as benchmarks for the subsequent experiments.[16]

Example 1 (Free Trade Equilibria): Let countries $i = 1, ..., n_1$ $(n_1 + 1, ..., N)$ be symmetric with parameters $(\widehat{L}_1, \upsilon_1, c_1)$ $((\widehat{L}_2, \upsilon_2, c_2))$. Let $\alpha = 0.39$, $\phi = 1.12$, $\beta = 0.90$, $\sigma = 0.22$, $\widehat{L}_1 = 0.0092$, $\upsilon_1 = c_1 = 1$; $N = 3$ with $n_1 = 2$, $n_2 = 1$; $\Gamma'(x) = (0.1)x^2 + 0.2$. First row of Table 1: $\upsilon_2 = \upsilon_1, c_2 = c_1,\ \widehat{L}_2 = \widehat{L}_1$; second row: $\upsilon_2 = 1.22,\ c_2 = 1,\ \widehat{L}_2 = 0.0070$; third row: $\upsilon_2 = 1.22,\ c_2 = 1.20,\ \widehat{L}_2 = 0.0070$.[17]

TABLE 1: Free Trade Equilibria

θ	Y_1/Y_2	Z_1/Z_2	C_1/C_2	p_z	r	g_z	U_1/U_2
1	1	1	1	0.2012	0.137	1.112	1
0.36	1.31	2.77	1.59	0.2002	0.122	1.044	1.44
0.22	1.31	5.01	1.80	0.2002	0.120	1.037	1.58

$x_{11}/\theta x_{22}$	x_{21}/x_{11}	x_{12}/x_{22}	NX_1/Y_1	NX_2/Y_2
1	1	1	0	1
3.64	0.76	1.31	0.07	-0.17
7.91	0.76	1.81	0.10	-0.26

The first row of Table 1 gives a symmetric balanced growth solution $(N = 3)$, while on the second and third rows one country is smaller and/or less productive than others $(\widehat{L}_2 < \widehat{L}_1,\ c_2 > c_1)$ with higher innovation costs $(\upsilon_2 > \upsilon_1)$. Under symmetry, all countries innovate equally $(\theta = 1)$ and attain equal output, consumption, and aggregate investment and the intra-industry trade in capital goods balances. In Table 1, column $x_{11}/\theta x_{22}$ compares the volumes of intra-industry trade within the two country groups, n_1 and n_2;[18] columns x_{21}/x_{11} and x_{12}/x_{22} measure intra-industry trade between countries n_1 and n_2; and columns NX_1/Y_1 and NX_2/Y_2 measure inter-industry trade between n_1 and n_2 (NX_i/Y_i gives the ratio of total value of capital net exports and aggregate output in $n_i,\ i = 1, 2$).[19]

[16] These numerical examples do not represent a full calibration of the model. Mathematica routines for all examples are available from the second author upon request.

[17] Table 1 gives ratios of variables so as to avoid dependence on the initial value of A_{1t}.

[18] Since countries n_1 (n_2) are symmetric, x_{11} (x_{22}) gives the output of each capital variety developed in n_1 (n_2) for each market in n_1 (n_2).

[19] Due to symmetry, there is no inter-industry trade within each country group.

Two income clubs form when structural asymmetries appear. Due to the higher cost of innovation in n_2 (second row of Table 1), innovation is reallocated toward the lower cost source ($\theta = 0.36 < 1$). Aggregate investment expands in n_1 ($Z_1/Z_2 = 2.77 > 1$) and intra-industry trade is stimulated ($x_{11}/\theta x_{22} = 3.64 > 1$). Countries n_1 become net exporters of capital goods ($NX_1/Y_1 > 0$), leading to higher consumption ($C_1/C_2 = 1.59 > 1$) both because group n_2 must export consumption as payment for capital imports ($NX_2/Y_2 = -0.17 < 0$) and because $Y_1 > Y_2$ due to higher productivity in n_1 ($\widehat{L}_2 < \widehat{L}_1$). Growth is slower when a less productive and less innovative country group is present ($g_z = 1.044 < 1.112$) and all countries experience a lower long run level of well-being than under symmetry. However, due to their higher consumption levels, countries n_1 are considerably better off than the rest of the world ($U_1/U_2 = 1.44 > 1$). Differences in productivity in capital goods production (the third row of Table 1) yield changes similar to those arising from innovation cost asymmetries.

3 Learning Dynamics

We apply adaptive learning dynamics to determine the stability properties of long run equilibria and to obtain a description of the short run adjustment process that follows a structural shift (e.g., the imposition of trade tariffs). A model of these dynamics consists of (i) a mapping from initial expectations to a temporary (short run) equilibrium, and (ii) a learning rule that describes the updating of expectations based on the observed past. The short run equilibria and the learning rule are constructed as follows.

At a given time period, producers hold expectations about the rate of future growth, $g^e_{it} \in \mathbb{R}$, and relative profitability of innovation, $\widehat{k}^e_{it} \in \mathbb{R}$, in their location. Expectations about profitability implicitly reflect expectations about growth in other regions.[20] Investment plans are formulated subject to these expectations, taking into account the expected revenue from and the expected cost of innovation. Corresponding to the expected zero profitability

[20] This is because, due to (22), (13) and (9), $\widehat{k}^e_{it}$ for all i depend on perceptions about future levels of technology (θ_{jt}) and pricing (R_{jt}) in all locations j. Learning dynamics could be formulated by assuming that producers in each country hold expectations about growth (g_{jt}) in every location ($j = 1, ..., N$). The specification we follow is technically more convenient and allows us to solve a temporary equilibrium without assuming that individuals in different countries are aware of each other's expectations.

of invention at the given expectations, the technology arbitrage equations $(i = 1, ..., N)$

$$g_{it}^{e} = \left[1 + r_{it} - \Omega \widehat{k}_{it}^{e} r_{it}^{\frac{\alpha}{\alpha-1}} \Gamma'(g_{it}^{e} - 1)^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}}, \tag{26}$$

can be solved for the required return for innovation in each location, $r_{it}(g_{it}^{e}, \widehat{k}_{it}^{e})$. When combined with the consumers' Euler equations, these $r_{it}(.)$ determine the realized rate of growth

$$g_{it} = \left[\beta(1 + r_{it}(g_{it}^{e}, \widehat{k}_{i}^{e}))\right]^{1/\sigma} \equiv G_i(g_{it}^{e}, \widehat{k}_{it}^{e}), \quad i = 1, ..., N, \tag{27}$$

in each country at a temporary equilibrium and, given g_{it}, we obtain the realized price of capital

$$p_{it}^{z} = \Gamma'(g_{it} - 1), \quad i = 1, ..., N. \tag{28}$$

The temporary equilibrium value of $\widehat{k}_i$ equals $(i = 1, ..., N)$

$$\widehat{k}_{it}(\theta; \tau, \widehat{L}, \upsilon, c, R) = \frac{1}{c_i^{\frac{\alpha}{1-\alpha}} \upsilon_i \theta_{it}{}^{\xi}} \left[\sum_{j=1}^{N} \widehat{L}_j S_{jt}^{\xi} \tau_{ji}^{\frac{1}{\gamma-1}} \left(\frac{R_{it}}{R_{jt}}\right)^{\frac{\gamma\xi}{(1-\gamma)}}\right], \tag{29}$$

where terms S_{jt} are determined by (9),

$$R_{it} = \frac{c_i}{\gamma} r_{it}(g_{it}^{e}, \widehat{k}_{i}^{e}) \Gamma'(G_i(g_{it}^{e}, \widehat{k}_{it}^{e}) - 1), \quad i = 1, ..., N, \tag{30}$$

and, for $i = 2, ..., N$,

$$\theta_{i,t+1,} = \theta_{it} \left[\frac{G_i(g_{it}^{e}, \widehat{k}_{it}^{e})}{G_1(g_{1t}^{e}, \widehat{k}_{1t}^{e})}\right]^{1/(1+\xi)} \equiv F_i(\theta_{it;}, g_{1t}^{e} . g_{it}^{e}; \widehat{k}_{1t}^{e}, \widehat{k}_{it}^{e}). \tag{31}$$

Temporary equilibrium levels of aggregate output and consumption are found using (10) and (24).

By (26)-(27), country-specific rates of growth can differ during transition dynamics; the extent of realized growth in each location depends on short run business expectations and consumer preferences regarding saving and consumption, as well as the policy, productivity, and cost parameters. The opportunity cost of aggregate capital and the return to innovation also vary depending on regional short run growth. The allocation of technological attainment adjusts to reflect new innovation as expressed in (31). Given initial expectations $(g^e_{it}, \widehat{k}^e_{it})$ and technology levels θ_{it} for $i = 1, ..., N$, equations (26)-(31) define a unique temporary equilibrium solution for each g_{it}, p^z_{it}, $i = 1, ..., N$, and $\theta_{i,t+1}$, $i = 2, ..., N$.[21]

During the short run transition, expectations are adjusted according to specific learning rules. Using the vector notation $\theta_t = (\theta_{2t}, ...\theta_{Nt})^T$ introduced in connection with equation (22) and

$$g_t \equiv (g_{1t}, ..., g_{Nt})^T, \ \widehat{k}_t \equiv (\widehat{k}_{1t}, ..., \widehat{k}_{Nt})^T, \tag{32}$$

$$G(g^e_t; \widehat{k}^e_t) \equiv (G_1(.)\,, ..., G_N(.))^T \ , \tag{33}$$

$$F(\theta_t; g^e_t, \widehat{k}^e_t) \equiv (F_2(.), ..., F_N(.))^T \,, \tag{34}$$

equations (27) and (31) yield

$$g_t = G(g^e_t; \widehat{k}^e_t), \quad \theta_{t+1} = F(\theta_t; g^e_t, \widehat{k}^e_t). \tag{35}$$

Further, we define

$$K(g^e_t, \widehat{k}^e_t; \theta_t; \tau) \equiv (K_1(.)\,, ..., K_N(.))^T, \tag{36}$$

where $K_i(g^e_t, \widehat{k}^e_t; \theta_t; \tau) = \widehat{k}_{it}(\theta_t; \tau, \widehat{L}, \upsilon, c, S_t, R_t)$ is obtained using (29) when all realized values of S_{jt}, θ_t, R_{it} and R_{jt} have been substituted in for all $i, j = 1, ..., N$.

Based on (35) and (36), an adaptive learning system expresses the changes in expectations that reflect deviations of realized variables at a temporary equilibrium from their expected values, i.e., we write

$$g^e_{t+1} = g^e_t + \gamma_p[G(g^e_t, \widehat{k}^e_t) - g^e_t], \tag{37}$$

$$\widehat{k}^e_{t+1} = \widehat{k}^e_t + \gamma_p[K(g^e_t, \widehat{k}^e_t; \theta_t; \tau) - \widehat{k}^e_{it}], \tag{38}$$

$$\theta_{t+1} = F(\theta_t; g^e_t, \widehat{k}^e_t). \tag{39}$$

Equation (37) is N-dimensional and specifies the change in growth expectations in each location. Analogously, each of the N equations in (38) states the

[21] The $r_{it}(g^e_{it}, \widehat{k}^e_{it})$ obtained from (26) are unique because the right-hand side of each equation is upward sloping in r_{it}. For a given r_{it}, (27)-(31) yield unique g_{it}, p^z_{it}, and θ_{it} for all $i = 1, ..., N$.

revised expectations regarding profitability of innovation. The gain parameter γ_p in (37)-(38) determines the extent to which expectations are affected by observed errors in the past.[22] Equation (39) is $(N-1)$-dimensional and must be included in the learning system because vector θ_t is a state variable which appears in (38).

Equations (37)-(39) specify a system of learning dynamics for which the balanced growth equilibria characterized in (18)-(20) are the fixed points (see the Appendix). Stability of equilibria is determined by the local stability of the system (37)-(38) near each fixed point. For tractability, the following proposition states the necessary condition for stability when there are two learners ($i = 1, 2$).[23]

Proposition 1: *If a balanced growth equilibrium, $(\theta^*; g_z^*, r^*)$, is characterized by equations (18)-(20) and is locally stable under adaptive learning dynamics defined in (37)-(38) for all sufficiently small gain parameters, then*

$$\mathcal{B} \equiv \frac{\partial r_{1z}(g_z^*)}{\partial g_1^e} - \frac{dr_{1c}(g_z^*)}{dg_1} \left(= \frac{\partial r_{2z}(g_z^*)}{\partial g_2^e} - \frac{dr_{2c}(g_z^*)}{dg_2}\right) < 0, \tag{40}$$

where $r_{iz}(g_z)$ and $r_{ic}(g_z)$ are the functions describing the solutions for the interest rate from equations (26) and (27), respectively, for each $i = 1, 2$.

Condition (40) is a consistency requirement on producers' reactions to growth expectations (in (26)) and the consumers' willingness to invest in new innovation in each country (in (27)); in Fig. 1, the producers' arbitrage curve TT must cut the consumers' CC curve from above as shown at equilibria $E1$ and $E3$ (see the Proof of Proposition 1 in the Appendix).

FIGURE 2 HERE

Figure 2 illustrates the beginning of the transition process in one location (Country 1); analogous figures can be used to depict the adjustment elsewhere. Curve CC in Fig. 2 represents equation (27) and is the same everywhere. As in Fig. 1, the initial steady state (g_z^*, r^*) is identified by

[22] For simplicity, we assume that gain parameters are identical across countries. One could allow for heterogeneity in gains, see Honkapohja and Mitra (2006) and Pfajfar and Santoro (2008).

[23] We have not been able to derive theoretically a useable sufficient condition for stability (see the Appendix). In numerical simulations we check that the dynamics converge to a steady state after an exogenous shift.

the intersection of CC and the (common) technology relation, $TT(\widehat{k}^*)$, that is obtained from (26) by setting the expected profitability levels equal to their steady state values ($\widehat{k}_{it}^e = \widehat{k}^*$ for all i). Now suppose growth expectations in Country 1 rise in time period t ($g_{1t}^e > g_z^*$ in Fig. 2). The effect of these positive expectations on realized growth depends, according to (26), on the perceptions about relative profitability of innovation ($\widehat{k}_{it}^e$) that reflect expected changes in innovation elsewhere. If profitability is expected to improve ($\widehat{k}_{it}^e > k^*$), curve TT shifts up (e.g., position $TT_1(\widehat{k}_{1t}^e)$).[24] The realized temporary equilibrium (g_{1t}, r_{1t}) can then be read off the $TT_1(\widehat{k}_{1t}^e)$ and CC curves as specified in (26)-(27). After observing g_{1t} and k_{1t}, expectations are adjusted as specified in (37)-(39) and a new temporary equilibrium is obtained following an analogous process.

The magnitude of the change in realized growth depends on the initial change in expectations (whether positive or negative) but is also affected by the shift in the $TT_1(\widehat{k}_{1t}^e)$ curve (due to expectations about profitability) and the slope of this curve. To illustrate the shift effect, (g_{1t}', r_{1t}') in Fig. 2 gives the temporary equilibrium that is obtained if technological growth is not expected to alter relative profitability ($\widehat{k}_{1t}^e = k^*$); in this case, expectations for new profit opportunities are small (no shift in the TT curve) and the increase in growth is modest ($g_{1t}' << g_{1t}$). Realized growth would be slower still if growth expectations are such that curve $TT_1(\widehat{k}_{1t}^e)$ is located below its steady state position $TT(k^*)$ (not shown in Fig. 2). However, even in this case, assuming that curve $TT_1(\widehat{k}_{1t}^e)$ is sufficiently steep, the short run growth rate can be higher than the previously observed g_z^*. The slope of the relevant TT curve is the larger the less the opportunity cost of capital responds to additional capital investment (the smaller the term $\Gamma'' > 0$ in (51)). In other words, the smaller the cost pressure from new investment and the better the growth expectations, the larger the increase in growth in the short run. The slope of the CC curve also contributes: the more willing the consumers to save an invest (the flatter the CC curve), the larger the realized growth expansion.

[24] In (26), keeping r_{1t} and $p_{1t}^{\tilde{z}}$ fixed, the right-hand side of the equation is decreasing in $\widehat{k}_1$. Therefore, g_{1t} must also decrease, implying that, in Fig. 2, curve TT shifts up. (Because $p_{1t}^{\tilde{z}}$ does not decrease when g_{1t} increases, the adjustment of $p_{1t}^{\tilde{z}}$ reinforces the change.)

4 Dynamics, Expectations, and Tariffs

4.1 Balanced growth equilibria with tariffs

Economic intuition and previous results suggest that restrictions on international exchange of capital goods slow down growth in the long run.[25]

Proposition 2: *Near stable long run equilibria, an increase in any tariff on trade in intermediate capital goods lowers the balanced growth rate.*

The extent of the growth reduction depends on the size of the tariff increase, where the policy change is implemented, and how the technology trade patterns are altered (openness factors $S_i(.)$ affect growth through $\widehat{k}_1(.)$ in (20)). Complementarity of capital goods supports coordination to lower growth in the long run. This is because a reduction in demand for some capital goods, caused by new trade restrictions, reduces demand for all capital varieties, slowing down innovation everywhere.

Example 2 (Tariffs): The parameters are as in Example 1 on each row of Table 2a; countries n_1 form a customs union and adopt a uniform tariff ($\tau_1 = 1.1$) against capital goods imports from the rest of the world (n_2).

TABLE 2a: Equilibria with Tariffs, $\tau_1 = 1.1$.

θ	Y_1/Y_2	Z_1/Z_2	C_1/C_2	p_z	r	g_z	U_1/U_2
0.62	0.99	1.78	1.09	0.2009	0.134	1.097	1.07
0.21	1.31	5.21	1.80	0.2001	0.120	1.037	1.58
0.13	1.31	9.42	1.96	0.2001	0.119	1.033	1.69

$x_{11}/\theta x_{22}$	x_{21}/x_{11}	x_{12}/x_{22}	NX_1/Y_1	NX_2/Y_2
1.62	1.00	0.86	0.07	-0.14
6.17	0.76	1.13	0.10	-0.27
13.39	0.76	1.13	0.12	-0.32

Comparing Tables 1 (free trade) and 2a (customs union) the negative effect of the tariff on long run growth is apparent. The largest reduction in growth occurs when all countries are symmetric (first row of Tables 1

[25] For example, in Walz (1997: Theorem 2), a liberalization of capital goods trade speeds up growth in the long run; in HTR (2002), common tariffs on trade in intermediate capital goods shift the long run equilibrium toward lower growth.

and 2a). In this case, the tariff creates two income clubs with different output, investment, consumption, and utility levels. When two income clubs exist under free trade (rows two and three of Tables 1 and 2a), the tariff exacerbates the asymmetries in level variables but the growth reduction is smaller. In all cases, the share of the customs union of the world innovation, total investment, and consumption increases. Intra-industry trade within the customs union expands while the rest of the world increasingly exports aggregate consumption so as to purchase capital goods from the customs union. The customs union gains relatively (U_1/U_2 increases) but, because long run well-being mainly depends on growth (in (25)), all countries suffer a loss when a tariff is employed. Table 2b shows the aggregate utility levels under alternative tariff configurations when all countries are symmetric.[26]

Table 2b: Long Run Utility (Symmetry)

	$\tau_2 = 1$		$\tau_2 = 1.1$
$\tau_1 = 1$	$6.86, 6.86$		$5.09, 5.27$
$\tau_1 = 1.1$	$4.41, 4.13$		$3.49, 3.33$

The diagonal entries of Table 2b show the free trade utility levels ($\tau_i = 1,\ i = 1, 2$) and the outcome when both the customs union and the rest of the world impose a tariff ($\tau_1 = \tau_2 = 1.1$). The off-diagonal entries state the long run utility levels when trade restrictions are asymmetric. Because a smaller proportion of technology trade is affected in these cases, utility losses are smaller, but never is there a utility-based argument for interfering with technology trade: all restrictions make everyone worse off in the long run.

4.2 Short run dynamics

Despite the negative effect of trade barriers on balanced growth, one may wonder about the short run adjustment toward a new equilibrium. Adjustment paths may significantly vary, country by country, depending on policy specifics and structural parameters. Perhaps asymmetric gains of some duration are perceived to exist from policy that is unhelpful from the long run perspective?

[26] In Table 2b, the initial technology level has been set at $A_{10} = 1$. When countries are asymmetric (rows two and three of Tables 1 and 2a), results analogous to Table 2b are obtained.

Example 3 (Tariff Induced Learning Dynamics): A customs union (CU) ($n_1 = 2$) adopts a uniform tariff ($\tau = 1.1$) against the rest of the world (ROW) ($n_2 = 1$). The parameters in Tables 3a and 3b are as in Example 1, rows 1 and 2, respectively; $A_{10} = 1$; $\gamma_p = 0.1$, $i = 1, 2$.[27]

TABLE 3: Initial Parameters for Short Run Dynamics

Table 3a: Symmetric

	CU	ROW
$g^{FT}_{i,t-1}$	1.112	1.112
$U^{FT}_{i,t-1}$	6.864	6.864
$\theta^{FT}_{(t-1)}$	1	1
$\widehat{k}^{FT}_{i,t-1}$	0.034	0.034
g^e_{it}	1.142	1.052
$\widehat{k}^e_{it}$	0.034	0.032

Table 3b: Asymmetric

	CU	ROW
$g^{FT}_{i,t-1}$	1.044	1.044
$U^{FT}_{i,t-1}$	2.335	1.624
$\theta^{FT}_{(t-1)}$	1	0.361
$\widehat{k}^{FT}_{i,t-1}$	0.030	0.030
g^e_{it}	1.074	1.014
$\widehat{k}^e_{it}$	0.030	0.028

We assume that a free trade equilibrium has prevailed up to time period $(t-1)$; the first four rows of Tables 3 state the corresponding growth rate, long run utility, relative allocation of innovation, and the relative profitability of innovation (see Table 1). Between time periods $(t-1)$ and t, the change in trade barriers is announced and implemented. We treat this change as being partly anticipated by economic agents, i.e., aspects of the new policy were publicly proposed and discussed, and this leads to some immediate changes in expectations.[28] The fifth rows of Tables 3 give the new growth expectations for time period t: the customs union is expected to grow faster ($g^e_{1t} > g^{FT}$), whereas a slowdown is expected in the rest of the world ($g^e_{2t} < g^{FT}$). While the direction of these expectations can be justified by appealing to the anticipated reallocation of investment and innovation from the rest

[27] The gain parameter is larger than observed in empirical studies where $\gamma_p \in (0.02, 0.05)$ (e.g., Evans, Honkapohja and Mitra (2009), p. 937). This is because the model does not include random shocks which would be filtered out by agents, justifying lower values of γ_p.

[28] Shifts in initial expectations that are due to changes in policy have appeared in macroeconomic literature. Feldstein (1982) emphasizes changes in expectations that may be caused by fiscal policy. Eggertsson (2008) argues that recovery from the Great Depression was driven by a large shift in expectations caused by President Roosevelt's monetary and fiscal policies. Evans, Honkapohja and Mitra (2009) analyze learning dynamics arising from anticipated policy changes in cases where the structure of the change is fully known in advance.

of the world toward the customs union, the particular values of g_{it}^e can be thought of as imprecise guesses about future growth that producers of capital goods may form based on rough projections of output (see the Appendix for the calculations). Expectations about the relative profitability of innovation (last row of Tables 3) do not need to be consistent with growth expectations but we have chosen approximate values using the given g_{it}^e in (31) and (29) for $i = 1, 2$. In Table 3a, $\widehat{k}_{1t}^e/\widehat{k}_{2t}^e = 1.065$ showing an expected rise in relative profitability of innovation in the customs union following the imposition of the tariff. In Table 3b, $\widehat{k}_{1t}^e/\widehat{k}_{2t}^e = 1.071$.

FIGURES 3 HERE

Figures 3 show the features of the short run adjustment paths that are most interesting (solid line = CU, thin line = ROW, dotted line = free trade).[29] Whether the structural parameters are symmetric (Fig. 3a, i)) or asymmetric (Fig. 3b, i)), the customs union experiences an acceleration in growth while growth in the rest of world slows down significantly; over time, both approach the new balanced growth state (see Table 2) but fast growth in the customs union, exceeding the growth rate under free trade, persists for many time periods. During the transition, innovation and investment expand in the customs union (θ_t declines while the relative capital stock Z_1/Z_2 increases). There is an initial dip in the consumption level of the customs union (Figures 3a, ii) and 3b, ii)) that is due to higher investment but, because of fast growth and expanding imports from the rest of the world, consumption eventually attains and exceeds the free trade level. Consumption in the rest of the world expands over time because local investment spending ebbs.

Figures 3a, iii) and 3b, iii) aggregate the impact on growth and consumption using utility as perceived in a location at each point in time.[30] The utility levels are calculated using equation (25) where consumption (C_{it}) and growth (g_{it}) as realized at temporary equilibria are substituted in ($i = 1, 2$). This method of determining aggregate well-being in the short run is consistent with the learning dynamics formulation: at every point in time, the current temporary equilibrium is viewed by decision makers as a new balanced growth state and the corresponding utility index is given in (25).[31] We

[29] In Figs. 3, the change in tariffs occurs between time periods 0 and 1.

[30] This notion of anticipated utility was suggested by Kreps (1995) and is further analyzed, for example, in Cogley and Sargent (2008).

[31] Utility along the free trade path is also obtained from (25): g_z is given by the free trade equilibrium and $C_{it} = g_z C_{i,t-1}$ for all t.

obtain

Result 3: *Along the transition path, countries that restrict trade in capital goods can experience both higher growth and higher perceived utility than under free trade.*

When all countries are initially symmetric (Fig. 3a, iii)), the gains for the customs union persist for sixteen time periods but, under initial asymmetry (Fig. 3b, iii)), the perceived utility of the customs union never dips below the free trade path even after one hundred and forty time periods. These utility comparisons differ from Table 2b where each utility number is calculated once, at the beginning of an equilibrium path. In Figs. 3, every temporary equilibrium (at each t) defines a new starting point for a utility evaluation, yielding a path of perceived utility over time.

The perceived utility paths of the customs union and the rest of the world emphasize the asymmetric impact of the tariff in different locations. When structural symmetry is assumed, two income clubs quickly emerge with very different transition paths. While the customs union does better in utility terms than under free trade, the rest of the world suffers a large and prolonged loss, both with respect to the free trade path and in comparison to the customs union. Even when growth equalizes in the long run, differences in level variables remain and convert to long-lasting relative gains for the customs union (see column U_1/U_2 in Tables 1 and 2a). On the other hand, if the rest of the world begins at a disadvantage, slow growth there exacerbates the difference in aggregate utility levels. Comparison of Figs. 3a, iii) and 3b, iii) yields

Result 4: *Gains in perceived utility (over free trade) persist longer when the tariff is imposed by countries that have an advantage in market size, productivity, or innovation costs (larger* $\widehat{L}_i$*, lower* υ_i*).*

Interestingly, the loss for the rest of the world, when compared to its free trade (utility) path, is less severe in Fig. 3b, iii). This is because, under the asymmetry assumption, innovation is more expensive there. Given this, the tariff induced reallocation of innovation toward the customs union makes it possible for consumption level in the rest of the world to rise faster, alleviating the impact on local utility.

The previous example demonstrates that, in a growth model, there can exist perceived gains of some duration that are due to barriers on capital goods trade. These gains appear on the transition path toward a new steady state as growth expectations are altered in view of the new policy. The perceived gains may serve as a basis of policy making by shortsighted decision makers for whom economic growth is important (because it largely determines utility). Such policy makers perceive that a tariff against imports in the innovating, growth inducing sector affects producers' growth expectations, leading to faster local expansion at the subsequent temporary equilibria. With initial perceptions so confirmed, learning dynamics can create a self-referential cycle of local optimism, observed faster growth, and sustained positive expectations that lasts for some time.

The duration and size of the utility gains to the customs union are of special interest. In determining these, the initial reactions in expectations that follow a tariff proposal, the subsequent adjustment of expectations, the nature and size of the change in the trade barriers, and the structural parameters of the country economies all play a role. Figure 2 of Section 3 can be used to understand the relative effects.

First, as to expectations, we can state[32]

Result 5: *(i) The larger the initial change in expectations (g_{it}^e), the larger the effect on observed growth and perceived utility in the short run,* ceteris paribus, *but expected changes in relative profitability of innovation ($\widehat{k}_{it}^e$) also matter. (ii) The slower the pace of learning (the smaller the gain parameter γ_p), the larger and longer lasting the perceived gains for the customs union.*

In Fig. 2, high growth expectations (large g_{it}^e vs. g_z^*) raise the rate of observed growth in the next time period (g_{it}). However, a tariff is likely to reduce the innovation share of other countries (lowering θ_t) and this may work toward lower expected profitability ($\widehat{k}_{it}^e$) everywhere, shifting down $TT(\widehat{k}_{it}^e)$ and slowing down growth at temporary equilibria. On the other hand, expected profitability of innovation may rise in the customs union because the relative price of capital goods, R_{1t}/R_{2t}, will increase with higher local growth. Thus, for countries in the customs union, expectations about local growth and future profitability may work in the same or opposite directions depending on the relative importance of innovation in the rest of the world (for countries in the rest of the world, both expectations work toward lower

[32] The numerical examples that support the subsequent results are described in a technical appendix that is available from the authors upon request.

growth). If the rest of the world is smaller and/or less innovative, any adverse effects arising from the tariff are less likely to affect expectations in the customs union, raising the growth spike there.

The gain parameters of the learning dynamics are important in shaping the transition. E.g., if gain parameters are reduced by half, the number of time periods over which the customs union enjoys faster growth and higher perceived utility nearly triples (assuming symmetry). If the gain parameters are smaller in the rest of the world, the period of disproportionate gains for the customs union is somewhat shorter.[33]

Two aspects of the tariff policy matter:

Result 6: (*i) The perceived gains from the tariff decline as the tariff rate increases,* ceteris paribus, *but, because initial growth expectations may also change and structural asymmetries play a role, the overall effect is uncertain. (ii) For the same set of initial expectations, a country gains more in perceived utility by imposing a tariff as a member of a customs union than by choosing the tariff unilaterally.*

In Fig. 2, a higher tariff tends to shift down curves $TT(\widehat{k_{it}^e})$ and this lowers observed growth. The negative effect works through changes in expectations about profitability of innovation (θ_{it} is likely to decline, reducing $S_i(.)$ and $\widehat{k_{it}^e}$). But, these negative effects may be counterbalanced by higher growth expectations (a higher tariff is likely to cause a larger reallocation of innovation and investment in favor of the countries with the tariff) and in such a case, a higher tariff barrier may increase gains for the customs union ($\widehat{k_{1t}^e}$ may also rise because R_{1t}/R_{2t} can be expected to increase). Structural asymmetries create additional room for countries with an advantage to raise tariffs.[34] Experiments suggest that, when the rest of the world is smaller with higher innovation costs, gains in perceived utility persist in the customs union for many periods even when trade barriers are raised but disappear more quickly if structural symmetry is assumed.

Membership in a customs union creates larger perceived gains because of the larger distortion of trade that occurs in the case of a unilateral tariff

[33]The magnitude of the gain parameters has been empirically linked to population income (Pfajfar and Santoro (2008)). This suggests applying lower gain parameters in the rest of the world if this area is assumed to be smaller and less productive.

[34]Recall that perceived utility gains persist longer when the tariff is imposed by countries that have an advantage in market size, productivity or innovation costs (Result 4).

(within a customs union, instead of all imports of capital goods being affected by the tariff, only those from nonmember countries are). Being a member of a customs union may also help in that initial expectations about profitability of innovation are more positive when the rest of the world is smaller.

On the other hand, if a unilateral tariff is imposed by a country that is at a structural disadvantage (small, less productive, with high innovation costs), any perceived utility gains are likely to be very short-lived and small. In experiments, even a strong initial acceleration in growth cannot significantly raise perceived utility if the high costs of innovation necessitate a large sacrifice in local consumption. Among structural parameters, technological complementarity of capital goods supports profitability of all investment and creates an incentive for new innovation to be synchronized. We observe that when initial expectations are kept fixed, the gain in perceived utility is the larger and lasts the longer the higher the degree of complementarity between capital inputs.

Because aggregate utility in a growth model is determined by the level of consumption and the rate at which consumption possibilities expand, the above results are not based on domestic reallocation of production and changes in the distribution of income (Stolper and Samuelson effects), nor are changes in the terms of trade central (the optimal tariff argument).[35] Rather, utility gains are created along the dynamic transition path that includes a reallocation of new innovation and investment toward countries that restrict imports of intermediate capital goods. Tariffs raise local growth expectations and accelerate growth, thus supporting self-referential cycles of positive expectations and high growth.

5 Conclusions

We show that transition dynamics can create asymmetric transitional growth and utility gains in countries that choose to restrict trade in capital goods. The magnitude and duration of these gains depend on many factors and their existence is not guaranteed. Gains are more likely to exist if growth

[35] Ossa (2009) has observed that a tariff in an imperfectly competitive (static) model can be motivated by a relocation effect (a utility increase that occurs when the variety of domestically produced (differentiated) consumption goods that are not subject to the tariff increases). The relocation effect does not exist in the present model because production of capital goods only occurs in the location of their invention.

expectations react strongly to the tariff and if learning dynamics are not fast. Gains are also more likely when the tariff distortion affects a smaller fraction of trade in capital goods, i.e., when the tariff is imposed by a customs union (with free internal trade) or if the tariff is used by a large and productive economy with low innovation costs against a smaller and less productive rest of the world. Negative effects of a tariff are reduced by complementarity of capital goods.

Transition paths of different countries are not alike. In all experiments and even assuming complete structural symmetry, a modest tariff against capital exports of the rest of the world results in an immediate and relatively large reduction in local growth and utility in rest of the world that persists longer than gains to the countries with the tariff. If structural asymmetries are present, disadvantages of the rest of the world are widened. Somewhat disturbingly, the incentive for a tariff to be imposed appears to be the stronger the larger the asymmetries in favor of the countries that would limit trade. In the long run, the pace of growth equalizes but, even assuming initial symmetry, this does not remove differences in level variables that remain and convert to long-lasting relative utility losses for the rest of the world.

We have assumed that the basic technology parameters of all countries remain the same over time. If, for example, the local cost of innovation were to rise as less new innovation takes place in the rest of the world (due to some learning by doing effects that we have not modeled), then the impact of tariffs could be worse in these countries (the world as a whole would grow slower in the long run). On the other hand, if organizational innovation or diffusion of such organization were to occur, productivity and cost differences could decline over time, allowing for more complete convergence in level outcomes.[36]

There is also the possibility that the adjustment to the long run steady state may include a large downward jump in growth and well-being (HTR (2002)). But irrespective of the magnitude of the eventual effect, results indicate that patterns of trade in capital goods can be a determinant of the aggregate growth state in the world economy. Fluctuations in trade policy may cause variations in growth that include both asymmetric short run effects and long run changes that conform to the usual intuition.

[36]Trade restrictions may slow such diffusion and if differences in social infrastructure explain initial asymmetries, convergence of parameters may be very slow.

6 Appendix

Lemma A.1: *Let countries* $i = 1, 2, ..., n_1$ $(n_1 + 1, ...N)$ *be symmetric with parameters* $(\widehat{L}_1, \upsilon_1, c_1)$ $((\widehat{L}_2, \upsilon_2, c_2))$. *Let the first* n_1 *countries impose a common tariff* τ (≥ 1) *against capital goods from the rest of the world. Then, for any set of parameters and tariff, there is a unique (positive) solution* $\theta_i \equiv \theta,\ i = n_1 + 1, ..., N$, *for equations* (21).

Proof: By (9) and (13) at a steady state,

$$k_1 = n_1 \widehat{L}_1 S_1^{\xi} + n_2 \widehat{L}_2 S_2^{\xi}\ (c_1/c_2)^{\frac{\gamma\xi}{1-\gamma}}, \tag{41}$$

$$k_2 = n_1 \widehat{L}_1 S_1^{\xi} \tau^{\frac{1}{\gamma-1}} (c_2/c_1)^{\frac{\gamma\xi}{1-\gamma}} + n_2 \widehat{L}_2 S_2^{\xi}\ , \tag{42}$$

$$S_1 = n_1 + n_2 \theta (\tau c_2/c_1)^{\frac{\gamma}{\gamma-1}}, \quad S_2 = n_1 (c_1/c_2)^{\frac{\gamma}{\gamma-1}} + n_2 \theta. \tag{43}$$

Given (41)-(43), (21) yields

$$f(\theta) g(\theta) = \frac{n_2 \widehat{L}_2}{n_1 \widehat{L}_1}, \tag{44}$$

$$f(\theta) \equiv \left(\frac{S_1}{S_2} \right)^{\xi} = \left(\frac{n_1 + n_2 \theta (\tau c_2/c_1)^{\frac{\gamma}{\gamma-1}}}{n_1 (c_1/c_2)^{\frac{\gamma}{\gamma-1}} + n_2 \theta} \right)^{\xi} \ (> 0), \tag{45}$$

$$g(\theta) \equiv \frac{M\theta^{\xi} - \tau^{\frac{1}{\gamma-1}} (c_2/c_1)^{\frac{\gamma\xi}{1-\gamma}}}{1 - M\theta^{\xi} (c_1/c_2)^{\frac{\gamma\xi}{1-\gamma}}}, \ M \equiv (c_1/c_2)^{\frac{\alpha}{\alpha-1}} (\upsilon_2/\upsilon_1)^{\varsigma}. \tag{46}$$

Because $f(\theta) > 0$ and the right-hand side of (44) is positive, $g(\theta) > 0$ and so

$$0 < \tau^{\frac{1}{\gamma-1}} (c_2/c_1)^{\frac{\gamma}{\gamma-1}} (\upsilon_1/\upsilon_2)^{\zeta} < \theta^{\xi} < (c_2/c_1)^{\frac{\gamma}{\gamma-1}} (\upsilon_1/\upsilon_2)^{\zeta}. \tag{47}$$

At the lower limit of this interval, $f(\theta)g(\theta) = 0$, and $f(\theta)g(\theta)$ grows arbitrarily large as θ approaches the upper limit. By continuity, there is a $\theta > 0$ that solves (44).

If $f(\theta)g(\theta)$ is monotonic in interval (47), the solution is unique. Differentiation yields

$$\frac{df(\theta)}{d\theta} = \xi \left(\frac{S_1}{S_2} \right)^{\xi - 1} \frac{n_1 n_2 (\tau^{\frac{\gamma}{\gamma-1}} - 1)}{S_2^2} < 0, \ \frac{d^2 f(\theta)}{d\theta^2} > 0, \tag{48}$$

$$\frac{dg(\theta)}{d\theta} = \frac{\xi\theta^{\xi-1}M(1-\tau^{\frac{1}{\gamma-1}})}{\left(1-M\theta^{\xi}(c_1/c_2)^{\frac{\gamma\xi}{1-\gamma}}\right)^2} > 0, \ \frac{d^2g(\theta)}{d\theta^2} > 0. \tag{49}$$

Thus, $f(\theta)$ is decreasing in θ but at a decreasing rate, and $g(\theta)$ increases at an increasing rate ($g'(\theta)$ becoming arbitrarily large as θ approaches its upper limit). When θ takes its smallest value in (47),

$$f'(\theta)g(\theta) + f(\theta)g'(\theta) = \xi\left(\frac{S_1}{S_2}\right)^{\xi-1}\left[\frac{1}{S_2\left(1-M\theta^{\xi}(c_1/c_2)^{\frac{\gamma\xi}{1-\gamma}}\right)}\right] * \tag{50}$$

$$\left[\frac{n_1n_2(\tau^{\frac{\gamma}{\gamma-1}}-1)\left(M\theta^{\xi}-\tau^{\frac{1}{\gamma-1}}(c_2/c_1)^{\frac{\gamma\xi}{1-\gamma}}\right)}{S_2} + \frac{M\theta^{\xi-1}S_1(1-\tau^{\frac{1}{\gamma-1}})}{1-M\theta^{\xi}(c_1/c_2)^{\frac{\gamma\xi}{1-\gamma}}}\right] > 0,$$

i.e., $f(\theta)g(\theta)$ is increasing. For larger values of θ, $f(\theta)g(\theta)$ remains increasing because $f''(\theta) > 0$ so that $f(\theta)$ decreases at a lower rate than at the lower limit for θ but $g(\theta)$ grows at an ever faster rate as θ increases. ■

Slope of the TT curve: Given the parameters and θ_i, $i = 2, ..., N$, differentiation of equations (18) and (20) yields:

$$\frac{dr(g_z)}{dg_z} = \frac{1+\mathcal{CD}\Gamma''}{\mathcal{C}(1-\mathcal{E})}, \tag{51}$$

$$\mathcal{C} \equiv \left(\frac{1+\xi}{\xi}\right)\left[1+r-\Omega\widehat{k}_1(.)r^{\frac{\alpha}{\alpha-1}}(p^z)^{\frac{1}{\alpha-1}}\right]^{\frac{1}{\xi}} \ (>0), \quad \frac{1+\xi}{\xi} = \frac{\phi-\alpha}{\phi-1}, \tag{52}$$

$$\mathcal{E} \equiv \frac{\widehat{k}_1(.)\alpha\Omega}{\alpha-1}r^{\frac{1}{\alpha-1}}(p^z)^{\frac{1}{\alpha-1}} \ (<0), \tag{53}$$

$$\mathcal{D} \equiv \frac{\Omega\widehat{k}_1(.)}{\alpha-1}r^{\frac{\alpha}{\alpha-1}}(p^z)^{\frac{2-\alpha}{\alpha-1}} \ (<0), \tag{54}$$

The TT curve can have a negative slope if the numerator in (51) is negative ($\Gamma'' > 0$ is large relative to ϕ). See HTR (2002: p. 502-503 and the Appendix) for a further discussion.

Fixed points of (37)-(39) satisfy (18)-(20): At a fixed point,

$$g^e_{it} = \left[\beta(1+r_{it}(g^e_{it}, \widehat{k}^e_{it})\right]^{1/\sigma} = g_{it}, \ i = 1, ..., N, \tag{55}$$

$$\widehat{k}^e_{it} = K_i(g^e_t, \widehat{k}^e_{it}; \theta_{t-1}; \tau), \quad i = 1, ..., N, \tag{56}$$

$$\theta_{i,t+1} = \theta_{i,t} \left[\frac{G_i(g^e_{it}, \widehat{k}^e_{it})}{G_1(g^e_{1t}, \widehat{k}^e_{1t})} \right]^{1/(1+\xi)} = \theta_{i,t} \left[\frac{g_{it}}{g_{1t}} \right]^{1/(1+\xi)}, \tag{57}$$

where $r_{it}(g^e_{it}, \widehat{k}^e_{it})$ are obtained using (26) - (27). Equations (55)-(56) imply that all realized g_{it} and $\widehat{k}_{it}$ are equal to their respective expected values. Thus, by (37)-(38), all expectations remain unchanged and so do the realized values of g_{it} and $\widehat{k}_{it}$. But, if none of these variables can change, none of θ_{it} can change and then $g_{it} = g_z$ for all i and, by (55), $r_{it} = r_t$ for all i; finally, by (26), $\widehat{k}_i(.) = \widehat{k}_1(.)$ which determines the equilibrium values of θ_t.

Proof of Proposition 1: Assume that there are two learners (a customs union and the rest of the world). The customs union (U) consists of n_1 symmetric countries ($i = 1, 2, ..., n_1$) and imposes a uniform tariff τ (> 1) against imported capital goods from the rest of the world (R) (n_2 ($\equiv N - n_1$) symmetric countries); trade within U is free and countries R impose no trade interventions.

Given (g^e_{1t}, g^e_{2t}) and ($\widehat{k}^e_{1t}, \widehat{k}^e_{2t}$) and using (37)-(39), we obtain the difference equation system

$$g^e_{1,t+1} = g^e_{1t} + \gamma_p(\left[\beta(1 + r_{1t}(g^e_{1t}, \widehat{k}^e_{1t})\right]^{1/\sigma} - g^e_{1t}), \tag{58}$$

$$g^e_{2,t+1} = g^e_{2t} + \gamma_p(\left[\beta(1 + r_{2t}(g^e_{2t}, \widehat{k}^e_{2t})\right]^{1/\sigma} - g^e_{2t}), \tag{59}$$

$$\widehat{k}^e_{1,t+1} = \widehat{k}^e_{1t} + \gamma_p \left[K_1(g^e_t, \widehat{k}^e_t; \theta_t; \tau, L, \upsilon, c) - \widehat{k}^e_{1t}\right], \tag{60}$$

$$\widehat{k}^e_{2,t+1} = \widehat{k}^e_{2t} + \gamma_p \left[K_2(g^e_t, \widehat{k}^e_t; \theta_t; \tau, L, \upsilon, c) - \widehat{k}^e_{2t}\right], \tag{61}$$

$$\theta_{t+1} = \theta_t \left(\frac{1 + r_{2t}(g^e_{2t}, \widehat{k}^e_{2t})}{1 + r_{1t}(g^e_{1t}, \widehat{k}^e_{1t})} \right)^{\frac{1}{\sigma(1+\xi)}}. \tag{62}$$

As the model is non-stochastic, this system can be analyzed a vector difference equation, and local stability of a steady state can be examined using its linearization.

Before proceeding to linearization, we note the following explicit structure (dropping constant parameters from argument, changing arguments to scalar notation and dropping time subscripts):

$$
\begin{aligned}
K_1(g_1^e, g_2^e, \widehat{k}_1^e, \widehat{k}_2^e, \theta) &= N_1[n_1 \widehat{L}_1 S_1^{\xi} + n_2 \widehat{L}_2 S_2^{\xi}\ (R_1/R_2)^{\frac{\gamma\xi}{1-\gamma}}] && (63)\\
K_2(g_1^e, g_2^e, \widehat{k}_1^e, \widehat{k}_2^e, \theta) &= \frac{N_2}{\theta^{\xi}}[n_1 \widehat{L}_1 S_1^{\xi} \tau^{\frac{1}{\gamma-1}} (R_1/R_2)^{\frac{\gamma\xi}{\gamma-1}} + n_2 \widehat{L}_2 S_2^{\xi}], && (64)
\end{aligned}
$$

where $N_i = c_i^{\alpha/(\alpha-1)} v_i^{-1}$. In addition,

$$
\begin{aligned}
S_1 &= n_1 + n_2\theta \left(\frac{R_1}{R_2}\right)^{\frac{\gamma}{1-\gamma}} \tau^{\frac{\gamma}{\gamma-1}}, && (65)\\
S_2 &= n_1 \left(\frac{R_1}{R_2}\right)^{\frac{\gamma}{\gamma-1}} + n_2\theta. && (66)
\end{aligned}
$$

We have the derivatives

$$
\begin{aligned}
\frac{\partial S_1}{\partial(R_1/R_2)} &= \theta n_2 \left(\frac{\gamma}{1-\gamma}\right) \left(\frac{R_1}{R_2}\right)^{\frac{\gamma}{1-\gamma}-1} \tau^{\frac{\gamma}{\gamma-1}} > 0, && (67)\\
\frac{\partial S_2}{\partial(R_1/R_2)} &= n_1 \left(\frac{\gamma}{\gamma-1}\right) \left(\frac{R_1}{R_2}\right)^{\frac{1}{\gamma-1}-1} < 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial S_1}{\partial\theta} &= n_2 \left(\frac{R_1}{R_2}\right)^{\frac{\gamma}{1-\gamma}} \tau^{\frac{\gamma}{\gamma-1}} > 0, && (68)\\
\frac{\partial S_2}{\partial\theta} &= n_2 > 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial(R_1/R_2)}{\partial g_1^e} &= \frac{1}{R_2}\frac{\partial R_1}{\partial g_1^e} = \frac{c_1}{\gamma R_2}\left[r^*\Gamma'' \mathcal{A}^* + \Gamma'\right] \frac{\partial r_1}{\partial g_1^e}, && (69)\\
\frac{\partial(R_1/R_2)}{\partial \widehat{k}_1^e} &= \frac{c_1}{\gamma R_2}\left[r^*\Gamma'' \mathcal{A}^* + \Gamma'\right] \frac{\partial r_1}{\partial \widehat{k}_1^e} > 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial(R_1/R_2)}{\partial g_2^e} &= -\left(\frac{R_1}{R_2}\right) \frac{c_2}{\gamma R_2}\left[r^*\Gamma'' \mathcal{A}^* + \Gamma'\right] \frac{\partial r_2}{\partial g_2^e}, && (70)\\
\frac{\partial(R_1/R_2)}{\partial \widehat{k}_2^e} &= -\left(\frac{R_1}{R_2}\right) \frac{c_2}{\gamma R_2}\left[r^*\Gamma'' \mathcal{A}^* + \Gamma'\right] \frac{\partial r_2}{\partial \widehat{k}_2^e} < 0,
\end{aligned}
$$

where $\mathcal{A}^*$ is defined below in (81). Differentiating (26) we obtain, at a given fixed point,

$$\frac{\partial r_{1t}}{\partial g_{1t}^e} = \frac{\partial r_{2t}}{\partial g_{2t}^e} = \frac{1+\mathcal{C}\mathcal{D}\Gamma''}{\mathcal{C}(1-\mathcal{E})}, \tag{71}$$

$$\frac{\partial r_{1t}}{\partial \widehat{k}_{1t}^e} = \frac{\partial r_{2t}}{\partial \widehat{k}_{2t}^e} = \frac{\mathcal{C}\mathcal{F}}{\mathcal{C}(1-\mathcal{E})} > 0, \tag{72}$$

with terms $\mathcal{C}$, $\mathcal{E}$, and $\mathcal{D}$ defined in (52)-(54) and

$$\mathcal{F} \equiv \ \Omega r^{\frac{\alpha}{\alpha-1}} (p^z)^{\frac{1}{\alpha-1}} \quad (>0). \tag{73}$$

The sign of $\partial r_{it}/\partial g_{it}^e$ is positive if $\Gamma''=0$ but can be negative if $\mathcal{C}\mathcal{D}\Gamma''$ in (71) is large in absolute value.

Next, we obtain

$$\begin{aligned} \frac{\partial K_1}{\partial(R_1/R_2)} \quad &= \quad N_1 \left\{ n_1 \widehat{L}_1 \xi S_1^{\xi-1} \frac{\partial S_1}{\partial(R_1/R_2)} + \right. \\ & \left. n_2 \widehat{L}_2 \left[\xi S_2^{\xi-1} (\frac{R_1}{R_2})^{\frac{\gamma\xi}{1-\gamma}} \frac{\partial S_2}{\partial(R_1/R_2)} + S_2^{\xi} \left(\frac{\gamma\xi}{1-\gamma} \right) (\frac{R_1}{R_2})^{\frac{\gamma\xi}{1-\gamma}-1} \right] \right\} > 0, \end{aligned} \tag{74}$$

$$\frac{\partial K_1}{\partial \theta} = N_1 \left[n_1 \widehat{L}_1 \xi S_1^{\xi-1} \frac{\partial S_1}{\partial \theta} + n_2 \widehat{L}_2 \xi S_2^{\xi-1} (R_1/R_2)^{\frac{\gamma\xi}{1-\gamma}} \frac{\partial S_2}{\partial \theta} \right] > 0, \tag{75}$$

$$\begin{aligned} \frac{\partial K_2}{\partial(R_1/R_2)} \quad &= \quad \frac{N_2}{\theta^\xi} \left\{ n_2 \widehat{L}_2 \xi S_2^{\xi-1} \frac{\partial S_2}{\partial(R_1/R_2)} + \right. \\ & \left. n_1 \widehat{L}_1 \tau^{\frac{1}{\gamma-1}} \left[\left(\frac{R_1}{R_2} \right)^{\frac{\gamma\xi}{\gamma-1}} \xi S_1^{\xi-1} \frac{\partial S_1}{\partial(R_1/R_2)} + S_1^{\xi} \left(\frac{\gamma\xi}{\gamma-1} \right) (\frac{R_1}{R_2})^{\frac{\gamma\xi}{\gamma-1}-1} \right] \right\} < 0. \end{aligned} \tag{76}$$

$$\begin{aligned} \frac{\partial K_2}{\partial \theta} \quad &= \quad \frac{N_2}{\theta^\xi} \left[n_1 \widehat{L}_1 \xi S_1^{\xi-1} \tau^{\frac{1}{\gamma-1}} (\frac{R_1}{R_2})^{\frac{\gamma\xi}{\gamma-1}} \frac{\partial S_1}{\partial \theta} + n_2 \widehat{L}_2 \xi S_2^{\xi-1} \frac{\partial S_2}{\partial \theta} \right] - \\ & \frac{N_2 \xi}{\theta^{\xi+1}} [n_1 \widehat{L}_1 S_1^{\xi} \tau^{\frac{1}{\gamma-1}} (R_1/R_2)^{\frac{\gamma\xi}{\gamma-1}} + n_2 \widehat{L}_2 S_2^{\xi}] < 0. \end{aligned} \tag{77}$$

The linearized system, evaluated at a fixed point, equals

$$\begin{pmatrix} g_{1,t+1}^e \\ g_{2,t+1}^e \\ \widehat{k}_{1,t+1}^e \\ \widehat{k}_{2,t+1}^e \\ \theta_{t+1} \end{pmatrix} = \Omega \begin{pmatrix} g_{1t}^e \\ g_{2t}^e \\ \widehat{k}_{1t}^e \\ \widehat{k}_{2t}^e \\ \theta_t \end{pmatrix}, \tag{78}$$

$$\Omega \equiv \begin{pmatrix} \gamma_p(A_g-1)+1 & 0 & \gamma_p A_k & 0 & 0 \\ 0 & \gamma_p(A_g-1)+1 & 0 & \gamma_p A_k & 0 \\ \gamma_p B_1 & \gamma_p B_2 & \gamma_p B_3+1 & \gamma_p B_4 & \gamma_p B_5 \\ \gamma_p C_1 & \gamma_p C_2 & \gamma_p C_3 & \gamma_p C_4+1 & \gamma_p C_5 \\ D_1 & D_1 & D_2 & D_2 & 1 \end{pmatrix}. \tag{79}$$

The elements of Ω are (superscript $*$ denotes values at the fixed point):

$$A_g = \mathcal{A}^* \frac{\partial r_{1t}}{\partial g_{1t}^e} = \mathcal{A}^* \frac{\partial r_{2t}}{\partial g_{2t}^e}, \quad A_k = \mathcal{A}^* \frac{\partial r_{1t}}{\partial \widehat{k}_{1t}^e} = \mathcal{A}^* \frac{\partial r_{2t}}{\partial \widehat{k}_{2t}^e} > 0, \tag{80}$$

$$\mathcal{A}^* \equiv \frac{1}{\sigma} \beta^{\frac{1}{\sigma}} (1+r^*)^{\frac{1-\sigma}{\sigma}} > 0, \tag{81}$$

$$\begin{aligned} B_1 &= \frac{\partial K_1}{\partial(R_1/R_2)} \frac{\partial(R_1/R_2)}{\partial g_1^e} > 0, \\ B_2 &= \frac{\partial K_1}{\partial(R_1/R_2)} \frac{\partial(R_1/R_2)}{\partial g_2^e} < 0, \\ B_3 &= \frac{\partial K_1}{\partial(R_1/R_2)} \frac{\partial(R_1/R_2)}{\partial \widehat{k}_1^e} - 1, \\ B_4 &= \frac{\partial K_1}{\partial(R_1/R_2)} \frac{\partial(R_1/R_2)}{\partial \widehat{k}_2^e} < 0, \\ B_5 &= \frac{\partial K_1}{\partial \theta} > 0; \end{aligned}$$

$$\begin{aligned} C_1 &= \frac{\partial K_2}{\partial(R_1/R_2)} \frac{\partial(R_1/R_2)}{\partial g_1^e} < 0, \\ C_2 &= \frac{\partial K_2}{\partial(R_1/R_2)} \frac{\partial(R_1/R_2)}{\partial g_2^e} > 0, \\ C_3 &= \frac{\partial K_2}{\partial(R_1/R_2)} \frac{\partial(R_1/R_2)}{\partial \widehat{k}_1^e} < 0, \\ C_4 &= \frac{\partial K_2}{\partial(R_1/R_2)} \frac{\partial(R_1/R_2)}{\partial \widehat{k}_2^e} - 1, \\ C_5 &= \frac{\partial K_2}{\partial \theta} < 0, \end{aligned}$$

$$D_1 = \frac{\theta^*}{\sigma(1+\xi)(1+r^*)} \frac{\partial r_{1t}}{\partial g_{1t}^e} > 0,\ D_2 = \frac{\theta^*}{\sigma(1+\xi)(1+r^*)} \frac{\partial r_{1t}}{\partial \widehat{k}_{1t}^e} > 0. \qquad (82)$$

Next, we obtain some relationships among the elements of Ω. One has

$$\begin{aligned}
B_1 + B_2 &= \frac{\partial K_1}{\partial (R_1/R_2)} \left(\frac{\partial (R_1/R_2)}{\partial g_1^e} + \frac{\partial (R_1/R_2)}{\partial g_2^e} \right) = 0, \\
C_1 + C_2 &= \frac{\partial K_2}{\partial (R_1/R_2)} \left(\frac{\partial (R_1/R_2)}{\partial g_1^e} + \frac{\partial (R_1/R_2)}{\partial g_2^e} \right) = 0, \\
B_3 + 1 + B_4 &= \frac{\partial K_1}{\partial (R_1/R_2)} \left(\frac{\partial (R_1/R_2)}{\partial \widehat{k}_1^e} + \frac{\partial (R_1/R_2)}{\partial \widehat{k}_2^e} \right) = 0, \\
C_3 + C_4 + 1 &= \frac{\partial K_2}{\partial (R_1/R_2)} \left(\frac{\partial (R_1/R_2)}{\partial \widehat{k}_1^e} + \frac{\partial (R_1/R_2)}{\partial \widehat{k}_2^e} \right) = 0,
\end{aligned}$$

and

$$\begin{aligned}
(A_k/A_g)B_1 &= B_3 + 1, \\
(A_k/A_g)C_1 &= C_3.
\end{aligned}$$

Using these relationships, the matrix Ω has two eigenvalues equal to $1 - \gamma_p$, one eigenvalue equal to $1 + (A_g - 1)\gamma_p$ while the two remaining eigenvalues satisfy a quadratic. With $\gamma_p \in (0, 1)$ the first two eigenvalues are inside the unit circle. The third eigenvalue is also inside if and only if

$$A_g - 1 < 0, \qquad (83)$$

or, equivalently,

$$\frac{\partial r_{it}}{\partial g_{it}^e} < \frac{1}{\mathcal{A}^*}, \quad i = 1, 2. \qquad (84)$$

Thus, the necessary condition for stability of (37)-(39) is (40). In (84), $1/\mathcal{A}^*$ gives the slope of curve CC in Fig. 1, whereas $\partial r_{it}/\partial g_{it}^e$ is the slope of the TT curve.■

We remark that the remaining two eigenvalues may or may not be stable, i.e., they may lie inside or outside the unit circle. Therefore, in the numerical examples we have verified convergence of the dynamics directly by simulating the nonlinear difference equation system (58)-(62).

Proof of Proposition 2: It suffices to show that an increase in any tariff on intermediate capital goods reduces $\widehat{k}_1(.)$. By (22) and (13) and at a steady state,

$$\widehat{k}_1 = \left(\frac{c_1^{\frac{\alpha}{\alpha-1}}}{\upsilon_1^{\varsigma}}\right)\left[\widehat{L}_1 S_1^{\xi} + \widehat{L}_2 S_2^{\xi}\tau_{21}^{\frac{1}{\gamma-1}}\left(\frac{c_1}{c_N}\right)^{\frac{\gamma\xi}{(1-\gamma)}} + ... + \widehat{L}_N S_N^{\xi}\tau_{N1}^{\frac{1}{\gamma-1}}\left(\frac{c_1}{c_N}\right)^{\frac{\gamma\xi}{(1-\gamma)}}\right], \tag{85}$$

where S_i are defined in (9). Thus, $\partial\widehat{k}_1/\partial\theta_i > 0$ for all $i = 1, ..., N$ ($\theta_1 \equiv 1$). By (21), technology levels at equilibrium satisfy

$$\theta_i^{\xi} = M_i \frac{\left[\widehat{L}_1 S_1^{\xi}\tau_{1i}^{\frac{1}{\gamma-1}} + \widehat{L}_2 S_2^{\xi}\tau_{2i}^{\frac{1}{\gamma-1}}\left(\frac{c_i}{c_2}\right)^{\frac{\gamma\xi}{(1-\gamma)}} + ... + \widehat{L}_N S_N^{\xi}\tau_{Ni}^{\frac{1}{\gamma-1}}\left(\frac{c_i}{c_N}\right)^{\frac{\gamma\xi}{(1-\gamma)}}\right]}{\left[\widehat{L}_1 S_1^{\xi} + \widehat{L}_2 S_2^{\xi}\tau_{21}^{\frac{1}{\gamma-1}}\left(\frac{c_1}{c_2}\right)^{\frac{\gamma\xi}{(1-\gamma)}} + ... + \widehat{L}_N S_N^{\xi}\tau_{N1}^{\frac{1}{\gamma-1}}\left(\frac{c_1}{c_N}\right)^{\frac{\gamma\xi}{(1-\gamma)}}\right]}, \tag{86}$$

$$M_i \equiv \frac{(c_i^{\frac{\alpha}{\alpha-1}}/\upsilon_i^{\varsigma})}{(c_1^{\frac{\alpha}{\alpha-1}}/\upsilon_1^{\varsigma})}. \tag{87}$$

By (86)-(87), an exogenous increase in any tariff τ_{ji}, $j = 1, ..., N$ $(j \neq i)$, that reduces exports of country i to country j must lower the equilibrium value of θ_i. (For example, if country 1 imposes a uniform tariff against all capital imports and then raises that tariff, all θ_i $(i = 2, ..., N)$ must decline.) Then, since $\partial\widehat{k}_1/\partial\theta_i > 0$ for all i, an increase in any tariff must reduce $\widehat{k}_1(.)$ and therefore, near stable steady states, the long run growth rate must decline.■

Determination of (g_{1t}^e, g_{2t}^e) and $(\widehat{k}_{1t}^e, \widehat{k}_{2t}^e)$: Consider producers in the proposed CU. They have observed x_{11} and x_{21} and $x_{11} = x_{21} = x_{22} = x_{12}$ initially. The total output of each producer in both CU countries equals $2x_{11} + x_{21}$ $(= 2(0.1559) + 0.1559 = 0.4677)$ (when $A_{1t} = 1$). Given (8), the effect of the tariff on imports from the rest of the world (x_{12}) can be approximated by

$$\frac{x_{11}}{x_{12}} = \tau^{\frac{1}{1-\gamma}} = 1.1575, \tag{88}$$

so that $x_{12} \approx 0.1347$ when $\tau = 1.1$. Assuming that domestic production takes the place of imports, $x_{11} \approx 0.1559 + 0.021 = 0.1771$. Exports to the other CU country are not expected to change due to the internal free trade of the

CU. The CU exports to the rest of the world can be approximated (using (8)) by

$$\frac{x_{21}}{x_{11}} = \frac{x_{22}}{x_{11}} = \left(\frac{S_2}{S_1}\right)^{\xi} = \left(\frac{2+\theta}{2+\theta\tau^{\frac{\gamma}{\gamma-1}}}\right)^{\xi} \approx \left(\frac{3}{2+\tau^{\frac{\gamma}{\gamma-1}}}\right)^{\xi} = 1.0033, \qquad (89)$$

keeping θ, p_z and r constant. Thus, the estimated total output of each CU producer equals $0.1771 + 0.1559 + 0.1564 = 0.4894$ which corresponds to a 4.64% increase in the output of capital goods. To guess the impact on growth of aggregate capital (g_z), by (8) and (17), capital goods output grows at rate $g_x = g_A^{\xi} < g_A^{1+\xi} = g_z$ at a steady state. Thus, an initial growth estimate larger than approximately +4% can be sustained.

In the ROW, producers estimate $x_{12} \approx 0.1347$. The total output of capital goods per producer is $0.1559 + 2(0.1347) = 0.4253$ which corresponds to a 9.06% reduction. The growth expectations in Table 3b have been determined analogously. The expected values of $\widehat{k}^e_{1t}$ and $\widehat{k}^e_{2t}$ are obtained using (29) where θ_t, r_{it}, and p^z_{it} are solved using (g^e_{1t}, g^e_{2t}).

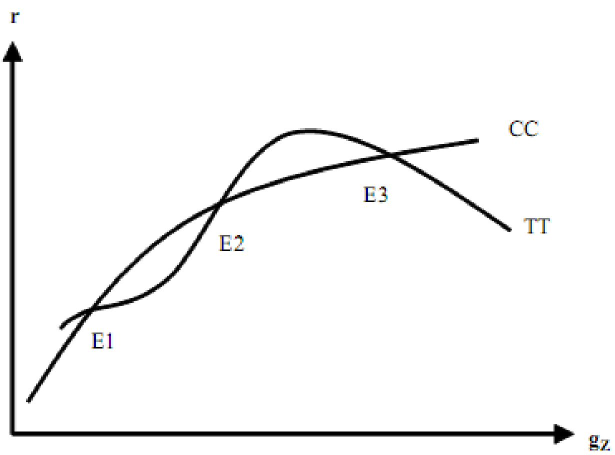

FIGURE 1: Steady States (Long Run Equilibria)

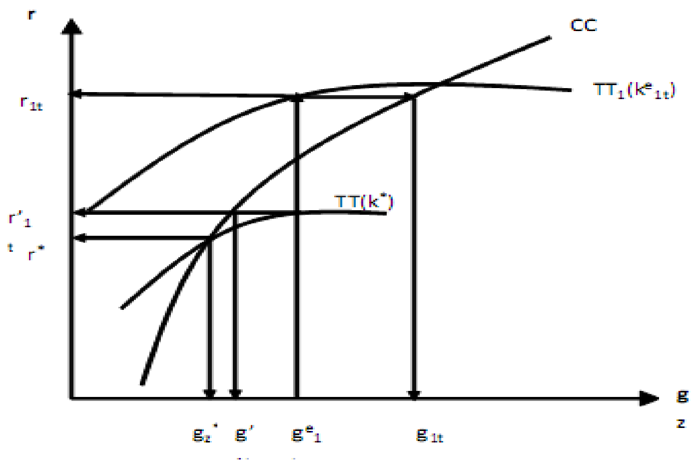

FIGURE 2: Short Run Adjustment and Expectations

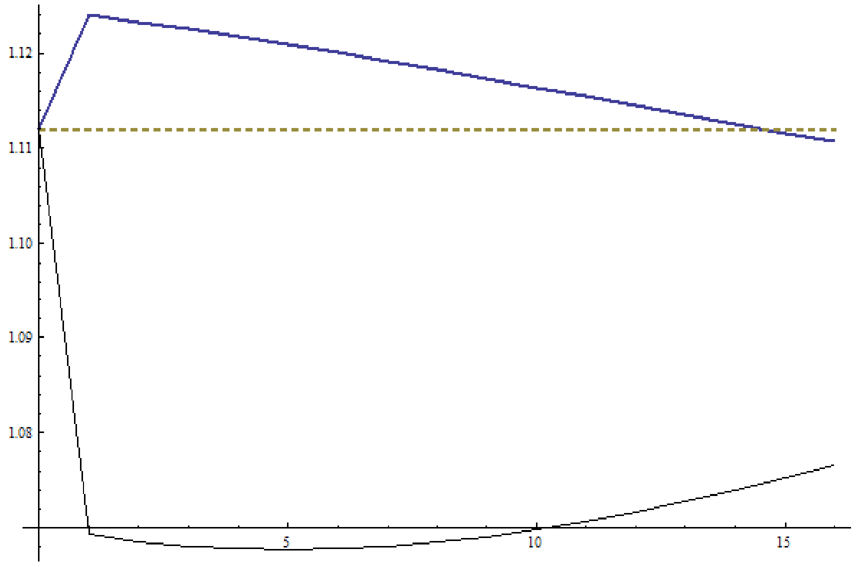

FIGURE 3a, i): Growth in the Customs Union and the Rest of the World Compared to Free Trade

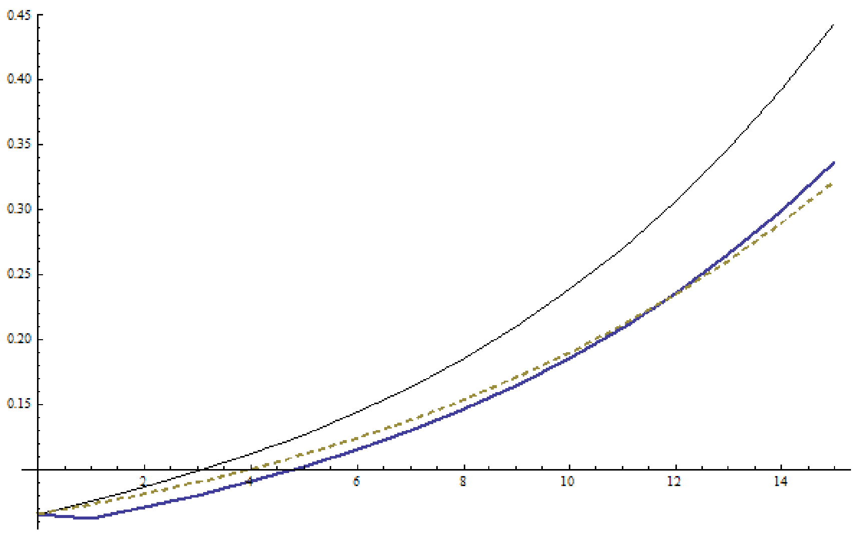

FIGURE 3a, ii): Consumption in the Customs Union and the Rest of the World Compared to Free Trade

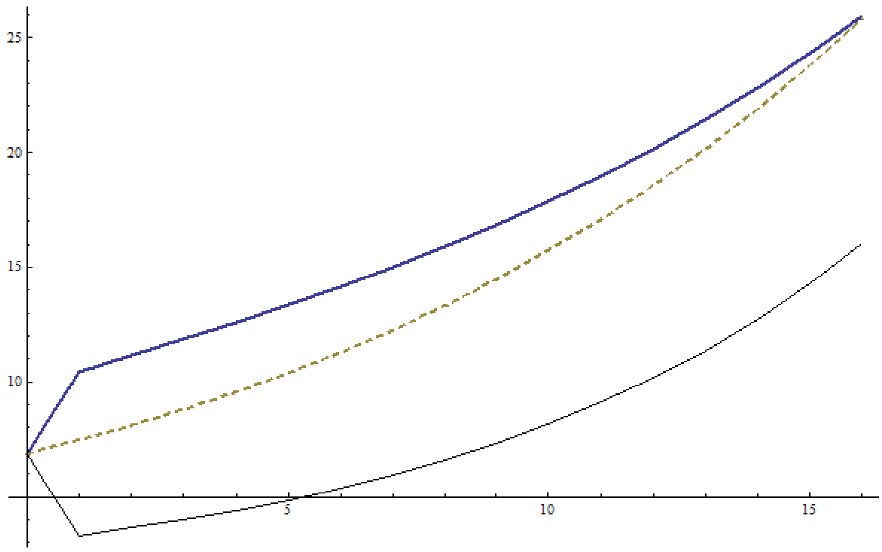

FIGURE 3a, iii): Perceived Utility in the Customs Union and the Rest of the World Compared to Free Trade

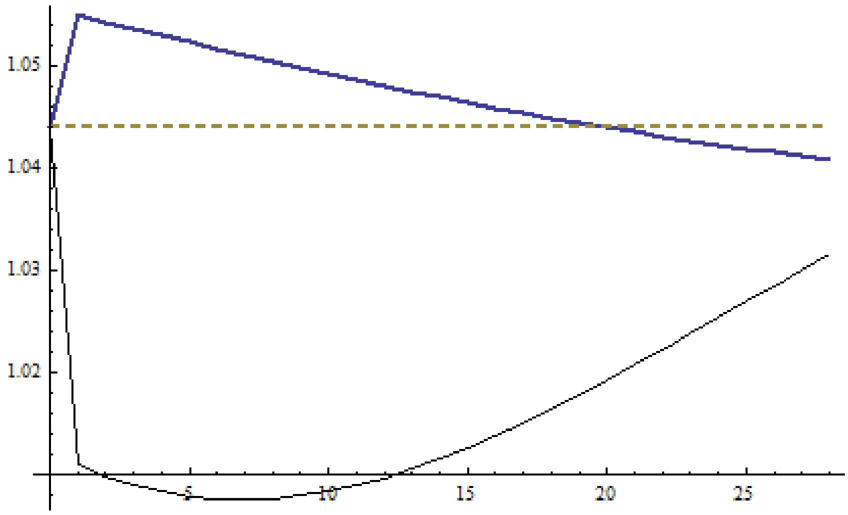

FIGURE 3b, i): Growth in the Customs Union and the Rest of the World Compared to Free Trade

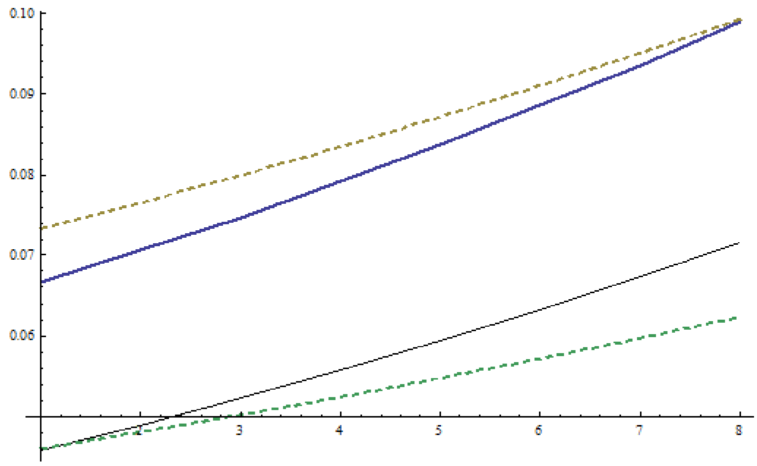

FIGURE 3b, ii): Consumption in the Customs Union and the Rest of the Word Compared to Free Trade

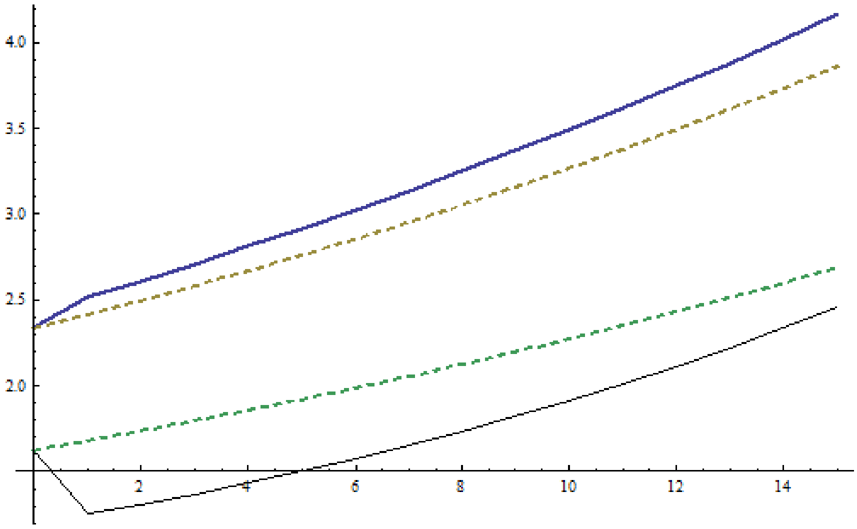

FIGURE 3b, iii): Utility in the Customs Union and the Rest of the World Compared to Free Trade

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