CYCLES AND INDETERMINACY IN OVERLAPPING GENERATIONS ECONOMIES WITH STONE-GEARY PREFERENCES

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Abstract

We investigate dynamics in an overlapping generations economy with Stone-Geary preferences. We show that a steady state exists, and furthermore and importantly, that there can be a multitude of two cycles even though intertemporal elasticity of substitution in consumption exceeds unity.

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1. Introduction

Necessities of consumption characterize a large set of goods. They are a key part of the linear expenditure system, which can be derived from the Stone-Geary preferences originally developed by Geary (1950-51). Stone (1954) utilized them in empirical work. These preferences have been used also in theoretical work (see e.g. Auerbach and Hines, 2002, and Azariadis, 1996).

Consumer with the Stone-Geary preferences gets utility from that part of consumption, which exceeds the subsistence level. These preferences are closely related to preferences with habit persistence. Lahiri and Puhakka (1998) have studied the implications of these preferences in an overlapping generations (OG) model with pure exchange. Among other things they showed that the possibility for cycles, with relatively weaker concavity of the utility function than in conventional specifications.

Koskela and Puhakka (2006) introduced logarithmic Stone-Geary preferences into a standard overlapping generations economy with pure exchange, and studied stability, indeterminacy and cycles. They showed that dynamics depend on the relative necessities of current and future consumption. In particular, a stable, and at the same time indeterminate, nontrivial steady state exists for parameter values, for which there is no such equilibrium in the model with purely logarithmic utility function. Moreover, the features of saving behavior lead to the possibility of period-doubling bifurcation (a two-cycle).

In this paper we generalize the model in Koskela and Puhakka (2006). Their results also hold with more general Stone-Geary preferences with an important exception that there are a multitude of two cycles although intertemporal elasticity of

substitution exceeds unity. As shown first by Grandmont (1985) (see also Gale, 1973) an overlapping generations economy with pure exchange can exhibit a stable monetary steady states and cycles, if intertemporal elasticity of substitution is smaller than unity.

Section 2 specifies the more general Stone-Geary utility function and presents its implications for saving behavior. Section 3 explores the dynamic properties of equilibrium. There is also a brief concluding section.

2. General Stone-Geary utility function and saving behavior

We analyze the dynamic implications of a perfect foresight overlapping generations economy under pure exchange, where consumers live for two periods. The general Stone-Geary preferences of the consumer are

$$(1) \qquad v(c_1^t, c_2^t) = \frac{(c_1^t - a_1)^{1-\sigma}}{1-\sigma} + \beta \frac{(c_2^t - a_2)^{1-\sigma}}{1-\sigma},$$

where $1/\sigma$ is the intertemporal elasticity of substitution calculated for consumption over the subsistence level, $c - a$,[1] β is the discount factor, and a_i is the exogenous level of subsistence. The measure of concavity of the periodic utility function is $-cu''/u' = \sigma c/(c-a)$. It is decreasing in consumption and increasing in subsistence level.

A representative consumer maximizes (1) subject to the periodic budget constraints

$$(2i) \qquad c_1^t + s_t = y_1$$

$$(2ii) \qquad c_2^t = R_{t+1}s_t + y_2,$$

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[1] This formulation of preferences was originally not due to Geary and Stone. Literature calls these more general preferences Stone-Geary preferences, since they also lead to linear expenditure system (see e.g. Barro and Sala-i-Martin, 2004, p. 178).

where y_1 (y_2) is the endowment in the first (second) period, s_t the saving, R_{t+1} the

interest factor from period t to period $t+1$, and c_1^t (c_2^t) the first (second) period

consumption, respectively. For the problem to make sense the following inequality must

be satisfied for the equilibrium interest factor

(3) $\qquad a_1 + \dfrac{a_2}{R} < y_1 + \dfrac{y_2}{R}$.

This means that the point (a_1, a_2) should lie inside the budget line. For (3) to hold it is

not necessary that $a_1 < y_1$ and $a_2 < y_2$ at the same time. If $a_1 < y_1$ and $a_2 < y_2$, we have

the decision problem with well-known properties.

The decision problem becomes different when either $a_1 > y_1$ or $a_2 > y_2$. Figure

1 describes the budget set, when $y_1 - a_1 > 0$ and $y_2 - a_2 < 0$, so that saving is positive.

In Figure 1 we need to have $R > \underline{R}$ for the decision problem to make sense. Inspecting

Figure 1 reveals that $\underline{R} = (a_2 - y_2)/(y_1 - a_1)$.
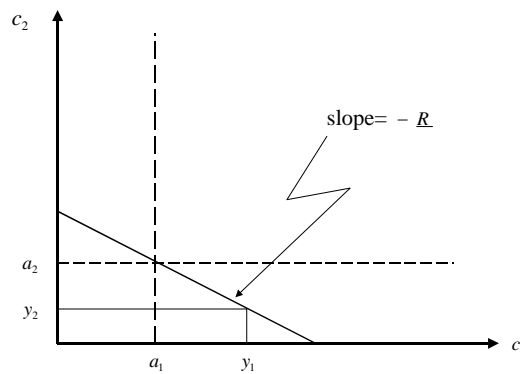

Figure 1.

The first-order condition for consumer's optimum, $(c_2^t - a_2)^\sigma / \beta(c_1^t - a_1)^\sigma = R_t$,

leads to the saving function

$$(4) \qquad s(R; \bullet) = \frac{R^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}}}{R + R^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}}}(y_1 - a_1) - \frac{y_2 - a_2}{R + R^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}}},$$

with the following property

$$(5) \qquad \frac{\partial s}{\partial R} = \frac{(\frac{1}{\sigma} - 1)R^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}}}{\left[R + R^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}} \right]^2}(y_1 - a_1) + \frac{1 + \frac{1}{\sigma}R^{\frac{1}{\sigma}-1} \beta^{\frac{1}{\sigma}}}{\left[R + R^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}} \right]^2}(y_2 - a_2)$$

If $a_1 < y_1$ and $a_2 < y_2$ we have the same result as with $a_1 = a_2 = 0$, i.e. with typical

CRRA preferences. In this case it is necessary to have $\sigma > 1$ for saving to be a

decreasing function of the interest factor. But if $a_1 > y_1$ or $a_2 > y_2$, then saving can be a

decreasing function of the interest factor even though $\sigma < 1$.

3. Dynamic Equilibrium

We introduce an outside asset into the economy by assuming that government borrows

(lends) from (to) the public. Government debt (or assets) at the beginning of the period is

denoted by b_t, and the primary deficit by d_t so that government's budget constraint is

$$(6) \qquad b_{t+1} = d_t + R_{t+1}b_t.$$

Since we want to concentrate on the fundamental dynamic implications of the model

with Stone-Geary preferences, we assume the primary deficit to be zero, i.e. $d_t = 0$ for

all t, and study the case with nonnegative government debt, i.e. $b_t \geq 0$. If we had

positive deficits, there would be two stationary equilibria in this economy.[2] Thus in the asset market equilibrium $b_t = s_t$, which leads to the following difference equation

$$(7) \qquad s_{t+1} \equiv s(R_{t+2}) = R_{t+1}s(R_{t+1}) \equiv R_{t+1}s_t.$$

We study the case presented in Figure 1, when $y_1 - a_1 > 0$ and $y_2 - a_2 < 0$, so that saving is positive. This is a more interesting case than the other one (i.e. $a_2 < y_2$) since the restriction ($a_2 > y_2$) seems to be quite plausible in economies with retirement systems, where the subsistence consumption exceeds the second period endowment. Because saving is always positive, there is no interest factor such that saving is zero. Hence in (7) there is only one steady state such that $R_t = 1$ for all t, which is possible when $\underline{R} = (a_2 - y_2)/(y_1 - a_1) < 1$, i.e. under $a_2 - y_2 > 0$, and when the first period endowment is sufficiently higher than the subsistence consumption.

We analyze equation (7) by the geometric techniques of the reflected generational offer curves developed by Cass, Okuno and Zilcha (1979). Inverting the saving function and substituting for R_{t+1} in (7) we obtain the reflected generational offer curve.

To derive the equibrium dynamics we take into account the periodic budget constraints and the fact that under zero primary deficit $R_{t+1} = s_{t+1}/s_t$. We can then rewrite the first-order condition as an equilibrium condition

$$(8) \qquad s_t^{\frac{1}{\sigma}}(s_{t+1} + y_2 - a_2) = \beta^{\frac{1}{\sigma}} s_{t+1}^{\frac{1}{\sigma}}(y_1 - a_1 - s_t),$$

which implicitly defines the reflected generational offer curve. The steady state saving is

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[2]   The appearance of more than one steady state in models with positive deficits is basically known since Bailey (1956), see pp. 102-105 and especially his Figure 2.

$$(9) \quad s^* = \frac{(a_2 - y_2) + \beta^{\frac{1}{\sigma}}(y_1 - a_1)}{1 + \beta^{\frac{1}{\sigma}}}.$$

Using equation (8) and totally differentiating gives

$$(10) \quad \frac{ds_{t+1}}{ds_t} = \frac{-\beta^{\frac{1}{\sigma}} s_{t+1}^{\frac{1}{\sigma}} - (s_{t+1} + y_2 - a_2)\frac{1}{\sigma}s_t^{\frac{1}{\sigma}-1}}{s_t^{\frac{1}{\sigma}} - \beta^{\frac{1}{\sigma}}(y_1 - a_1 - s_t)\frac{1}{\sigma}s_{t+1}^{\frac{1}{\sigma}-1}} = \frac{-\dfrac{1}{y_1 - a_1 - s_t} - \dfrac{1}{\sigma s_t}}{\dfrac{1}{s_{t+1} - (a_2 - y_2)} - \dfrac{1}{\sigma s_{t+1}}}.$$

Because the maximum amount consumer can save is $y_1 - a_1$, the numerator of (10) is negative. Since $a_2 - y_2 > 0$, we can see that (10) is negative for $\sigma \geq 1$. If a_2 is large enough, (10) can be negative for $\sigma < 1$, which is not possible when $a_1 = a_2 = 0$.

Evaluating (10) at the steady state defined in (9) gives the following stability condition

$$(11) \quad \left[\frac{1}{2}\beta^{\frac{2}{\sigma}} + (\frac{1}{\sigma} - \frac{1}{2})\beta^{\frac{1}{\sigma}}\right](y_1 - a_1) < \left[\frac{1}{2} + (\frac{1}{\sigma} - \frac{1}{2})\beta^{\frac{1}{\sigma}}\right](a_2 - y_2).^3$$

Suppose now that $\sigma < 1$, $y_2 = 0$ and the discount factor, β, is very small. Then the left-hand side gets very small, but the right-hand side approaches the value of $(1/2)a_2$. In particular, as $\beta \to 0$, the steady state is stable, and at the same time indeterminate.[4] As $\beta \to 1$ the stability condition becomes $y_1 - a_1 < a_2 - y_2$ and is thus independent of the coefficient, σ.

Moreover, the stability of the economy is more likely, the smaller is $y_1 - a_1$ and the higher is $a_2 - y_2$. As previously noted the condition, $a_2 - y_2 > 0$, is plausible in

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[3]   For stability the value of the slope must be greater than minus unity. To derive (11) we have assumed that the offer curve is downward sloping, and initially $\sigma > 1$.

[4]   Guesnerie and Woodford (1992) discuss thoroughly the concept of indeterminacy in OG models, see chapter 5.

economies with retirement systems. The small difference, $y_1 - a_1 > 0$, describes an aspect of poor economies. Furthermore, it can be seen from equation (11) that higher subsistence requirements a_1 and a_2 make the economy more stable, and decrease the intertemporal elasticity of substitution ($(c-a)/\sigma c$).

We present an example by specifying $y_2 = 0$, $a_1 = a_2 = a$, and where $a = \alpha\, y_1$. Then we can rewrite the stability condition as

$$(12)\quad LHS(\beta) \equiv \frac{\frac{1}{2}\beta^{\frac{2}{\sigma}} + (\frac{1}{\sigma} - \frac{1}{2})\beta^{\frac{1}{\sigma}}}{\frac{1}{2} + (\frac{1}{\sigma} - \frac{1}{2})\beta^{\frac{1}{\sigma}}} < \frac{\alpha}{1-\alpha}.$$

The left-hand side is an increasing function of the discount factor, if $\sigma \le 1$, and $LHS(1) = 1$, $LHS(0) = 0$, $0 < LHS(\beta) < 1$ for $0 < \beta < 1$. Then the condition in (12) implies an upper bound for the discount factor. For that upper bound to be less than unity, it must be the case that $\alpha < 1/2$ (see Figure 2). For discount factors $\beta < \beta'$ $LHS(\beta) < \alpha/(1-\alpha)$, and the stability condition thus holds.
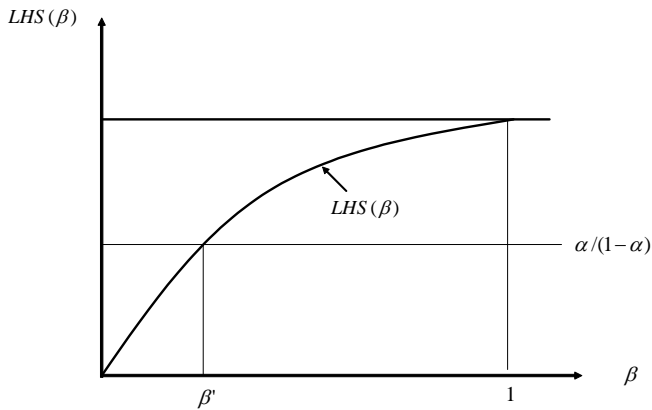

Figure 2.

Cycles can emerge in equilibrium under preferences without subsistence consumption, if intertemporal elasticity of substitution is smaller than unity (see Gale 1973, and in particular Grandmont 1985). We explore the existence of a period-doubling bifurcation (a two-cycle) for the case when intertemporal elasticity of substitution is higher than unity, $\sigma < 1$, by resorting to a numerical example.

We fix the parameters such that the slope of the reflected generational offer curve is slightly less than minus unity at the steady state. Then we find a pair of numbers, x and z (different from $s*$) fulfilling (8) for $\sigma < 1$ such that the following equations hold

(13) $\quad \dfrac{z + y_2 - a_2}{y_1 - a_1 - x} = \beta^{\frac{1}{\sigma}} \left(\dfrac{z}{x}\right)^{\frac{1}{\sigma}}$ and $\dfrac{x + y_2 - a_2}{y_1 - a_1 - z} = \beta^{\frac{1}{\sigma}} \left(\dfrac{x}{z}\right)^{\frac{1}{\sigma}}$.

The slope at the steady state is

(14) $\quad \left. \dfrac{ds_{t+1}}{ds_t} \right|_{s_t = s*} = \dfrac{-\beta^{\frac{1}{\sigma}} - \dfrac{\beta^{\frac{1}{\sigma}}(y_1 - a_1 - s*)}{\sigma s*}}{1 - \dfrac{\beta^{\frac{1}{\sigma}}(y_1 - a_1 - s*)}{\sigma s*}}$.

We choose the parameters as follows: $\beta = 1/2$, $\sigma = .95$ and $y_1 = 1$. We know from (14) that the slope is sensitive to the values of necessities. We set the slope to be -1.001, and choose the value of a_1 such that the slope indeed equals -1.001. Such a value for a_1 (and indeed we assume the same value for a_2) is .332782. $\underline{R}$ (in this case $a_2/(y_1 - a_1)$) is .4987. Using these values the steady state, $s*$, will be .441566. Solving equations in (13) for these parameter values we get $x = .441564$ and $z = .441568$. The respective equilibrium interest factors are .99999 and 1.000009 so that the computed two-cycle is

feasible in the case of general Stone-Geary preferences as well. Of course the steady

state is one solution for (13).

4. Conclusion

We have used an overlapping generations model with pure exchange and Stone-Geary

preferences to study the dynamic properties of equilibrium in economies with positive

levels of saving and public debt. We have shown that a stable nontrivial steady state

exists. Furthermore, and importantly there can be a multitude of period-doubling

bifurcations (two-cycles) in equilibrium even though the intertemporal elasticity of

substitution is greater than one.

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