TESTING SUSTAINABILITY OF GERMAN FISCAL POLICY: EVIDENCE FOR THE PERIOD 1960 - 2003

ALFRED GREINER
UWE KOELLER
WILLI SEMMLER

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Abstract

In this paper we test whether German public debt has been sustainable by resorting to a test proposed by Bohn (1998). We apply non-parametric and semi-parametric regressions with time depending coefficients. This test shows that the mean of the coefficient relevant for sustainability has been significantly positive over the time period considered. However, there is a negative trend in that coefficient which seems to have ceased to decline only in the middle to late 1990s. Further, we find evidence that the response of the primary deficit is a U-shaped function of the debt ratio which first declines and then rises after a certain threshold of the debt ratio is exceeded.

JEL Code: H63, E62.

Keywords: public debt, intertemporal budget constraint, varying coefficient model, non-parametric estimation.

Alfred Greiner
Center for Empirical Macroeconomics
Department of Economics
University of Bielefeld
P.O. Box 100131
33501 Bielefeld
Germany
agreiner@wiwi.uni-bielefeld.de

Uwe Koeller
Center for Empirical Macroeconomics
Department of Economics
University of Bielefeld
P.O. Box 100131
33501 Bielefeld
Germany
ukoeller@uni-bielefeld.de

Willi Semmler
New School University
65 Fifth Avenue
New York, NY 10003
USA
semmlerw@newschool.edu

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1 Introduction

The issue of public debt has become a primary interest of both economists and politicians since the 1990s in the Euro-area. Most of the OECD countries have revealed a chronic government deficit since the middle of the 1970s which has led to an increase in the debt to GDP ratio. Looking at time series data the major cause of the increase of the public debt in the 1970s seems to be related to the two oil crises. In the later periods, with moderate or low growth rates and secularly rising unemployment rates in the Euro-area countries, welfare state expenditures seem to have caused high level of public deficit and rising public debt until the middle of the 1990s.[1] Yet, since then the effort to reduce the public debt in preparation of the start of the European Monetary Union 1999 has led to declining public deficits. Only recently, in the recession 2001-2003, has the deficit increased again for some countries, in particular Germany and France.

In the academic debate on the deficit and debt of the Euro-area countries it has been argued that fiscal policy has been threatened to become unsustainable and that fiscal policy may neither be an instrument nor effective in stabilizing the macroeconomy any longer. This paper is concerned with formal econometric procedures that allow one to test for sustainability of fiscal policy. What we are thus concerned with in this paper is not the short run violation of the stability and growth pact of the European Monetary Union, but rather the long-run sustainability of fiscal policy of the member countries. We here study a particular country, namely Germany, yet our proposed test can be applied to other Euro-area countries as well.

From the theoretical point of view the question of how large a private agent's debt can become is usually answered as follows. Private households are subject to the borrowing constraint which means that, given no initial debt, the expected present value of expenditures (exclusive of interest payments) should not exceed the expected present

[1] In Germany, it was in particular the unification of East and West Germany that has given rise to a debt to GDP ratio from about 44 percent in 1990 to roughly 58 percent in 1995.

value of receipts, known as the no-Ponzi game condition. This condition implies that a private household cannot continually borrow and pay the interest by borrowing more. There are limits of borrowing capacity of economic agents which are usually defined by the intertemporal budget constraints of the agents.[2]

For government debt this question has somewhat been left unsettled from the theoretical point of view. If a government could borrow and pay the interest by borrowing more any fiscal policy would be sustainable and in some suggested models this is indeed possible.[3] However, that possibility is not given any longer when the economy is dynamically efficient.[4] Then, the government faces a present-value borrowing constraint stating that the current value of public debt must equal the discounted sum of future surpluses exclusive of interest payments.[5] Bohn (1995) has proved that in an exchange economy with infinitely lived agents the government must always satisfy the no-Ponzi game condition. In a series of papers, Bohn (1995, 1998) not only presents a theoretical advancement of studying sustainability of public debt but also provides a new econometric approach to estimate sustainability of public debt.

Empirical studies which help to clarify whether governments follow the intertemporal budget constraint or not are indeed desirable. For the US there exist numerous studies starting with the paper by Hamilton and Flavin (1986). In this paper they propose a framework for analyzing whether governments can run a Ponzi scheme or not and apply the test to US time series data. They find sustainability of fiscal policy in the US. Other papers followed which also have investigated this issue for the US and other countries, but

[2]A model for sustainability of private debt is developed and studied in Grüne et al. (2004).

[3]In overlapping generations models, for example, which are dynamically inefficient a government can borrow in order to pay interest on outstanding debt (see Diamond, 1965), i.e. it may run a Ponzi scheme.

[4]For an empirical study analyzing whether the US economy is dynamically efficient, see Abel et al. (1989).

[5]McCallum (1984) has studied a perfect foresight version of the competitive equilibrium model of Sidrauski (1967) and proved that permanent primary deficits are not possible if the deficit is defined exclusive of interest payments.

partly reached different conclusions (see e.g. Kremers, 1988, Wilcox, 1989, or Trehan and Walsh, 1991 and Greiner and Semmler, 1999). However, these tests have been criticized by Bohn (1995, 1998) because they make assumptions about discount rates as well as future states of nature. The latter are difficult to estimate from a single set of observed time series data. In a recent paper Bohn (1998) proposes a new test that is not open to this criticism.

In our paper we extend the approach by Bohn by developing a time varying coefficient model and by applying it to German time series data. Other time series methods have been employed to the German economy in Greiner and Semmler (1999). The latter contribution resorted to the tests proposed by Hamilton and Falvin (1986), Trehan and Walsh (1991) and Wilcox (1989). It turned out that the results are in part not robust which was one motivation to apply the test proposed by Bohn (1998) in this paper. For example, the number of lags is crucial as to the question of whether the budget surplus is a stationary time series or not.

The remainder is organized as follows. Section 2 elaborates on some theoretical considerations concerning the intertemporal budget constraint and discusses the relevant literature. Section 3 presents our estimation results for Germany. Section 4 concludes the paper.

2 Some theoretical considerations

Usually, it is postulated that governments too have to fulfill an intertemporal budget constraint. In economic terms, this constraint states that public (net) debt at time zero must equal the expected value of future present-value primary surpluses. This requirement is also often referred to as the no-Ponzi game condition. Neglecting stochastic effects and assuming that the interest rate is constant the intertemporal budget constraint can be written as

$$B(0) = \int_0^\infty e^{-r\tau}\, S(\tau)\, d\tau, \tag{1}$$

with r the constant interest rate, $B(0)$ public debt at time zero and S the primary surplus.[6] Equivalent to equation (1) is the following equation

$$\lim_{t\to\infty} e^{-rt} B(t) = 0, \tag{2}$$

with $B(t)$ public debt at time t, stating that the present value of public debt converges to zero for $t \to \infty$.

In the economics literature numerous studies exist which explore whether (1) and (2) hold in real economies (see for example Hamilton and Flavin, 1986, Kremers, 1988, Wilcox, 1989, Trehan and Walsh, 1991). For example, Hamilton and Flavin (1986) suggest to test for the presence of a bubble term in the time series of public net debt which would indicate that a given fiscal policy is not sustainable. Trehan and Walsh (1991) proposed to test whether the budget deficit is stationary or to test whether the primary budget deficit and the public debt series are cointegrated and $(1 - \lambda L)S_t$ is stationary, with $0 \leq \lambda < 1 + r_t$. Another test, proposed by Wilcox (1989), is to test whether the series of undiscounted debt displays an unconditional mean of zero. If this holds the intertemporal government constraint will be fulfilled because the intertemporal budget constraint requires the discounted debt to converge to zero.[7]

One aspect of these tests which has given rise to criticism is that they need strong assumptions because the transversality condition involves an expectation about states in the future that are difficult to obtain from a single set of time series data and because assumptions about the discount rate have to be made (see e.g. Bohn, 1995, 1998).

A test procedure which circumvents that problem is to look at the time series of the debt ratio, i.e. on the ratio of public debt to GDP. If this series is constant the

[6]For a derivation see e.g. Blanchard and Fischer (1989), ch. 2.

[7]As to these tests applied to Germany see Greiner and Semmler (1999).

intertemporal budget constraint is fulfilled for dynamic efficient economies. To see this let $B/Y = c_1$ be the constant debt ratio, with Y the GDP and c_1 a positive constant. Inserting $B/Y = c_1$ in (2) yields

$$\lim_{t \to \infty} c_1 \, Y_0 \, e^{(\gamma - r)t} = 0, \tag{3}$$

for $\gamma < r$, with $\gamma > 0$ the constant growth rate of GDP. The condition $\gamma < r$ characterizes a dynamic efficient economy and is likely to hold in real economies. For example, in EU countries this seems to be obvious if one compares interest rates with GDP growth rates. But even the US, where growth rates have exceeded interest rates on safe government bonds, is a dynamic efficient economy (for details see Abel et al., 1989). Therefore, in the following we will limit our considerations to the case of dynamic efficient economies and assume that the discount rate of government debt exceeds the GDP growth rate.[8]

However, testing for stationarity of the debt ratio is characterized by some shortcomings, too. So, it is difficult to distinguish between a time series which is stationary about a positive intercept and one that shows a trend. This holds because standard unit root regressions have low power against autoregressive alternatives if the AR coefficient is close to one. As a consequence, the hypothesis that a given fiscal policy is sustainable has been rejected too easily.

Therefore, Bohn (1998) suggests to test whether the primary deficit to GDP ratio is a positive linear function of the debt to GDP ratio. If this holds, a given fiscal policy will be sustainable. The intuitive reasoning behind this argument is as follows: If a government raises the primary surplus as the public debt ratio rises, it takes a corrective action which stabilizes the debt ratio and makes public debt sustainable. Before we undertake empirical tests which apply this concept we make some theoretical considerations about the relevance of this test where we limit our considerations to deterministic economies.

[8]In dynamic inefficient economies the government budget constraint is irrelevant because in that case the government can play a Ponzi game.

Assuming that the primary surplus to GDP ratio depends on a constant and linearly on the debt to GDP ratio this variable can be written as

$$\frac{S(t)}{Y(t)} = \frac{T(t) - G(t)}{Y(t)} = \alpha + \rho \left(\frac{B(t)}{Y(t)} \right), \tag{4}$$

with $S(t)$ the primary surplus at time t, $T(t)$ tax revenue at t, $G(t)$ public spending at t, α and ρ are constants which can be negative or positive. α is a systematic component which determines how the level of the primary deficit reacts to variations in GDP. α can also be interpreted as other (constant) economic variables which affect the surplus ratio.

The coefficient ρ can be called a reaction coefficient since it gives the response of the primary surplus ratio to an increase in the debt ratio. Inserting (4) in the differential equation giving the evolution of public debt the latter equation is given by

$$\dot{B}(t) = r\,B(t) + G(t) - T(t) = (r - \rho)\,B(t) - \alpha\,Y(t). \tag{5}$$

Solving this equation we get

$$B(t) = \left(\frac{\alpha}{r - \gamma - \rho} \right) Y(0)\, e^{\gamma\, t} + e^{(r-\rho)\, t}\, C_1, \tag{6}$$

with $B(0) > 0$ debt at time $t = 0$ which is assumed to be strictly positive and with C_1 a constant given by $C_1 = B(0) - Y(0)\,\alpha/(r - \gamma - \rho)$. Multiplying both sides of (4) by e^{-rt} leads to

$$e^{-rt} B(t) = \left(\frac{\alpha}{r - \gamma - \rho} \right) Y(0)\, e^{(\gamma - r)t} + e^{-\rho\, t}\, C_1. \tag{7}$$

The first term on the right hand side in (7) converges to zero in dynamic efficient economies and the second term converges for $\rho > 0$ and diverges for $\rho < 0$. These considerations show that $\rho > 0$ guarantees that the intertemporal budget constraint of the government holds.

It must also be pointed out that in a stochastic economy dynamic efficiency does not necessarily imply that the interest rate on government debt exceeds the growth rate of the economy, i.e. $\gamma > r$ may occur. This holds because with risky assets the interest rate on safe government bonds can be lower than the marginal product of capital. If the stochastic economy is dynamically efficient and the growth rate exceeds the interest rate on government bonds, a positive β is nevertheless also sufficient for the intertemporal budget constraint to be fulfilled if $\alpha = 0$ holds which is immediately seen froom (7).

For Germany the average difference between the growth rate of GDP and the interest rate has been negative implying that the first term in (7) converges to zero. In this case, the sign of α is irrelevant. On average, the difference between the growth rate of GDP and the interest rate was -1.02 percent in Germany for the period from 1960-2003. Therefore, a positive sign for ρ would be sufficient for sustainability in Germany.

Further, with equation (4) the debt to GDP ratio evolves according to

$$\dot{b} = b\left(\frac{\dot{B}}{B} - \frac{\dot{Y}}{Y}\right) = b\,(r - \rho - \gamma) - \alpha, \tag{8}$$

Solving this differential equation we get the debt to GDP ratio b as

$$b(t) = \frac{\alpha}{(r - \rho - \gamma)} + e^{(r-\gamma-\rho)t}\,C_2, \tag{9}$$

where C_2 is a constant given by $C_2 = b(0) - \alpha/(r - \rho - \gamma)$, with $b(0) \equiv B(0)/Y(0)$ the debt-GDP ratio at time $t = 0$.

Equation (9) shows that the debt to GDP ratio remains bounded if $r - \gamma - \rho < 0$ holds. This shows that a positive ρ does not assure boundedness of the debt-GDP ratio although the intertemporal budget constraint of the government is fulfilled in this case. Only if ρ is larger than the difference between the interest rate and the GDP growth rate the debt ratio remains bounded. These considerations demonstrate that sustainability of public debt may be given even if the debt-GDP ratio rises over time. A situation which seems to hold for Germany.

In the next section, we perform some empirical tests based on our theoretical considerations of this section in order to get evidence about sustainability of German fiscal policy.

3 Empirical Evidence for Germany

The previous section has highlighted an alternative estimation strategy to test for sustainability of fiscal policy. We here pursue this test to Germany and empirically analyze how the primary surplus reacts to the debt-GDP ratio in order to see whether a given fiscal policy is sustainable where we consider the period from 1960-2003.

As outlined above the main idea in testing for sustainability is to estimate the following equation

$$ps_t = \rho b_t + \alpha^\top \mathbf{Z_t} + \epsilon_t \tag{10}$$

where ps_t and b_t is the primary surplus and debt ratio respectively, $\mathbf{Z_t}$ is a vector which consists of the number 1 and of other factors related to the primary surplus and ϵ_t is an error term which is i.i.d. $N(0, \sigma^2)$.

As concerns the other variables contained in $\mathbf{Z_t}$, which are assumed to affect the primary surplus, we include the real long-term interest rate (r) and a variable reflecting the business cycle ($YVAR$). YVAR is calculated by applying the HP-filter on the GDP-series. Further, in one version of (10) the social surplus ratio (Soc) is subtracted from the primary surplus ratio and is considered as exogenous in order to catch possible effects of transfers between the social insurance system and the government.[9] In the second equation to be estimated the social surplus ratio is included in the primary surplus ratio.

In addition, we decided that it is more reasonable to include the lagged debt ratio b_{t-1} instead of the instantaneous b_t, although theory says that the response of the surplus on higher debt should be immediate. We do this, because interest payments on debt and repayment of the debt occurs at later periods.

[9] Soc_t is computed by subtracting Social Benefits Paid By Government from the Social Security Contributions Received By Government.

Summarizing our discussion the equations to be estimated are as follows:

$$ps_t = \alpha_0 + \rho b_{t-1} + \alpha_1 Soc_t + \alpha_2\, r_t + \alpha_3 YVAR_t + \epsilon_t \quad (11)$$

$$ps_t^{soc} = \alpha_0 + \rho b_{t-1} + \alpha_2 r_t + \alpha_3 YVAR_t + \epsilon_t \quad (12)$$

where ps_t is the primary surplus ratio exclusive of the social surplus and ps_t^{soc} denotes the primary surplus ratio including the social surplus.

As already mentioned, we estimate equation (10) both with the primary surplus exclusive and inclusive of the social surplus. We do this because, on the one hand, the social deficit does not affect public debt if the deficit is financed by possible surpluses of previous periods. However, if there are no surpluses in previous periods the government has to finance the deficit which raises public debt, although not directly because public social insurances do not borrow at the capital market. In addition, the public health insurance in Germany is not allowed to make deficits. However, if it does run deficits public debt rises but the debt has to be repaid in the next period so that the increase in debt is only transitory. Because of this, the social deficit only contributes in part to public debt or the contribution is transitory so that estimation of (10) both inclusive and exclusive of public debt seems to be justified.

Before we estimate equations (11)-(12) we take a look at the evolution of the public debt-GDP ratio as well as the primary surplus ratio depicted in figure 1.

Figure 1: Debt-GDP ratio and primary surplus-ratio, exclusive of social surplus (Surplus) and inclusive of social surplus (Surplus+Soc).

As figure 1 shows the German government was confronted with high debt ratios accompanied with permanent primary deficits at the beginning of the mid-seventies. Furthermore, in figure 1 two episodes of a sharp rise in the growth rate of public debt can be observed followed by periods with budgetary discipline and lower increasing debt ratios. In the mid-seventies the debt ratio increases very rapidly, due to the oil shock, which also caused a recession with the rise of the unemployment rate. This fact is highlighted in figure 1 by the dotted line for the debt to GDP ratio and by the solid and dashed lines for the primary surplus exclusive and inclusive of the social surplus. The second sharp increase of the debt ratio was caused by the German unification and began in the early nineties as the GDP growth rates slowed down.

Further, figure 1 shows there might be possible structural breaks in the time series of the debt ratio. The first oil price shock in 1974 is one of them. Secondly, plotting the surplus against the debt level (cf. figure 4 below) indicates a breakpoint at a debt level of about 40 %, i.e. in 1980 the reaction of the surplus of growing debt changes and presents now a positive slope. At last, German unification in 1990 has lead to a sharp increase of public indebtedness.

The next table presents the results of the Chow breakpoint test and the F-test on equal variances.

Year	*Debt Ratio*	*Chow Test (p-value)*	*Variance-Ratio-Test (p-value)*
1974	0.180	7.286 (0.000)	4.053 (0.017)
1980	0.311	7.896 (0.000)	2.746 (0.027)
1989	0.409	4.035 (0.004)	1.999 (0.085)

Table 1: Structural Breaks for Germany

For 1980 we get the highest Chow F-test statistic and the hypothesis of no structural break must be rejected. The variance ratio test indicates that the variances of both subsamples are significantly different. The same holds for the breakpoint in 1974. Only in 1989 the null for equal variances cannot be rejected at the 5% level. Summarizing our findings, we must conclude that there are at least more than one structural break in the data.

These findings suggest that the response of the primary surplus ratio to higher debt ratios has not been constant but, instead, varied with time. Therefore, we estimate equations (11)-(12) assuming a time dependent coefficient ρ_t.

3.1 Estimating the coefficient ρ_t

In this section we estimate the coefficient ρ_t which may vary over time. Generally, the regression model with l varying coefficients is given by

$$y_i = x_{1i}\beta_1(t_{1i}) + x_{2i}\beta_2(t_{2i}) + ... + x_{li}\beta_l(t_{li}) + \epsilon_i, \tag{13}$$

where we have denoted the set of n observations for the response variable Y, for the predictor variable X_j and for the modifying variable T_j by y_i, x_{ij}, and t_{ij}, respectively. The problem in the varying coefficient model (13), then, is to fit the data by finding the solution of the following penalized least-squares criterion (cf. Ryan, 1997)

$$\min_{\beta_k} \sum_{i=1}^{n} \left(y_i - \sum_{j=1}^{l} x_{ji}\beta_j(t_j) \right)^2 + \sum_{j=1}^{l} \lambda_j \int_a^b \left(\frac{d^2\beta_j(t_j)}{dt_j^2} \right)^2 dt_j, \tag{14}$$

where the simplifying assumption $a < x_{j1} < ... < x_{jn} < b$ can be made. λ is the smoothing parameter and the choice of that parameter plays an important role. Small values for λ reduce the variance of the fit but raise the bias so that one can speak of a bias-variance trade-off (see Hasti and Tibshirani, 1990).

One way to determine the parameter λ is to resort to the Generalized Cross Validation (GCV) criterion (see e.g. Hasti and Tibshirani, 1990, chap. 3). According to that criterion λ is chosen such that

$$GCV(\lambda) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i - \hat{\beta}_\lambda(x_i)}{1 - tr(\mathbf{S})/n} \right)^2 \tag{15}$$

is minimized, where $\mathbf{S}$ is the smoother matrix and $\hat{\beta}_\lambda(x_i)$ denotes the fit at x_i. GCV is similar to the Ordinary Cross Validation (OCV) criterion which works as follows: The model is fitted to the data with one observation left out. Then, the squared difference between the missing data point and the prediction of the model for that missing observation is measured. This process is repeated for each datum and the mean square difference between the models (fitted to all data except for one missing datum) and the missing data is computed. This difference is an objective which should be minimized. The idea is that a model that is too smooth will not yield a good prediction for the observation left out in the process of fitting the model. The same holds for a model that is too wiggle and overfits the model. The difference between OCV and GCV is that GCV replaces the diagonal elements of the smoother matrix by its average value, tr$\mathbf{S}$/n, which is easier to compute.

Assuming that the reaction parameter ρ_t depends on time equation (11) in our approach becomes

$$ps_t = \alpha_0 + (\bar{\rho} + s(t))b_{t-1} + \alpha_1 Soc_t + \alpha_2\, r_t + \alpha_3 YVAR_t + \epsilon_t. \tag{16}$$

$\bar{\rho}$ in equation (16) gives the mean of that coefficient over the whole period and $s(t)$ gives the deviation from that mean depending on time. Estimating (16) for Germany gives the following results.[10]

	Coeff.	*Std. Error (t-stat.)*	$\Pr(>\|t\|)$
constant	-0.07	0.017 (-3.982)	$3.5{\cdot}10^{-4}$
b_{t-1}	0.313	0.073 (4.297)	$1.4{\cdot}10^{-4}$
Soc_t	0.643	0.258 (2.49)	0.018
r_t	-0.084	0.189 (-0.447)	0.658
$YVAR_t$	-0.073	0.165 (-0.439)	0.664
$sm(t)$	edf: 4.859	chi-sq.: 65.322	p-value: $4.08{\cdot}10^{-7}$
Deviance explained	84%		
R^2(adj.)	0.803		
GCV score	$8.0{\cdot}10^{-5}$		

Table 2: Estimates for equation (16)

Table 2 shows that the mean of the reaction coefficient $\bar{\rho}$ is positive and highly significant. That is, on average assuming a sustainable fiscal policy in Germany is supported by the data. The interest rate and the business cycle variable $YVAR$, however, are not statistically significant. On the other side, the social surplus has a significantly positive effect on the primary surplus (exclusive of the social surplus).

Further, table 2 also shows that the time varying smooth term $s(t)$ is significant. In figure 2 we show the graph of the estimated $s(t)$.

[10] All estimation were performed using the mgcv package in R (cf. Wood, 2001).

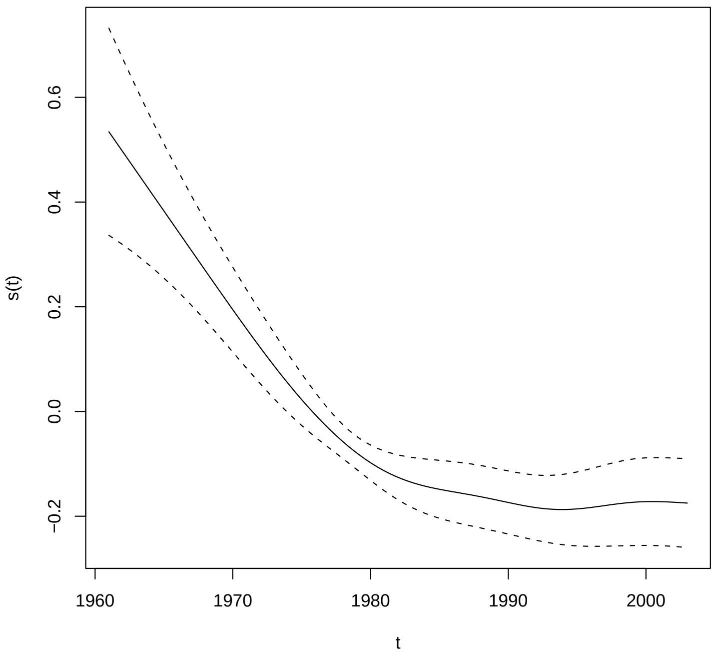

Figure 2: Time varying component $s(t)$ giving the deviation from the mean value $\bar{\rho}$ in equation (16).

Figure 2 shows that the reaction coefficient $\rho_t = (\bar{\rho} + s(t))$ declined over time. This means that over time the government had less scope for its spending leading the government to put less importance on the question of sustainability of public debt. Reasons for this may be deficits in the public social insurances. So, on the one hand the government has to finance deficits in public social insurances, on the other hand, it cannot use possible surplus made at some times. Nevertheless, it should be noticed that ρ_t remains strictly positive over the whole period. Further, it seems that the negative trend was stopped in the mid 1990s although this statement should be made with care since additional observations are needed to learn more about more recent periods.

When we allow for other time varying coefficients the main results do not change. Then, the qualitative outcome is basically the same as in table 2. For example, assuming that all coefficients are time dependent we get statistically significant effects of the debt ratio and of the social surplus while the coefficients of the interest rate and of $YVAR$ are not statistically significant. Further, the assumption of a time varying coefficient ρ_t is still statistically significant and ρ_t becomes smaller over time as in figure 2. But the hypothesis that other coefficients are time dependent must be rejected, with the exception of the interest rate. The result of the estimation is shown in table 3.

	Coeff.	*Std. Error (t-stat.)*	$\Pr(>\|t\|)$
constant	-0.07	0.022 (-3.207)	0.003
b_{t-1}	0.285	0.088 (3.222)	0.003
Soc_t	1.257	0.412 (3.052)	0.005
r_t	0.154	0.25 (0.614)	0.545
$YVAR_t$	0.009	0.688 (0.013)	0.99
$s(t): b_{t-1}$	edf: 2.05	chi-sq.: 14.45	p-value: 0.003
$s(t): Social_t$	edf: 1	chi-sq.: 1.114	p-value: 0.3
$s(t): r_t$	edf: 6.569	chi-sq.: 18.673	p-value: 0.025
$s(t): YVAR_t$	edf: 1	chi-sq.: $2.4{\cdot}10^{-4}$	p-value: 0.988
Deviance explained	90.2%		
R^2(adj.)	0.849		
GCV score	$7.6{\cdot}10^{-5}$		

Table 3: Estimates for equation (11) with all coefficients time dependent.

When we allow for a time varying coefficient ρ_t in the equation with the primary surplus including the social surplus as the dependent variable we can rewrite equation (12) as follows

$$ps_t^{soc} = \alpha_0 + (\bar{\rho} + s(t))b_{t-1} + \alpha_2\, r_t + \alpha_3 YVAR_t + \epsilon_t. \tag{17}$$

Table 4 shows the outcome of this estimation

	Coeff.	*Std. Error (t-stat.)*	$\Pr(>\|t\|)$
constant	-0.049	0.028 (-1.767)	0.087
b_{t-1}	0.216	0.112 (1.923)	0.064
r_t	-0.323	0.258 (-1.253)	0.219
$YVAR_t$	0.419	0.456 (0.919)	0.365
$s(t)$	edf: 7.217	chi-sq.: 30.324	p-value: 0.002
Deviance explained	64.2%		
R^2(adj.)	0.526		
GCV score	$1.4 \cdot 10^{-4}$		

Table 4: Estimates for equation (17).

Table 4 shows that the average of the coefficient ρ_t is statistically significant at roughly the 7 percent significance level. Further, the assumption that ρ_t is constant can again clearly be rejected. The statistical significance of the average of the coefficient ρ_t rises when we allow all coefficients to be time varying as demonstrated in table 5.

	Coeff.	*Std. Error (t-stat.)*	$\Pr(>\|t\|)$
constant	-0.066	0.031 (-2.127)	0.042
b_{t-1}	0.305	0.131 (2.324)	0.027
r_t	-0.232	0.288 (-0.804)	0.428
$YVAR_t$	3.346	2.026 (1.652)	0.11
$s(t): b_{t-1}$	edf: 7.628	chi-sq.: 28.493	p-value: 0.005
$s(t): r_t$	edf: 1.921	chi-sq.: 2.936	p-value: 0.234
$s(t): YVAR_t$	edf: 1	chi-sq.: 2.08	p-value: 0.16
Deviance explained	72.3%		
R^2(adj.)	0.591		
GCV score	$1.3 \cdot 10^{-4}$		

Table 5: Estimates for equation (11) with all coefficients time dependent.

Table 5 yields slightly better results than table 4 implying that the estimation with all coefficients being time dependent reflects the behaviour of German government definitely better. So, both R^2(adj.) and the deviance explained rise while the GCV score decreases by about 3.7 percent. The mean of the reaction coefficient, $\bar{\rho}$, is positive and statistically significant at about the 3 percent level. The assumption of time dependence of this coefficient is also statistically significant. The time path of the deviation of ρ_t from its mean is shown in figure 3.[11]

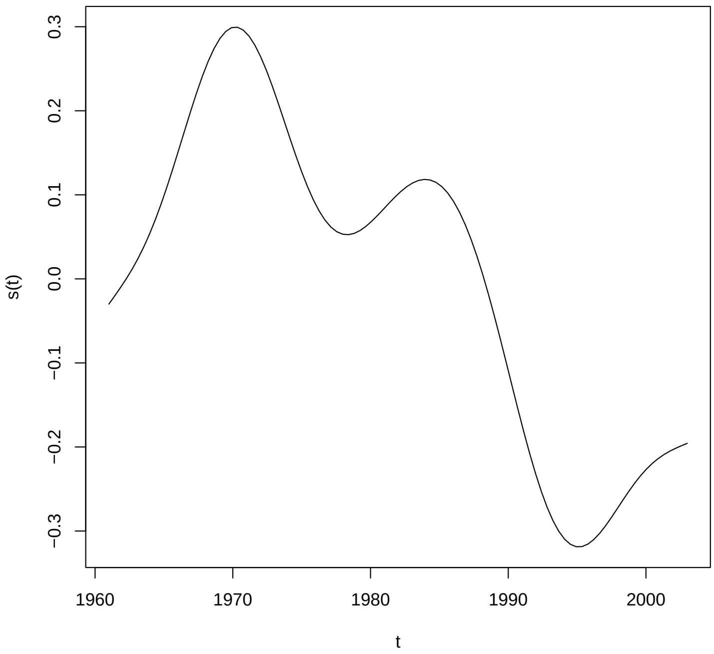

Figure 3: Time varying component $s(t)$ giving the deviation from the mean value $\bar{\rho}$ in equation (17).

It seems that the social surplus plays an important although not the only role in determining the deviation of ρ_t from its mean. So up to the mid late 1970s the deficit in the social insurances relative to GDP had monotonously decreased until it became a surplus around 1972 (cf. figure 1). This had a positive effect on ρ_t and made this

[11]We leave out the confidence interval so that the maximum and the minimum can be better recognized.

coefficient increase since the government did not have to finance deficits of the public social insurances any longer. Since the late 1980s and, especially, with German unification in 1990 sustainability issues seem to have become smaller leading to a sharp decline in the reaction coefficient ρ_t. Only since the mid or late 1990s ρ_t has increased again.

But, as in the estimations with the primary surplus exclusive of the social surplus, the reaction coefficient remains positive over the whole time period, except for the mid 1990s possibly. Further, in contrast to the previous estimations, where the primary surplus exclusive of the social surplus was the independent variable, the reaction coefficient seems to show a clear tendency to rise since the mid or late 1990s.

Overall, our estimations yield evidence that German fiscal policy has been sustainable over the time period we consider. This conclusion is based on the sign of the reaction coefficient which has been strictly positive for most of the time period. However, there is a clear negative trend in the reaction coefficient up to the mid 1990s. Only in the mid 1990s this trend could be stopped.

3.2 A non-parametric estimation

In the last section we have estimated a semi-parametric model, where parameters were present which, however, depend on time in a non-parametric way. In this section we want to estimate a non-parametric model where we assume an additive structure. We do this because we are interested in the question of whether the coefficient ρ possibly is a nonlinear function of b_{t-1} as suggested by figure 4 which shows the primary surplus and the debt ratio of the previous period.

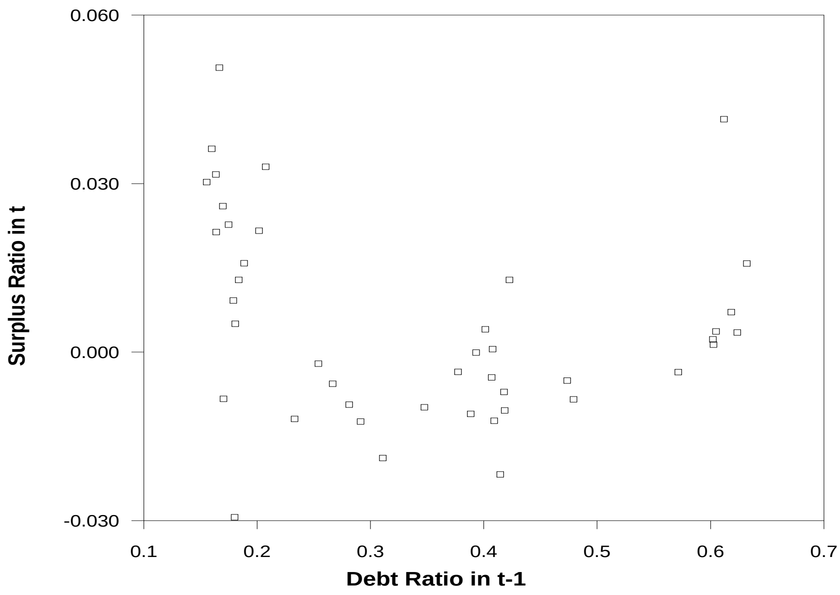

Figure 4: Primary surplus ratio exclusive of social surplus and debt ratio.

Assuming an additive model, equation (13), then, becomes

$$y_i = \alpha_0 + f_1(x_{1i}) + ... + f_l(x_{li}) + \epsilon_i. \tag{18}$$

Equivalently to (14) the problem now is to find the function $\hat{f}(x)$ with two continuous derivatives that minimizes the following penalized sum of squares:

$$\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{l} f_j(x_{ji})\right)^2 + \sum_{j=1}^{l}\lambda_j \int_a^b \left(\frac{d^2 f_j(x_j)}{dx_j^2}\right)^2 dx_j, \tag{19}$$

where again the simplifying assumption $a < x_{j1} < ... < x_{jn} < b$ can be made and λ is the smoothing parameter and is chosen according to GCV as in the last section.

The equation to be estimated in our approach, then, becomes[12]

$$ps_t = \alpha_0 + s(b_{t-1}) + f_1(r_t) + f_2(YVAR_t) + \epsilon_t. \tag{20}$$

[12]We delete the variable Soc_t because it is insignificant and leads to worse results in terms of the deviance explained, R^2(adj.) and the GCV score.

Estimating (20) for Germany we get the results shown in table 6 which demonstrate that the assumption of a nonlinear response of the primary surplus ratio to a higher debt ratio is statistically significant.

	Coeff.	*Std. Error (t-stat.)*	$\Pr(>\lvert t\rvert)$
constant	0.005	0.002 (2.692)	0.011
$s(b_{t-1})$	edf: 2.379	chi-sq.: 20.667	p-value: $4.2{\cdot}10^{-4}$
$f_1(r_t)$	edf: 1	chi-sq.: 7.765	p-value: 0.008
$f_2(YVAR_t)$	edf: 1.007	chi-sq.: 12.88	p-value: $9.6{\cdot}10^{-4}$
Deviance explained		59.3%	
R^2(adj.)		0.545	
GCV score		$1.7{\cdot}10^{-4}$	

Table 6: Estimates for equation (20).

Figure 5 shows the function $s(b_{t-1})$ and confirms that for small values of b_{t-1} the primary surplus ratio negatively depends on b_{t-1} and only for values of b_{t-1} larger than about 0.4 a positive relation can be observed where the response of the primary surplus increases with higher debt ratios. This means that the German government obviously did not pay attention to the question of sustainability of public debt as long as the debt ratio was below a certain threshold. Only when this threshold was exceeded the government positively reacted to higher debt ratios.

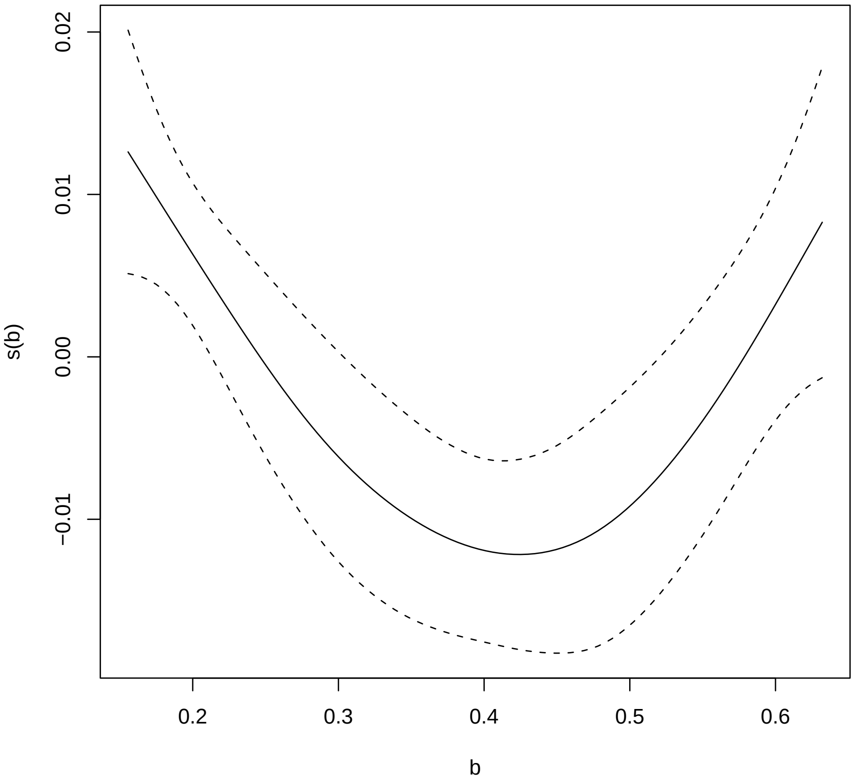

Figure 5: Response of the primary surplus as a function of the lagged debt ratio b_{t-1}.

Taking the primary surplus ratio inclusive of the social surplus ratio the equation to be estimated is given by

$$ps_t^{soc} = \alpha_0 + s(b_{t-1}) + f_1(r_t) + f_2(YVAR_t) + \epsilon_t. \tag{21}$$

The results of this estimation are given in table 7.

	Coeff.	*Std. Error (t-stat.)*	$\Pr(>\vert t\vert)$
constant	0.001	0.002 (0.645)	0.523
$s(b_{t-1})$	edf: 1	chi-sq.: 3.106	p-value: 0.086
$f_2(r_t)$	edf: 1	chi-sq.: 3.686	p-value: 0.062
$f_3(YVAR_t)$	edf: 1	chi-sq.: 10.762	p-value: 0.002
Deviance explained		26.6%	
R^2(adj.)		0.209	
GCV score		$1.9 \cdot 10^{-4}$	

Table 7: Estimates for equation (21).

Table 7 shows that the assumption of a nonlinear response of the primary surplus ratio to a higher debt ratio must be rejected because the estimated degrees of freedom (edf) are equal to 1. Further, the statistical significance of a constant linear response of the primary surplus ratio to a higher debt ratio is small. But, this does not mean that the primary surplus ratio does not react to changes in the debt ratio. All that can be said is that linear models with constant coefficients perform poor, which is also seen by the small value of R^2(adj.) among other things. But models with time varying coefficients, as shown in tables 4 and 5, yield good results and seem to better fit reality.

Further, one also realizes that the business cycle variable has a relatively high statistical significance. This may be explained by the fact that the government has less scope in controlling the primary surplus when the social surplus is included because the latter is strongly correlated with the business cycle.

Only when the debt ratio is the sole explanatory variable this variable is statistically significant at roughly the 5% level. 8 gives the outcome of this estimation.

	Coeff.	*Std. Error (t-stat.)*	$\Pr(>\lvert t \rvert)$
constant	0.001	0.002 (0.623)	0.537
$s(b_{t-1})$	edf: 3.434	chi-sq.: 8.978	p-value: 0.058
Deviance explained	22.1%		
R^2(adj.)	0.152		
GCV score	$2.1 \cdot 10^{-4}$		

Table 8: Estimates for equation (21) with b_{t-1} as the only variable.

The graph obtained is qualitatively the same as the one shown in figure 5 except that the response of the primary surplus ratio to the debt ratio becomes positive for b larger than about 0.3. However, the estimation with b as the only variable yields a poor outcome with an R^2(adj.) of 0.152, an explained deviance of 22.1 percent and with a higher GCV score compared to the one shown in table 7.

4 Conclusion

In this paper we have developed and apply a new empirical test that allows us to study the question of whether public debt in Germany is sustainable. In our time series study we employ the sample period 1960-2003. Estimating a semi-parametric model with time varying reaction coefficients has revealed that the primary surplus to GDP ratio positively reacts to higher debt ratios implying sustainability of a given path of public debt. However, we have also shown that there is a negative trend in that coefficient which seems to have ceased to decline only in the middle or late 1990s.

Assuming that the response of the primary surplus to GDP ratio is a nonlinear function of the debt ratio and of other variables suggests that there exists a U-shaped relationship between the primary surplus and the debt ratio. Only when the debt ratio exceeds a certain threshold the primary surplus increases as a result of a higher debt ratio while it declines when the debt ratio is below that threshold. This demonstrates that the German government seems to have paid attention to the problem of debt sustainability only for sufficiently high debt ratios.

Overall, the conclusion we draw from our empirical estimations is that there is significant evidence that German fiscal policy has been sustainable. Primarily, this is based on the outcome of our estimations with time varying reaction coefficients. In almost all of these estimations the reaction coefficients have been significantly positive.

However, we should also like to point out an important drawback of our conclusion. We know that the German demographic development is characterized by an increasing life expectancy and declining birth rates implying a large implicit public debt in the future. Even if the retirement age is increased and the average pension is reduced this will pose a tremendous challenge for German fiscal policy. Therefore, the answer of the question whether German public debt will remain on a sustainable path in the future will significantly depend on how the government meets the latter challenge.

5 Appendix: Data

Source: OECD Economic Outlook Statistics and Projections

We use the Data Set corresponding to those published in the June 2003 issue of the OECD Economic Outlook. Especially, we take the entire Data set for the Government Account and the series for Gross Domestic Product at Market prices (GDP)

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