EXCHANGE RATE DYNAMICS: WHERE IS THE SADDLE PATH?

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Abstract

A strand of exchange rate models postulate exchange rate fluctuations are driven by saddle-path dynamics and the related overshooting behavior. Using a bivariate system, the paper illustrates the relationship of the cointegration, saddle-path, and stationarity dynamics. Monte Carlo results indicate that the Johansen tests have reasonable power to discriminate saddle-path behavior from cointegration dynamics. Using monthly data from five major industrial countries, we find that exchange rates and prices are cointegrated. The cointegration result casts doubt on the use of saddle-path dynamics and the associated overshooting behavior to elucidate exchange rate variations.

Keywords: overshooting, cointegration, Johansen test, simulation, convergence behavior.

JEL Classification: F31, C12.

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1. Introduction

Undeniably, exchange rate behavior is one of the most intensely studied topics in the international finance literature. The overshooting model *à la* Dornbusch provides a prominent explanation for high variability of (real) exchange rates. Since its publication in the 1970s (Dornbusch, 1976), the overshooting model occupies a key position in modeling exchange rate dynamics (Frankel and Rose, 1995). A notable feature of the model is the saddle-path dynamics, which follows from the assumption that the price of goods and the exchange rate have different adjustment speeds. Under the sticky price assumption, the exchange rate overshoots its new equilibrium level in response to shocks so that the system reaches a new saddle-path trajectory and converges to the new equilibrium position. Strictly speaking, "overshooting dynamics" is the consequence of the presence of "saddle-path dynamics." [1] In the literature, nonetheless, "overshooting" is commonly used to describe this class of exchange rate models. Thus, for convenience, in the following sections the terms "overshooting dynamics" and "saddle-path dynamics" are used interchangeably.

Several approaches have been adopted to test the overshooting model. For instance, some empirical studies are based on the reduced form exchange rate equation derived from the model. Despite the initial success of the model to describe observed data, the subsequent evidence is far from supportive (Frankel, 1979; Driskill, 1981; Driskill and Sheffrin, 1981). Other studies examine the relationship between real interest rate differentials and real exchange rates. Again, the empirical evidence is usually not in favor of the model (Meese and Rogoff, 1988; Edison and Pauls, 1993).

Engel and Morley (2001) consider a modified overshooting model that does not require exchange rates and prices to have the same adjustment speed.

Using an unobserved component specification, the authors find prices adjust faster toward their equilibrium values – a result that lends support to the modified overshooting model. Cheung, Lai and Bergman (2004), on the other hand, compare the individual contributions of exchange rate and price movements to real exchange rate dynamics. It is found that real exchange rate dynamics are mainly driven by exchange rate adjustments while the reversion to real exchange rate equilibrium is attributable to price adjustments. Also, exchange rate movements tend to amplify and prolong deviations from the equilibrium real exchange rate. The finding is at odds with the adjustment mechanism predicted by the standard overshooting model.

Several studies directly evaluate the effect of monetary shocks on exchange rates and, hence, infer the validity of the overshooting hypothesis. Eichenbaum and Evans (1995), for instance, find that exchange rate overshooting exists but the maximal impact of a monetary shock on exchange rates occurs with a lag of two to three years. The finding is not entirely consistent with the overshooting model *à la* Dornbusch, which predicts exchange rate overshooting is instantaneously triggered by the shock. The non-instantaneous overshooting phenomenon appears to be a common empirical regularity (Cheung and Lai, 2000; Clarida and Gali, 1994). Faust and Rogers (1999), however, argue that the observed non-instantaneous overshooting effect derived from a vector autoregression (VAR) system can be spurious. These authors point out that the timing of the maximum monetary shock effect depends on the assumptions used to identify the VAR system. They show that the identification scheme proposed by Faust (1998) can be used to obtain the almost immediate overshooting effect.

This study offers an alternative perspective to evaluate the validity of the overshooting hypothesis. Essentially, we exploit the implication of the saddle-path mechanism, which is the driving force of the overshooting result, for data dynamics. The intertemporal dynamics of a given system are governed by the roots of its characteristic polynomial. In the exchange rate literature, the saddle-path property that yields the overshooting phenomenon is defined by the presence of both explosive and stationary roots. Typically, some transversality conditions are imposed to limit the effects of explosive roots so that the system can settle on the saddle path that leads to the steady state.

To certain extent, the characterization of saddle-path dynamics is comparable to, but different from, that of cointegration. Both saddle-path and cointegration dynamics depend upon the roots of the system's characteristic polynomial. Such a similarity suggests that a test for cointegration may be adopted to test for the presence of saddle-path dynamics.

This paper explores whether the Johansen procedure, a standard approach to test for cointegration, is a useful tool to detect saddle-path dynamics. Instead of testing for non-stationary behavior directly, the Johansen test exploits the implications of cointegration for the rank of the coefficient matrix defined by the characteristic polynomial and uses the rank condition to infer system dynamics. By using rank conditions, the Johansen test sidesteps some technical issues of hypothesis testing in the presence of non-stationarity. Indeed, it can be shown that the saddle-path and cointegration dynamics have different implications for the rank of the coefficient matrix defined by the characteristic polynomial. Specifically, the presence of cointegration is not consistent with saddle-path dynamics. Thus, the Johansen procedure can be used to discriminate between the two types of system dynamics.

When we apply the Johansen procedure to study the interaction between exchange rates and relative prices, we find that exchange rates and relative prices are cointegrated. The empirical results are suggestive of the absence of the Dornbusch-type overshooting behavior in the data.

A canonical Dornbusch-type overshooting is presented in the next section. Section 3 describes the design of the Monte Carlo experiment and reports the empirical power of the Johansen procedure for detecting saddle-path dynamics. The results of testing for cointegration in monthly data from five industrial countries are presented in Section 4. Section 5 offers some concluding remarks.

2. An Overshooting Model

For illustrative purposes, we present a standard overshooting model *à la* Dornbusch. The sticky-price assumption is a key element of the standard Dornbusch model. Although the purchasing power parity is assumed to hold in the long run, prices are assumed to be inflexible in the short run and do not react instantaneously to a shock. The overshooting phenomenon occurs because, in respond to a monetary shock, the exchange rate has to adjust to clear not just the foreign exchange market but also the goods market to attain a short-run equilibrium. The gradual price adjustment is the mechanism bringing the system to the long-run equilibrium.

A stochastic version of Dornbusch's overshooting model can be formulated as follows (Azariadis, 1993, chapter 5):

$$i_t - i_t^* = E_t e_{t+1} - e_t \tag{1}$$

$$m_t - p_t = \phi y_t - \eta i_t + u_t \tag{2}$$

$$y_t = \delta(e_t + p_t^* - p_t) - \sigma(i_t - E_t(p_{t+1} - p_t)) + \varepsilon_t \tag{3}$$

$$E_t p_{t+1} - p_t = \alpha(y_t - \bar{y}) \tag{4}$$

where all variables (except the interest rates) are in logarithms and all parameters are non-negative. Equation (1) captures the uncovered interest rate parity condition: with e_t being the nominal exchange rate defined as the domestic price of foreign currency and $i_t (i_t^*)$ being the domestic (foreign) interest rate. The domestic nominal interest rate can exceed the foreign rate when the market anticipates a depreciation of the domestic currency. Equation (2) describes a money-market equilibrium relationship, where m_t is the nominal money supply, p_t is the price level and y_t is the real national income. The shock to the monetary equilibrium is given by u_t. Equation (3) states that the income level is demand determined. A real depreciation raises demand and so does a fall in the real interest rate. ε_t is a real demand shock. Equation (4) governs the price adjustment scheme. Although prices are predetermined and do not respond instantly to current realizations of other variables, they adjust gradually over time in response to the excess of aggregate demand over the natural/full employment output level ($\bar{y}$)

Conceptually, the model generates overshooting behavior in the following manner. With short-run price stickiness, an unanticipated monetary expansion induces a fall in domestic interest rates and leads to a capital outflow that will lead to the overshooting of the domestic currency to the point where the expected rate of appreciation exactly offsets the interest differential. Moreover, aggregate demand is boosted by the currency depreciation and lower interest rates. In response to higher aggregate demand, prices begin to rise slowly, thereby reducing the real money supply and pushing domestic interest rates back up. The domestic currency then appreciates gradually over time, along with rising prices. The gradual price adjustment will drive both the exchange rate and the real exchange rate to converge asymptotically to their corresponding equilibrium levels.

The implications of the equations (1) to (4) for exchange rate dynamics can be seen from the solution of the model. Following the standard procedure, we assume the foreign interest rate, the foreign price, and the domestic money supply are constant; that is, $i_t^* = i^*$, $p_t^* = p^*$, $m_t = m$. The resulting solutions of the exchange rate and price paths are given by a system of first-order simultaneous difference equations:

$$e_{t+1} - e_t = (1/\eta)(p_t - \bar{p}) + v_{1t+1} \tag{5}$$

$$\begin{aligned} p_{t+1} - p_t &= (\alpha\delta/(1-\alpha\sigma))(e_t - \bar{e}) \\ &\quad -(\alpha(\delta + (\sigma/\eta))/(1-\alpha\sigma))(p_t - \bar{p}) + v_{2t+1} \end{aligned} \tag{6}$$

where $\bar{e}$ and $\bar{p}$ are the respective steady-state values of the exchange rate and the price level. The zero mean disturbance terms v_{1t+1} and v_{2t+1} are combinations of monetary shocks, real shocks and prediction errors. The system can be compactly written as

$$\Delta X_{t+1} = \mu + AX_t + V_{t+1} \tag{7}$$

where $X_{t+1} = (e_{t+1}, p_{t+1})'$, $\Delta = (1-L)$, L is the lag operator, the constant μ is a function of the parameters and the steady-state values of the exchange rate and price, and

$$A = \begin{pmatrix} 0 & \dfrac{1}{\eta} \\ \dfrac{\alpha\delta}{1-\alpha\sigma} & -\dfrac{\alpha(\delta + (\sigma/\eta))}{1-\alpha\sigma} \end{pmatrix}. \tag{8}$$

Let $\theta_1 < \theta_2$ be the two roots of the characteristic equation $|A - \theta I| = 0$.[2] Depending on parameter configuration, the model can generate different types of dynamics.

For instance, under the assumption that $\alpha\sigma < 1$, then the determinant of A, $|A| = \theta_1\theta_2 = -(\alpha\delta/(1-\alpha\sigma)\eta) < 0$, the two roots have different algebraic signs, implying that $\theta_1 < 0 < \theta_2$. Therefore, θ_2 is the explosive root and θ_1 is the stationary root. In this case, the system exhibits saddle-path dynamics and the associated overshooting behavior.

The popular cointegration dynamics are also encompassed under (7). Note that equation (7) is already in an error correction format. For cointegration to take place, the rank of matrix A should be equal to one and $|A| = 0$. For instance, the rank condition is satisfied when $\delta = 0$; that is, aggregate demand does not respond to the real exchange rate. When $\delta = 0$, the roots are $\theta_1 = -(\alpha\sigma/(1-\alpha\sigma)\eta) < 0$ and $\theta_2 = 0$. Therefore, cointegration dynamics can be viewed as a limiting case of saddle-path behavior, in particular, when δ tends to zero.

Figures 1 and 2 illustrate the saddle-path and cointegration dynamics respectively. Technically speaking, the saddle-path dynamics are described by the unique manifold that leads the system towards its steady state. Appropriate initial conditions place the economy on the saddle-path manifold. Figure 1 gives a canonical phase diagram for a saddle-path system. The arrows indicate the system dynamics. The unique trajectory that brings the system to its steady state is the saddle-path line, denoted by the SP line in Figure 1.

For a cointegrated system, only one common I(1) process drives the evolution of the system components. The system converges to its steady state disregarding the initial conditions. Under cointegration, there are an infinite number of trajectories that bring the system to its equilibrium. Notice that the slope of the phase line $\Delta p_t = 0$ is equal to $\alpha\delta/(\alpha(\delta+(\sigma/\eta)))$. Therefore as δ tends to zero, the phase line $\Delta p_t = 0$ rotates clock-wise until it overlaps with the $\Delta e_t = 0$

phase line. Figure 2 depicts the phase diagram of a cointegrated system, where the two lines overlap. When δ tends to zero, the explosive region in Figure 1 disappears and there is an infinite number of paths leading to the line where the two phase lines overlap. The manifold where the two phase lines overlap is known as the “attractor” of the system.

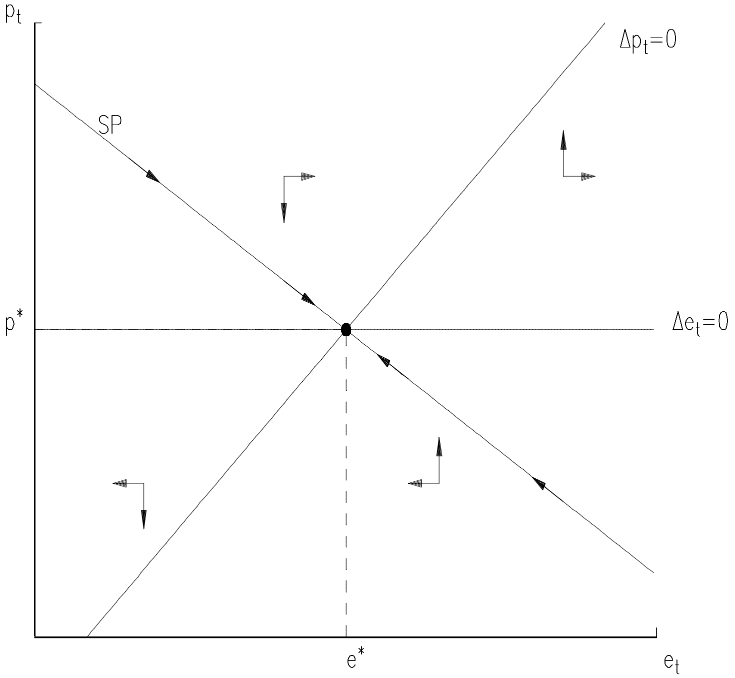

Figure 9.1 Saddle Path Dynamics

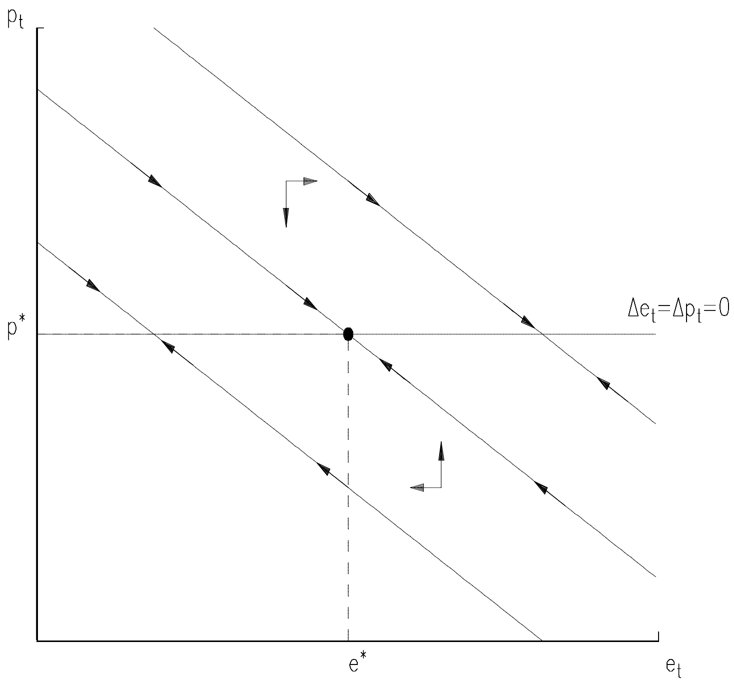

Figure 2 Cointegration Dynamics

There is another case that deserves attention. When $\alpha = 0$, the price level follows a random walk (a martingale difference process, to be precise), the rank of matrix A is null, and there is no cointegration between prices and exchange rates. Apparently, this case is not relevant to empirical data on exchange rates and prices examined in Section 4 because these data are typically non-stationary and prices do not follow a martingale.

3. Detecting Saddle-path Behavior

The discussion in the previous section suggests that the rank of A can be used to infer the dynamics of the system. Instead of deriving a new testing method, we observe that the Johansen's procedure, which is a standard test for cointegration, uses the rank of A to infer the system dynamics. Thus, we explore the possibility of using the Johansen's procedure to discriminate the saddle-path and stationary systems from a cointegrated system.

For a bivariate difference-stationary system, the Johansen procedure is usually implemented as follows. First, the maximum eigenvalue statistic of the Johansen procedure tests the null hypothesis $H_0 : rank(A) = 0$ against the alternative $H_1 : rank(A) = 1$. Under H_0, the unit root components of two individual series are driven by two different $I(1)$ processes and there is no cointegration. Under H_1, the variables are cointegrated and the two variables are driven by one common $I(1)$ process and one stationary process. If H_0 is rejected, the procedure then considers the hypothesis $H_1 : rank(A) = 1$ against the alternative $H_2 : rank(A) = 2$. While the cointegration dynamics is consistent with the non-rejection of H_1, either a stationary system or a saddle-path system implies A has full rank. It is interesting to recall that, in the previous section, it is shown that a cointegration system can be interpreted as a limiting case of either a saddle-path system or a stationary system.

In the literature, there are several studies examining the empirical performance of the Johansen procedure (Cheung and Lai, 1993b; Gonzalo 1994). Typically these studies consider the cointegration rather than the saddle-path alternative. The Johansen procedure is constructed to test for the rank of A and, at least theoretically, can be used to detect saddle-path behavior. The natural question to ask is - "What is the empirical power of the Johansen's tests against the saddle-path alternative?" A Monte Carlo experiment is designed to shed some insights on the power issue. Again, a bivariate system that has the form of (7) is used to illustrate the point.

TABLE 1. The Empirical Power of the Johansen Maximum Eigenvalue Statistic Against Saddle-Path Alternatives

	Roots		T = 100		T = 300	
	θ_2	θ_1	$H_0 vs H_1$	$H_1 vs H_2$	$H_0 vs H_1$	$H_1 vs H_2$
σ_0=0.0	0.20	-0.20	1.0	0.7845	1.0	1.0
	0.20	-0.10	1.0	0.5119	1.0	0.9944
	0.20	-0.01	1.0	0.1027	1.0	0.2023
	0.10	-0.20	1.0	0.5186	1.0	0.9936
	0.10	-0.10	1.0	0.3703	1.0	0.9508
	0.10	-0.01	1.0	0.1087	1.0	0.1977
	0.01	-0.20	1.0	0.1120	1.0	0.2004
	0.01	-0.10	1.0	0.1244	1.0	0.1938
	0.01	-0.01	1.0	0.1686	1.0	0.1566
σ_0=0.1	0.20	-0.20	0.9995	0.8355	1.0	1.0
	0.20	-0.10	0.9715	0.5779	1.0	0.9980
	0.20	-0.01	0.9283	0.1114	1.0	0.1956
	0.10	-0.20	0.9981	0.5565	1.0	0.9974
	0.10	-0.10	0.8976	0.4221	1.0	0.9640
	0.10	-0.01	0.7304	0.1391	1.0	0.1867
	0.01	-0.20	0.9943	0.1148	1.0	0.2052
	0.01	-0.10	0.7893	0.1313	1.0	0.2028
	0.01	-0.01	0.4326	0.1301	0.7625	0.1688
σ_0=1.0	0.20	-0.20	0.8881	0.9573	1.0	1.0
	0.20	-0.10	0.5943	0.8082	1.0	0.9995
	0.20	-0.01	0.5732	0.0343	1.0	0.0521
	0.10	-0.20	0.8253	0.7037	1.0	0.9997
	0.10	-0.10	0.3279	0.5974	1.0	0.9939
	0.10	-0.01	0.2619	0.0672	0.9487	0.0544
	0.01	-0.20	0.8304	0.1163	1.0	0.2014
	0.01	-0.10	0.2393	0.2173	1.0	0.1997
	0.01	-0.01	0.2429	0.0543	0.2119	0.1369

TABLE 1 Continued.

	Roots		T = 100		T = 300	
	θ_2	θ_1	$H_0 vs H_1$	$H_1 vs H_2$	$H_0 vs H_1$	$H_1 vs H_2$
σ_0=10.0	0.20	-0.20	0.7946	0.9911	1.0	1.0
	0.20	-0.10	0.4929	0.9503	1.0	1.0
	0.20	-0.01	0.4770	0.0027	1.0	0.0057
	0.10	-0.20	0.7024	0.7809	1.0	1.0
	0.10	-0.10	0.2123	0.7612	1.0	0.9999
	0.10	-0.01	0.1951	0.0046	0.9017	0.0065
	0.01	-0.20	0.7479	0.1223	1.0	0.2087
	0.01	-0.10	0.1465	0.2874	1.0	0.2077
	0.01	-0.01	0.2180	0.0234	0.1244	0.0748

Note to Table 1: The empirical rejection frequencies of applying the Johansen maximum eigenvalue test to artificial data generated according to saddle-path dynamics are reported. The rejection frequencies are based on 10,000 replications and a 5% critical value. Two sample sizes; T = 100 and T = 300, are considered. The hypotheses are defined by H_0: rank(A) = 0, H_1: rank(A) = 1, and H_2: rank(A) = 2. Two rejection frequencies are recorded. The first one reported under the column "$H_0 vs H_1$" is the frequency of H_0 being rejected. The second one reported under "$H_0 vs H_1$" is the frequency of H_1 being rejected conditioning on the rejection of H_0. The characteristic roots of the system are given by θ_1 and θ_2. The standard deviation of the initial condition is given by σ_0. See the text for a more detailed description of the simulation.

The Monte Carlo experiment is conducted as follows. First, T observations of X_t are generated according to saddle-path dynamics. The Appendix contains information on the procedure used to generate the data. Second, the maximum eigenvalue statistic is used to test the hypothesis $H_0 : rank(A) = 0$ against the alternative $H_1 : rank(A) = 1$. If H_0 is rejected in favor of H_1, then H_1 is tested against the alternative $H_2 : rank(A) = 2$. Third, the procedure is repeated N times. Two rejection frequencies are recorded. The first one is the frequency of H_0 being rejected. The second one is the frequency of H_1 being rejected, conditioning on the rejection of H_0. The following parameter values are considered: T = (100, 300), N = 10,000, θ_1 = (-0.20, -0.1, -0.01), θ_2 = (0.20, 0.1, 0.01), and $\sigma_i^2 = \text{var}(\nu_{it}) = 0.01$. An additional parameter is the variance of the distribution (σ_0^2) from which the initial observation is drawn. The values of σ_0 used in the experiment are 0, 0.1, 1, and 10.

Because the Johansen's methodology is a standard test procedure, we refer the reader to, for example, Johansen and Juselius (1990), for a detailed discussion of the procedure and of the construction of the maximum eigenvalue statistic.

The simulations results are reported in Table 1. The rejection frequencies are derived using the 5% critical value. One relatively easy to interpret result is that the power increases with the sample size. Conditional on the other parameter values, the rejection frequency increases with the sample size – that is, the test is consistent. The implications of the roots θ_1 and θ_2 for the rejection frequencies are quite intuitive. In general, the further away the roots are from zero, the higher is the rejection frequency. Exceptions occur when T = 100, θ_1 = -0.01 and θ_2 = 0.01. In some of these cases, the rejection frequency for H_1 against H_2 is higher than in some other parameter combinations in which the roots are further away from zero. However, the apparent odd result disappears when the rejection frequency for H_1 against H_2 is computed without conditioning on the rejection of H_0.

It is interesting to observe that, for the two tests H_0 against H_1 and H_1 against H_2, both θ_1 and θ_2 have comparable effects on the rejection frequencies. The observation is consistent with the fact that the Johansen procedure is a test for the rank of the relevant coefficient matrix. When either θ_1 or θ_2 is approaching zero, the rank of the relevant matrix is approaching one, the system dynamics are shifting towards H_1, and it is getting more and more difficult to reject H_1. As a general rule, when both θ_1 and θ_2 are close to zero, the rank is close to zero and the test has low power to reject H_0. It is not too surprising to observe the limited power of the test, especially when θ_1 = -0.01 and θ_2 = 0.01. Statistical tests always have low power for alternatives that are very close to the null hypothesis.

The effect of σ_0 appears intricate. When $\sigma_0 = 0$, all simulated time series are initially at the steady state. The system moves away from the steady state in the presence of random shocks and, then, follows the saddle-path to the new steady state. When $\sigma_0 > 0$, the initial position of the system is not necessarily at the steady state. The greater σ_0, the more likely the system is initially far away from the steady state. In fact, the σ_0 parameter can have two opposite effects on the empirical power. On the one hand, when σ_0 is large, the initial shock moves the system far away from the steady state and, hence, the system stays for a long time on the converging saddle path. Intuitively, it would be easier for the test to reveal the saddle-path dynamics. On the other hand, a large σ_0 introduces a high level of noise and, subsequently, makes it more difficult to reject the nonstationarity (null) hypothesis and less easy to uncover saddle-path dynamics.

The results in Table 1 indicate that the effect of σ_0 depends on the roots θ_1 and θ_2. It is instructive to compare the two extremes cases (θ_1 = -0.20 and θ_2 = 0.20) and (θ_1 = -0.01 and θ_2 = 0.01). In the former case, the roots are quite different from zero and the system is far away from H_0 and H_1. An increase in the value of σ_0 from 0 to 1 is accompanied with an increase in the number of cases in which favorable evidence is gardened for H_2. The result holds when either the conditional rejection frequency (the one reported in the table) or the total rejection frequency is considered. Thus, for this parameter configuration, an increase in the value of σ_0 from 0 to 1 improves the ability to detect saddle-path dynamics. The rejection frequency falls, however, when σ_0 is increased from 1 to 10. Thus, when the noise level associated with σ_0 is high (relative to the distance from H_0 and H_1), σ_0 negatively affects the power of the test to detect saddle-path dynamics. For the case θ_1 = -0.01 and θ_2 = 0.01, the system is very close to having two zero roots. Under this situation, an increase in the value of σ_0 makes it more difficult to discern the saddle-path dynamics and, thus, lowers the ability of the test to reject H_0 and H_1. The positive (negative) effect of σ_0 on ability to reveal saddle-path dynamics dominates when the system dynamics is far away from (close to) those implied by H_0 and H_1.

TABLE 2. The Empirical Power of the Johansen Maximum Eigenvalue Statistic Against Stationary Alternatives

Roots		T = 100		T = 300	
θ$_2$	θ$_1$	Ho Vs H1	H1 Vs H2	Ho Vs H1	H1 Vs H2
-0.20	-0.20	1.0	0.8349	1.0	1.0
-0.20	-0.10	1.0	0.5641	1.0	0.9974
-0.20	-0.01	1.0	0.1042	1.0	0.1972
-0.10	-0.20	1.0	0.5485	1.0	0.9975
-0.10	-0.10	1.0	0.3980	1.0	0.9613
-0.10	-0.01	1.0	0.1135	1.0	0.1862
-0.01	-0.20	1.0	0.1064	1.0	0.2028
-0.01	-0.10	1.0	0.1128	1.0	0.1939
-0.01	-0.01	1.0	0.1326	1.0	0.1551

Notes to Table 2: The empirical rejection frequencies of applying the Johansen maximum eigenvalue test to artificial data generated according to stationary dynamics are reported. The rejection frequencies are based on 10,000 replications and a 5% critical value. Two sample sizes;

T = 100 and T = 300, are considered. The hypotheses are defined by H_0: rank(A) = 0, H_1: rank(A) = 1, and H_2: rank(A) = 2. Two rejection frequencies are recorded. The first one reported under the column "$H_0 vs H_1$" is the frequency of H_0 being rejected. The second one reported under "$H_1 vs H_2$" is the frequency of H_1 being rejected conditioning on the rejection of H_0. The characteristic roots of the system are given by θ_1 and θ_2. See the text for a more detailed description of the simulation.

In conducting the simulation experiment, the Johansen trace statistics were also computed. However, the empirical power estimates based on the trace statistic are very similar to those based on the maximum eigenvalue statistic. Different values of σ_1^2 and σ_2^2 were also included in the experiment. It turns out that the simulation results are quite insensitive to a) the value of σ_1^2 and σ_2^2, and b) the relative size of of σ_1^2 and σ_2^2. Thus, the simulation results related to the trace statistic and different combinations of σ_1^2 and σ_2^2 are not reported for brevity. These results are available from the authors upon request.

While the results indicate that the Johansen procedure has a reasonable power to uncover saddle-path behavior, it is noted that a stationary bivariate system can lead to similar rejection results. It is instructive to assess the power of the test in the presence of stationary data. To this end, we apply the Johansen procedure to data generated under stationary alternatives. The stationary roots considered are θ_1, θ_2 = (-0.2, -0.10, -0.01). The other parameters are the same as those considered in Table 1. Table 2 reports the power of the Johansen procedure against the stationary alternatives when we set $\sigma_0 = 0$.

Similar to the saddle-path experiment, the empirical power in Table 2 is increasing with the sample size and the distance of the roots from zero. Compared with results in Table 1, results in Table 2 indicate that the Johansen maximum eigenvalue statistic has reasonable power in detecting the full rank condition – no matter it is generated by saddle-path or stationary dynamics.

4. Exchange Rate Dynamics

In this section, we use the Johansen procedure to infer whether the saddle-path and the related overshooting dynamics are an appropriate description of exchange rate dynamics. Four dollar-based exchange rates namely British pound, French franc, German mark, and Italian lira are included in the sample. Monthly data of nominal exchange rates and consumer price indexes from April 1973 to December 1998 were retrieved from the International Financial Statistics data CD-ROM. These data are expressed in logarithms. As commonly conceived, the individual exchange rate and price series display I(1) non-stationarity. Following the literature, the bivariate system comprising of the nominal exchange rate and the relative price is employed to study the cointegration relationship between exchange rates and relative prices.

For notational purposes, a bivariate system as (7) is re-writen in its general form:

$$\Delta X_t = \mu + AX_{t-1} + \sum\nolimits_{i=1}^{k-1} A_i X_{t-i} + V_t \qquad (9)$$

where no parameter restriction is imposed on matrices A and A_i. The lagged X_t's are included to ensure that V_t follows a white noise process and that the Johansen result is not distorted by serial correlation in the error term. In implementing the test, the lag parameter k is selected using the Akaike information criterion. Both the Johansen maximum eigenvalue and trace statistics are calculated. Again we refer readers to Johansen and Juselius (1990) for the construction of these test statistics.

TABLE 3. Johansen cointegration test results

	Max. Eigenvalue Stat.		Trace Stat.	
	Rank(A) = 0	Rank(A) = 1	Rank(A) = 0	Rank(A) = 1
British Pound				
Lag = 2	21.3878*	5.3478	26.7356*	5.3478
French Franc				
Lag = 4	34.7165*	4.3357	39.0522*	4.3357
German Mark				
Lag = 1	15.9658**	3.9937	19.9595*	3.9937
Italian Lira				
Lag = 1	26.6205*	4.9276	31.5481*	4.9276

Notes to Table 3: The Johansen tests for cointegration between nominal exchange rates and relative prices are presented. Both the maximum eigenvalue statistic "Max. Eigenvalue Stat." and the trace statistics "Trace Stat." are reported. The null hypotheses are given underneth the statistic labels. The alternatives for the maximum eigenvalue statistic are Rank(A) = 1 and Rank(A) = 2 and the those for the trace statistic are Rank(A) > 0 and Rank(A) > 1. The lag paramter "Lag =" is selected using the Akaike information criterion. Significance at the 5% and 1% levels are indicated by "**" and "*" according to the finite sample critical values in Cheung and Lai (1993b). The hypothesis of Rank(A) = 0 is rejected by both statistics but the hypothesis of Rank(A) = 1 is not rejected.

The results of the Johansen tests are reported in Table 3. Both the maximum eigenvalue and trace statistics reject the null hypothesis $H_0 : rank(A) = 0$ but not the null hypothesis $H_1 : rank(A) = 1$. Thus, the exchange rate and the relative price are cointegrated and the two series in each bivariate system are driven by a common I(1) process. Individually, each series evolves as a non-stationary I(1) process. However, a unique combination of the two series governed by the cointegrating vector is stationary. Typically, the cointegration result is interpreted as the evidence of the presence of an empirical long-run relationship between exchange rates and prices, which constitutes a necessary condition for long-run purchasing power parity (Cheung and Lai, 1993a; Kugler and Lenz, 1993).

The results in the previous sections allow us to use the rank of A to infer the system dynamics from a different perspective. In addition to the long-run relationship interpretation, our results also indicate that neither the notion of saddle-path nor stationary dynamics are consistent with the inference that the rank of A is equal to one. Because exchange rates and relative prices are I(1) processes, the bivariate system consisting of these two variables is not stationary. Thus, the strength of the result is its implications for the irrelevance of using saddle-path and the related overshooting dynamics to describe exchange rate behavior.

There are a few caveats in generalizing the cointegration results. First, the empirical illustration includes only a few countries even though these are the key industrial countries. It is fair to say that a more definite inference on the relevancy of saddle-path dynamics still awaits additional results from a larger set of dollar-based exchange rates and cross-rates. Second, as indicated in the simulation experiment, the ability to detect saddle-path dynamics is severely handicapped when the explosive root is very close to one. Further analyses are required to rule out this possibility. Nonetheless, the cointegration results in Table 3 cast doubt on the general validity of saddle-path/overshooting exchange rate dynamics.

5. Concluding Remarks

The overshooting model *à la* Dornbusch is a prominent explanation for the volatile exchange rate behavior in the current floating period. Assuming prices are sticky, the model displays a saddle-path pattern and yields overshooting dynamics that induces high short-term exchange rate volatility. Using a bivariate system, this study illustrates the implications of saddle-path, cointegration, and stationary dynamics for the characteristic roots that determines the system's intertemporal behavior. It is shown that a cointegration system can be interpreted as a limiting case of a system that displays either saddle-path or stationarity dynamics. A Monte Carlo experiment is designed to illustrate the usefulness of the Johansen tests to uncover saddle-path dynamics. The simulation results indicate that the Johansen tests have a) reasonable power to detect saddle-path dynamics, and b) similar power to reject the cointegration hypothesis in favor of saddle-path or stationarity alternatives.

Our empirical example shows that exchange rates and prices are cointegrated. Because the variables in a saddle-path system are not supposed to display a cointegrating relationship, the empirical evidence is indicative of the absence of saddle-path dynamics in the data under investigation. Exchange rate models that do not rely on saddle-path properties and over-shooting dynamics may deserve some more serious attention.

It is conceivable that the implications of the current study go beyond the exchange rate saddle-path behavior. There are models in different areas in economics exhibiting saddle-path properties. For instance, the neo-classical growth model (Cass, 1965) is an early example in which saddle-path dynamics are used to elaborate balanced-growth. Other models that utilize saddle-path dynamics to elucidate relationships between economic variables include those of Bruno and Fischer (1991) for interest rates and inflation, Evans and Yarrow (1981) for real money balances and inflation. The saddle-path property in these models, however, is not commonly subject to direct empirical test.

Nonetheless, it is noted that some studies report cointegrating relationship between a) output, investment, and consumption (King *et al.*, 1991)[3] and between interest rates and inflation (Bonham; 1991). These cointegration results imply the saddle-path models may not be appropriate for these variables.

While the Johansen procedure, as illustrated in previous sections, can be used to test for saddle-path dynamics, further studies on other testing procedures for saddle-path dynamics are warranted; especially given the widespread use of saddle-path models in economics.

Appendix: Generating Data that Exhibit Saddle-Path Dynamics

The simulation experiment dealing with saddle-path dynamics is conducted as follows. First, we find a solution to equation (7) under the saddle-path hypothesis. Second, using the saddle-path solution, we simulate X_t of length T. T = 100 and T = 300 are considered in the exercise. Third, the Johansen test statistic is calculated from the simulated data. The above steps are repeated N times and N is set to 10,000. The N sample Johansen statistics are then compared with the 5% critical value to tally the rejection frequency.

We follow the standard procedure to obtain the saddle-path solution to equation (7). Let B be a (2x2) matrix whose columns contain the eigenvectors of $(A+I)$. Pre-multiplying system (7) by B^{-1}, we obtain $Z_t = \Lambda Z_{t-1} + U_t$ where $Z_t = B^{-1} X_t$, $\Lambda = B^{-1}(A+I)$ is a diagonal matrix with the eigenvalues of $(A+I)$ along the diagonal and $U_t = B^{-1} V_t$. Then, we solve each of the first-order difference equations $z_{it} = (1+\theta_i) z_{it-1} + u_{it}$; $i = 1,2$ where $Z_t = (z_{1t}, z_{2t})'$ and $U_t = (u_{1t}, u_{2t})'$. Under the saddle-path hypothesis, $\theta_1 < 0$ and $\theta_2 > 0$. We solve the first equation backward and the second equation forward. The solutions can be expressed as the sum of two terms:

$$z_{it} = z_{it}^* + (1+\theta_i)^t \, c_{i0}$$

where

$$z_{1t}^* = \sum_{i=0}^{\infty} (1+\theta_1)^i \, u_{1t-i} \ ,$$

$$z_{2t}^* = \sum_{i=0}^{\infty} \left(\frac{1}{1+\theta_2} \right)^{i+1} u_{2t+i+1}$$

and $c_{i0} = z_{i0} - z_{i0}^*$. In economics, these two terms are usually labeled the "steady state" and the "bubble." The saddle-path solution is obtained by setting the terminal condition $c_{20} = 0$ so that the resulting sequence is not explosive. The original variables of the system are then recovered using $X_t = BZ_t$.

The steady state $Z_t^* = (z_{1t}^*, z_{2t}^*)'$ is approximated by the sum of a finite number of elements. We first generate the series U_t of length $3T$ using a normal random number generator. The first T simulated numbers are used to generate z_{11}^*, the first $T+1$ simulated numbers are used to generate z_{12}^*, ..., and so on. The last T simulated numbers are used to generate z_{2T}^*, the last $T+1$ simulated numbers are used to generate z_{2T-1}^*, …, and so on. In addition, the initial condition c_{10} is required to calculate the solution. In the experiment, the initial condition c_{10} is drawn from a normal distribution with zero mean and variance σ_0^2. The idea of the random choice is to capture the existence of a continuum of equilibria (each indexed by a different initial condition) lying on the unique stable manifold converging to the steady state.

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Notes.

[1] Strictly speaking, overshooting implies saddle-path but the opposite is not true.

[2] Notice that if equation (7) is written as $X_{t+1} = \mu + \Pi X_t + V_{t+1}$, where $\Pi = A + I$, then the roots of Π, say λ_1 and λ_2, are related to the roots of A according to $\lambda_i = 1 + \theta_i$, i=1,2. Therefore, a unit root of Π is equivalent to a zero root of A.

[3] King *et al*. (1991) show in a neoclassical growth framework that (the logs of) output, consumption and investment are cointegrated when thechnology shocks follow an I(1) process, whereas certain ratios characterizing the balanced-growth path (for instance, the consumption-output and the investment-output *great ratios*) exhibit saddle-path dynamics.

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