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Unemployment and the Direction of Technical Change

Abstract

I construct and analyze a growth model in which technical change can increase unemployment. I first analyze the forces that deliver a constant steady state unemployment rate in this setting. Labor-saving technical change increases unemployment, which lowers wages and creates incentives for future investment in labor-using technologies. In the long run, this interaction generates a balanced growth path that is observationally equivalent to that of the standard neoclassical growth model, except that it also incorporates a positive steady state level of unemployment and a falling relative price of investment. I also study the effects of a permanent increase in the ability of R&D to improve labor-saving technologies. In the long run, this change leads to faster growth in output per worker and wages, but it also yields higher unemployment and a lower labor share of income. In the short run, this change exacerbates existing inefficiencies and slows economic growth.

JEL-Codes: E240, O330, O400.

Keywords: growth, unemployment, directed technical change.

Gregory Casey
Department of Economics
Williams College
Schapiro Hall, 24 Hopkins Hall Dr.
USA – Williamstown, MA 01267
gpc2@williams.edu

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1 Introduction

From the Luddites to Keynes (1930) to Silicon Valley, there has long been fear that labor-saving technical change would lead to increased unemployment and even the obsolescence of labor in production (Autor, 2014, 2015). Existing theoretical analyses of labor-saving technical change, however, abstract from unemployment and instead focus on income distribution, labor supply, and other related outcomes (e.g., Acemoglu and Restrepo, 2018b; Hémous and Olsen, 2021).

I study a model in which increases in labor productivity lead to increases in unemployment, holding all else constant. The key features of the model are Leontief production, directed technical change, and a reduced-form representation of wage bargaining. Despite the assumption that labor productivity can cause unemployment, the resulting model has a balanced growth path that has a constant, non-zero rate of unemployment and is consistent with standard stylized growth facts (Jones, 2016). I use the model to investigate the impacts of an exogenous increase in the ability of research and development (R&D) to improve labor-saving technologies. I pay special attention to trade-offs between equity and efficiency and between short- and long-run outcomes.

Improvements in technology can decrease marginal labor productivity when the elasticity of substitution between capital and labor is sufficiently low (Acemoglu and Restrepo, 2018a). To construct a tractable model where improvements in productivity increase unemployment, I consider the case where production is Leontief in the short run. In other words, holding technology fixed, inputs must be combined in fixed proportions. Embodied technological characteristics determine the input requirements of creating and operating each capital good, and substitution between capital and labor occurs via the choice of technology (Jones, 2005; Caselli and Coleman, 2006; Leon-Ledesma and Satchi, 2018).[1] For a given set of technologies and quantity of installed capital, the economy can only support a finite number of workers with positive marginal product. Insufficient labor demand can generate unemployment when labor market frictions ensure a positive wage.[2] In the presence of such frictions, technical change can push workers into unemployment.

The standard labor-augmenting technology lowers labor input requirements and, holding all else constant, reduces employment. This type of technology is *labor-saving*. A second

[1]Casey (2024) analyzes a model with Leontief production and directed technical change to investigate the response of energy use to environmental policy. In distinguishing between the rigidity of production after capital goods are installed and the flexibility in choosing types of capital goods, the formulation in this paper is similar to a putty-clay model (e.g., Akerlof and Stiglitz, 1969; Gilchrist and Williams, 2000). Models of putty-clay production, however, generally focus on different vintages of capital goods and the irreversibility of investment. This paper instead focuses on the evolution of cutting-edge technologies.

[2]This channel is emphasized, but not modeled, in Korinek and Stiglitz (2017).

type of technology lowers the cost of producing new capital goods (i.e., reduces the relative price of investment). This type of technology is *labor-using*. The two types of technology are embodied in capital goods and evolve over time according to profit-maximizing R&D investments (Acemoglu, 2002). Firms have greater incentive to invest in labor-saving technology when labor inputs costs are a large fraction of total costs. Bargaining creates a direct link between unemployment, wages, and R&D incentives. I assume full depreciation and focus on the evolution of cutting-edge technologies.

A balanced growth path with a positive unemployment rate emerges from the interaction between directed technical change and wage bargaining. Given a set of technologies and amount of installed capital, the economy can only support a finite number of workers with positive marginal product. When labor becomes more efficient, fewer workers can be profitably employed in the short run, holding all else constant. This tendency towards increased unemployment is offset by economic expansion. In particular, the accumulation of capita – via investment or technological progress that lowers the cost of creating new capital – increases the number of jobs in the economy. The long-run unemployment rate depends on the relative growth rates of the labor-saving and labor-using forces. Improvements in labor-saving technologies lead to higher unemployment, which lowers wages and consequently increases future R&D in labor-using technologies. A balanced growth path exists with R&D in both types of technology, as well as capital accumulation and labor force growth.[3] These results suggest that directed technical change and wage bargaining play important roles in explaining unemployment along the balanced growth path.

The forces of balanced growth continue to operate after an increase in the productivity of R&D in labor-saving technologies. In the short run, labor-saving technology grows at a faster rate and unemployment increases. The increase in unemployment lowers wages and increases the return to capital. As a result, the fall in wages leads to faster capital accumulation and greater investment in labor-using technologies. The economy eventually converges to a new balanced growth path with a higher, but constant, unemployment rate and faster economic growth. In this way, the model suggests that increases in the growth rate of labor-saving technology will create winners and losers within homogeneous groups of workers, not just between workers with different skill levels as highlighted by the existing literature on labor market polarization (e.g., Autor, 2015; Hémous and Olsen, 2021). Also,

[3]The Uzawa steady state theorem implies that, with a standard two-factor neoclassical production function, all technological progress must be labor-augmenting on a balanced growth path (Uzawa, 1961; Schlicht, 2006; Jones and Scrimgeour, 2008). Following Casey and Horii (2023, 2024), the new model generates balanced growth with capital-augmenting technical change by incorporating land into the production function. See Grossman et al. (2017) for a discussion of the evidence that capital-augmenting technical change has been a feature of balanced growth in the United States.

the growth rate of output per capita increases, but the labor share decreases. Thus, there are also equity-efficiency trade-offs between different factors of production.

Despite boosting long-run economic growth, increases in labor-saving R&D productivity slow economic growth in the short run. In the presence of unemployment, investment in labor-saving technology is socially wasteful, because it saves a resources that is not scarce. Improvements in labor-saving technology push workers out of the labor force and leave effective labor inputs – and, therefore, output – unchanged. In other words, they increase output per worker, but not output per person. In the presence of positive unemployment and positive R&D in labor-saving technology, total output could always be increased by substituting unemployed workers for R&D in labor-saving technology and employing the newly available R&D inputs to improve labor-using technologies. Increases in labor-saving R&D productivity cause a reallocation of R&D towards labor-saving technologies, exacerbating this inefficiency and slowing the rate of economic growth in the short run.

While the main goal of this paper is to examine the relationship between technology and unemployment, I also consider the impact of changes in worker bargaining power, which introduce a different set of equity-efficiency trade-offs. Increases in worker bargaining power increase both unemployment and the labor share of income, while permanently lowering total output. Changes in bargaining power have no long-run effect on technological progress or economic growth.

Existing Literature. This paper contributes to a growing literature that studies labor-saving technical change (e.g., Graetz and Michaels, 2018; Korinek and Stiglitz, 2017). The most closely related paper is that of Acemoglu and Restrepo (2018b) who also focus on substitution between capital and labor in a model with a homogeneous set of workers. This paper complements the existing theoretical literature in two key ways. First, it models the relationship between labor-saving technical change and unemployment. Earlier works examine income distribution or changes in labor supply (e.g., Acemoglu and Restrepo, 2018b; Hémous and Olsen, 2021). Second, this paper examines the role of factor-augmenting technologies, whereas Acemoglu and Restrepo (2018b) and related work build on the tasked-based framework of Zeira (1998).[4] Despite the difference in structure, the model studied here highlights economic forces closely related to those in Acemoglu and Restrepo (2018b), reinforcing the

[4] Peretto and Seater (2013) consider a growth model where innovation changes the exponents in a Cobb-Douglas production function. The authors focus on the role of labor-saving technical change in offsetting diminishing returns and generating endogenous growth, rather than effect of automation on labor market outcomes. Boldrin and Levine (2002) examine the role of factor-saving innovation in generating endogenous growth in a setting with perfect competition. Their model has endogenous cycles with unemployment during downturns. Berg et al. (2018) focus on models with different types of capital and different patterns of savings between capitalists and workers.

generality of the underlying intuition. The focus on factor-augmenting technical change also demonstrates the connection between the relative price of investment and labor-using technical change.

Stiglitz (2014) also considers the role of innovation in generating constant unemployment in a setting with Leontief production. As in this paper, the long-run direction of technical change depends on wages, which in turn depend on the unemployment rate. The current paper complements his work in three important ways. First, I study a fully-specified model where the direction of technical change is determined by profit-maximizing firms, while in Stiglitz (2014) innovation is cost-less. Second, I examine the impacts of an increase in the effectiveness of R&D into new labor-saving technologies. Third, I develop a model where the relative price of investment decreases on the balanced growth path, which is consistent with data and implies that both labor-saving and labor-using technical change exist on the balanced growth path.

Most contemporary work examines unemployment through the lens of search and matching frictions (e.g., Pissarides, 2000). Aghion and Howitt (1994) incorporate job search into a Schumpeterian growth model to study the relationship between growth and unemployment. In this setting, technical change can increase unemployment, because creative destruction eliminates existing firm-worker matches. I build on this literature by studying an alternate mechanism, insufficient labor demand, that creates a link between technical change and unemployment.

There is a long history within the growth literature of studying unemployment in models with Leontief production (Johansen, 1959; Solow et al., 1966; Akerlof and Stiglitz, 1969). This paper connects directly with this older literature. It shows how adding directed technical change to the Harrod (1948)-Domar (1946) framework can help overcome the criticisms of these models posed by Solow (1956, 1994) and uses the resulting model to provide insight into relationship between unemployment and the direction of technical change. In his seminal contribution that demonstrated how to 'synthesize' the fixed-factor and substitution approaches to production functions, Johansen (1959) addresses the importance of considering technological change in this context: "[i]n conclusion it is suggested that the proposed hypothesis would be particularly appropriate in studying the introduction of new techniques and the relationship between labor force growth, the rate of saving and 'structural' unemployment" (p. 157). This is an apt description of the current paper, which examines how the evolution of cutting-edge technologies interacts with labor force growth and capital accumulation to determine long-run unemployment.

Roadmap. The paper proceeds as follows. Section 2 discusses how the model studied here connects with an older literature in growth theory. Section 3 presents the model, while Section 4 presents the calibration and quantitative exercises. Section 5 concludes.

2 Context and Background

In this section, I briefly review the core concepts of this paper in the context of an aggregate production function with exogenous technological progress. I also highlight the continuity of the model with an older strand of the growth theory literature. To capture the notion that productivity improvements can increase unemployment, I consider an aggregate production given by

$$Y_t = \min\Big[\big(A_{K,t}K_t\big)^{\alpha} M^{1-\alpha},\ A_{L,t}L_t\Big], \tag{1}$$

where where Y_t is output, K_t is the aggregate capital stock, M is a fixed factor of production such as land, and L_t is employed workers. The variables $A_{L,t}$ and $A_{K,t}$ capture two different kinds of technology. I use N_t to denote the size of the labor force, which implies the constraint $L_t \leq N_t$.

The Leontief production function implies that the maximum quantity of workers with positive marginal product is

$$L_t^{\max} = \frac{\big(A_{K,t}K_t\big)^{\alpha} M^{1-\alpha}}{A_{L,t}}, \tag{2}$$

and these workers all have marginal product $A_{L,t}$. For a given a wage $w_t \in (0, A_{L,t})$, therefore, firms will be willing to hire $L_t^{\max}$ workers and the unemployment rate will be

$$u_t \equiv 1 - \frac{L_t^{\max}}{N_t} = 1 - \frac{\big(A_{K,t}K_t\big)^{\alpha} M^{1-\alpha}}{N_t A_{L,t}}. \tag{3}$$

Remark. *Consider the definition of unemployment given in equation (3). Holding all else constant,* $\frac{\partial u_t}{\partial A_{L,t}} > 0$ *and* $\frac{\partial u_t}{\partial A_{K,t}} < 0$.

So, $A_{L,t}$ captures the state of *labor-saving* technology, and $A_{K,t}$ captures the state of *labor-using* technology. In the fully specified model, $A_{K,t}$ will be the inverse of the relative price of investment. Through $A_{L,t}$, the Leontief model creates the direct link between increases in productivity and increases in unemployment. The full model will combine the short-run Leontief production function with a model of directed technical change. As a result, substitution between capital and labor will occur via the choice over a set of potential technologies.

While the Leontief model is particularly tractable for investigating the relationship between technology and unemployment, the intuition is more general. As explained by Acemoglu and Restrepo (2018a), improvements in labor-augmenting technology decrease the marginal product of labor when the elasticity of substitution, holding technology fixed, is sufficiently low. If wages do not fully adjust, this implies that improvements in labor-augmenting technology can lead to increases in unemployment. Leontief production is the limiting case of perfect complementarity, implying that all substitution takes place through the choice of technology.

Early models of balanced growth with Leontief production – e.g., Harrod (1948) and Domar (1946, 1947) – were abandoned in large part because they were unable to explain the fact that unemployment was constant in the long run (Solow, 1956, 1994). For the case of exogenous technological progress, the same is true for the structure of production considered in this paper.

Remark. *Let output be given by the aggregate production function (1). Consider a balanced growth path were K_t/Y_t is constant and the technology growth rates are constant, $\frac{A_{J,t}}{A_{J,t-1}} = (1+g_{A_J})\ \forall t$, $J = K, L$. In this case, $u_t > 0$ is constant only if $(1+g_{A_K}) = ((1+n)(1+g_{A_L}))^{\frac{1-\alpha}{\alpha}}$, where n is the growth rate of the labor force.*[5]

Thus, when technological progress is exogenous, unemployment is only constant in a knife-edge case. In this way, the model also captures the fear that increases in the growth rate labor-saving technical change can lead to mass unemployment (Keynes, 1930; Leontief, 1952).

The purpose of this paper is to build and analyze a model that captures the relationship between labor-saving technical change and unemployment. First, I show that the model is consistent with stylized facts observed in macroeconomic data. In particular, I show that a constant long-run unemployment rate is the endogenous outcome in a model with Leontief production, directed technical change, and wage bargaining, addressing the criticisms of Solow (1956, 1994). Moreover, the new model explains all of the standard balanced growth facts that are frequently used to discipline growth models. Second, I use the model to investigate the impact of an increase in the productivity of R&D into labor-saving technologies.

[5]

Proof. If u_t is constant, then equation (3) implies that $\left(A_{K,t}K_t\right)^{\alpha} M^{1-\alpha}$ grows at factor $(1+g_{A_L})(1+n)$. Since $L_t < N_t$, Y_t grows at this same factor, which in turn implies that K_t does as well. Thus, $(1+g_{A_K})^{\alpha} \cdot \left((1+n)(1+g_{A_L})\right)^{\alpha} = (1+n)(1+g_{A_L}) \Rightarrow (1+g_{A_K}) = \left((1+n)(1+g_{A_L})\right)^{\frac{1-\alpha}{\alpha}}$. □

3 Model

3.1 Environment

3.1.1 Production

The market for final output is perfectly competitive. As is standard in the endogenous growth literature, there is a continuum of capital goods. These capital goods are combined with land and labor to produce final output. The aggregate production function is given by

$$Y_t = \int_0^1 \min\left[X_t(i)^\alpha M^{1-\alpha}, A_{L,t}(i)L_t(i)\right] di, \tag{4}$$

where $X_t(i)$ is the quantity of capital good i, M is natural capital (e.g., land), and $L_t(i)$ is labor hired to work with capital good i. The stock of natural capital is assumed to be fixed. For the remainder of the paper, I normalize $M = 1$. Labor-augmenting technical change, $A_{L,t}(i)$, determines the labor input requirement of operating capital good i. This is the *labor-saving* technology. Technology is embodied in the capital goods. There is no substitution between capital and labor after capital goods are installed. The price of output is normalized to one.

Each type of capital good is produced by a single monopolist. The productivity of investment is given by $A_{K,t}(i)$, and there is full depreciation of capital goods within a period. The market clearing condition for capital is given by

$$\int_0^1 \frac{X_t(i)}{A_{K,t}(i)}\, di \leq K_t, \tag{5}$$

where K_t is the quantity of output saved in the period $t-1$, which is also the aggregate capital stock. A long literature examines the existence and implications of this investment-specific technical change (e.g., Greenwood et al., 1997; Grossman et al., 2017). Given the Leontief production function for the final good, $A_{K,t}(i)$ is *labor-using* technology.[6]

Aggregate employment is given by

$$L_t \equiv \int_0^1 L_t(i)di \leq N_t, \tag{6}$$

where N_t is the size of the labor force at time t. The unemployment rate is $u_t \equiv 1 - \frac{L_t}{N_t}$. The labor force grows at a constant rate $n \geq 0$.

[6] The model presented here is isomorphic to one in which $A_{K,t}(i)$ appears in the final good production function and the productivity of capital good production is constant.

3.1.2 Research and Development

Monopolists can hire R&D inputs to improve either labor-saving or labor-using technology. In either case, technology evolves according to

$$A_{J,t}(i) \quad = \quad (1 + \eta_J R_{J,t}(i))\, A_{J,t-1}, \tag{7}$$

where $J = K, L$, $R_{J,t}(i)$ are research inputs hired by firm i to improve technology characteristic J, and $A_{J,t-1} = \int_0^1 A_{J,t-1}(i)\ di$. As in Fried (2018), patents last for one period, after which technology flows freely between firms.[7] Thus, the properties of new technologies depend on the amount of research inputs hired, as well as the aggregate state of technology. In addition, the terms η_L and η_K give the inherent ease of improving labor-saving and labor-using technology, respectively. Later in the paper, I will examine the effects of an exogenous increase in η_L, which will lead to a permanent increase in the growth rate of labor-saving technology. For each i, the level of technology prior to period 0 R&D, denoted $A_{L,-1}(i)$ and $A_{K,-1}(i)$, is given.

The goal of this paper is to study aggregate dynamics. This R&D specification implies that all firms have identical productivity from period 0 onwards. In other words, it abstracts from firm heterogeneity. The benefit of this specification is that it greatly simplifies the analytic and computational analysis of the aggregate outcomes in the model. The cost of this specification is that it provides no insight into firm-level outcomes.

There is a unit mass of research inputs in all periods, and these inputs are perfectly mobile across firms and technologies. The market clearing is given by

$$\int_0^1 R_{L,t}(i)di + \int_0^1 R_{K,t}(i)di = 1. \tag{8}$$

I also define $R_{J,t} \equiv \int_0^1 R_{J,t}(i)di,\ J = K, L$.[8]

[7] I write the expression in terms of technology at time t, rather than $t+1$, to highlight the fact that there is no forward-looking component to the monopolists' decisions.

[8] The assumption of a fixed set of research inputs is commonly made in the directed technical change literature (e.g., Acemoglu, 2003; Acemoglu et al., 2012). It is a stand in for two offsetting forces, an increase in aggregate research inputs and an increase in the cost of generating a given aggregate growth rate (Jones, 2002; Bloom et al., 2020).

3.1.3 Representative Household

The representative household has lifetime utility

$$U = \sum_{t=0}^{\infty} \beta^t N_t \frac{\tilde{c}_t^{1-\xi}}{1-\xi}, \tag{9}$$

where $\tilde{c}_t = C_t/N_t$ is consumption per person. I focus on the decentralized equilibrium where the household takes prices and technology levels as given. The relevant budget constraint is given by

$$C_t + K_{t+1} = w_t L_t + r_t K_t + p_{R,t}^K + p_{R,t}^L + \Pi_t + p_{M,t} M = Y_t, \tag{10}$$

where w_t is wages, r_t is the rental rate on capital, $p_{R,t}^J$ is the rental rate for R&D inputs used to improve technology $J = K, L$, $p_{M,t}$ is the rental rate for land, and Π_t is total profits of the capital good producers.

Capital must be non-negative in all periods, and the initial stock (K_0) is given. Since there is full depreciation and no uncertainty, the representative household will never save output that goes unused in the next period. Thus, the market clearing condition (5) holds with equality for $t > 0$. I assume that K_0 is low enough that it also holds with equality at time 0.

3.1.4 Wage Determination

Firms bargain with workers through a union to set wages, $w_t(i)$, which are specific to the type of capital good that a worker uses. The outcome of the bargaining process is captured by the following reduced-form expression:

$$w_t(i) = \chi A_{L,t}(i) + \chi\nu(1 - u_t)A_{L,t}. \tag{11}$$

Each worker produces a quantity $A_{L,t}(i)$ of the final good. Workers receive higher wages when they (a) are more productive (i.e., higher $A_{L,t}(i)$) or (b) have better outside options. Outside options improve when labor markers are tighter (i.e., higher $1 - u_t$) or average productivity in other jobs ($A_{L,t}$) is higher. To ensure an equilibrium labor share between 0 and 1, I assume that $\nu > 0$ and $\chi \in (0, \frac{1}{1+\nu})$.

The reduced-form wage equation is designed to capture the forces from the standard search and matching model of unemployment with Nash bargaining (e.g., Pissarides, 2000). In that model, workers are paid a fraction of their marginal product plus their reservation wage, which is determined in part by the value of unemployment. The value of unemployment, in turn, depends on how difficult it is to get a new job and the potential wage that could be negotiated in that job. Expression (11) captures this intuition.

Within such a model, χ captures forces, like bargaining institutions, that determine the fraction of surplus paid to workers.[9] Similarly, ν reflects factors that determine how much weight workers place on outside options when bargaining. In other words, it captures institutional factors, like the degree of search frictions, that affect a worker's reservation wage, conditional on labor market tightness and average productivity. The important difference between the model presented here and a full search model is that here there is an explicit expression for wages in terms of productivity and unemployment. This allows for a tractable investigation of directed technical change.

All of the key qualitative results hold with a more general function $h(u_t)$ replacing $(\nu(1-u_t))$, as long as $h'(u_t) < 0 \; \forall u_t \in (0,1)$. The goal of this paper is to highlight the relevant intuition. This simplified expression will allow for more tractable analysis of the dynamics and steady state results, as well as a more transparent calibration. Given that the functional forms are picked for convenience, I focus on qualitative results even when studying the simulated model. The benefit of the simulated model is that it allows for an analysis of the full transition dynamics.

Appendix Section A.8 presents a very simple, illustrative model of bargaining to motivate the expression for wages given in the text. The limited the purpose of this appendix section is to provide intuition for why firm-level productivity ($A_{L,t}(i)$), aggregate productivity ($A_{L,t}$), and the unemployment rate (u_t) are included in equation (11). In doing so, it also provides some intuition for which institutional forces would affect the exogenous parameters (χ and ν).[10]

3.2 Optimization

As demonstrated in Appendix Section A.1, the inverse demand for reproducible capital has the familiar iso-elastic form,

$$p_{X,t}(i) \quad = \quad \alpha \left[1 - \frac{w_t(i)}{A_{L,t}(i)}\right] X_t(i)^{\alpha-1}. \tag{12}$$

Intuitively, this occurs because reproducible capital is combined with natural capital in a Cobb-Douglas manner. The overall demand for capital maintains this functional form, but

[9] I study the qualitative implications of changes in χ, interpreted as changes in bargaining institutions, in Section 4.3.

[10] The illustrative model yields a non-linear function for $h(u_t)$. I do not employ this function in the main analysis, because the model is meant to be illustrative and is not sufficiently rich as to permit careful quantitative analysis.

is adjusted for payments to labor, since the final good producer must employ labor to run each unit of the capital good.

Appendix Section A.2 derives the behavior of capital good producers. The iso-elastic demand yields convenient analytic expressions. Capital good producers set prices as a constant markup over unit costs,

$$p_{X,t}(i) \quad = \quad \frac{1}{\alpha}\frac{r_t}{A_{K,t}(i)}. \tag{13}$$

This equation demonstrates how $A_{K,t}(i)$ affects the relative price of investment. Conditional on technology levels, profits from capital good production are given by

$$\bar{\pi}_{X,t}(i) = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} A_{K,t}(i)^{\frac{\alpha}{1-\alpha}} r_t^{\frac{-\alpha}{1-\alpha}} \left[1 - \chi - \chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}\right]^{\frac{1}{1-\alpha}}. \tag{14}$$

This is again a standard expression adjusted for payments to labor. Capital good producers choose research inputs to maximize these profits, subject to the research production function given in equation (7).

The research arbitrage equation is given by

$$\frac{p^R_{L,t}}{p^R_{K,t}} \quad = \quad \underbrace{\frac{\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}}{\alpha\left[1-\chi-\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}\right]}}_{\text{Bargaining}} \cdot \underbrace{\frac{A_{K,t}(i)}{A_{L,t}(i)}}_{\text{Leontief}} \cdot \underbrace{\frac{\eta_L A_{L,t-1}}{\eta_K A_{K,t-1}}}_{\text{R\&D Productivity}}. \tag{15}$$

Given that R&D inputs are mobile across sectors, factor payments are equal in equilibrium, as long as the allocation is interior.

In some dimensions, the research arbitrage equation resembles the standard directed technical change model (Acemoglu, 2002). The first term is closely related to relative input prices, capturing the notion of 'price effects' in the standard approach. Relative input prices are determined by the outcome of bargaining. Specifically, $\chi+\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}$, is the effective labor cost per unit of output produced with capital good i and $1-\chi-\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}$ is the effective capital cost per unit of output produced with capital good i. The parameter α is the share of expenditure on effective capital that is paid to capital good producers. As expected, higher labor cost increase incentives for labor-saving technical change. But, the numerator is not exactly equal to the labor share. From (11), firms know that workers receive a share χ of increases in output generated by higher $A_{L,t}(i)$. Thus, this share of output does not contribute to their demand for capital goods and, therefore, does not affect the profits of the capital good producer. The second term captures the impact of the low elasticity of substitution between inputs. Higher values of labor-using technology, $A_{K,t}(i)$, increase the return to R&D directed toward labor-saving technology and vice versa. This is closely related to the 'market size' effect in more standard models. Input quantities are inversely related to factor-specific productivity with Leontief production. Finally, the last term captures the relative productivity of R&D between the two technologies. This is the 'research productivity effect' (e.g., Acemoglu et al., 2012).

The representative household maximizes lifetime utility (9) subject to the budget constraint (10). This yields the following Euler equation

$$\left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{\xi} = \beta r_{t+1}, \tag{16}$$

and the tranversality condition,

$$\lim_{T\to\infty} \beta^T \tilde{c}_T^{-\xi} K_{T+1} = 0 \tag{17}$$

which are standard.[11]

3.3 Aggregate Substitution between Capital and Labor

In this section, I discuss how the model captures substitution between capital and labor. This discussion serves two purposes. First, it highlights the connection to the standard neoclassical growth model. Despite the Leontief production function, the new model still features substitution between capital and labor, which occurs via the choice of technology. Second, the discussion of substitution is relevant for the comparison with the task-based models of labor-saving technology. Acemoglu and Restrepo (2018b,a) argue that factor-augmenting technologies are not the ideal way to capture labor-saving technical change, because improvements in technology can only decrease the marginal product of labor when the elasticity of substitution between capital and labor is low.[12] Importantly, this is the elasticity of substitution when holding technology fixed. In the model presented here, this elasticity is zero, even though there is non-zero aggregate substitution between capital and labor when taking the choice of technology into account, as in Jones (2005), (Caselli and Coleman, 2006), and Leon-Ledesma and Satchi (2018). Unfortunately, it is notoriously difficult to separately estimate the elasticity of substitution holding technology fixed and

[11] The equality in the transversality condition captures the non-negativity constraint on capital.

[12] In a model with perfect competition, the requirement is that the elasticity between capital and labor is less than the the capital share of income. More generally, the condition is that the elasticity between capital and labor is less than the elasticity of output with respect to capital.

the process of technology choice (Diamond et al., 1978). Most empirical analyses estimate the elasticity assuming away endogeneity in the direction of technical change (León-Ledesma et al., 2010). As a result, it is difficult to know whether the true elasticity is low enough to generate labor-saving technological change in a model with factor-augmenting technologies.[13] I investigate a model that assumes this to be true. The qualitative results are quite similar to those in Acemoglu and Restrepo (2018b), reinforcing the generality of the findings from both approaches.

I now turn to demonstrating the existence of substitution in the Leontief model. To study substitution at the aggregate level, it is necessary to integrate across capital good producers. From (15), each capital producer has an identical research arbitrage equation, implying that they make identical R&D decisions. Since there are a unit mass of capital good producers, $R_{J,t}(i) = R_{J,t}$ and $A_{J,t}(i) = A_{J,t}$ $\forall J, t, i$. From (11), this implies that wages are equalized, i.e., $w_t(i) = w_t$ $\forall i, t$. This, in turn, implies that all capital good producers face identical inverse demand curves (12). As a result, they will choose identical production quantities, i.e., $X_t(i) = X_t$ $\forall i, t$. I will drop the i subscripts for everything that follows. Since there is full depreciation of capital within a period, the representative household will never save capital that goes unused in the next period, implying that the market clearing condition for capital (5) holds with equality in all periods, i.e., $\int_0^1 X_t di = A_{K,t}K_t$. Thus, the aggregate production function is $Y_t = \min\left[(A_{K,t}K_t)^\alpha, A_{L,t}L_t\right]$.

After capital goods are installed, technology parameters are fixed. There is a minimum amount of labor and capital needed to produce a given amount of output. The isoquant for producing a given amount of output, $\bar{Y}$, is given by the following correspondence:

$$\bar{K} = \begin{cases} [\frac{\bar{Y}^{\frac{1}{\alpha}}}{A_{K,t}}, \infty] & \text{if } L_t = \frac{\bar{Y}}{A_{L,t}} \\ \frac{\bar{Y}^{\frac{1}{\alpha}}}{A_{K,t}} & \text{if } L_t > \frac{\bar{Y}}{A_{L,t}} \\ \emptyset & \text{otherwise.} \end{cases} \tag{18}$$

Panel (a) of Figure 1 plots the isoquants in (K, L) space. It demonstrates the defining property of Leontief production: there is no substitution between capital and labor when holding technology fixed. The presence of land implies that the corners of the isoquants do not follow a straight line out from the origin.

[13]Recent work by Leon-Ledesma and Satchi (2018) develops a model with endogenous technology choice to examine medium-run variation factor shares. Holding technology fixed, they find an elasticity that is consistent with labor-saving technology in a factor-augmenting model and, importantly, is also consistent with standard econometric estimates that do not account for endogenous technology responses. Oberfield and Raval (2021) find the opposite result when using firm-level micro data to estimate the aggregate elasticity.

Prior to the installation of capital goods, there exists a menu of potential technology pairs. In particular, the resource constraint for R&D inputs implies that $1 = R_{L,t} + R_{K,t}$. Together with the law of motion for technology, equation (7), this can be rewritten as

$$\tilde{\eta} A_{K,t-1} = A_{K,t} + \frac{A_{K,t-1}}{A_{L,t-1}} \frac{\eta_K}{\eta_L} A_{L,t}, \tag{19}$$

where $\tilde{\eta} = \eta_K(1 + \frac{1}{\eta_K} + \frac{1}{\eta_L})$ is a constant. Capital good producers take lagged technology levels as given. Thus, the technology menu captures the feasibility constraint in allocating R&D resources. Given that technology must be strictly increasing, there is an additional constraint that $A_{J,t} \geq A_{J,t-1}\ \forall t, J$. In each period, $A_{K,t-1}$ increases, which demonstrates the expansion of the production possibility frontier for technologies. In the long run, the two technologies will not grow at the same rate, implying that the trade-off between technologies changes over time.

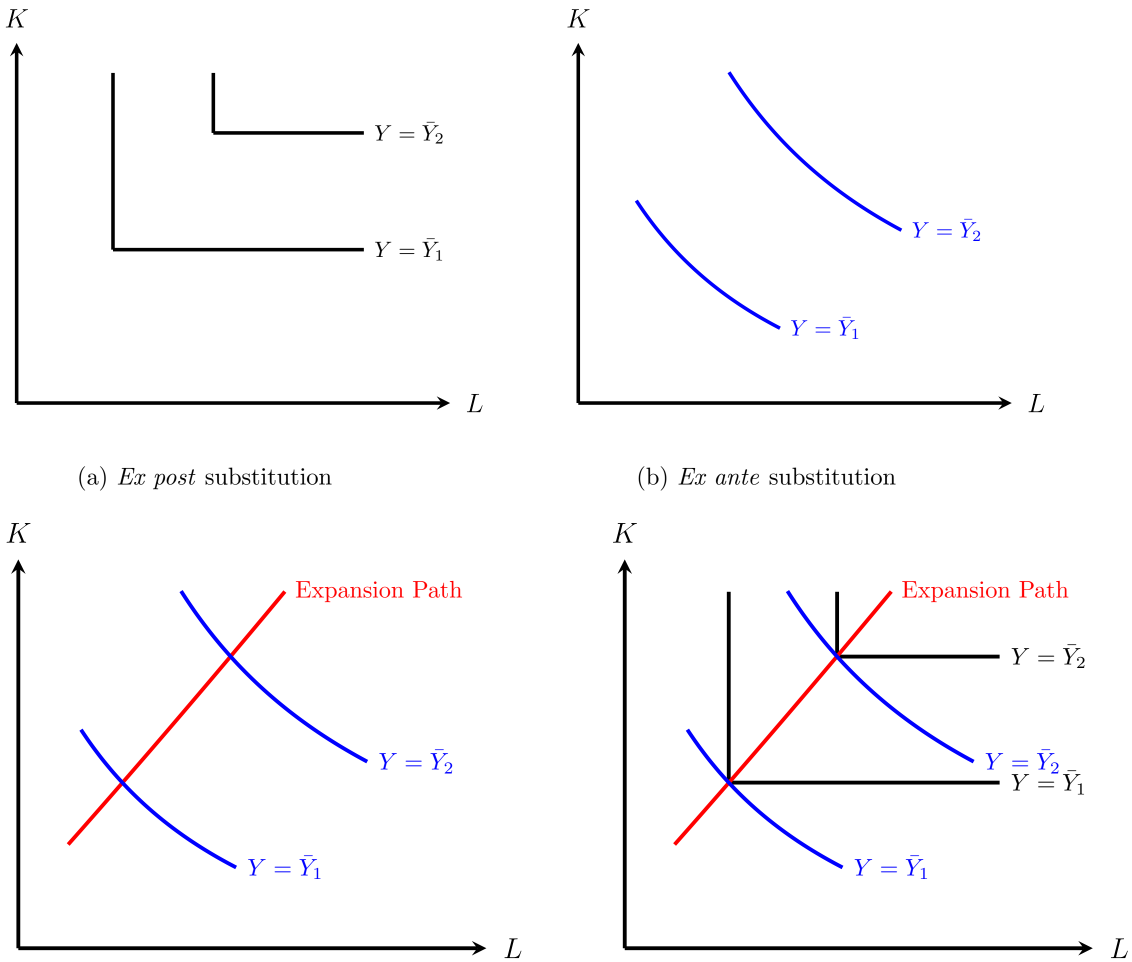
Equation (19) highlights how the new model resembles earlier literature with a distribution of different types of capital goods (e.g., Jones, 2005; Caselli and Coleman, 2006; Leon-Ledesma and Satchi, 2018). Unlike the existing literature, the entire menu of cutting-edge technologies is not freely available. Instead, capital good producers must hire R&D inputs to create the new technologies. As a result, only one technology pair will materialize. Since R&D inputs are distinct from production inputs, it will always be profitable to use a cutting-edge technology.

The technology menu demonstrates the possibility for *ex ante* substitution between capital and labor. When considering the choice of technology, the relationship given in (18) becomes

$$\bar{K} = \begin{cases} \frac{\bar{Y}^{\frac{1}{\alpha}}}{\tilde{\eta} A_{K,t-1} - \frac{A_{K,t-1}}{A_{L,t-1}} \frac{\bar{Y}}{L_t}} & \text{if } L_t \in [\frac{\bar{Y}}{(1+\eta_L) A_{L,t-1}}, \frac{\bar{Y}}{A_{L,t-1}}] \\ \emptyset & \text{otherwise,} \end{cases} \tag{20}$$

which defines the smooth curves given in panel (b). Thus, the model allows for smooth substitution between capital and labor, much like the standard neoclassical growth model.

Capital good producers choose technology pairs to maximize profits. The optimal decision is given in equation (15). Once these technological parameters are set, the expansion path is given by $K_t = \frac{A_{L,t}^{\frac{1}{\alpha}}}{A_{K,t}} L_t^{\frac{1}{\alpha}}$, which follows from the fact that $Y_t = A_{L,t} L_t = \big(A_{K,t} K_t\big)^{\alpha}$. This is shown in panel (c). Once capital good producers have chosen a point on the technology menu, they sell specific capital goods to the final good producer. The final good producers only have access to the Leontief technology, as depicted in panel (d).



(a) *Ex post* substitution

(b) *Ex ante* substitution

(c) Expansion path

(d) *Ex post* and *ex ante* substitution

Figure 1: This figures demonstrates that, despite the Leontief production function, there is still substitution between capital and labor. Panel (a) presents the isoquants for the final good producer, who takes technology as given. Panel (b) presents the isoquants when considering the ability of the capital good producers to choose from the technology menu. The expansion path, presented in panel (c), is determined by the profit maximizing behavior of the capital good producer. Finally, panel (d) combines the results.

3.4 Intensive Form

In this subsection, I analyze the dynamics of the economy. Compared to the standard neoclassical set-up, the new model adds unemployment and endogenous technological progress, while removing *ex post* substitution between labor and installed capital. For the remainder of the paper, I restrict attention to the case where the unemployment rate is interior, $u_t \in (0, 1)$. This condition is consistent with U.S. historical data and will hold in the simulations. Similarly, I restrict attention to the case where the research allocations are interior, $g_{A_L,t},\ g_{A_K,t} > 0$. This implies that output per worker rises and the relative price of investment falls.

As explained in the previous subsection, the aggregate production function is given by

$$Y_t = \min\left[(A_{K,t}K_t)^{\alpha}, A_{L,t}L_t\right]. \tag{21}$$

The first argument of the production function looks quite similar to the standard neoclassical model, except that land is replacing labor and capital-augmenting technology is replacing labor-augmenting technology. As a result, the usual intensive-form approach to solving the model will be useful. Let $k_t \equiv \frac{K_t}{A_{K,t}^{\frac{\alpha}{1-\alpha}}}$, $c_t \equiv \frac{C_t}{A_{K,t}^{\frac{\alpha}{1-\alpha}}}$, and $g_{A_J,t} \equiv \frac{A_{J,t}}{A_{J,t-1}} - 1$.

Equation (10) implies that the law of motion for capital is given by $K_{t+1} = Y_t - C_t$. Substituting in (21) and putting everything in intensive form gives

$$k_{t+1} = \frac{k_t^{\alpha} - c_t}{(1 + g_{A_K,t})^{\frac{\alpha}{1-\alpha}}}. \tag{22}$$

The Euler equation, (16), gives

$$c_{t+1} = \frac{\beta^{\frac{1}{\xi}} r_{t+1}^{\frac{1}{\xi}}(1+n)}{(1 + g_{A_K,t+1})^{\frac{\alpha}{1-\alpha}}} c_t. \tag{23}$$

These expressions are relatively standard. Labor force growth appears in the numerator of (23), because the intensive form is not normalized by population. Moreover, capital-augmenting, rather than labor-augmenting, technology determines the evolution of the dynamical system. This highlights the benefit of including land in the production function, which implies that there are diminishing returns to reproducible capital within the first argument of the production function and that the model generates tractable and intuitive expressions, despite Leontief production.

Note that, in equilibrium, $\frac{w_t}{A_{L,t}(i)} = \chi + \chi\nu(1 - u_t)\ \forall i$. From (5), (12) and (13), the real interest rate is given by

$$r_t = \alpha^2\left[1 - \chi - \chi\nu(1 - u_t)\right] k_t^{\alpha-1}, \tag{24}$$

which is again similar to the standard neoclassical growth model with monopolistic competition. The difference is that the real interest rate must be adjusted for payments to labor, which depend on the unemployment rate. Higher unemployment leads to lower wages, increasing the value of capital from the perspective of the final good producer.

Now, I turn to the more unique portions of the model, the dynamics of technology and unemployment. Considering (21) and noting that the minimum function is met with equality,

$$u_t \equiv 1 - \frac{L_t}{N_t} = 1 - \frac{(A_{K,t}K_t)^\alpha}{A_{L,t}N_t}. \tag{25}$$

Unemployment exists because of the finite quantity of capital and specific labor-input requirements. Unemployment decreases when economic production expands via the accumulation of capital goods, which results from saving or capital-augmenting technological progress. Conversely, labor-saving technology increases the unemployment rate, as does labor force growth. For the dynamics, this yields

$$\frac{1-u_{t+1}}{1-u_t} = \frac{(1+g_{A_K,t+1})^{\frac{\alpha}{1-\alpha}}}{(1+g_{A_L,t+1})(1+n)} \cdot \big(\frac{k_{t+1}}{k_t}\big)^\alpha. \tag{26}$$

Now, I consider the dynamics of technology. Noting that all monopolists make identical decisions, research arbitrage equation (15) yields

$$g_{A_L,t} = \frac{1}{1+\Gamma(u_t)}\left(\frac{\eta_L}{\eta_K} + \eta_L - \Gamma(u_t)\right), \tag{RD-RA}$$

where $\Gamma(u_t) = \frac{\alpha\left(1-\chi-\chi\nu(1-u_t)\right)}{\chi\nu(1-u_t)}$, for any interior solution.[14] For convenience, I define

$$\psi \equiv \left(\alpha(1-\chi(1-\nu))\right)^{-1}\chi\nu\left(\frac{\eta_L}{\eta_K}+\eta_L\right) > 0,$$

which is the right-hand side of (RD-RA) when $u_t = 0$. Since $\Gamma'(u_t) > 0$, the growth rate of labor-saving technological change is decreasing in the unemployment rate. When unemployment falls, wages rise, generating greater incentives for producers to economize on labor inputs. Finally, combining the R&D market clearing condition (8) and the law of motion for technology (7) gives

$$g_{A_K,t} \;=\; \eta_K - \frac{\eta_K}{\eta_L} g_{A_L,t}. \tag{27}$$

Equations (22) – (27) describe the period-to-period dynamics of $\{k_t, c_t, u_t, r_t, g_{A_L,t}, g_{A_K,t}\}$. These equations will be utilized in the theoretical and computational analyses discussed below. I now turn to discussing the boundary conditions. Initial conditions K_0, $A_{L,-1}$, $A_{K,-1}$ are given. Appendix Section A.5 shows how u_t, $g_{A_L,t}$, and $g_{A_K,t}$ are jointly determined in ev-

[14] See Appendix Section A.3 for a derivation.

ery period. Applied to period 0, this gives u_0, $g_{A_L,0}$, and $g_{A_K,0}$ as a function of the given initial conditions for K_0, $A_{L,-1}$, and $A_{K,-1}$. This also implies that $k_0 = K_0/\left(A_{K,-1}(1+g_{K,0})\right)^{\frac{\alpha}{1-\alpha}}$ is fully determined by the initial conditions. Now, (24) implies that r_0 is pinned down. The final boundary condition comes from transversality condition (17), which can be re-written as[15]

$$\lim_{T\to\infty}(\beta(1+n)^{\xi})^T\left(\prod_{\tilde{t}=0}^{T}(1+g_{A_K,\tilde{t}})^{\frac{\alpha}{1-\alpha}}\right)^{1-\xi}(1+g_{A_K,T+1})\,c_T^{-\xi}k_{T+1}=0. \tag{28}$$

Together, the period-to-period dynamics and boundary conditions fully determine the path of the intensive-form variables, which are in turn sufficient to determine the path of all the endogenous variables in the model.

3.5 Balanced Growth Path

3.5.1 Characterization

In this subsection, I characterize the balanced growth path and highlight the key forces in the model. Throughout the remainder of the paper, I use asterisks ($*$) to denote balanced growth levels.

Definition 1. *A* balanced growth path (BGP) *occurs when output, consumption, capital and technology all grow at constant rates.*

Definition 2. *A BGP is* unique *if the constant equilibrium growth rates of the aggregate variables are unique.*

One of the primary goals of this paper is to understand the forces that generate constant unemployment in the presence of labor-saving technical change.

Lemma 1. *On any BGP, the unemployment rate is constant.*

Proof. Follows from equation (RD-RA) and the definition of a BGP. □

Since the direction of technological progress is determined by profit-maximizing firms, investment in labor-saving technology will increase when labor costs are high. Labor costs, in turn, depend on unemployment. On a BGP, relative incentives for R&D in the two types of technology must be constant, implying that the unemployment rate must also be constant. Incentives for R&D are captured by research arbitrage equation (RD-RA), which is derived from (15).

[15] See Appendix Section A.6 for a derivation.

As discussed in Section 2, constant unemployment is only possible under knife-edge conditions when technical change is exogenous. When considering directed technical change, however, constant unemployment is an endogenous outcome on any balanced growth path. This result suggests that directed technical change is important for understanding balanced growth and constant unemployment in an economy with labor-saving technical change. This result also demonstrates how to overcome the criticism of Leontief growth models that served as motivation for the now standard neoclassical growth model (Solow, 1956, 1994).

With constant unemployment and technological growth rates, equations (22) – (24) are essentially identical to the standard neoclassical growth model with monopolistic competition. As a result, the steady state of this system will also exhibit the usual properties.

Lemma 2. *If the economy is on a BGP, then the intensive-form variables, k_t and c_t, the unemployment rate, u_t, and the real interest rate, r_t, and are constant.*

Proof. The previous lemma shows that the unemployment rate must be constant on a BGP. By definition, the technology growth rates are also constant. By equation (26), therefore, k_t must also be constant. With both k_t and u_t constant, r_t must also be constant by equation (24), which implies that c_t must be constant by equation (23). □

Lemma 3. *If the economy is on a BGP, then the relative price of investment falls at rate g_K^*.*

Proof. Follows from equation (13) and the fact that the real interest rate is constant (Lemma 2). □

Lemma 4. *If the economy is on a BGP, then aggregate variables K_t, Y_t, and C_t grow at factor $(1 + g_{A_K}^*)^{\frac{\alpha}{1-\alpha}}$.*

Proof. The growth rates of K_t and C_t follow from Lemma 2 and the definitions of $k_t \equiv \frac{K_t}{A_{K,t}^{\frac{\alpha}{1-\alpha}}}$ and $c_t \equiv \frac{C_t}{A_{K,t}^{\frac{\alpha}{1-\alpha}}}$. Then, since $Y_t = (A_{K,t}K_t)^\alpha$, output also grows at a constant rate. □

As noted in the previous subsection, these results highlight the importance of including land in the production function. The diminishing returns to reproducible capital within the first argument of the production function imply that the dynamical system has all of the usual properties, even before considering the role of labor inputs.

I now turn to examining the link between the two types of technical change.

Lemma 5. *On any BGP,*

$$(1 + n)(1 + g_{A_L}^*) = (1 + g_{A_K}^*)^{\frac{\alpha}{1-\alpha}}, \tag{29}$$

and aggregate variables K_t, Y_t, and C_t grow at factor $(1+n)(1+g^*_{A_L})$ as in the standard neoclassical growth model.

Proof. The Lemma follows from equation (26), Lemma 1, and Lemma 4. □

As noted in Section 2, this condition must hold for the unemployment rate to be constant. The BGP endogenously conforms to this requirement, and the model is consistent with the standard stylized macroeconomic facts. In particular, the capital-output ratio, savings rate, and real interest rate will all be constant. Moreover, output per worker and capital per worker will grow at the same constant rate.

This result further highlights the importance of directed technical change. The BGP relationship between the two types of technical change is driven by the requirement that unemployment be constant, which is guaranteed by research arbitrage. Thus, directed technical change is essential to explaining both the constant unemployment rate and the standard BGP stylized facts.

I now turn to showing that the BGP is unique. For the remainder of the analysis, I impose the following assumption.

Assumption. *The rate of labor force growth and efficiency of R&D investment in capital-augmenting technical change are such that*

$$(1+n) < (1+\eta_K)^{\frac{\alpha}{1-\alpha}}. \tag{A1}$$

Rearranging equation (29) yields a relationship between technology growth rates, and implicitly R&D allocations, that must hold on the BGP:

$$1+g^*_{A_K} = \left[(1+n)(1+g^*_{A_L})\right]^{\frac{1-\alpha}{\alpha}}. \tag{RD-BGP}$$

This curve gives a positive relationship between $(1+g^*_{A_K})$ and $(1+g^*_{A_L})$. Evaluating the R&D market clearing condition (27) on the BGP gives a negative relationship:

$$1+g^*_{A_K} = 1+\eta_K - \frac{\eta_K}{\eta_L} g^*_{A_L}. \tag{RD-MC}$$

Putting (RD-BGP) and (RD-MC) together gives

$$\left(1+\eta_K - \frac{\eta_K}{\eta_L} g^*_{A_L}\right)^{\frac{\alpha}{1-\alpha}} = (1+g^*_{A_L})(1+n). \tag{30}$$

The determination of the technology growth rates is show in panel (a) of Figure 2. Assumption (A1) guarantees that there is a solution to these equations and the resulting BGP has $g^*_{A_L}, g^*_{A_k} > 0$.

Now, research arbitrage equation (RD-RA) implies that the BGP unemployment rate is unique. In particular,

$$u^* = \text{arg solve} \left\{ g^*_{A_L} - \frac{1}{1+\Gamma(u^*)} \left(\frac{\eta_L}{\eta_K} + \eta_L - \Gamma(u^*) \right) = 0 \right\}, \tag{31}$$

where $g^*_{A_L}$ is the implicit solution to equation (30). The determination of u^* is shown in panel (b) of Figure 2. Intuitively, equation (30) pins down the BGP growth rates of technology using only the R&D market clearing condition and the law of motion for unemployment. Then, unemployment must adjust so that BGP growth rates are consistent with the incentives for R&D.

With unique rates of unemployment and technological progress, equations (22) – (24), the more standard aspects of the dynamical system, also have a unique steady-state.

Lemma 6. *The balanced growth path is unique.*

Proof. Proof in the text. □

Thus far, I have assumed that the transversality condition is satisfied and lifetime utility is finite.

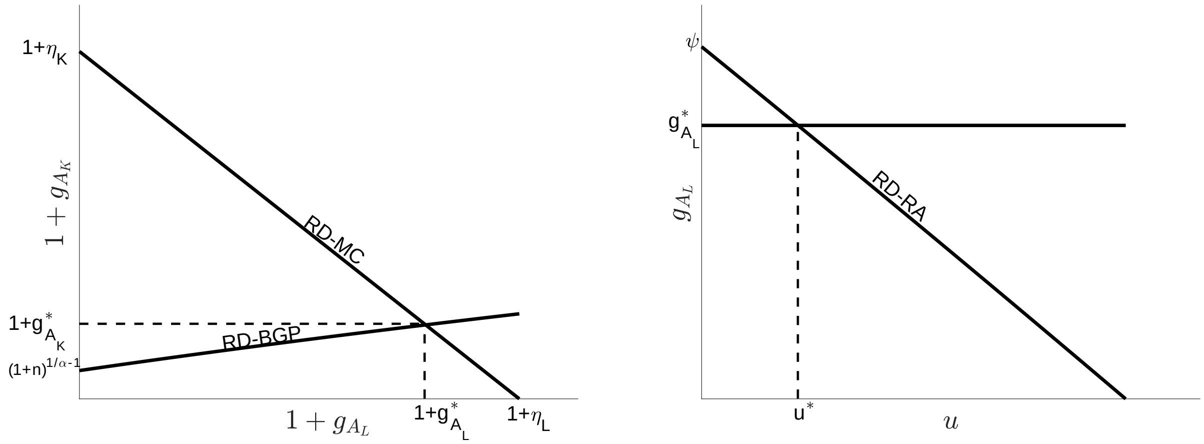
Lemma 7. *If β is sufficiently low, the transversality condition (17) is satisfied and lifetime utility (9) is finite on the unique balanced growth path.*

Proof. As shown in Appendix Section A.6, evaluating (28) on the balanced growth path and using the results from Lemma 5 implies that the transversality condition (17) is satisfied when $\beta < (1+n)^{-1}(1+g^*_{A_L})^{\xi-1}$. Appendix Section A.6 shows that this condition also guarantees finite lifetime utility. The derivation of Lemma 6 shows that $g^*_{A_L}$ does not depend on β. So, the condition must be satisfied for a sufficiently low β. □

This condition will be satisfied in the calibrated model.

To examine equity-efficiency trade-offs, I consider the labor share of income, which I denote with $\kappa_{L,t}$. At all times, $Y_t = A_{L,t}L_t$. Together with the fact that all capital good producers make identical decisions, this yields

$$\kappa_{L,t} = \frac{w_t}{A_{L,t}} = \chi + \chi\nu(1-u_t), \tag{32}$$



(a) Technology (b) Unemployment

Figure 2: Determination of the BGP

which is constant on the BGP. Intuitively, the fraction of output paid to workers is constant when the worker bargaining position is constant. This occurs when unemployment reaches its steady state value. Appendix Section A.4 derives the factor shares of all other inputs and shows that they are constant on a BGP.

All of the results discussed above are summarized in Proposition 1.

Proposition 1. *Let Assumption (A1) hold. There exists a* unique balanced growth path*, where each of the following holds true: (1) The relationship between technological growth rates is given by* $(1+n)(1+g^*_{A_L}) = (1+g^*_{A_K})^{\frac{\alpha}{1-\alpha}}$*; (2) The unemployment rate is constant and aggregate employment grows at factor* $(1+n)$*; (3) The real interest rate is constant and the relative price of investment falls at rate* g^*_K*; (4)* K_t*,* Y_t *and* C_t *grow at factor* $(1+n)(1+g^*_{A_L})$*, the capital-output ratio is constant, and both capital per worker and output per worker grow at constant rate* $g^*_{A_L}$*; (5) Factor shares are constant.*

Proof. Follows from the preceding discussion. □

Panel (b) of Figure 2 illustrates the relationship between two important endogenous variables, $g^*_{A_L}$ and u^*, for a given set of parameters. In particular, it is the shape of (RD-RA) that determines how the two variables are related. Since the equation is roughly linear across this small parameter space, I will summarize the shape in terms of the slope. Equation (RD-RA) is simply the research arbitrage equation, after imposing equilibrium conditions for wages and productivity growth rates. The slope is determined by the parameters from the wage and R&D equations. In addition, the slope is affected by the assumed linearity of the wage equation. As noted above, all of the qualitative results of this paper hold with a more general monotonic functional form $h(u_t)$ replacing $1 - u_t$ in the wage equation. This change would affect the precise relationship between $g^*_{A_L}$ and u^* shown in panel (b) of Figure 2. It would not affect the equilibrium growth rates, as shown in panel (a) of Figure 2. In the next section, I discuss how changes in exogenous parameters affect these two important outcomes, as well as the labor share of income.

3.5.2 Long-run impacts of an increase in η_L

In this section, I discuss the long-run effects of an increase in η_L. Equation (30) describes a necessary condition for constant unemployment that is independent of incentives for R&D. From this equation, it is immediate that the increase in η_L will increase the growth rate of both technologies. Geometrically, an increase in η_L is equivalent to increasing the horizontal intercept of (RD-MC) in panel (a) of Figure 2.

An increase in η_L has two countervailing effects on unemployment. First, as noted above, it increases $g^*_{A_L}$. Geometrically, this is an upward shift in the horizonal line in panel (b) of Figure 2. In equilibrium, the relative growth rates of technology must be consistent with the R&D incentives of capital good producers. The fall in η_L increases the incentive for R&D in labor saving technology holding u^* fixed. This is equivalent to shifting (RD-RA) up in panel (b) of Figure 2. Without further parameter restrictions, it is not clear which effect dominates. A sufficient (endogenous) condition for a decrease u^* is given below.

Assumption. *On the BGP, the endogenous relationship between wages and the efficiency of R&D in capital-augmenting technology is given by*

$$\left(1 + \frac{1}{\eta_K}\right) \frac{1}{1 + \Gamma(u^*)} > 1. \tag{A2}$$

In Section 4.1, I check whether this condition is satisfied in the calibrated model.

As demonstrated in equation (32), an increase in η_L affects the labor share of income only via unemployment. When unemployment rises, the bargaining position of workers deteriorates, leading to a fall in the labor share. All of the results from this section are summarized in the following proposition.

Proposition 2. *Let Assumption (A1) hold. An increase in the productivity of labor-saving R&D leads to faster growth in labor-productivity, i.e.* $\frac{dg^*_{A_L}}{d\eta_L} > 0$. *Also, if Assumption (A2) holds, then* $\frac{du^*}{d\eta_L} > 0$ *and* $\frac{d\kappa^*_L}{d\eta_L} < 0$.

Proof. Follows from applying the implicit function theorem to equations (30), (31), and (32). Details are provided in Appendix Section A.7. □

3.6 Transition Impacts of a Increase in η_L

When there is unemployment, labor-saving technical change is socially wasteful because it saves a resource, labor, that is not scarce. Within a period, increases in $A_{L,t}$ decrease L_t one-for-one, implying that they do not increase effective labor inputs or output. Policy could increase total output within a period by redirecting R&D inputs towards labor-using technology and employing a greater number of workers.

Lemma 8. *Consider an economy at some point $\tilde{t}$ where $K_{\tilde{t}}$, $N_{\tilde{t}}$, and $A_{J,\tilde{t}-1}$, $J = K, L$ are given. If $L_{\tilde{t}} < N_{\tilde{t}}$ and $R_{L,\tilde{t}} > 0$, then it is possible to increase total output and decrease unemployment by shifting some R&D resources to $R_{K,\tilde{t}}$.*

Proof. It will always be the case that $\left(A_{K,\tilde{t}}(i)X_{\tilde{t}}(i)\right)^{\alpha} = A_{L,\tilde{t}}(i)L_{\tilde{t}}(i)\ \forall i, \tilde{t}$. Let $\epsilon > 0$ be arbitrarily small. Decreasing $R_{L,\tilde{t}}(i)$ by ϵ for all i and adding these resources to the corresponding $R_{K,\tilde{t}}(i)$ increases potential output by

$$\epsilon \cdot \frac{\partial Y_{\tilde{t}}}{\partial R_{K,\tilde{t}}(i)} = \epsilon \cdot \frac{\partial Y_{\tilde{t}}}{\partial A_{K,\tilde{t}}(i)} \frac{\partial A_{\tilde{t}}}{\partial R_{K,\tilde{t}}(i)} = \epsilon \alpha A_{K,\tilde{t}}(i)^{\alpha-1} X_t(i)^{\alpha-1} \eta_K A_{K,t-1} > 0$$

for each i. Then, to increase actual output, it is necessary to employ more workers to ensure that the two arguments of the Leontief production function (4) are equal. Since $L_t < N_t$, this must be feasible for an arbitrarily small ϵ. □

The task-based model of Acemoglu and Restrepo (2018b) also yields inefficiently high investment in labor-saving technologies.

Increases in η_L create incentives for capital good producers to invest more heavily in labor-saving technologies. This will boost the long-run growth rate of the economy, but exacerbate short-run inefficiencies.

Proposition 3. *Let Assumption (A1) hold. Consider an economy on a BGP at some point $\tilde{t}$, where $K_{\tilde{t}}$, $N_{\tilde{t}}$, and $A_{J,\tilde{t}-1}$ $J = K, L$ are given. An exogenous increase in η_L at time $\tilde{t}$ lowers output and increases unemployment in the short run, relative to a baseline scenario without this exogenous shock. It also increases the growth rate of labor productivity and decreases the labor share of income.*

Proof. Assumption (A1) ensures that research allocations are interior on the BGP. Thus, equation (RD-RA) must hold before the shock. This implies that an increase in η_L leads to an increase in both $u_{\tilde{t}}$ and $g_{A_L,\tilde{t}}$. By the resource constraint for R&D inputs, this leads to a decrease in $g_{A_K,\tilde{t}}$, as demonstrated in equation (27). Since $Y_{\tilde{t}} = \left(A_{K,\tilde{t}}K_{\tilde{t}}\right)^{\alpha}$, output decreases in the short run. □

4 Simulation

In this section, I present the calibration and simulation of the model. Compared to the theoretical results presented in Section 3.6, the simulation allows for a more thorough investigation of the qualitative transition path following an increase in η_L. I also use the simulation to study the impact of changes in bargaining institutions. While the primary goal of the simulation is to better understand qualitative results, it also makes it possible to confirm that Assumption (A2) is satisfied in the data, at least for the functional forms chosen here.

4.1 Calibration

As demonstrated in Section 3.5, the BGP of the model closely resembles the standard neoclassical growth model. To calibrate the model, I use aggregate data from the United States. Details on data sources can be found in Appendix Section B.

I take the period length to be ten years. Three parameters can be determined exogenously. I start by assuming log preferences, i.e., $\xi = 1$. I take $\beta = 0.86 = 0.985^{10}$. Using data on the size of the labor force from the Bureau of Labor Statistics (BLS), I take $n = 0.15$ (1.4% per year). With log preferences, the transversality condition is satisfied if $\beta < (1+n)^{-1}$, which holds at these parameter values.

The calibration procedure leaves one free parameter. Thus, I set the ratio $\frac{\eta_K}{\eta_L}$ exogenously. To separately identify these two parameters, it help to observe how R&D inputs within firms are divided between labor-saving and labor-using technical change. For the baseline analysis, I assume that $\eta_K = \eta_L$. I also show that none of the qualitative results are driven by this assumption and discuss how the quantitative results change when making alternate assumptions.

Together with the exogenous ratio $\frac{\eta_L}{\eta_K}$, a simple calibration procedure uses four equations to calibrate the remaining five unknown parameters, $\{\eta_K, \eta_L, \chi, \nu, \alpha\}$. As usual, α can be identified from factor shares. Valentinyi and Herrendorf (2008) estimate that the land share of income in the United States is approximately $\kappa_m^* = 5\%$, while the labor share is approximately $\kappa_m^* = 67\%$. Combining equations (32) and (A.28) yields: $\kappa_M^* = (1-\alpha)(1-\kappa_L^*)$. In other words, the land share of income is a fraction of $(1-\alpha)$ of the non-labor income. Taking the values cited above gives $\alpha = 0.85$, implying that 85% of non-labor income is paid to capital producers. The structure of the model yields the relationship between the technological growth rates on the BGP given in equation (30). Taking $g_{A_L}^* = 0.24$ (2.2% per year) from BLS data on labor productivity yields $\eta_K = \eta_L = 0.31$.

Table 1: Calibrated Parameters

Parameter	Value	Description	Target	Source
α	0.85	Physical capital share (excl. labor)	κ_M^*	Valentinyi and Herrendorf (2008)
$\eta_k = \eta_L$	0.31	R&D efficiency	$g_{A_L}^*$	BLS
$\chi\nu$	0.41	Bargaining Institutions	u^*	BLS
ν	1.43	Weight on outside options	κ_L^*	Valentinyi and Herrendorf (2008)

The remaining parameters, χ and ν, capture bargaining institutions and the importance of outside options in the bargaining process. I re-write equations (RD-RA) and (32) as

$$g_{A_L}^* = \frac{1}{1 + \frac{\alpha(1-\kappa_L^*)}{\chi\nu(1-u^*)}} \left(\frac{\eta_L}{\eta_K} + \eta_L - \frac{\alpha(1-\kappa_L^*)}{\chi\nu(1-u^*)} \right). \tag{33}$$

Combining the previous results with an estimate of $u^* = 5.5\%$ from the BLS data implies that $\chi\nu = 0.35$. Finally, rearranging (32) yields $\chi = \kappa_L^* - \chi\nu(1-u^*)$, which implies that $\chi = 0.34$ and $\nu = 1.01$.

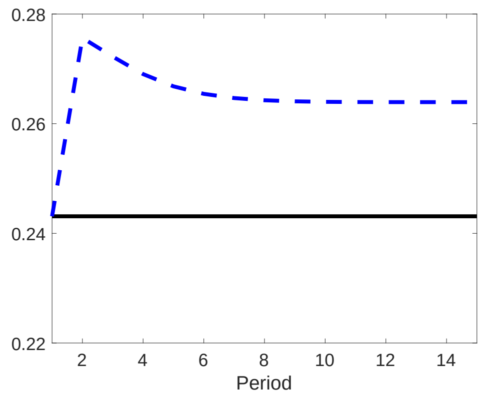
Table 1 summarizes the results. It is important to note that Assumption (A2) holds. Specifically, $\left(1 + \frac{1}{\eta_K}\right)\frac{1}{1+\Gamma(u^*)} = 2.28 > 1$, which implies that $\frac{du^*}{d\eta_L} > 0$ and $\frac{d\kappa_L^*}{d\eta_L} < 0$. Thus, an increases in η_L leads to an increase in unemployment and a decrease in the labor share of income. For robustness, I also consider alternate assumptions about research productivity, $\frac{\eta_L}{\eta_K} \in \{0.67, 2\}$. Assumption (A2) continues to hold in these alternate scenarios and all of the dynamics are qualitatively similar.

4.2 Simulation Results

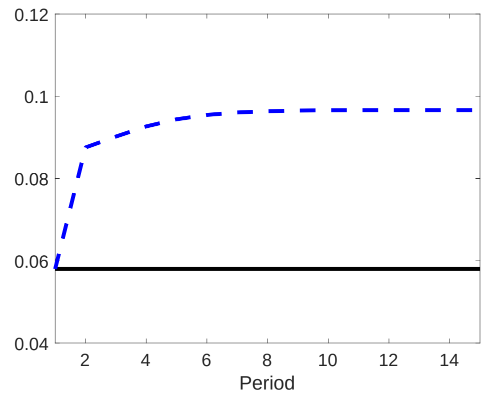
In this section, I use the calibrated model to trace out the dynamic impacts of changes in the productivity of labor-saving R&D (i.e., an increase in η_L) and a change in bargaining institutions that raises wages (i.e., an increase in χ). The primary goal of this analysis is to highlight the results of the comparative static analyses and better understand qualitative features of the transition path. Appendix Section C discusses the details of the simulation method.

4.2.1 Increase in η_L

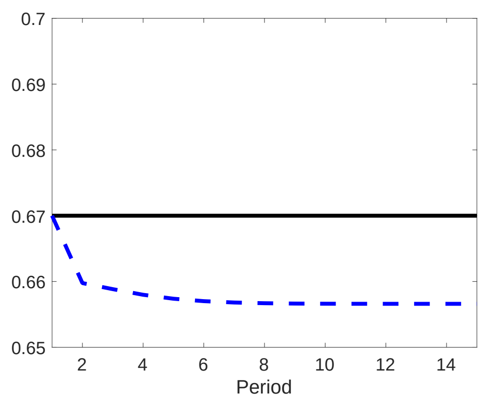
I use the calibrated model to trace out the dynamic impacts of a 10% increase in η_L. This represents a permanent increase in the ability of R&D to improve labor-saving technologies. The results are presented in Figure 3.



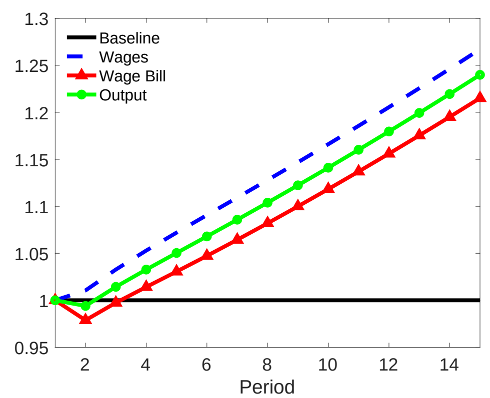
(a) Labor Prod. Growth



(b) Unemployment



(c) Labor Share



(d) Output and Wages

Figure 3: Increase in productivity of labor-saving R&D

Note: This figures traces the impact of a 10% increase η_L. All results are presented relative to a baseline scenario in which in the economy remains on its initial balanced growth path.

An increase in η_L leads to greater investment in labor-saving technologies. Since labor-saving technology also makes workers more productive, the growth rate of worker productivity increases, as demonstrated in panel (a). An increase in the rate of labor-saving technical change also leads to higher unemployment, as shown in panel (b). The unemployment rate eventually converges to a new constant level. This occurs because the forces pushing the economy towards balanced growth continue to operate even after the change in R&D productivity. Mass unemployment cannot be an equilibrium outcome because labor becomes cheaper relative to capital as unemployment rises. The falling labor costs eventually make it profitable to invest in labor-using technologies at a rate that stabilizes the economy on a new balanced growth path.

The model of Acemoglu and Restrepo (2018b) also suggests that labor-saving technical change does not lead to a collapse in employment. In their model, balanced growth is possible because of the assumption that workers have an inherent advantage in new tasks. In the current model, by contrast, balanced growth exists because of endogenous investment in labor-using technologies.

Panels (a) and (b) highlight the fundamental trade-off associated with labor-saving technical change: when worker productivity increases, employed workers see faster wage growth, but a greater fraction of workers cannot find a job. Thus, there is an equity-efficiency trade-off associated with a change in R&D productivity, even when looking within a homogeneous group of workers. When considering the labor-market impacts of new technologies, therefore, the model suggests that policy should be concerned with winners and losers within narrow subsets of the labor force and not just the forces of polarization between workers with different skill levels that play a large role in academic discussions (e.g., Autor, 2015; Hémous and Olsen, 2021).

Panel (c) demonstrates the impact of a change in R&D productivity on the labor share of income. Since higher unemployment hurts the bargaining position of workers, the labor share of income decreases. In this way, labor-saving technical change also induces an equity-efficiency trade-off when considering the distribution of income between factors of production.

Panel (d) shows results from three important aggregate variables that are relevant to the well-being of workers: total output, wages, and the total wage bill. As demonstrated in Section 3.6, an increase in η_L slows growth in the short run. This is consistent with empirical evidence presented in Berg et al. (2018). In the simulation, the growth slowdown is brief. Since the the increase in η_L permanently increases the growth rate of labor productivity, output eventually grows faster than in the original steady state, demonstrating a trade-off between the short- and long-run outcomes.

The short-run reductions are even larger for the wage bill, since both output and the labor share decrease. The effect on wages, however, is more complicated. The increase in productivity pushes up wages, but the increase in unemployment undermines worker bargaining position, pushing wages down. The net effect cannot be determined from the theoretical analysis. In the main analysis, where the research efficiencies are equal prior to the increase in η_L, wages are essentially unchanged in the period of the shock, implying that these two forces offset. When the ratio $\frac{\eta_L}{\eta_K}$ is high, wages increase rapidly following a shock (Figure D.1). When $\frac{\eta_L}{\eta_K}$ is low, wages decrease slightly following the shock (Figure D.3).

So far, I have focused on the qualitative features of the results. It is worth nothing that these baseline results suggest that large changes in unemployment can occur alongside small changes in labor productivity growth rates. This result is sensitive to the exogenously determined ratio $\frac{\eta_L}{\eta_K}$. As noted in Sections 3.5 and 3.5.2, the relative change in $g^*_{A_L}$ and u^* is determined in part by the absolute value of the slope of (RD-RA) around the initial equilibrium. This slope is increasing in $\frac{\eta_L}{\eta_K}$. When the ratio is high, the slope is steeper and u^* becomes less sensitive to changes in $g^*_{A_L}$. Consistent with this intuition, the robustness results in Figure D.3 show smaller changes in u^*.[16] If there were data on this sensitivity, it would help pin down $\frac{\eta_L}{\eta_K}$. Given the speculative nature of the analysis, which focuses on permanent changes in R&D productivity, I am not aware of any data that could be used to perform this calibration.[17]

4.3 Bargaining Institutions

The model developed in this paper also presents a natural environment in which to study the effect of changes in bargaining institutions. In particular, I examine the effect of a 4% increase in χ, which almost doubles the steady state unemployment rate. The results are presented in Figure 4. See Appendix Section A.7 for a formal analysis of the comparative statics.

Equation (30) demonstrates that a change in bargaining institutions has no long-run effect on the growth rate of labor productivity. The BGP condition for relative rates of technological progress is unrelated to incentives for R&D investment. Panel (a) suggests that changes in bargaining institutions – even when large enough to nearly double the steady state unemployment rate – do not necessarily have a large impact on productivity growth along the transition path. Interestingly, the impact on labor productivity growth in non-monotonic. After the shock, the jump in unemployment reduces incentives for subsequent investment in labor-saving technical change.

While bargaining institutions do not affect long-run productivity growth rates, they do alter wages and incentives for R&D investment, as shown in equation (31). A change in χ induces a change in long-run unemployment such that long-run R&D incentives return to their original BGP levels.

The increase in χ also increases the labor share of income. The rise in u^* has the opposite effect. Panel (c) demonstrates that the former effect is stronger for the calibrated model. Figures D.2 and D.4 present the robustness exercises. The shocks are scaled to deliver similar impacts on unemployment. All results are qualitatively similar.

[16] This discussion focuses on the role of the R&D parameters. This relationship is also affected by the wage specification (11). The term $1 - u_t$ in (RD-RA) comes from the wage equation. If the wage equation was non-linear in u_t, this would affect the slope of the equation around the initial equilibrium.

[17] Cross-country data could be used to estimate the correlation between $g^*_{A_L}$ and u^*, but given that technology can flow between countries, cross-country variation is unlikely to reflect differences in R&D productivity.

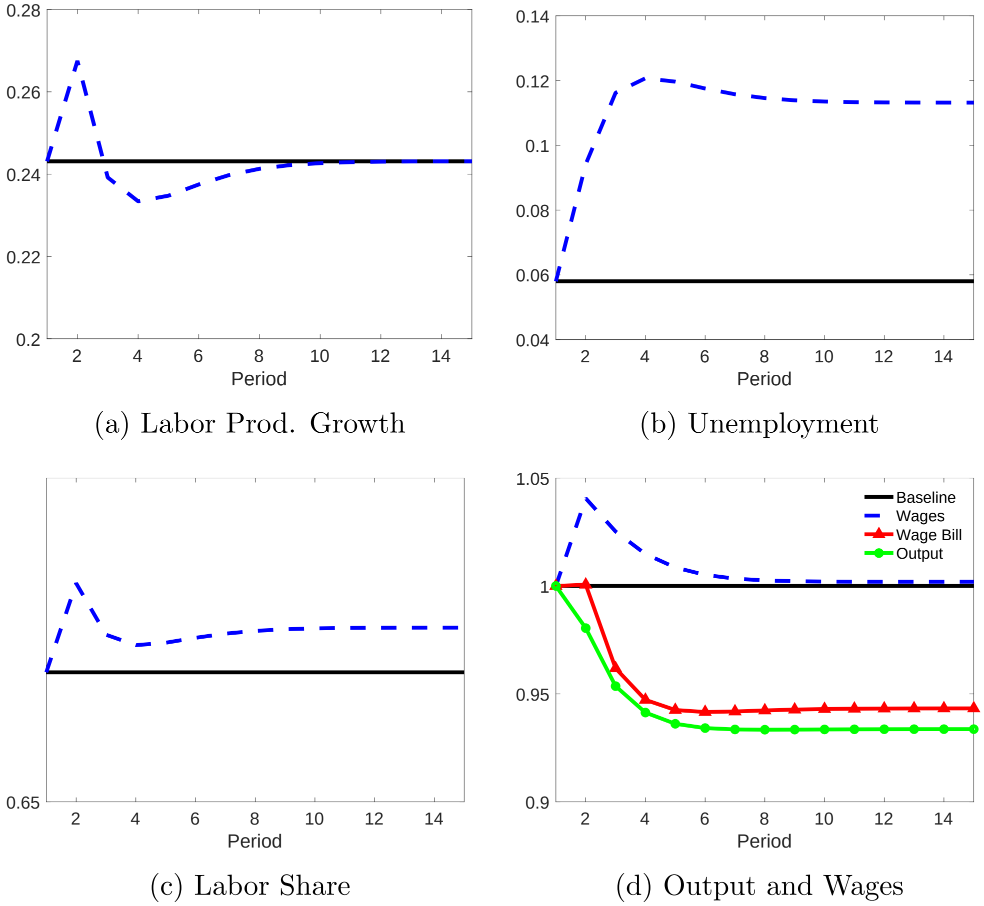


Figure 4: Changes in bargaining institutions

Note: This figures traces the impact of a 4% increase in χ, which nearly doubles technology-driven unemployment. All results are presented relative to baseline where the economy remains on its initial balanced growth path.

The inefficiencies in the model depend on labor market bargaining. Since there is surplus labor and no opportunity cost to working, wages would be zero in a perfectly competitive labor market with unemployment. With zero wages, there would be no incentive to invest in inefficient labor-saving technologies. Increases in the exogenous component of worker bargaining power exacerbate this inefficiency, increasing unemployment and decreasing output.

As in the previous section, panel (d) examines output, wages, and the wage bill. Since output and the labor share move in opposite directions, the impact on the wage bill is ambiguous. In the calibrated model, the change is wages is quite small, implying that even employed workers do not benefit significantly from the improvement in bargaining institutions.

5 Conclusion

I examine a growth model in which technological progress can increase unemployment. I ask two primary questions. First, I examine the forces that lead to a constant long-run rate of unemployment in the presence of labor-saving technical change. Together with labor market bargaining, directed research activity plays a pivotal role in this regard. Second, I examine the consequences of a permanent increase in the productivity of labor-saving R&D. Labor does not become obsolete following the change in R&D productivity, but there are welfare-relevant trade-offs. The long-run growth rate of wages and output increase, but unemployment also increases and the labor share of income falls. The increase in labor-saving R&D productivity also exacerbates existing inefficiencies, leading to slower growth in the short run.

I study a simple model that highlights the relevant intuition, but is not designed to be quantitatively realistic. Future work can further this line of inquiry by developing quantitative models to determine the quantiative consequences of policy interventions. In particular, microfounding the process of wage determination with a search and matching model would allow for a more meaningful quantitative analysis at the cost of eliminating the simple closed-form expressions for R&D incentives. Another fruitful direction would be to generalize the model presented here to focus on cases where the short-run elasticity of substitution between capital and labor is low, but not zero. While this paper uses a Leontief production function, this is not central to the intuition. Another interesting extension would be to combine the model developed here, which focuses on factor-augmenting technical change, with the task-based approach of Acemoglu and Restrepo (2018b) and Hémous and Olsen (2021). This would allow for a quantitative comparison of how different types of technological progress contribute to unemployment. The hybrid model would be a natural environment in which to study the effects of labor-saving technology on workers with different types of skills and education.

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Online Appendix

"Unemployment and the Direction of Technical Change"
by Gregory Casey
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A Derivations

A.1 Final Good Producer

After the resolution of the bargaining process, the representative final good producer takes all factor prices – including wages – as given and chooses inputs $L_t(i)$, $X_t(i)$ and M to maximize profits subject to the constraint imposed by the Leontief production function. Since the final good producer would never hire excess workers or rent excess capital, it is immediate that $A_{L,t}(i)L_t(i) = X_t(i)^\alpha M^{1-\alpha}$ $\forall i, t$. Let $\mathcal{L}_{FG}$ be the Lagrangian for the final good producer's maximization problem:

$$\mathcal{L}_{FG,t} = \int_0^1 A_{L,t}(i)L_t(i)di - \int_0^1 w_t L_t(i)di - \int_0^1 p_{X,t}(i)X_t(i)di - p_{M,t}M - \int_0^1 \lambda_t(i)\left[A_{L,t}(i)L_t(i) - X_t(i)^\alpha M^{1-\alpha}\right]di. \quad \text{(A.1)}$$

The first order conditions are

$$w_t = (1-\lambda_t(i))\, A_{L,t}(i),\ \forall i, \quad \text{(A.2)}$$

$$p_{X,t}(i) = \alpha\lambda_t(i)X_t(i)^{\alpha-1}M^{1-\alpha},\ \forall i, \quad \text{(A.3)}$$

$$p_{M,t} = (1-\alpha)\int_0^1 \lambda_t(i)X_t(i)^\alpha M^{-\alpha}di. \quad \text{(A.4)}$$

Rearranging (A.2) and plugging in to (A.3) and (A.4) yields

$$p_{X,t}(i) = \alpha\left[1 - \frac{w_t(i)}{A_{L,t}(i)}\right]X_t(i)^{\alpha-1}M^{1-\alpha}, \quad \text{(A.5)}$$

$$p_{M,t} = (1-\alpha)\int_0^1 \left[1 - \frac{w_t(i)}{A_{L,t}(i)}\right]X_t(i)^\alpha M^{-\alpha}di. \quad \text{(A.6)}$$

A.2 Capital Good Producers

Capital good producers choose prices ($p_{X,t}(i)$), production quantities ($X_t(i)$), R&D inputs ($R_{K,t}(i), R_{L,t}(i)$), and technological characteristics ($A_{K,t}(i), A_{L,t}(i)$) to maximize profits, subject to constraints on inverse demand (A.5) and research productivity constraints (7). Let $\mathcal{L}_{CG}$ be the Lagrangian for the capital good producer problem. Then,

$$\begin{aligned}\mathcal{L}_{CG,t} = p_{X,t}(i)X_t(i) - \frac{r_t}{A_{K,t}(i)}X_t(i) - p^K_{R,t}R_{K,t}(i) + p^L_{R,t}R_{L,t}(i)\\ - \sum_{J=L,K}\mu_{J,t}\left[A_{J,t}(i) - \big(1+\eta_J R_{J,t}(i)\big)A_{J,t-1}\right]\\ - \upsilon_t(i)\left[p_{X,t}(i) - \alpha\left[1-\chi-\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}\right]X_t(i)^{\alpha-1}\right],\end{aligned} \tag{A.7}$$

which applies the facts that $w_t(i) = \chi A_{L,t}(i) + \chi\nu(1-u_t)A_{L,t}$ and $M \equiv 1$. The first order conditions are

$$\upsilon_t(i) = X_t(i), \tag{A.8}$$

$$p_{X,t}(i) = \frac{r_t}{A_{K,t}(i)} - \upsilon_t(i)\alpha(\alpha-1)\left[1-\chi-\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}\right]X_t(i)^{\alpha-2}, \tag{A.9}$$

$$\mu_{K,t}(i) = r_t A_{K,t}(i)^{-2}X_t(i), \tag{A.10}$$

$$\mu_{L,t}(i) = \upsilon_t(i)\alpha\chi\nu(1-u_t)A_{L,t}A_{L,t}(i)^{-2}X_t(i)^{\alpha-1}, \tag{A.11}$$

$$p^K_{R,t} = \mu_{K,t}(i)\eta_K A_{K,t-1}, \tag{A.12}$$

$$p^L_{R,t} = \mu_{L,t}(i)\eta_L A_{L,t-1}. \tag{A.13}$$

Plugging (A.8) into (A.9) and applying (A.5) yields,

$$p_{X,t}(i) = \frac{1}{\alpha}\frac{r_t}{A_{K,t}(i)}, \tag{A.14}$$

$$X_t(i) = \alpha^{\frac{2}{1-\alpha}}A_{K,t}^{\frac{1}{1-\alpha}}r_t^{\frac{-1}{1-\alpha}}\left[1-\chi-\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}\right]^{\frac{1}{1-\alpha}}. \tag{A.15}$$

Putting these together yields

$$\bar{\pi}_{X,t}(i) \equiv \left(p_{X,t}(i) - \frac{r_t}{A_{K,t}(i)}\right)X_t(i) = \left(\frac{1}{\alpha}-1\right)\alpha^{\frac{2}{1-\alpha}}A_{K,t}^{\frac{\alpha}{1-\alpha}}r_t^{\frac{-\alpha}{1-\alpha}}\left[1-\chi-\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}\right]^{\frac{1}{1-\alpha}}, \tag{A.16}$$

where $\bar{\pi}_{X,t}(i)$ is profits before considering payments to R&D inputs.

Taking the ratio of (A.11) and (A.10) yields

$$\frac{\mu_L(i)}{\mu_K(i)} = \frac{\alpha\chi\nu(1-u_t)A_{L,t}X_t(i)^{\alpha-1}M^{1-\alpha}A_{K,t}(i)}{\frac{r_t}{A_{K,t}(i)}A_{L,t}(i)^2}. \tag{A.17}$$

Applying (A.5), (A.8), and (A.14) gives

$$\frac{\mu_L(i)}{\mu_K(i)} = \frac{\chi\nu(1-u_t)A_{L,t}A_{K,t}(i)}{\alpha A_{L,t}(i)^2\left[1-\chi-\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}\right]}. \tag{A.18}$$

Similarly, taking the ratio of (A.13) and (A.12) yields

$$\frac{p^L_{R,t}}{p^K_{R,t}} = \frac{\mu_{L,t}(i)\eta_L A_{L,t-1}}{\mu_{K,t}(i)\eta_K A_{K,t-1}}. \tag{A.19}$$

Combining these expressions yields

$$\frac{p^L_{R,t}}{p^K_{R,t}} = \frac{\chi\nu(1-u_t)A_{L,t}A_{K,t}(i)}{\alpha\left[1-\chi-\chi\nu(1-u_t)\frac{A_{L,t}}{A_{L,t}(i)}\right]A_{L,t}(i)^2}\frac{\eta_L A_{L,t-1}}{\eta_K A_{K,t-1}}. \tag{A.20}$$

Multiplying both the numerator and denominator on the RHS by $A_{L,t}(i)^{-1}$ gives (15) in the main text.

A.3 Research Allocations

Equation (A.20) is the same for all i, implying that all capital good producers make identical decisions. This, in turn, implies that $A_{J,t}(i) = A_{J,t}\ \forall i,t,J$. Since there is a unit mass of research inputs, this also implies that $R_{J,t}(i) = R_{J,t}\ \forall i,t,J$. Since R&D inputs are freely mobile across technologies, $p^L_{R,t} = p^K_{R,t}$ whenever the allocation is interior. Applying these results to (A.20) and rearranging yields

$$\Gamma(u_t)\frac{A_{L,t}}{A_{L,t-1}} \quad = \quad \frac{A_{K,t}}{A_{K,t-1}}\frac{\eta_L}{\eta_K}, \tag{A.21}$$

where $\Gamma(u_t) = \frac{\alpha\left(1-\chi-\chi\nu(1-u_t)\right)}{\chi\nu(1-u_t)}$. Applying (7) and (8) yields

$$\Gamma(u_t)\big(1+\eta_L R_{L,t}\big) = \big(1+\eta_K(1-R_{L,t})\big)\frac{\eta_L}{\eta_K}. \tag{A.22}$$

Rearranging yields

$$\eta_L R_{L,t} = \frac{1}{1+\Gamma(u_t)}\left(\frac{\eta_L}{\eta_K}+\eta_L-\Gamma(u_t)\right), \tag{A.23}$$

and noting that $\eta_L R_{L,t} = g_{L,t}$ gives (RD-RA) in the main text.

A.4 Factor Shares

Throughout this section, I take advantage of the fact that all capital good producers make identical decisions. In addition, since there is full depreciation, the representative household will never save capital that goes unused in the subsequent period. Implying that the market clearing condition for capital holds in every period.

As described in the main text

$$\frac{w_t L_t}{Y_t} = \frac{w_t}{A_{L,t}} = \chi + \chi \Gamma(u_t). \tag{A.24}$$

Using equations (5) and (A.14),

$$\begin{aligned} r_t K_t &= \alpha p_{X,t} A_{K,t} K_t & \text{(A.25)} \\ &= \alpha^2 \left[1 - \chi - \chi\nu(1-u_t)\right] \left(A_{K,t} K_t\right)^{\alpha}. & \text{(A.26)} \end{aligned}$$

So, since $Y_t = \left(A_{K,t} K_t\right)^{\alpha}$,

$$\frac{r_t K_t}{Y_t} = \alpha^2 \left[1 - \chi - \chi\nu(1-u_t)\right]. \tag{A.27}$$

Similarly, equation (A.6) implies

$$\frac{p_{M,t}}{Y_t} = (1-\alpha)\left[1 - \chi - \chi\nu(1-u_t)\right]. \tag{A.28}$$

When unemployment is constant, as on the BGP, then all of these factor shares are constant.

Combining equations (A.8), (A.10), and (A.12) with market clearing condition (5) gives

$$p_{R,t} = \alpha^2 \left[1 - \chi - \chi\nu(1-u_t)\right] \left(A_{K,t} K_t\right)^{\alpha} \eta_K \frac{1}{(1 + g_{A_K,t})}. \tag{A.29}$$

Noting that there is a unit mass of R&D inputs, this yields

$$\frac{p_{R,t}}{Y_t} = \alpha^2 \left[1 - \chi - \chi\nu(1-u_t)\right] \eta_K \frac{1}{(1 + g_{A_K,t})}. \tag{A.30}$$

On a BGP, both unemployment and technology growth rates are constant, implying that the R&D factor share is constant as well.

The remainder of output is paid to capital good producers as profits. Given that all other factors shares are constant on a BGP, this share must also be a constant.

A.5 Equilibrium R&D Allocations

In this section, I discuss how the R&D allocations and unemployment are determined. Taking logs of (25) and applying small value approximations , the definition of u_t can be written as

$$u_t = g_{A_L,t} - \alpha g_{A_K,t} - \ln\left(\frac{A^{\alpha}_{K,t-1}K^{\alpha}_t}{A_{L,t-1}N_t}\right).$$

All the variables within the last term are predetermined in period t. Using market clearing condition (27),

$$u_t = \left(1+\frac{\alpha}{\eta_L}\right) g_{A_L,t} - \ln\left(\frac{e^{\alpha\eta_K}A^{\alpha}_{K,t-1}K^{\alpha}_t}{A_{L,t-1}N_t}\right),$$

which can be rearranged to get

$$g_{A_L,t} = \left(1+\frac{\alpha}{\eta_L}\right)^{-1}\left(u_t + \ln\left(\frac{e^{\alpha\eta_K}A^{\alpha}_{K,t-1}K^{\alpha}_t}{A_{L,t-1}N_t}\right)\right). \tag{u-DEF}$$

Equation (u-DEF) defines an upward sloping relationship in $(g_{A_L,t}, u_t)$ space. The research arbitrage equation (RD-RA) defines a downward sloping relationship. Thus, there is at most one intersection between the two points. Conditional on $g_{A_L,t}$, $g_{A_K,t}$ is determined by (27). As discussed in Section (3.4), I focus on the case of interior allocations, which is consistent with US data. Equation (u-DEF) demonstrates that allocations are interior if and only if the evolution of the endogenous object $\ln\left(\frac{A^{\alpha}_{K,t-1}K^{\alpha}_t}{A_{L,t-1}N_t}\right)$ satisfies certain conditions along the equilibrium path. I find that these conditions hold in all of the simulations discussed in Section 4.

A.6 Transversality Condition

To write the transversality condition (17) in intensive form, note that

$$\beta^T(C_T/L_t)^{-\xi}K_{t+1} = L_0\left(\beta(1+n)^{\xi}\right)^T A_{K,t}^{\frac{-\xi\alpha}{1-\alpha}} c_T^{-\xi} A_{K,t+1}^{\frac{\alpha}{1-\alpha}} k_{t+1} \tag{A.31}$$

$$= L_0\left(\beta(1+n)^{\xi}\right)^T A_{K,T}^{(1-\xi)\frac{\alpha}{1-\alpha}}(1+g_{A_K,T+1})^{\frac{\alpha}{1-\alpha}} c_T^{-\xi} k_{T+1} \tag{A.32}$$

$$= A_{K,-1}^{(1-\xi)\frac{\alpha}{1-\alpha}} L_0\left(\beta(1+n)^{\xi}\right)^T \left(\prod_{\tilde{t}=0}^{T}(1+g_{A_K,\tilde{t}})^{\frac{\alpha}{1-\alpha}}\right)^{1-\xi}(1+g_{A_K,T+1})^{\frac{\alpha}{1-\alpha}} c_T^{-\xi} k_{T+1}. \tag{A.33}$$

Plugging the final expression into (17) and dividing by $A_{K,-1}^{(1-\xi)\frac{\alpha}{1-\alpha}} L_0$ gives (28).

Evaluated on the BGP, (28) gives

$$\lim_{T\to\infty}\left(\beta(1+n)^{\xi}(1+g^*_{A_K})^{\frac{(1-\xi)\alpha}{1-\alpha}}\right)^T(1+g^*_{A_K})^{\frac{\alpha}{1-\alpha}}(c^*)^{-\xi}k^*=0, \tag{A.34}$$

which holds as long as

$$\beta(1+n)^{\xi}(1+g^*_{A_K})^{\frac{(1-\xi)\alpha}{1-\alpha}}<1. \tag{A.35}$$

Using Lemma 5, this condition can be written as

$$\beta(1+n)(1+g^*_{A_L})^{1-\xi}<1, \tag{A.36}$$

which is standard for growth models. In the calibrated model, $\xi=1$, and the required condition is $\beta<(1+n)^{-1}$, which is satisfied.

For finite lifetime utility, let $\bar{t}$ be some point after which the economy is on the BGP. Then, use the Proposition 1 to write (9) as

$$U=\sum_{t=0}^{\bar{t}}\beta^t L_t\frac{(C_t/L_t)^{1-\xi}}{1-\xi}+L^{\xi}_{\bar{t}+1}(\beta(1+n)^{\xi})^{\bar{t}+1}\sum_{k=0}^{\infty}\left(\beta(1+n)^{\xi}\right)^k\frac{(c^*)^{1-\xi}}{1-\xi}\left((1+g^*_{A_k})^{\frac{\alpha}{1-\alpha}}\right)^{k(1-\xi)}, \tag{A.37}$$

which is again finite if $\beta(1+n)^{\xi}(1+g^*_{A_K})^{\frac{(1-\xi)\alpha}{1-\alpha}}<1$.

A.7 Comparative Statics

In this section, I prove the results presented in Section 3.5.2. I also investigate the comparative statics for changes in χ, which informs the analysis presented in Sections 4.3.

A.7.1 Labor Productivity Growth

I start by considering the determinants of the growth rate of labor productivity ($g^*_{A_L}$). Using equation (30), I define

$$G(g^*_{A_L},\eta_L,\eta_K,n,\chi)\equiv(1+\eta_K-\frac{\eta_K}{\eta_L}g^*_{A_L})^{\frac{\alpha}{1-\alpha}}-(1+n)(1+g^*_{A_L})=0. \tag{A.38}$$

By the implicit function theorem,

$$\frac{\partial g^*_{A_L}}{\partial \eta_L} = \frac{-G_{\eta_L}}{G_{g^*_{A_L}}} = \frac{-\Xi \eta_K g^*_{A_L} \eta_L^{-2}}{-\Xi \frac{\eta_K}{\eta_L} - (1+n)} \tag{A.39}$$

$$= \frac{\Xi \eta_K g^*_{A_L}}{\Xi \eta_K \eta_L + (1+n)\eta_L^2} \in (0,1), \tag{A.40}$$

where $\Xi = \frac{\alpha}{1-\alpha}(1+\eta_K - \frac{\eta_K}{\eta_L} g^*_{A_L})^{\frac{2\alpha-1}{1-\alpha}} > 0$. The sign of ψ and the upper bound on $\frac{\partial g^*_{A_L}}{\partial \eta_L}$ follow from the fact that $g^*_{A_L} \leq \eta_L$. Moreover, since all of the arguments of G, other than $g^*_{A_L}$, are exogenous parameters, $\frac{\partial g^*_{A_L}}{\partial \eta_L} = \frac{d g^*_{A_L}}{d \eta_L}$. This will be true for all subsequent analyses as well.

Also,

$$\frac{d g^*_{A_L}}{d\chi} = \frac{-G_\chi}{G_{g^*_{A_L}}} = \frac{0}{-\Xi \frac{\eta_K}{\eta_L} - (1+n)} = 0. \tag{A.41}$$

A.7.2 Unemployment

Next, I investigate the determinants of the steady state rate of unemployment (u^*). I use equation (31) to define

$$H\big(u^*, \eta_L, \eta_K, n, \chi\big) \equiv g^*_{A_L}(\eta_L, \eta_K, n, \chi) - \frac{1}{1+\Gamma(u^*, \chi)}\left(\frac{\eta_L}{\eta_K} + \eta_L - \Gamma(u^*, \chi)\right) = 0, \tag{A.42}$$

where $\Gamma(u,\chi) = \frac{\alpha(1-\chi-\chi\nu(1-u_t))}{\chi\nu(1-u_t)}$ is the same function as in the main text, with updated notation to highlight the dependence on the exogenous parameter χ. The function $g^*_{A_L}(\eta_L, \eta_K, n, \chi)$ is implicitly defined in the previous subsection. It is helpful to note that $H_{\Gamma(u^*,\chi)} > 0$ and $\Gamma_{u^*} > 0$. Thus, $H_{u^*} > 0$. Moreover, $\Gamma_\chi < 0$, implying that $H_\chi < 0$. By the implicit function theorem,

$$\frac{du^*}{d\chi} = \frac{-H_\chi}{H_{u^*}} > 0 \tag{A.43}$$

and

$$\frac{du^*}{d\eta_L} = \frac{-H_{\eta_L}}{H_{u^*}} = \frac{\frac{-d g^*_{A_L}}{d\eta_L} + \big(1 + \frac{1}{\eta_K}\big)\frac{1}{1+\Gamma(u^*,\chi)}}{H_{u^*}}. \tag{A.44}$$

As a result,

$$\frac{du^*}{d\eta_L} > 0 \iff \tag{A.45}$$
$$\frac{dg^*_{A_L}}{d\eta_L} < \left(1+\frac{1}{\eta_K}\right)\frac{1}{1+\Gamma(u^*,\chi)}, \tag{A.46}$$

where $\frac{dg^*_{A_L}}{d\eta_L} < 1$, as shown in the previous section. Assumption (A2), therefore, is a sufficient condition for $\frac{du^*}{d\eta_L} > 0$.

A.7.3 Labor Share of Income

Finally, I examine the determinants of the labor share of income (κ_L^*). I use equation (32) to define

$$I(\kappa_L^*, \eta_L, \eta_K, n, \chi) \equiv \kappa_L^* - \chi - \chi\nu\left(1 - (u^*(\eta_L, \eta_K, n, \chi)\right) = 0, \tag{A.47}$$

where $u^*(\eta_L, \eta_K, n, \chi)$ is implicitly defined in the previous subsection.

By the implicit function theorem,

$$\frac{d\kappa_L^*}{d\eta_L} = \frac{-I_{\eta_L}}{I_{\kappa_L^*}} = \frac{-\chi\nu u^*_{\eta_L}}{1}. \tag{A.48}$$

So, $\frac{d\kappa_L^*}{d\eta_L} > 0 \iff \frac{du^*}{d\eta_L} < 0$ as described in Proposition (2). Also,

$$\frac{d\kappa_L^*}{d\chi} = \frac{-I_{\chi}}{I_{\kappa_L^*}} = -\frac{-1+\chi\nu u^*_{\chi}}{1}. \tag{A.49}$$

So, $\frac{d\kappa_L^*}{d\chi} > 0 \iff \chi\nu u^*_\chi < 1$. The direct effect of χ is to increase the labor share of income, but χ also increases unemployment, which decreases the labor share. As demonstrated in Figures 4, D.2 and D.4, $\frac{d\kappa_L^*}{d\chi} > 0$ in all of the quantitative exercises.

A.8 Intuition for Wage Equation (11)

As explained in the main text, the expression for wages given in equation (11) is designed to capture the main features of wage determination in a search and matching model of labor markets with wage bargaining. At the same time, it is also convenient for modeling endogenous and directed technical change. In this section, I present a simple model of bargaining to motivate the expression for wages given in the text. The goal is not to provide a comprehensive, realistic, or elegant model of bargaining. Instead, the limited the purpose of this exercise is to provide intuition for why firm-level productivity ($A_{L,t}(i)$), aggregate productivity ($A_{L,t}$), and the unemployment rate (u_t) are included in equation (11). In doing so, it also provides some intuition for which economic or institutional forces would affect the exogenous parameters (χ and ν).

A.8.1 Structure of Illustrative Model

I consider a model of bargaining that occurs within each period of the growth model. Each period is split into two sub-periods of equal duration. At the beginning of the first sub-period, firms purchase capital that lasts throughout the full period. After this capital purchase, there is a finite quantity of workers, $L_t^{\max}(i)$, that can work with capital good i with positive marginal product:

$$L_t^{\max}(i) = \frac{X_t(i)^\alpha}{A_{L,t}(i)}. \tag{A.50}$$

Each of these workers has marginal product $A_{L,t}(i)$. I consider $L_t^{\max}(i)$ as a continuum of potential jobs to be filled. Since there are a unit mass of capital goods and all capital good producers face an identical problem, the total quantity of jobs is $L_t^{\max} = L_t^{\max}(i) = \frac{A_{K,t}K_t^\alpha}{A_{L,t}}$.

There is a continuum of $N_t > L_t^{\max}$ homogeneous workers in the economy who might fill a job. They have no dis-utility of labor. The number of employed workers is L_t. The unemployment rate is

$$u_t \equiv 1 - \frac{L_t}{N_t}. \tag{A.51}$$

At the beginning of the first sub-period, $L_t^{\max}(i)$ workers are randomly matched to work with each capital good. The engage in Nash bargaining with the final good producer over the wage they will earn. They bargain via a risk-neutral intermediary, such as a union. In equilibrium, all workers will reach an agreement with the firm. But, if they did not, they would be unemployed for the first sub-period and the job would be vacant. At the end of the first sub-period, a continuum of $\lambda L_t^{\max}(i)$ workers in each line are separated from their jobs.

At the beginning of the second sub-period, there are $\lambda L_t^{\max}$ jobs available. These jobs can be filled by the $\lambda L_t^{\max}$ workers who just lost their jobs or the $N_t - L_t^{\max}$ workers who were not matched at the beginning of the first sub-period. Each open job is matched with a randomly-selected worker from this pool. They then engage in Nash bargaining with the firm to determine the wage. Once again, all workers and firms will reach an agreement in equilibrium. If they did not, then they would be unemployed for the remainder of the period and the job would be left vacant.

All agents have perfect information and foresight when bargaining. Neither workers nor the firm can break contracts once they have been set. This completes the description of the model.

Before I turn to solving the model, it is helpful to highlight the role of the endogenous variables that show up in equation (11). Capital-good-specific productivity ($A_{L,t}(i)$) will affect the wage, because this is the surplus to be divided between firms and workers. The average productivity level ($A_{L,t}$) and the unemployment rate (u_t) are relevant, because they determine the outside options of workers, which in turn determine the reservation wage. If workers do not reach an agreement with firms during the first round of bargaining, the unemployment rate (u_t) affects the likelihood that they will be matched to a firm in the second round of bargaining. If they are matched to a firm, the expected surplus of the match is the average of firm-level productivity ($A_{L,t}$). Equation (11) captures all of this intuition in a simple and transparent way.

In addition, equation (11) has two exogenous parameters, χ and ν. While these are reduced-form parameters, the structure of the illustrative model provides some intuition for what sorts of institutional changes would affect them. Parameter χ influences how both capital-good-specific labor productivity ($A_{L,t}(i)$) and average labor productivity ($A_{L,t}$) affect wages. In doing so, it represents the state of bargaining institutions and the degree to which they favor workers over firms. Parameter ν gives the relative weight of capital-good-specific productivity and worker's outside options in determining the equilibrium wage. Put differently, it plays a role in determining the reservation wage, conditional on average productivity and the unemployment rate. Among other things, the degree of search frictions in the economy would affect ν. If search frictions are high, then workers will have lower reservation wages, corresponding to a lower ν in the reduced-form specification.

A.8.2 Second sub-period bargaining

To start, I consider the bargaining process that occurs when workers meet firms at the beginning of the second sub-period. This only occurs for firms that receive the shock at the end of the first sub-period and for workers who either received the shock at the end of the first sub-period or were not matched to a firm at the beginning of the first sub-period.

As noted in the main text, the firm rents capital before bargaining with workers. The rental costs, therefore, are sunk and not relevant to the bargaining surplus. Of course, firms take the bargaining outcome into account when purchasing capital. All agents take the outcome of second sub-period bargaining as given at the beginning of the first sub-period.

There is no dis-utility from working, and workers bargain through a risk neutral intermediary. Let $w_{t,2}(i)$ be the wage per instant that is proposed – and eventually paid – in the second period. The worker gets payoff $\frac{1}{2}w_{t,2}(i)$ if they reach an agreement and 0 otherwise. From the firm's perspective, the net benefit of reaching an agreement is $\frac{1}{2}\big(A_L(i) - w_{t,2}(i)\big)$.

Nash bargaining maximizes the surplus. The negotiated wage is given by

$$w_{t,2}(i) = \text{argmax}\Big\{\delta \ln\big[A_L(i) - w_{t,2}(i)\big] + (1-\delta)\ln\big[w_{t,2}(i)\big]\Big\}, \tag{A.52}$$

where $\delta \in (0,1)$ is the bargaining power of the firm. This yields

$$w_{t,2}(i) = (1-\delta)A_{L,t}(i). \tag{A.53}$$

All workers and firms make this deal, implying that no job is left vacant. Both workers and the representative firm take this outcome as given when they undertake bargaining at the beginning of the first period. Since workers do not know the job to which they will be matched in the second sub-period, they will take into account the average wage, $w_{2,t} = (1-\delta)A_{L,t}$, when bargaining, where $A_{L,t} = \int_0^1 A_{L,t}(i)di$ as in the main text.

At the beginning of the second sub-period, there are two groups of workers looking for jobs: the $N_t - L_t^{\max}$ workers that were unemployed during the first sub-period and the $\lambda_t L_t^{\max}$ that experience the job separation shock. Each of these workers has an equal probability of being matched to a job at the beginning of the second sub-period. The probability that a searching worker finds a match, therefore, is given by[18]

$$\begin{aligned}
\frac{\#\text{ of openings}}{\#\text{ of searching workers}} &= \frac{\lambda L_t^{\max}}{(\lambda-1)L_t^{\max} + N_t} && \text{(A.54)}\\
&= \left[\frac{\lambda-1}{\lambda} + \frac{1}{\lambda}\frac{1}{1-u_t}\right]^{-1} && \text{(A.55)}\\
&= \left[\frac{\lambda(1-u_t)}{1-(1-\lambda)(1-u_t)}\right] && \text{(A.56)}\\
&\equiv h_2(u_t), && \text{(A.57)}
\end{aligned}$$

where $u_t = 1 - \frac{L_t^{\max}}{N_t}$. It is helpful to note that $h_2(u_t) \in (0,1)\ \forall u_t, \lambda \in (0,1)$ and that $h_2'(u_t) < 0$. When the unemployment rate is high, workers find it more difficult to find a job in the second sub-period.

If a worker does not have a job at the beginning of the second sub-period, their expected payoff per instant is $h_2(u_t)(1-\delta)A_{L,t}$, which is the probability of being matched to a firm

[18] There are a continuum of openings and workers. I use # informally to denote the mass.

multiplied the average wage paid in the second period. Given that the sub-period lasts for half a period, their total expected payoff is $\frac{1}{2}h_2(u_t)(1-\delta)A_{L,t}$.

A.8.3 First sub-period bargaining

Now, I consider the bargaining that occurs at the beginning of the first sub-period. Let $w_{t,1}(i)$ be the wage that is agreed upon in the first sub-period. Noting that there is no down time and a job is filled with certainty in the second sub-period, the payoff to the representative firm if it reaches a first sub-period deal is $(1-\frac{\lambda}{2})\big[A_{L,t}(i)-w_{t,1}(i)\big]+\frac{\lambda}{2}\delta A_{L,t}(i)$. The firm's payoff if a deal is not reached is $\frac{1}{2}\delta A_{L,t}(i)$, the value of second sub-period match. Thus, the surplus from reaching a deal is $\frac{1}{2}\Bigg[\big((2-\lambda)+\delta(\lambda-1)\big)A_{L,t}(i)-(2-\lambda)w_{1,t}(i)\Bigg]$.

Meanwhile, the payoff to the worker of reaching a deal is $(1-\frac{\lambda}{2})w_{t,1}(i)+\frac{\lambda}{2}h_2(u_t)(1-\delta)A_{L,t}$. The payoff if a deal is not reached is $\frac{1}{2}h_2(u_t)(1-\delta)A_{L,t}$, which is the reservation wage. Thus, the surplus from reaching a deal in the first sub-period is $\frac{1}{2}\Bigg[(2-\lambda)w_{t,1}(i)+(\lambda-1)h_2(u_t)(1-\delta)A_{L,t}\Bigg]$.

Now, Nash bargaining in the first sub-period yields

$$w_{1,t}(i)=\operatorname{argmax}\Bigg\{\delta\ln\big[\big((2-\lambda)+\delta(\lambda-1)\big)A_{L,t}(i)-(2-\lambda)w_{1,t}(i)\big] \\ +(1-\delta)\ln\big[(2-\lambda)w_{t,1}(i)+(\lambda-1)h_2(u_t)(1-\delta)A_{L,t}\big]\Bigg\}, \quad \text{(A.58)}$$

which yields

$$w_{t,1}(i)=\frac{(1-\delta)}{2-\lambda}\big[(2-\lambda)+\delta(\lambda-1)\big]A_{L,t}(i)+\frac{(1-\lambda)}{2-\lambda}\delta(1-\delta)h_2(u_t)A_{L,t}. \quad \text{(A.59)}$$

A.8.4 Expected wages

When firms demand capital, they care about the expected wage:

$$w_t(i) \quad = \quad \left(1-\frac{\lambda}{2}\right)w_{t,1}(i)+\frac{\lambda}{2}w_{t,2}(i). \quad \text{(A.60)}$$

Substituting (A.53) and (A.59) into (A.60) yields

$$w_t(i)=\frac{1}{2}(1-\delta)\big[(2-\lambda)+\delta(\lambda-1)+\lambda\big]A_{L,t}(i)+\frac{1}{2}\left((1-\lambda)\delta(1-\delta)h_2(u_t)\right)A_{L,t}. \quad \text{(A.61)}$$

This equation has a similar to form to u_t, except that the u_t enter non-linearly. In particular, we can define $\chi = \frac{1}{2}(1-\delta)\big[2-\delta(1-\lambda)+\lambda\big]$ and $h(u_t) = (\chi\nu)^{-1}\frac{1}{2}\left((1-\lambda)\delta(1-\delta)h_2(u_t)\right)$ and then re-write the equation as

$$w_t(i) = \chi A_{L,t}(i) + \chi\nu h(u_t) A_{L,t}, \tag{A.62}$$

with $h'(u_t) < 0$.

The central goal of this paper is to evaluate the qualitative behavior of the dynamical system. Thus, I use the alternate formulation from equation (11) for the quantitative analysis and comparative statics, because this yields simpler and more intuitive expressions.

Now that I have finished the description of the model, it is worth quickly revisiting why the various endogenous variables show up in equation (11) (or, similarly, in equations (A.61) and (A.62)). Capital-good-specific productivity ($A_{L,t}(i)$) enters (11) for two reasons. First, when a firm and worker are matched at the beginning of the second sub-period, they are unaffected by any outside considerations and they simply bargain over the match-specific productivity ($A_{L,t}(i)$), which is the surplus from a match. Second, when a firm and a worker meet at the beginning of the first sub-period, they are again interested in dividing match-specific output ($A_{L,t}(i)$). In the first sub-period, however, this productivity is not equal to the match surplus, because agreeing to a match in the first sub-period reduces the likelihood that a worker or firm will be subject to a new wage in the second sub-period. From the worker's point of view, the value of not reaching an agreement in the first sub-period (i.e., their reservation wage), is affected by the expected wage they would earn in the second sub-period bargaining. The expected wage is determined in part by the probably that they would be matched to a firm, which in turn depends labor market tightness as captured by the unemployment rate ($u_{L,t}$). Since workers are matched to firms at random, the expected wage also depends on the average productivity level across firms ($A_{L,t}$). All of this intuition is captured through the reduced form equation (11).

The illustrative model also provides some intuition for what institutional changes might affect χ and ν, the reduced-form parameters in (11). Parameter χ increases the coefficient on capital-good-specific productivity and on the reservation wage. I interpret this parameter as reflecting the state of bargaining institutions and the degree to which they favor workers over firms. Such institutions would directly affect the fraction of the surplus paid to workers in the first and second sub-periods. In addition, they would also affect the reservation wage of the workers in the first sub-period, since reservation wages are determined by second sub-period bargaining outcomes and match probabilities. Parameter ν increases the coefficient on outside options, but not capital-good-specific productivity. It captures institutions that affect the reservation wage, holding average productivity and unemployment fixed. For example, ν reflects search frictions, because reservation wages are lower when it is harder to find a new job.

B Calibration Data

Income Share of Labor. Definition: Payments to labor as a share of GDP. Value in model: κ_L^*. Value in data: 67%. Source: Table 2 of Valentinyi and Herrendorf (2008).

Income Share of Land. Definition: Payments to land as a share of GDP. Value in model: κ_M^*. Value in data: 5%. Source: Table 2 of Valentinyi and Herrendorf (2008).

Unemployment Rate. Definition: Fraction of labor force not employed. Value in model: u^*. Value in data: 5.8%, annual average of civilian unemployment rate, 1948 – 2016. Source: Bureau of Labor Statistics (BLS) via FRED. FRED Code: UNRATE.

Labor Productivity Growth. Definition: Growth rate of output per worker. Value in model: $g^*_{A_L}$. Value in data: 2.2%/year, geometric average of growth in real output per person in the nonfarm business sector, 1948 – 2016. Source: BLS via FRED. FRED Code: PRS85006092.

Labor Force Growth. Definition: Growth rate of the labor force. Value in model: n. Value in data: 1.4%/year, geometric average of growth in civilian labor force, 1948 – 2016. Source: BLS via FRED. FRED code: CLF16OV_PC1.

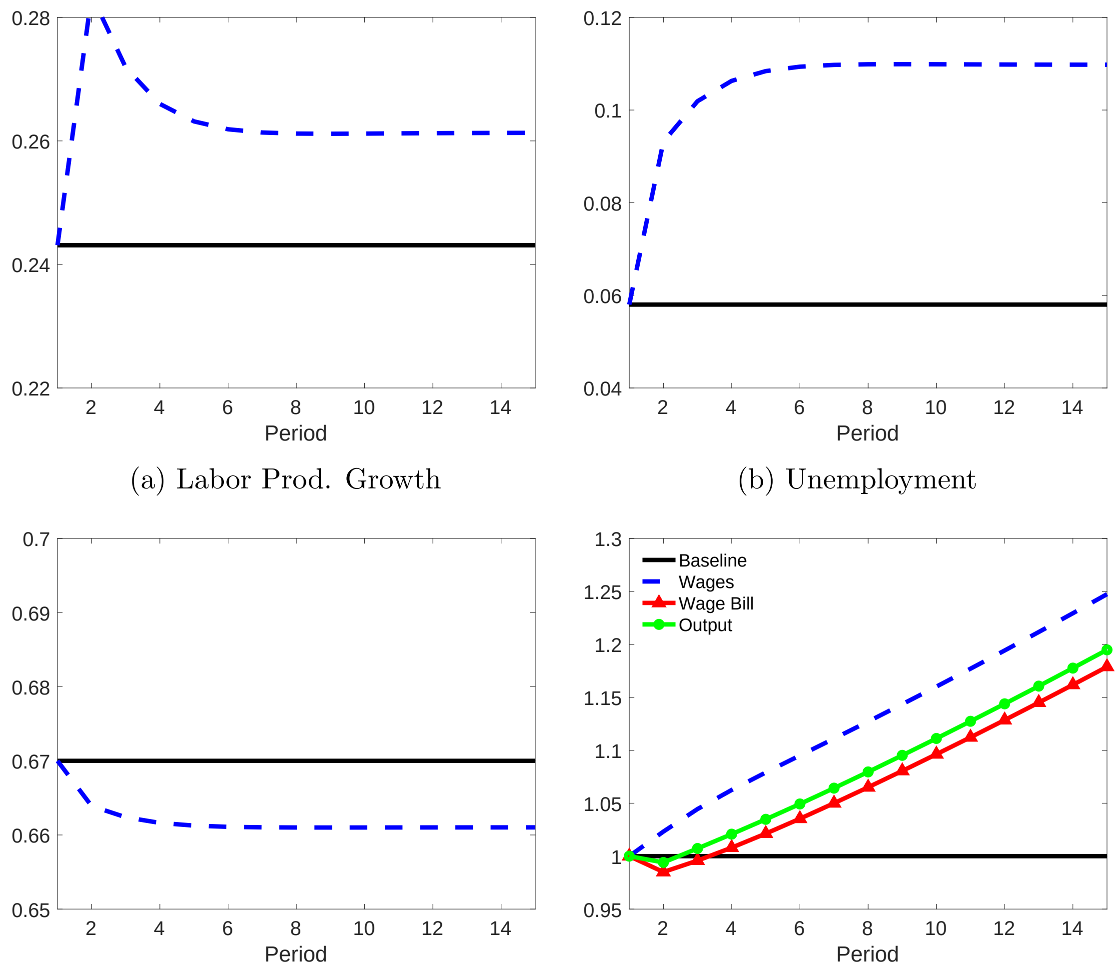
C Simulation Method

The model is simulated using the perfect foresight solver in *Dynare* (Adjemian et al., 2011). Dynare is fed the system of equations in Section 3.4. A few minor changes are made relative to the presentation in the main text. First, since the model solves quickly, intermediate equations for $y_t = k_t^\alpha$ and $\Gamma(u_t)$ are also included, which helps increase the readability of the code. Second, since I study the transition path following changes in exogenous parameters, the code is modified to account for the distinction between normalized savings in period $t-1$ ($s_{t-1} = \frac{Y_{t-1}-C_{t-1}}{A_{K,t-1}^{\frac{\alpha}{1-\alpha}}}$) and normalized capital in period t ($k_t = \frac{Y_{t-1}-C_{t-1}}{A_{K,t}^{\frac{\alpha}{1-\alpha}}}$). Normalized savings does not respond to changes in parameters in period t, while normalized capital adjusts through $A_{K,t}^{\frac{\alpha}{1-\alpha}}$. This is necessary to correctly capture the dynamics in the first period after the parameter change.

D Figures for Robustness Exercises

D.1 High η_L

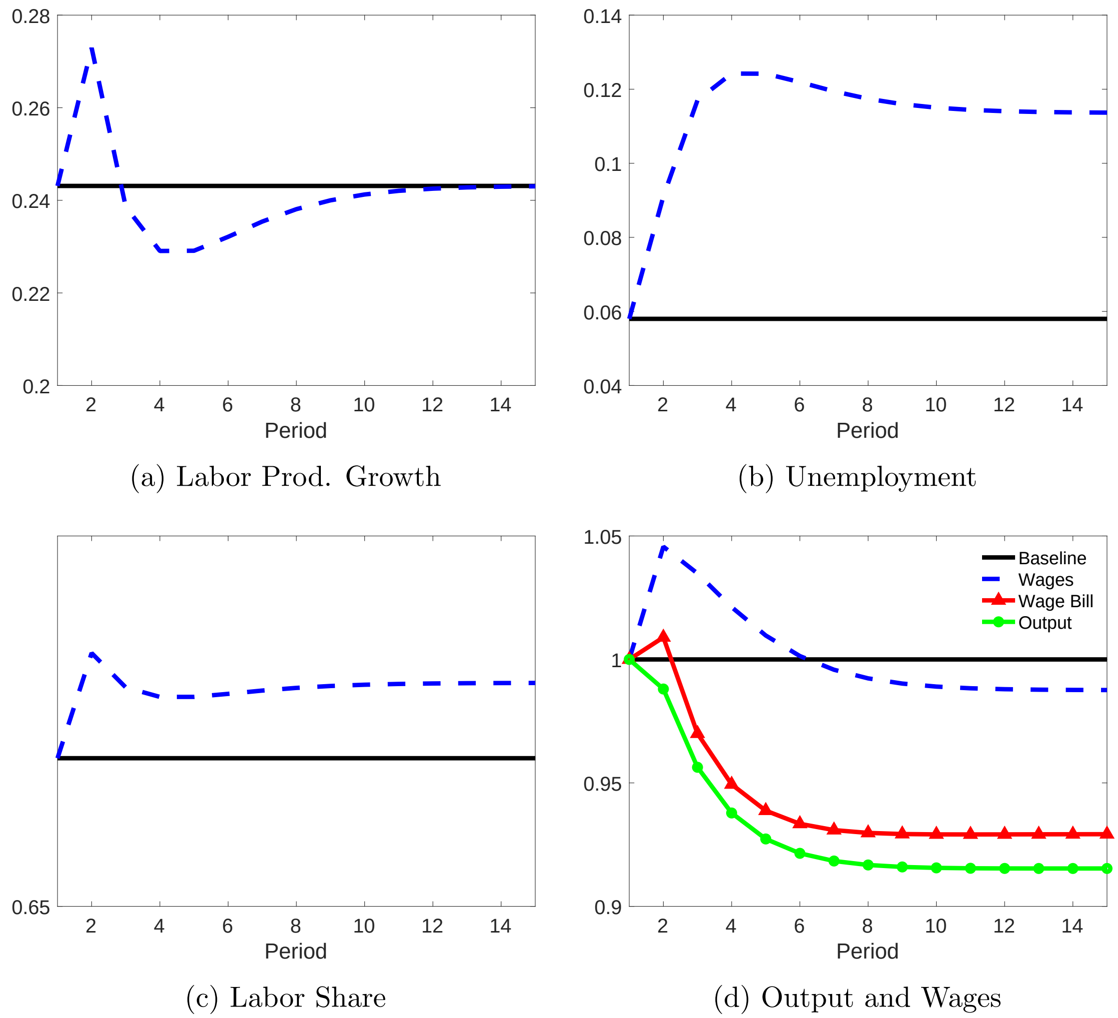
This section presents the results when assuming that $\frac{\eta_L}{\eta_K} = 2$. The qualitative results are the same as the baseline case.



(a) Labor Prod. Growth (b) Unemployment

(c) Labor Share (d) Output and Wages

Figure D.1: This figures traces the impact of a 10% increase in η_L. All results are presented relative to a baseline scenario where the economy remains on its initial balanced growth path.



(a) Labor Prod. Growth

(b) Unemployment

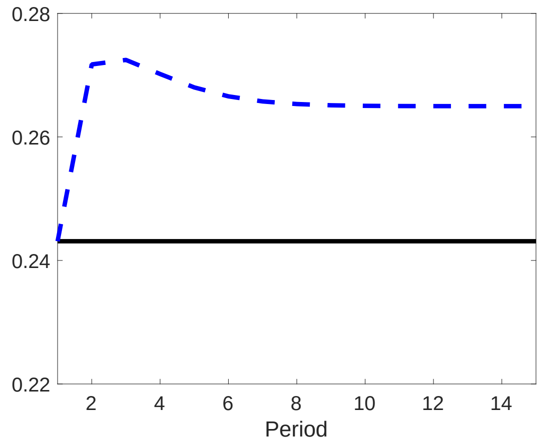
(c) Labor Share

(d) Output and Wages

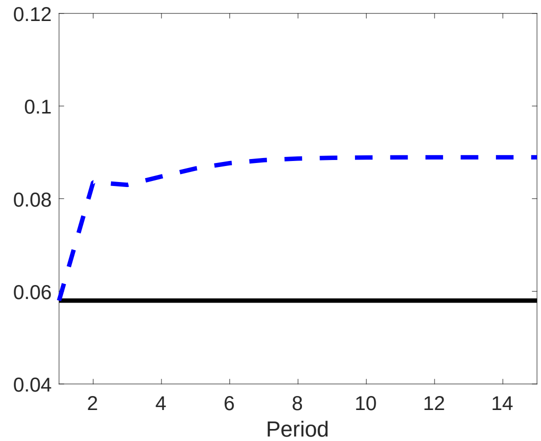
Figure D.2: This figures traces the impact of a change in bargaining institutions that increase χ by 3%, which doubles unemployment. All results are presented relative to a baseline scenario where the economy remains on its initial balanced growth path.

D.2 Low η_L

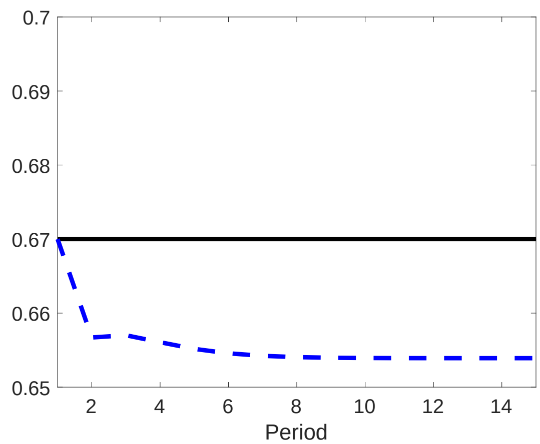
This section presents the results when assuming that $\frac{\eta_L}{\eta_K} = 0.67$. The qualitative results are the same as the baseline case.



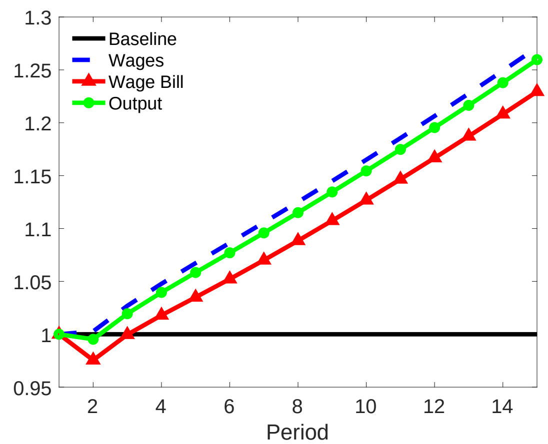
(a) Labor Prod. Growth



(b) Unemployment

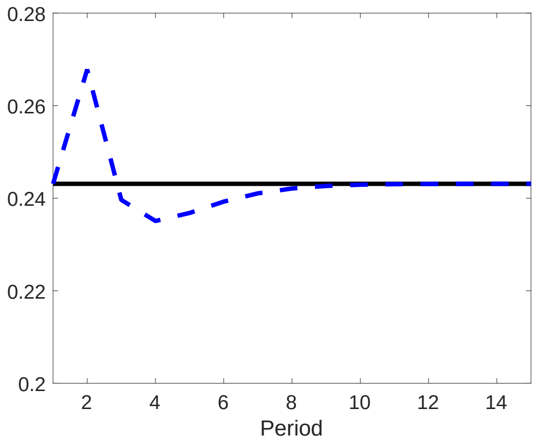


(c) Labor Share

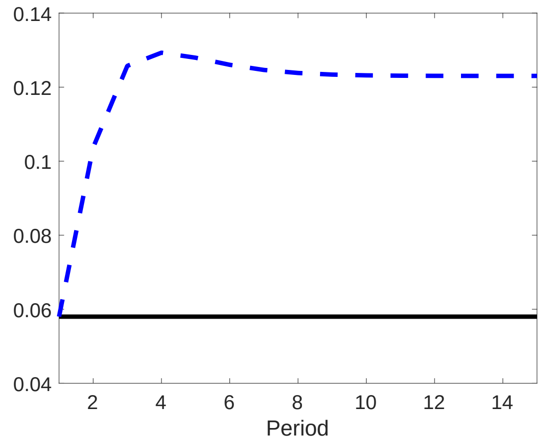


(d) Output and Wages

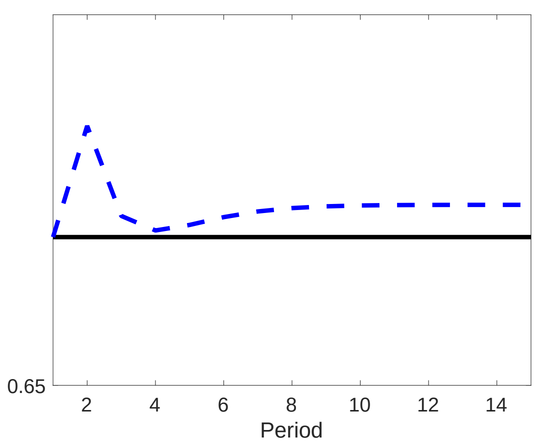
Figure D.3: This figures traces the impact of a 10% increase in η_L. All results are presented relative to a baseline scenario where the economy remains on its initial balanced growth path.



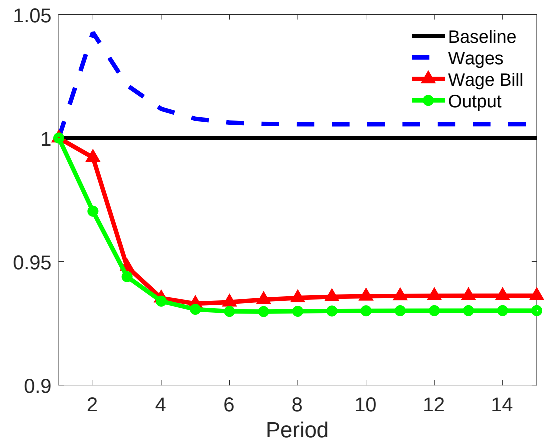
(a) Labor Prod. Growth



(b) Unemployment



(c) Labor Share



(d) Output and Wages

Figure D.4: This figures traces the impact of a change in bargaining institutions that increase χ by 6%, which doubles unemployment. All results are presented relative to a baseline scenario where the economy remains on its initial balanced growth path.