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## Abstract

We examine the effects of endogenous sequences on the fairness in round-robin tournaments with four players, multiple prizes, and general contest technologies. A tournament is called horizontally ex-ante fair if symmetric contestants have the same expected payoffs (odds) before the tournament starts. It is called perfectly fair if the winning probabilities in each match depend only on the players' characteristics but not on the position of the match in the course of the tournament. We show that there is no sequence which implies perfect fairness. By contrast, some endogenous sequences imply horizontal ex-ante fairness irrespective of the prize structure. In winner-take-all tournaments, additional endogenous sequences are horizontally ex-ante fair. Our findings question the prevailing use of exogenous sequences in four-player round-robin tournaments in commercial sports despite horizontally ex-ante fair alternatives.


JEL-Codes: C720, D720, Z200.

Keywords: sequential round-robin tournament, endogenous sequence, contest success function, multiple prizes, fairness.

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# 1 Introduction

A round-robin tournament is a form of competition in which all contestants face each other in a sequence of pairwise matches. Round-robin tournaments are widely-used for the organization of athletic competition. On a large scale, many sports leagues – such as the major European football leagues, including the English Premier League and Spanish La Liga – are organized as double round-robin tournaments where each team meets each other team twice (home and away). On a small scale, round-robin tournaments are prevalent as early stages of international championships. A famous example is the first round (group stage) of the FIFA World Cup (soccer) with four teams per group.[1] Other examples with four teams per group include the first round of the UEFA European Championship and the CONMEBOL Copa América (soccer), the first and second round of the FIBA Basketball World Cup, and the first round of the recent IHF Men's World Championship (handball).

Usually, round-robin tournaments employ an exogenous sequence of matches, i.e., the exact order of pairings is determined beforehand and common knowledge when the tournament starts. For instance, the 2022 FIFA World Cup schedule was fixed more than half a year before the opening match.[2] By contrast, with an endogenous sequence the order of later pairings depends on the results of previous matches. Endogenous sequences are canonical in other organizational forms like elimination tournaments (predominant, e.g., in professional tennis) or Swiss tournaments (predominant, e.g., in professional chess).

For a simple example within a round-robin tournament, consider one with three players. Call the opponents of the first match player 1 and player 2, and the remaining contestant player 3. Under an exogenous schedule, the order of the two remaining pairings is fixed, e.g., player 3 meets player 1 in match 2, and player 2 in match 3. By contrast, under an endogenous schedule, the order of the two remaining pairings depends on the result of the first match, e.g., player 3 meets the loser of the first match in match 2, and the winner of the first match in match 3. Unlike the exogenous sequence, the endogenous sequence allows to control for possible intermediate scores: in the example above, player 3 knows that the number of his opponent's wins is zero in match 2 and one in match 3. Using this kind of endogenous sequence, the organizer of the tournament can, e.g., maintain suspense: since no player is able to achieve two wins after two matches, the tournament winner cannot be decided before the final match.

Besides maintaining suspense, tournament design may pursue various goals. One particular important objective, on which we will focus in this paper, is fairness. As a minimum requirement, a fair tournament should provide equal chances for equally strong players from an ex ante perspective: if all players have the same characteristics, they should have the same expected payoffs (winnings) before the tournament starts. We call a tournament with this property *horizontally ex-ante fair*. A much stronger notion of fairness results from requiring that in each match of the tournament, the winning probabilities of the matched contestants should depend only on their characteristics but not on the position of the match in the schedule. We call a tournament with this property *perfectly fair*.

While the common wisdom deems "a round-robin tournament ... the fairest way to

[1]On 03/14/2023, the FIFA council discarded its earlier plan to switch from groups of four to groups of three in the first round of the FIFA World Cup 2026 in order to avoid collusion and ensure that all the teams play a minimum of three matches (https://www.fifa.com/about-fifa/organisation/fifa-council/media-releases/fifa-council-approves-international-match-calendars; accessed on 02/22/2024).

[2]https://www.fifa.com/tournaments/mens/worldcup/qatar2022/media-releases/final-match-schedule-for-the-fifa-world-cup-qatar-2022-tm-now-available-on; accessed on 02/13/2023

determine the champion from among a known and fixed number of contestants" (Wikipedia[3], 2024), the recent literature has demonstrated that this statement is usually not true (see below). In particular, fully sequential round-robin tournaments with four players are never fair if the schedule is exogenous (Laica et al., 2021). Intuitively, asymmetries in intermediate scores induce discouragement effects for contestants that are lagging behind and (possibly) lean-back effects for contestants in the lead. This precludes perfect fairness. Depending on the position of their matches in the schedule of the tournament, contestants assign different valuations to these anticipated intermediate asymmetries. This rules out horizontal ex-ante fairness as well.

In this paper, we address the question whether fairness in round-robin tournaments with four players can be restored by the use of endogenous sequences. To this end, we consider the round-robin tournament as a dynamic form of strategic competition. We model the sequence of pairwise matches as a series of two-player contests with a fixed, but rather general technology (including, e.g., the frequently used all-pay auction or Tullock contest), in which the contestants invest in order to maximize their expected payoffs. According to their final ranking, which is based on the number of matches won, they receive rank-dependent prizes. Backward induction allows to solve the resulting extensive-form game for its subgame perfect equilibrium.

We show that the answer to the question whether endogenous sequences can ensure fair tournaments depends on the notion of fairness: While there is no sequence which implies perfect fairness, some endogenous sequences imply horizontal ex-ante fairness irrespective of the prize structure. In winner-take-all tournaments, additional endogenous sequences are horizontally ex-ante fair. Intuitively, endogenous sequences do not allow to create enough symmetry such that asymmetries in intermediate evaluations can be avoided altogether. This would be required to guarantee perfect fairness. However, the use of suitable endogenous sequences allows to create sufficient symmetry such that all contestants evaluate the inevitable asymmetry in intermediate scores equally from an ex-ante perspective. Because it is easier to achieve the necessary extent of symmetry for the particular prize structure of winner-take-all tournaments, the set of appropriate endogenous sequences is larger in this case.

In an extension of our analysis, we consider a third notion of fairness: we call the tournament ex-ante fair if the expected payoffs (ranking probabilities) of the (possibly heterogeneous) contestants depend only on their characteristics, but not on the order of their matches in the schedule. We show that even sequences that ensure horizontally ex-ante fair tournaments usually do not imply ex-ante fairness. The intuitive reason is that the endogenous sequences do not allow to control for the additional asymmetries in intermediate evaluations that stem from the contestants' asymmetric characteristics.

The remainder of the paper is structured as follows. In Section 2, we briefly review the related literature on round-robin tournaments. Section 3 describes the general model of the tournament and its matches, as well as the different notions of fairness. In Section 4, we demonstrate the impossibility of perfectly fair tournaments. In Section 5 we identify the endogenous sequences that ensure horizontally ex-ante fair tournaments. As we show in Section 6, these sequences, however, do not generally satisfy the stronger notion of ex-ante fairness. Section 7 concludes.

[3]https://en.wikipedia.org/wiki/Round-robin_tournament; accessed on 02/22/24

## 2 Related Literature

Our paper contributes to the recent literature on strategic aspects in sequential round-robin tournaments. Indeed, this literature is predicated on the observation that the sequential structure of round-robin tournaments causes issues of discrimination. The models of the related studies share the assumptions that (i) the different pairwise matches of the round-robin tournament are two-player contests, which take place one after the other, and (ii) players are ranked according to the number of matches won and maximize their expected payoff from rank-dependent prizes. They differ, though, in the assumptions on the number and characteristics of players, the rank-dependent prize scheme, and the contest success function that describes competition on the match level.

Laica et al. (2021) conduct a general analysis of fully sequential round-robin tournaments with an arbitrary number of heterogeneous players, matches organized as general Tullock contests (including the perfectly discriminating all-pay auction as a limit case), and multiple arbitrary rank-dependent prizes. They show that a tournament with three players is perfectly fair if and only if the second prize equals half of the first prize, regardless of the use of exogenous or endogenous match schedules. In this case, the tournament is, a fortiori, (horizontally) ex-ante fair. Moreover, their numerical calculations suggest that this prize scheme will be not only sufficient but also necessary for the tournament to be (horizontally) ex-ante fair if the sequence of matches is exogenous. By contrast, if the sequence of matches is endogenous, there are certain combinations of the prize scheme and the discriminatory power of the contest success function for which the tournament will be horizontally ex-ante fair. The findings by Laica et al. (2021) encompass the respective results of earlier studies that have focused on particular cases; see Krumer et al. (2017a), Krumer et al. (2017b), Sahm (2019), Krumer et al. (2020).

Dagaev and Zubanov (2022) also consider round-robin tournaments with three players and arbitrary rank-dependent prizes but focus on exogenous schedules and matches organized as all-pay auctions. In contrast to the aforementioned articles, they assume that players have limited resources but face no real effort costs: each player just decides how to split her endowment between her two matches. Though the authors find a multiplicity of equilibria, which precludes unambiguous predictions about the extent and direction of discrimination, the case in which the second prize equals half of the first prize is capable to entail a fair tournament in their model as well.

By contrast, round-robin tournaments with more than three players and a fully sequential exogenous match schedule are always discriminatory: Laica et al. (2021) show that there is no prize scheme that ensures perfect fairness.

Due to the quickly growing number of required computations, so far, the analysis of (horizontal) ex-ante fairness in round-robin tournaments with more than three players is limited to tournaments with four players. On practical grounds (e.g., to guarantee sufficient recovery time between two matches of the same player), the six matches of round-robin tournaments with four players are usually scheduled in three consecutive rounds of two matches with distinct players. The literature has focused on this structure. We will call a round-robin tournament fully sequential if any two matches (of any round) take place one after another.

For matches organized as all-pay auctions or lottery contests, Krumer et al. (2017a) and Sahm (2019), respectively, show that fully sequential round-robin tournaments with a single prize, four players, and an exogenous match schedule are not (horizontally) ex-ante fair. Laica et al. (2021) provide numerical calculations suggesting that this negative result will hold for any prize scheme if matches are organized as all-pay auctions or lottery contests and the fully sequential match schedule is exogenous. In the case of all-pay auctions on the match level, certain prize schemes may induce even adverse ex-interim incentives such that players may prefer to lose certain matches.[4]

As opposed to fully sequential tournaments, Caglayan et al. (2022) show that round-robin tournaments with four players, a single prize, and matches organized as lottery contests will be horizontally ex-ante fair if the two matches of each round take place simultaneously (regardless of the use of exogenous or endogenous match schedules.). In practice, however, simultaneous matches per round are often not feasible (e.g., because the venue is restricted to a single playing field) or not desirable (e.g., because the organizer wants to grant the audience live access to all matches).

Our contribution to the theoretical literature is twofold. On the one hand, to the best of our knowledge, we present the first study of endogenous match schedules in fully sequential round-robin tournaments with four players. We demonstrate that endogenous sequences constitute a horizontally ex-ante fair alternative to the – in many cases unfeasible or undesirable – use of simultaneous matches in each round. On the other hand, unlike the previous studies, our formal analysis allows for more general contest success functions (including the all-pay auction and the Tullock contest) that shape competition on the match level. Because our reasoning is largely based on arguments of symmetry in expected valuations, the assumption of a particular contest success function is usually not required.

Recent empirical and experimental studies demonstrate that issues of fairness in round-robin tournaments are not only a theoretical artifact but also play a significant role in practice. Based on sports data from mega-events, Krumer and Lechner (2017) provide empirical evidence for discrimination in round-robin tournaments with three and four players.[5] Lauber et al. (2023) test the theoretical predictions of Laica et al. (2021) in the laboratory and provide experimental support for discrimination in round-robin tournaments with three players. While these studies document the practical relevance of fairness issues in round-robin tournaments, our analysis provides additional theoretical predictions that can be tested in a similar vein.

# 3 Model

We adapt the model by Laica et al. (2021) to round-robin tournaments with four players, endogenous schedules, and more general contest success functions that shape competition on the match level. Moreover, we introduce three notions of fairness that differ by strictness.

## 3.1 Tournament

Similar to Dagaev and Zubanov (2022), we consider a round-robin tournament $T = \{I, (k_t), p, (R_j)\}$ as a game in extensive form characterized by a set $I = \{1, \ldots, 4\}$ of four players, a sequence $(k_t)$ of six pairwise distinct two-player contests (called matches) with

[4]See also Krumer et al. (2023); without providing a formal proof, the authors argue that such adverse incentives could be avoided by making the match schedule contingent on the results – in other words, by using endogenous sequences. Particularly, they suggest that the match between the two winners of the first round should be delayed as much as possible.

[5]See also Deutscher et al. (2022) for an empirical analysis of directly observable strategic decisions in more complex round-robin tournaments.

contest success function $p$, and a vector of rank-dependent prizes $(R_j)$ where the players' ranking is based on the number of matches won in descending order.

We assume that the ranking of prizes is non-trivial (not all prizes are the same) and monotonic. Without loss of generality, we normalize the first prize to one ($R_1 = 1$) and the last prize to zero ($R_4 = 0$) and denote $R_2 = a$ and $R_3 = b$ with $1 \geq a \geq b \geq 0$.

Players are risk-neutral and may differ with respect to their motivation. The motivation of player $i \in I$ is modeled as an idiosyncratic weight $v_i > 0$ on the prizes.[6] Thus player $i$'s individual valuation for attaining rank $j$ equals $v_i R_j$.

We abstract from draws: in each match, one player wins and the other player loses. At the end of the tournament, players are ranked according to the number of matches won. Potential ties are broken randomly.[7] More concretely, if three players win two matches each (tie for ranking first), each of them will expect a prize of $\frac{1+a+b}{3} =: \Omega$. Similarly, if three players win one match each (tie for ranking second), each of them will expect a prize of $\frac{a+b}{3} =: \Theta$. Finally, if two players win two matches each and two players win one match each (ties for ranking first and third), each of the former players will expect a prize of $\frac{1+a}{2} =: \Delta$ and each of the latter players will expect a prize of $\frac{b}{2}$.

We focus on fully sequential round-robin tournaments. Successively, each player is matched one-to-one with each other player. We distinguish between exogenous sequences, where the exact order of matches is determined before the tournament starts, and endogenous sequences, where the order of later matches depends on the results of previous matches. With four players, 30 different exogenous sequences of matches may be considered (Sahm, 2019). Endogenous sequences expand this number even further.

For practical reasons (e.g., to guarantee sufficient recovery time between two matches of the same player), however, the six matches of real-world round-robin tournaments with four players are usually scheduled in three consecutive rounds of two matches with distinct players. On grounds of their empirical predominance of these schedules, we follow the related literature (Krumer et al., 2017a; Sahm, 2019; Krumer et al., 2023; Laica et al., 2021) and limit attention to such sequences. Given the subdivision of sequences into rounds, the subsequence of matches in round 1 is unique (except for renaming the players): w.l.o.g., player 1 meets player 2 in match 1 and player 3 meets player 4 in match 2. Based on this structure, Krumer et al. (2017a) identify two different exogenous sequences.

We now provide a taxonomy of available endogenous sequences. To this end, we distinguish between *simple* and *advanced* endogenous sequences. All endogenous sequences share the common feature that the subsequence of matches in round 2 (i.e., matches 3 and 4) depends on the results of round 1. In simple endogenous sequences, the subsequence of matches in round 3 (i.e., matches 5 and 6) also depends only on the results of round 1. Put differently, in simple endogenous sequences, the results of the first two matches determine all subsequent matches. By contrast, in advanced endogenous sequences, the subsequence of matches in round 3 (i.e., matches 5 and 6) depends on the results of round 2.

We denote each endogenous sequence by three characters $X, Y, Z \in \{W, L\}$ and an index $\iota \in \{1, 3, 4\}$. The first (second) character $X$ ($Y$) indicates whether the winner ($W$) or loser ($L$) of match 1 (2) takes part in match 3. The two other players meet in match 4. The third character $Z$ indicates whether the winner ($W$) or loser ($L$) of match $\iota$ takes part

[6]The assumption of rank-independent idiosyncratic weights on the prizes entails a restriction compared to a model with arbitrary rank-dependent player values $(v_1^i, \ldots, v_4^i)$. It is, however, equivalent to the assumption that players are heterogeneous with respect to individual abilities or effort costs (Cornes and Hartley, 2005; Ryvkin, 2013) and, therefore, a fairly general modelling choice.

[7]For risk-neutral players, random tie breaking is equivalent to the assumption that players with the same number of wins equally share the sum of the corresponding prizes.

in match 5 (and meets the player he has not been matched with before). Again, the two remaining players meet in match 6. For example, $WLW_3$ indicates the sequence in which the winner of the first match meets the looser of the second match in match 3 and the winner of the third match takes part in match 5. Obviously, $\iota = 1$ ($\iota \in \{3, 4\}$) indicates a simple (advanced) endogenous sequence.

Overall, there are 24 different endogenous sequences. Table 1 lists the eight available simple endogenous sequences and Table 2 lists the 16 available advanced endogenous sequences. In the Appendix, we provide game trees for the sequences $LLL_3$ (Figure 6), $LLL_4$ (Figure 3), $LLW_3$ (Figure 4), $LLW_4$ (Figure 5), $WLL_3$ (Figure 2), $WWL_3$ (Figure 7), $WWW_4$ (Figure 8), $WWW_3$ (Figure 9), and $WWL_4$ (Figure 10). The game trees illustrate the possible courses of the respective tournaments, each consisting of 63 different match constellations (i.e., nodes $k \in \{1, ..., 63\}$) and 64 possible outcomes.

| Match | $LLL_1$ | $LLW_1$ | $WWL_1$ | $WWW_1$ |
|---|---|---|---|---|
| 1 | P1 v P2 | P1 v P2 | P1 v P2 | P1 v P2 |
| 2 | P3 v P4 | P3 v P4 | P3 v P4 | P3 v P4 |
| 3 | L1 v L2 | L1 v L2 | W1 v W2 | W1 v W2 |
| 4 | W1 v W2 | W1 v W2 | L1 v L2 | L1 v L2 |
| 5 | L1 v W2 | W1 v L2 | L1 v W2 | W1 v L2 |
| 6 | W1 v L2 | L1 v W2 | W1 v L2 | L1 v W2 |

| Match | $WLL_1$ | $WLW_1$ | $LWL_1$ | $LWW_1$ |
|---|---|---|---|---|
| 1 | P1 v P2 | P1 v P2 | P1 v P2 | P1 v P2 |
| 2 | P3 v P4 | P3 v P4 | P3 v P4 | P3 v P4 |
| 3 | W1 v L2 | W1 v L2 | L1 v W2 | L1 v W2 |
| 4 | L1 v W2 | L1 v W2 | W1 v L2 | W1 v L2 |
| 5 | L1 v L2 | W1 v W2 | L1 v L2 | W1 v W2 |
| 6 | W1 v W2 | L1 v L2 | W1 v W2 | L1 v L2 |

Table 1: Simple endogenous sequences

## 3.2 Matches

Each match $k$ of the tournament is organized as a contest between two players, $A$ and $B$, with linear costs of effort and a fixed contest success function (CSF) that specifies the winning probability $p_i^k$ of player $i \in \{A, B\}$ as a function $p_i^k(x_A, x_B)$ of the players efforts $x_A^k$ and $x_B^k$; see e.g. Skaperdas (1996).

For $i, j \in \{A, B\}, i \neq j$, player $i$ chooses $x_i^k$ in order to maximize his expected payoff

$$E_i^k = p_i^k(x_i^k, x_j^k) \cdot (w_i^k - x_i^k) + \left(1 - p_i^k(x_i^k, x_j^k)\right) \cdot (\ell_i^k - x_i^k), \tag{1}$$

where $w_i^k$ denotes player $i$'s expected continuation payoff from winning match $k$ and $\ell_i^k$ denotes his expected continuation payoff from losing match $k$, with $w_i^k, \ell_i^k \geq 0$.

We assume that the CSF satisfies the following properties.

**Assumption 1.** *Let $i, j \in \{A, B\}, i \neq j$.*

*(a) Probability: $p_i^k(x_A^k, x_B^k) \geq 0$ and $p_A^k(x_A^k, x_B^k) + p_B^k(x_A^k, x_B^k) = 1$ for all $x_A^k, x_B^k \geq 0$.*

| Match | $LLL_3$ | $LLW_3$ | $WWL_3$ | $WWW_3$ |
|---|---|---|---|---|
| 1 | P1 v P2 | P1 v P2 | P1 v P2 | P1 v P2 |
| 2 | P3 v P4 | P3 v P4 | P3 v P4 | P3 v P4 |
| 3 | L1 v L2 | L1 v L2 | W1 v W2 | W1 v W2 |
| 4 | W1 v W2 | W1 v W2 | L1 v L2 | L1 v L2 |
| 5 | L3 | W3 | L3 | W3 |
| 6 | W3 | L3 | W3 | L3 |

| Match | $LLL_4$ | $LLW_4$ | $WWL_4$ | $WWW_4$ |
|---|---|---|---|---|
| 1 | P1 v P2 | P1 v P2 | P1 v P2 | P1 v P2 |
| 2 | P3 v P4 | P3 v P4 | P3 v P4 | P3 v P4 |
| 3 | L1 v L2 | L1 v L2 | W1 v W2 | W1 v W2 |
| 4 | W1 v W2 | W1 v W2 | L1 v L2 | L1 v L2 |
| 5 | L4 | W4 | L4 | W4 |
| 6 | W4 | L4 | W4 | L4 |

| Match | $WLL_3$ | $WLW_3$ | $LWL_3$ | $LWW_3$ |
|---|---|---|---|---|
| 1 | P1 v P2 | P1 v P2 | P1 v P2 | P1 v P2 |
| 2 | P3 v P4 | P3 v P4 | P3 v P4 | P3 v P4 |
| 3 | W1 v L2 | W1 v L2 | L1 v W2 | L1 v W2 |
| 4 | L1 v W2 | L1 v W2 | W1 v L2 | W1 v L2 |
| 5 | L3 | W3 | L3 | W3 |
| 6 | W3 | L3 | W3 | L3 |

| Match | $WLL_4$ | $WLW_4$ | $LWL_4$ | $LWW_4$ |
|---|---|---|---|---|
| 1 | P1 v P2 | P1 v P2 | P1 v P2 | P1 v P2 |
| 2 | P3 v P4 | P3 v P4 | P3 v P4 | P3 v P4 |
| 3 | W1 v L2 | W1 v L2 | L1 v W2 | L1 v W2 |
| 4 | L1 v W2 | L1 v W2 | W1 v L2 | W1 v L2 |
| 5 | L4 | W4 | L4 | W4 |
| 6 | W4 | L4 | W4 | L4 |

Table 2: Advanced endogenous sequences

*(b) Monotonicity:* $p_i^k$ *is (strictly) increasing in* $x_i^k$ *and (strictly) decreasing in* $x_j^k$ *(if* $x_i^k = x_j^k$*).*

*(c) Anonymity:* $p_A^k(x, y) = p_B^k(y, x)$*.*

*(d) Feasibility: For all* $w_i^k, \ell_i^k \geq 0$*,the strategic form game, in which players A and B choose their efforts as strategies to maximize their expected payoffs as given by equation (1),*

*(i) has a Nash equilibrium and*

*(ii) in any Nash equilibrium of the game,* $p_A^k = p_B^k$ *if and only if* $w_A^k - \ell_A^k = w_B^k - \ell_B^k$*.*

The first three properties represent the usual features of a CSF. The last property guarantees that (only) identical intermediate net continuation payoffs yield equal odds in equilibrium, which will be crucial for our results. Notice that Assumption 1 holds, e.g., for the commonly used CSFs of the Tullock contest and the all-pay auction.[8]

Assumption 1 also ensures that each round-robin tournament $T$ has a subgame perfect equilibrium (SPE), which can be found by backward induction. The literature provides explicit solutions for particular instances of four-player round-robin tournaments with exogenous schedules (Krumer et al., 2017a; Sahm, 2019; Laica et al., 2021; Krumer et al., 2023). Below, we apply the same procedure to tournaments with endogenous schedules – though, in Section 4 and large parts of Section 5, without reference to an explicit functional form of CSF on the match level. In the examples of Sections 5 and 6, we assume that matches are organized as all-pay auctions; the corresponding equilibrium is characterized in Appendix A.1.

## 3.3 Notions of Fairness

Fairness – or competitive balance – is a major concern when it comes to the evaluation of tournament structures. In this paper, we distinguish between horizontal ex-ante fairness, ex-ante fairness and perfect fairness.[9]

We will call a tournament *horizontally ex-ante fair* if all players have the same expected equilibrium payoffs (winnings[10]) before the tournament starts given that they have identical characteristics ($v_1 = \ldots = v_4 = v$). This notion may be understood as a minimum requirement: in any case, a fair tournament should provide equal chances for equally strong players from an ex-ante perspective.[11]

We will call a tournament *ex-ante fair* with respect to payoffs (winnings) if the players' ex-ante expected payoffs (winnings) depend only on their (possibly heterogeneous) characteristics but not on the order of matches in the sequence of the tournament.[12] The idea is that the schedule should not influence the players' overall success in the tournament.

We will call a tournament *perfectly fair* if, for each match of the tournament, the winning probabilities in this match depend only on the characteristics of the matched players but not on the position of the match in the schedule of the tournament. This definition accommodates the requirement that the outcome of a certain match between

[8]In some cases, a qualification of this statement is required. If matches are organized as Tullock contests or all-pay auctions, a Nash equilibrium in match $k$ always exists if $w_i^k > \ell_i^k$ for all $i \in \{A, B\}$. If instead $w_i^k \leq \ell_i^k$ for some $i \in \{A, B\}$, player $i$'s optimal effort choice is $x_i^k = 0$ for any effort level $x_j^k \geq 0$ of player $j \in \{A, B\}, j \neq i$, and player $j$ may have no best reply. To avoid the problem that no equilibrium may exist because some player has no incentive to win match $k$, Laica et al. (2021) introduce an additional prize $m > m^k := \max\{0, \ell_A^k - w_A^k, \ell_B^k - w_B^k\}$ on the match-level that guarantees positive winning incentives for both players. The authors then consider the limit of equilibrium values as $m \to m^k$ from above. It is straightforward to show that, for $\min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\} \leq 0$, the (expected) equilibrium efforts of both players converge to zero and player $i$'s winning probability converges to $p_i^k = 1/2$ if $w_i^k - \ell_i^k = w_j^k - \ell_j^k$ but to $p_i^k = 1$ if $w_i^k - \ell_i^k > w_j^k - \ell_j^k$, where $i, j \in \{A, B\}, j \neq i$. Obviously, if $\min\{w_A^k - \ell_A^k, w_B^k - \ell_B^k\} > 0$, the limit values will coincide with the equilibrium values for $m = 0$. We adopt this approach in the examples of Section 5 and 6 whenever necessary.

[9]We borrow the definitions of ex-ante fairness and perfect fairness from Laica et al. (2021). In the latter case, they just talk of fairness (instead of perfect fairness).

[10]Ex-ante expected winnings are defined as the sum of the ex ante probabilities to rank first, second, or third multiplied by the first, second, and third prize, respectively (Lauber et al., 2023), and sometimes referred to as the weighted qualification probabilities (Laica et al., 2021).

[11]Notice that, for example, the analysis of Krumer et al. (2017a) and Sahm (2019) is based on this notion of horizontal ex-ante fairness.

[12]The definitions of ex-ante fairness based on ex-ante expected payoffs on the one hand and ex-ante expected winnings on the other hand are generally not equivalent; see Laica et al. (2021).

two players should not be influenced by the order in which matches take place.

Obviously, the notion of horizontal ex-ante fairness is weaker than the notion of ex-ante fairness and, in turn, the notion of ex-ante fairness is weaker than the notion of perfect fairness: any perfectly fair tournament is, a fortiori, ex-ante fair, and any ex-ante fair tournament is, a fortiori, horizontally ex-ante fair.[13] While (horizontal) ex-ante fairness only imposes the condition that the schedule should not influence the overall tournament outcome (for symmetric players), perfect fairness requires that the schedule should not influence the outcome of any single match.

# 4 Impossibility of Perfect Fairness

In this section, we show that none of the endogenous sequences implies perfect fairness. In fact, even if the designer can choose both, the (endogenous) schedule and the prize scheme, he cannot ensure a perfectly fair tournament.

**Proposition 1.** *In the subgame perfect equilibrium of round-robin tournaments with four players and an endogenous sequence, there is no prize structure which allows perfect fairness.*

The proof can be found in Appendix A.2. Proposition 1 is a direct extension to the result by Laica et al. (2021) that four-player round-robin tournaments with an exogenous sequences cannot imply perfect fairness for any price structure. Notice that, for symmetric players, perfect fairness implies that in each match both players win with probability $\frac{1}{2}$. Under the assumption of feasibility, incentives to win must then be equal for both players. In other words, $w_A^k - l_A^k = w_B^k - l_B^k$ must hold in any node $k \in \{1, ..., 63\}$. For each possible endogenous sequence, however, it can be easily shown that this condition cannot be satisfied simultaneously in each node. Intuitively, perfect fairness cannot be achieved because of the sequential nature and the associated variety of potential intermediate scores in a four-player tournament. Even with endogenous schedules, the number of available instruments for contest design (i.e., the prizes and the sequences) is too small compared to the number of potential courses of the tournament. The prize structure required to guarantee fairness in one particular course of any (endogenous) schedule may not be suitable to guarantee fairness in a different course of the same schedule.

# 5 Horizontal Ex-ante Fairness

In this section, we show that the use of particular endogenous sequences ensures horizontal ex-ante fairness: if all players are symmetric, these sequences will guarantee identical ex-ante expected payoffs (winnings).

## 5.1 Single-prize round-robin tournaments

We first consider the particular prize scheme of single-prize round-robin tournaments where only the player who ranks first receives a positive prize ($1 > a = b = 0$).

**Proposition 2.** *In the subgame perfect equilibrium of a single-prize round-robin tournament with four players, an endogenous sequence will yield horizontal ex-ante fairness if and only if the winners (losers) of the first round are matched with each other in the second round.*

[13] As we show below, the converse does not hold generally.

Table 3 illustrates the statement of Proposition 2: it depicts the ex-ante expected payoffs in the 24 different endogenous sequences for the example of single-prize round-robin tournaments with symmetric players and matches organized as all-pay auctions. The formal proof of Proposition 2 can be found in Appendix A.3.

| Sequence | $LLL_1$ | $LLW_1$ | $WWL_1$ | $WWW_1$ |
|---|---|---|---|---|
| Player 1 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 2 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 3 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 4 | 0.000 | 0.000 | 0.000 | 0.000 |

| Sequence | $WLL_1$ | $WLW_1$ | $LWL_1$ | $LWW_1$ |
|---|---|---|---|---|
| Player 1 | 0.078 | 0.078 | 0.000 | 0.000 |
| Player 2 | 0.078 | 0.078 | 0.000 | 0.000 |
| Player 3 | 0.000 | 0.000 | 0.078 | 0.078 |
| Player 4 | 0.000 | 0.000 | 0.078 | 0.078 |

| Sequence | $LLL_3$ | $LLW_3$ | $WWL_3$ | $WWW_3$ |
|---|---|---|---|---|
| Player 1 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 2 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 3 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 4 | 0.000 | 0.000 | 0.000 | 0.000 |

| Sequence | $LLL_4$ | $LLW_4$ | $WWL_4$ | $WWW_4$ |
|---|---|---|---|---|
| Player 1 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 2 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 3 | 0.000 | 0.000 | 0.000 | 0.000 |
| Player 4 | 0.000 | 0.000 | 0.000 | 0.000 |

| Sequence | $WLL_3$ | $WLW_3$ | $LWL_3$ | $LWW_3$ |
|---|---|---|---|---|
| Player 1 | 0.078 | 0.078 | 0.000 | 0.000 |
| Player 2 | 0.078 | 0.078 | 0.000 | 0.000 |
| Player 3 | 0.000 | 0.000 | 0.078 | 0.078 |
| Player 4 | 0.000 | 0.000 | 0.078 | 0.078 |

| Sequence | $WLL_4$ | $WLW_4$ | $LWL_4$ | $LWW_4$ |
|---|---|---|---|---|
| Player 1 | 0.078 | 0.078 | 0.000 | 0.000 |
| Player 2 | 0.078 | 0.078 | 0.000 | 0.000 |
| Player 3 | 0.000 | 0.000 | 0.078 | 0.078 |
| Player 4 | 0.000 | 0.000 | 0.078 | 0.078 |

Table 3: Ex-ante expected payoffs for $(v_1, v_2, v_3, v_4) = (1, 1, 1, 1)$ and $(a, b) = (0, 0)$

To get an intuition, consider the basic mechanism of endogenous sequences. If symmetric players face each other in round 1 and their tournament paths depend similarly on the outcome of this match, their continuation payoffs will be the same. Hence, they cannot obtain a positional advantage over each other. However, this mechanism does not necessarily avoid discrimination between players of the two different matches of the first round. Hence, the main task of an endogenous sequence is to also create equal tournament paths for symmetric players who do not face each other in round 1.

In Appendix A.3, we demonstrate the effectiveness of sequence $LLW_1$. Analog arguments hold for all other endogenous sequences which satisfy the condition that the winners (losers) of the first round are matched with each other in the second round. These sequences ensure that - after match 4 has been played - one player has won two out of two matches, two players have won one out of two matches and one player has lost both his matches. Comparing potential tournament paths, each player's way to leading the tournament after 4 matches is the same: win the first round and the second round match. Due to the endogenous schedule, no player enjoys a positional advantage while becoming the leader.

By contrast, the same is not true for endogenous sequences which do not match the two winners (losers) of round 1 in round 2. Intuitively, these sequences fail to create symmetric tournament paths for players of match 1 and players of match 2. First of all, match 3 is inherently asymmetric in the sense that it matches players with different histories (one has won the first match, the other has not) early in the tournament. Second, even a simple endogenous sequence like, e.g., $LWL_1$ yields a positional advantage for the players of match 1. The winner of match 1 can observe whether the winner of match 2 loses or wins in round 2 and, given the result, adjust his own behavior. He gains an informational advantage over the players of match 2. Finally, in an advanced endogenous sequence like, e.g., $WLL_3$, the winner of match 1 can further influence the tournament's sequence of matches by playing in match 3, whereas the same is not true for the winner of match 2. Consequently, even a prize scheme with only a single prize affects players of match 1 and match 2 differently, thereby unbalancing the overall tournament.

Notice that Proposition 2 is limited to round-robin tournaments with a single prize. The absence of additional prizes reduces variation in possible continuation payoffs and implies a number of irrelevant matches in which players have no incentive to invest any effort at all because they cannot catch up with the leader anymore. Adding a second (and third) prize reduces the number of irrelevant matches and yields additional incentives for trailing players. At the same time, additional prizes decrease the relative value of the first prize and, hence, create a lean-back effect for leading players. Therefore, multiple prizes create more complex continuation payoffs and potentially unbalance sequences which are horizontally ex-ante fair for the single-prize case.

## 5.2 Tournaments with general prize schemes

We now relax the assumption of a single prize and allow for general prize schemes $(1 \geq a \geq b \geq 0)$.

**Proposition 3.** *In the subgame perfect equilibrium of a fully sequential round-robin tournament with four players and any prize scheme* $1 \geq a \geq b \geq 0$*, an endogenous sequence will yield horizontal ex-ante fairness if and only if it is advanced and the winners (losers) of the first round are matched with each other in the second round.*

Table 4 illustrates the statement of Proposition 3: for the example of round-robin tournaments with four symmetric players, matches organized as all-pay auctions, and a second prize that equals 60% of the first prize ($a = .6$), it depicts the ex-ante expected payoffs in the twelve different endogenous sequences that match the two winners (losers) of the first round in the second round. As we know from Proposition 2, only these sequences are the remaining candidates for the establishment of horizontal ex-ante fairness. But as we observe, among them, only the eight advanced endogenous sequences do the job. The formal proof of Proposition 3 can be found in Appendix A.4.

| Sequence | $LLL_1$ | $LLW_1$ | $WWL_1$ | $WWW_1$ |
|---|---|---|---|---|
| Player 1 | 0.000 | 0.014 | 0.002 | 0.081 |
| Player 2 | 0.000 | 0.014 | 0.002 | 0.081 |
| Player 3 | 0.014 | 0.000 | 0.081 | 0.002 |
| Player 4 | 0.014 | 0.000 | 0.081 | 0.002 |

| Sequence | $LLL_3$ | $LLW_3$ | $WWL_3$ | $WWW_3$ |
|---|---|---|---|---|
| Player 1 | 0.000 | 0.000 | 0.042 | 0.033 |
| Player 2 | 0.000 | 0.000 | 0.042 | 0.033 |
| Player 3 | 0.000 | 0.000 | 0.042 | 0.033 |
| Player 4 | 0.000 | 0.000 | 0.042 | 0.033 |

| Sequence | $LLL_4$ | $LLW_4$ | $WWL_4$ | $WWW_4$ |
|---|---|---|---|---|
| Player 1 | 0.000 | 0.000 | 0.042 | 0.033 |
| Player 2 | 0.000 | 0.000 | 0.042 | 0.033 |
| Player 3 | 0.000 | 0.000 | 0.042 | 0.033 |
| Player 4 | 0.000 | 0.000 | 0.042 | 0.033 |

Table 4: Ex-ante expected payoffs for $(v_1, v_2, v_3, v_4) = (1, 1, 1, 1)$ and $(a, b) = (.6, 0)$

In the if-part of the proof, we demonstrate that the advanced endogenous sequence $LLL_4$ yields a horizontally ex-ante fair round-robin tournament. Similar arguments hold for all advanced endogenous sequences in which the two winners (losers) of the first round are matched with each other in the second round. Intuitively, structural symmetry is the main mechanism which allows these sequences to be horizontally ex-ante fair for any prize structure. By employing multiple endogenous rules, the number of match variations and thereby the number of different ex-interim standings is heavily reduced. For symmetric players, these sequences yield maximally four unique variations of match 6, two unique variations of match 5 and one unique variation of matches 4, 3, 2 and 1. Additionally, matches 1, 2 and 3 yield similar continuation payoffs for the players involved, i.e., $p_A^k = p_B^k = \frac{1}{2}$ for all $k \in \{57, ..., 63\}$. Only from match 4 onward, the ex-interim standings unbalance the continuation payoffs of the players and offer advantages given the prize structure. However, these advantages can be equally likely attained by all players. Thus, the sequences offer symmetric tournament paths for all players which allows horizontal ex-ante fairness.

This kind of symmetry can also be recognized in the pattern of payoffs below the respective game trees. Consider, e.g., the game tree of $LLL_4$ in Figure 3. Below the game tree, the players' payoffs for all possible outcomes are denoted from left to right. One can subdivide these entire sequences of payoffs into smaller sequences of eight payoffs each (see Figure 11). Table 5 defines all subsequences of payoffs that can be identified in the game tree of $LLL_4$. Subdividing the complete sequences of payoffs into the defined subsequences reveals the pattern illustrated in Figure 12. Similar patterns can be found in any of the advanced endogenous schedules which are organized such that the two winners of the first round face each other in the second round.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A | 1 | $\Delta$ | 1 | $\Delta$ | $\Omega$ | $a$ | $\frac{b}{2}$ | $\Theta$ |
| B | $a$ | $\Delta$ | $b$ | $\frac{b}{2}$ | $\Omega$ | $b$ | $\Delta$ | $\Theta$ |
| C | $b$ | $\frac{b}{2}$ | $a$ | $\Delta$ | $\Omega$ | 1 | $\Delta$ | 1 |
| D | 0 | $\frac{b}{2}$ | 0 | $\frac{b}{2}$ | 0 | 0 | $\frac{b}{2}$ | $\Theta$ |
| E | 1 | $\Delta$ | 1 | $\Omega$ | $\Delta$ | $a$ | $\frac{b}{2}$ | $b$ |
| F | $\Theta$ | $\frac{b}{2}$ | 0 | 0 | $\frac{b}{2}$ | 0 | $\frac{b}{2}$ | 0 |
| G | $\Theta$ | $\frac{b}{2}$ | $a$ | $\Omega$ | $\Delta$ | 1 | $\Delta$ | 1 |
| H | $\Theta$ | $\Delta$ | $b$ | $\Omega$ | $\frac{b}{2}$ | $b$ | $\Delta$ | $a$ |

Table 5: Definition of sequences in payoffs in $LLL_4$

Now, comparing the sequences in payoffs in Table 5, notice that each sequence has a mirror-inverse counterpart, i.e. E is the inverse of C, F is the inverse of D, G is the inverse of A, and H is the inverse of B. By contrast, simple endogenous sequences which are organized such that the two first round winners (losers) play against each other in round 2 also showcase patterns in their payoff sequences, but without inversion. It is this back-to-front characteristic which provides the reduction in possible variations per match in terms of incentive structures and enables horizontal ex-ante fairness. For example, $LLW_1$ yields seven unique variations of match 6, four unique variations of match 5, two unique variations of match 4, and one unique variation each for matches 3, 2 and 1. The reason for that is the missing second endogenous rule for round 3 which ultimately leaves over too many eventualities. In other words, the possible tournament paths for players of match 1 and players of match 2 are still different. In $LLW_1$, if player 3 or 4 is the player with two wins after match 4, she will take part in match 6 for sure, whereas if player 1 or 2 is the player with two wins after match 4, she will take part in match 5 with certainty. Hence, one can argue that the simple endogenous sequences still inherit some exogenous leftovers in round 3 whereas the advanced endogenous sequences do not. That is why a simple endogenous schedule requires very specific prize schemes to balance out the tournament structure.

Laica et al. (2021) show that the particular prize structure for which the second prize equals half of the first prize ensures (horizontal ex-ante) fairness in round-robin tournaments with three players, independent from the match schedule. By contrast, as this section has shown, some particular advanced endogenous sequences ensure horizontal ex-ante fairness in round-robin tournaments with four players, independent from the prize structure. Put differently, in round-robin tournaments with three players, the prize scheme is a sufficient instrument to ensure horizontal ex-ante fairness, while the use of endogenous sequences is not. In fully sequential round-robin tournaments with four players, it is exactly the other way around.

# 6 Failure of Ex-ante Fairness

In this section, we demonstrate that the use of endogenous sequences will usually not ensure ex-ante fairness. To this end, we show that the advanced endogenous sequences, which have been shown to imply horizontal ex-ante fairness, do not generally imply the stronger concept of ex-ante fairness.[14]

[14] As horizontal ex-ante fairness is the weaker concept, sequences that do not imply horizontal ex-ante fairness will, a fortiori, neither imply ex-ante fairness.

**Proposition 4.** *In round-robin tournaments with four heterogeneous players and a general prize scheme* $1 \geq a \geq b \geq 0$*, the advanced endogenous schedules* $LLL_4$*,* $LLW_3$*,* $LLW_4$*,* $LLL_3$*,* $WWL_3$*,* $WWW_4$*,* $WWW_3$ *or* $WWL_4$ *do not generally imply ex-ante fairness.*

Proposition 4 highlights the limitations of advanced endogenous schedules in regard to achieving ex-ante fairness. As outlined, horizontal ex-ante fairness heavily rests on symmetry. Assuming asymmetric players, however, may lead to asymmetric tournament paths with ultimately unbalanced discouragement effects and lean-back effects. Depending on the sequence used, players may experience some form of positional advantage or disadvantage.

To formally prove Proposition 4, we provide several examples in which the tournament is not ex-ante fair despite the use of the respective advanced endogenous schedules. To this end, we consider matches organized as (contests that are equivalent to) all-pay auctions (as outlined in Appendix A.1) and the particular prize scheme $(a, b) = (.6, 0)$. We discuss three specific configurations of the players' heterogeneous valuations: $(v_1, v_2, v_3, v_4) \in \{(1, 1, .8, .8), (1, .8, .8, .8), (1, .8, 1, .8)\}$.

| Sequence | $LLL_4$ | $LLW_3$ | $LLW_4$ | $LLL_3$ |
|---|---|---|---|---|
| Player 1 | .150 | .115 | .115 | .150 |
| Player 2 | .020 | .091 | .091 | .020 |
| Player 3 | .000 | .000 | .000 | .000 |
| Player 4 | .000 | .000 | .000 | .000 |

| Sequence | $WWL_3$ | $WWW_4$ | $WWW_3$ | $WWL_4$ |
|---|---|---|---|---|
| Player 1 | .155 | .179 | .179 | .155 |
| Player 2 | .007 | .013 | .013 | .007 |
| Player 3 | .008 | .017 | .017 | .008 |
| Player 4 | .008 | .017 | .017 | .008 |

Table 6: Ex-ante expected payoffs for $(v_1, v_2, v_3, v_4) = (1, .8, .8, .8)$ and $(a, b) = (.6, 0)$

First, consider the example of one strong and three symmetrically weak players, i.e., $(v_1, v_2, v_3, v_4) = (1, .8, .8, .8)$ as depicted in Table 6. Players 3 and 4 share equal ex-ante expected payoffs as they face each other in round 1 and their subsequent tournament paths depend on their first round result. They substantially differ from those of player 2 although all three players have the same prize valuation/strength. Intuitively, player 2's pairing with the strong opponent in round 1 either serves as an advantage or an disadvantage over players 3 and 4. Interestingly, the strong players' ex-ante expected payoffs are not necessarily the highest among players depending on the sequence. An explanation may be that her high prize valuation/strength in combination with a sufficiently high second prize may induce an overly discouraging lean-back effect. In simple terms, if the second prize is sufficiently high, there is no need to compete for the first prize if it comes with too much cost. For instance, $LLW_3$ leads to player 2 being more motivated than player 1 in match 1 because player 1's continuation payoff of losing is too high. Hence, player 2 has a higher probability to win that match than his strong opponent and is more likely to enjoy a head start. If the strong player 1 loses this match, she will be in a tighter spot in rounds 2 and 3 to finish first or second such that players 3 and 4 will face a higher motivated player 1 than player 2 did. Hence, it becomes apparent that in this setting none of the advanced endogenous sequences leads to ex-ante fairness.

| Sequence | $LLL_4$ | $LLW_3$ | $LLW_4$ | $LLL_3$ |
|---|---|---|---|---|
| Player 1 | .130 | .110 | .110 | .130 |
| Player 2 | .130 | .110 | .110 | .130 |
| Player 3 | .000 | .000 | .000 | .000 |
| Player 4 | .000 | .000 | .000 | .000 |

| Sequence | $WWL_3$ | $WWW_4$ | $WWW_3$ | $WWL_4$ |
|---|---|---|---|---|
| Player 1 | .131 | .108 | .108 | .131 |
| Player 2 | .131 | .108 | .108 | .131 |
| Player 3 | .000 | .000 | .000 | .000 |
| Player 4 | .000 | .000 | .000 | .000 |

Table 7: Ex-ante expected payoffs for $(v_1, v_2, v_3, v_4) = (1, 1, .8, .8)$ and $(a, b) = (.6, 0)$

Second, Table 7 depicts ex-ante expected payoffs for the setting with two symmetrically strong players and two symmetrically weak players, i.e. $(v_1, v_2, v_3, v_4) = (1, 1, .8, .8)$ and $(a, b) = (.6, 0)$. Adding a second strong player may change the qualitative results of the advanced endogenous sequences analysed. All sequences analysed achieve equal ex-ante expected payoffs for similarly strong/weak players. Also, ex-ante expected payoffs of the two strong players are higher than those of the two weak players. Hence, ranking players according to their ex-ante expected payoffs indeed relates to their relative strength in this setting. In a broader sense, these results can thus be interpreted as ex-ante fairness. However, the difference in ex-ante winning probabilities between a weak and a strong player depends on the sequence employed. The fact that the amount of discrimination is controllable by the choice of sequence illustrates the problem of evaluating these sequences in terms of ex-ante fairness in the asymmetric setting as it remains unclear which extend of discrimination between weak and strong players may be perceived as *fair* and which not.

| Sequence | $LLL_4$ | $LLW_3$ | $LLW_4$ | $LLL_3$ |
|---|---|---|---|---|
| Player 1 | .029 | .039 | .039 | .029 |
| Player 2 | .000 | .000 | .000 | .000 |
| Player 3 | .034 | .044 | .044 | .034 |
| Player 4 | .000 | .000 | .000 | .000 |

| Sequence | $WWL_3$ | $WWW_4$ | $WWW_3$ | $WWL_4$ |
|---|---|---|---|---|
| Player 1 | .163 | .156 | .156 | .163 |
| Player 2 | .015 | .018 | .018 | .015 |
| Player 3 | .182 | .176 | .176 | .182 |
| Player 4 | .014 | .018 | .018 | .014 |

Table 8: Ex-ante expected payoffs for $(v_1, v_2, v_3, v_4) = (1, .8, 1, .8)$ and $(a, b) = (.6, 0)$

Third, Table 8 depicts ex-ante expected payoffs and relative aggregate effort for the setting with two symmetrically strong/weak players who are not matched with each other in round 1, i.e. $(v_1, v_2, v_3, v_4) = (1, .8, 1, .8)$, and the prize scheme $(a, b) = (.6, 0)$. First and foremost, none of the sequences leads to ex-ante expected payoffs which are equal for players with equal prize valuations. Consequently, none of the sequences can establish ex-ante fairness in this setting. Obviously, the seeding of strong and weak players also influences the outcome of a given match schedule. For instance, in the case of two symmetrically strong and two symmetrically weak players, a comparison of the results in Tables 7 and 8 suggests to match the two strong (weak) players in the first round to establish some symmetry in tournament paths and to avoid discrimination between two equally strong (weak) players.

# 7 Conclusion

The previous literature has found that fully sequential round-robin tournaments with four players and exogenous sequences are inherently unfair. The goal of this paper was to examine the effects of endogenous sequences on the fairness in fully sequential round-robin tournaments with four players, multiple prizes, and general contest technologies. Three different notions of fairness have been considered: a tournament is called horizontally ex-ante fair if symmetric contestants have the same expected payoffs (winnings) before the tournament starts; it is called ex-ante fair if the players' ex-ante expected payoffs (winnings) depend only on their characteristics but not on the order of matches in the course of the tournament; it is called perfectly fair if the winning probabilities in each match depend only on the players' characteristics but not on the position of the match in the course of the tournament.

We have shown that there is no endogenous sequence which implies perfect fairness. Similarly, ex-ante fairness is usually also out of reach. By contrast, some endogenous sequences imply horizontal ex-ante fairness irrespective of the prize structure. In winner-take-all tournaments, additional endogenous sequences are horizontally ex-ante fair.

Several reasons may explain the empirical predominance of tournaments with exogenous sequences. First, exogenous sequences can be intuitively understood making them more comprehensible for viewers, contestants, and organizers. In contrast, a highly complex endogenous sequence may feel overly artificial and bulky. Second, exogenous sequences offer planning certainty as date and location of the matches are known long beforehand such that viewers and contestants can make travel arrangements early. If the sequence was endogenous, ticket sales would either have to start on short notice or would imply that fans book tickets for matches without knowing whether their team will take place in the respective match at all. Similarly, contestants would need to prepare for potentially different locations and dates of their next match in parallel.

However, notice that endogenous structures are already used in many sports competitions which culminate in elimination tournaments. For instance, at the FIFA World Cup, teams play at different locations and times in the elimination phase of the tournament depending on whether they have finished first or second in their respective group. Also, tournament locations are often in close proximity. Some are even held in only one place. Other round-robin tournaments (e.g., UEFA Champions League group stage) yield several weeks in between matches for preparation and logistics. Under such circumstances, endogenous sequences are still applicable.

Given that professional sports competitions often involve huge prize funds and lots of prestige, our findings question the prevailing use of exogenous sequences in four-player round-robin tournaments: they are inherently unfair and horizontally ex-ante fair alternatives are available. Future research may focus on the interplay of prize schemes and effort provision in round-robin tournaments with (horizontally ex-ante fair) endogenous sequences to offer guidance for organizers who want to comply with this minimum requirement for fairness.

# A Appendix

## A.1 All-pay-auctions on the match level

In Section 5 we provide examples demonstrating that a round-robin tournament may not be horizontally ex-ante fair if its endogenous sequence does not meet the assumptions of Propositions 2 and 3, respectively. Similarly, in Section 6, we provide examples demonstrating that round-robin tournaments may not be ex-ante fair even if they use endogenous sequences that ensure horizontal ex-ante fairness. In all of these examples, we assume that the single matches of the tournament are organized as (contests that are equivalent to) all-pay auctions.

Following the notation used in Laica et al. (2021), player $A$ wins the all-pay auction in match $k$ against player $B$ with probability

$$p_A^k = \begin{cases} 1 & \text{if } x_A^k > x_B^k \\ \frac{1}{2} & \text{if } x_A^k = x_B^k, \\ 0 & \text{if } x_A^k < x_B^k. \end{cases} \tag{2}$$

Player $i$ chooses effort $x_i^k$ in order to maximize the expected payoff as defined by equation (1). Krumer et al. (2017a), Sahm (2019), Krumer et al. (2020), and Laica et al. (2021) apply the general solution to all-pay auctions by Baye et al. (1996) to characterize the Nash equilibrium in the current framework. Without loss of generality, assume that $(w_A^k - l_A^k) \geq (w_B^k - l_B^k)$. For $(w_B^k - l_B^k) > 0$, there will always be a unique mixed-strategy equilibrium in which players A and B randomize on the interval $[0, w_B^k - l_B^k]$ such that their expected payoffs are[15]

$$E_A^k = w_A^k F_B^k(x_A^k) + l_A^k[1 - F_B^k(x_A^k)] - x_A^k = w_A^k - [w_B^k - l_B^k], \tag{3}$$

$$E_B^k = w_B^k F_A^k(x_B^k) + l_B^k[1 - F_A^k(x_B^k)] - x_B^k = l_B, \tag{4}$$

where $F_i^k$ denotes their cumulative distribution functions which equal

$$F_A^k(x_A^k) = \frac{x_A^k}{w_B^k - l_B^k},$$

and

$$F_B^k(x_B^k) = \frac{l_B^k - l_A^k + w_A^k - w_B^k + x_B^k}{w_A^k - l_A^k}.$$

Hence, the expected efforts equal

$$E[x_A^k] = \frac{w_B^k - l_B^k}{2}, \tag{5}$$

$$E[x_B^k] = \frac{(w_B^k - l_B^k)^2}{2(w_A^k - l_A^k)}, \tag{6}$$

[15] If $(w_B^k - l_B^k) \leq 0$, the procedure described in Footnote 8 ensures the existence of an equilibrium.

and the resulting equilibrium winning probabilities equal

$$p_B^k = \frac{w_B^k - l_B^k}{2(w_A^k - l_A^k)}, \tag{7}$$

$$p_a^k = 1 - \frac{w_B^k - l_B^k}{2(w_A^k - l_A^k)}. \tag{8}$$

## A.2 Proof of Proposition 1

Consider the set of endogenous sequences

$$T_1 = \{LLL_4, LLL_3, WWL_3, WWL_4, WLL_3, LWW_3,\\ WLW_4, LWL_4, LLW_1, WWL_1, WLL_1, LWL_1\}.$$

Let endogenous sequence $t \in T_1$ be perfectly fair. If players are symmetric and the assumptions outlined in Section 3.2 hold, then it must be true that for all $k \in \{1, ..., 63\}$ $p_A^k = p_B^k = \frac{1}{2}$ and subsequently $w_A^k - l_A^k = w_B^k - l_B^k$. $t$ can lead to 32 different nodes defining all variations of match 6, two of which must resemble the following situations:

- Player $A \in I$ who has won one out of two matches plays against player $B \in I \setminus \{A\}$ who has won her first two matches, while the other two players have already played 3 matches each and have earned 1 win each. Therefore,

$$\frac{1+a}{2} - \frac{a+b}{3} = 1 - \frac{1+a}{2}, \\ \Leftrightarrow 2a = b. \tag{9}$$

- Player $A \in I$ who has won one out of two matches plays against player $B \in I \setminus \{A\}$ who has won his first two matches, while the other two players have already played 3 matches and have earned 2 wins and no wins, respectively. Therefore

$$\frac{1+a+b}{3} - b = 1 - \frac{1+a+b}{3}, \\ \Leftrightarrow 2a - b = 1. \tag{10}$$

Inserting (9) in (10) yields $b - b = 1$, a contradiction. Therefore, $t \in T_1$ cannot be perfectly fair.

Consider the set of endogenous sequences

$$T_2 = \{LLW_4, LLW_3, WWW_4, WWW_3, WLW_3, LWL_3,\\ WLL_4, LWW_4, LLL_1, WWW_1, WLW_1, LWW_1\}.$$

Let endogenous sequence $t \in T_2$ be perfectly fair. If players are symmetric and the assumptions outlined in Section 3.2 hold, then it must be true that for all $k \in \{1, ..., 63\}$ $p_A^k = p_B^k = \frac{1}{2}$ and subsequently $w_A^k - l_A^k = w_B^k - l_B^k$. $t$ can lead to 32 different nodes defining all variations of match 6, two of which must resemble the following situations:

- Player $A \in I$ who was won one out of two match yet plays against player $B \in I \setminus \{A\}$ who has lost both her previous matches, while the other players who have already played 3 matches have both won two times. Therefore,

$$\begin{aligned}\frac{1+a+b}{3}-\frac{b}{2}&=\frac{b}{2}-0,\\ \Leftrightarrow 1+a&=2b.\end{aligned} \tag{11}$$

- Player $A \in I$ who was won one out of two matches yet plays against player $B \in I \backslash \{A\}$ who has lost both her previous matches, while the other players who have already played 3 matches have 3 wins and 1 win, respectively. Therefore,

$$\begin{aligned}a-\frac{a+b}{3}&=\frac{a+b}{3}-0,\\ \Leftrightarrow a&=2b.\end{aligned} \tag{12}$$

Inserting (11) in (12) yields $a = 1 + a$, a contradiction. Therefore, $t \in T_2$ cannot be perfectly fair. Hence, we have shown that no endogenous sequence can ever be perfectly fair.

## A.3 Proof of Proposition 2

To prove the only-if-part of Proposition 2, we show that there is a single-prize round-robin tournament with four symmetric players in which at least two players will have different ex-ante expected payoffs if the endogenous sequence does not match the two winners (losers) of the first round in the second round. To this end, we consider the example of Table 3 for a round-robin tournament with matches organized as all-pay auctions. Applying the results of Appendix A.1 repeatedly, we solve the games resulting under the different endogenous sequences by backward induction for their subgame perfect equilibria.[16] We collect the players' respective equilibrium ex-ante expected payoffs in Table 3. They show that there are, indeed, always two players with different ex-ante expected payoffs if the endogenous sequence does not match the two winners (losers) of the first round in the second round.

To prove the if-part, we demonstrate the arguments for the sequence $LLW_1$; analog reasoning applies to all other sequences that match the two winners (losers) of the first round in the second round. Suppose that players are symmetric ($v_1 = v_2 = v_3 = v_4$) and there is a single prize only ($(a, b) = (0, 0)$). Assume, moreover, that the CSF that shapes competition on the match level satisfies Assumption 1. Then, continuation payoffs, winning probabilities, expected payoffs and expected efforts of the respective players in nodes 1 - 32 representing all possible alternatives of match 6 can be rewritten as follows.

[16]The details of this procedure are outlined, e.g., in Krumer et al. (2017a); Laica et al. (2021).

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $w_2^1 = 0$ | $w_3^1 = 0$ | $l_2^1 = 0$ | $l_3^1 = 0$ | $p_2^1 = \frac{1}{2}$ | $p_3^1 = \frac{1}{2}$ | $E[x_2^1]$ | $E[x_3^1]$ | $E_2^1 = 0$ | $E_3^1 = 0$ |
| 2 | $w_2^2 = \frac{1}{2}$ | $w_3^2 = \frac{1}{2}$ | $l_2^2 = 0$ | $l_3^2 = 0$ | $p_2^2 = \frac{1}{2}$ | $p_3^2 = \frac{1}{2}$ | $E[x_2^2]$ | $E[x_3^2]$ | $E_2^2$ | $E_3^2$ |
| 3 | $w_2^3 = \frac{1}{3}$ | $w_3^3 = 1$ | $l_2^3 = 0$ | $l_3^3 = \frac{1}{3}$ | $p_2^3$ | $p_3^3$ | $E[x_2^3]$ | $E[x_3^3]$ | $E_2^3$ | $E_3^3$ |
| 4 | $w_2^4 = \frac{1}{2}$ | $w_3^4 = 1$ | $l_2^4 = 0$ | $l_3^4 = \frac{1}{2}$ | $p_2^4$ | $p_3^4$ | $E[x_2^4]$ | $E[x_3^4]$ | $E_2^4$ | $E_3^4$ |
| 5 | $w_2^5 = 0$ | $w_3^5 = 0$ | $l_2^5 = 0$ | $l_3^5 = 0$ | $p_2^5 = \frac{1}{2}$ | $p_3^5 = \frac{1}{2}$ | $E[x_2^5]$ | $E[x_3^5]$ | $E_2^5 = 0$ | $E_3^5 = 0$ |
| 6 | $w_2^6 = 0$ | $w_3^6 = \frac{1}{3}$ | $l_2^6 = 0$ | $l_3^6 = 0$ | $p_2^6 = 0$ | $p_3^6 = 1$ | $E[x_2^6]$ | $E[x_3^6]$ | $E_2^6 = 0$ | $E_3^6 = \frac{1}{3}$ |
| 7 | $w_2^7 = 0$ | $w_3^7 = 1$ | $l_2^7 = 0$ | $l_3^7 = \frac{1}{2}$ | $p_2^7 = 0$ | $p_3^7 = 1$ | $E[x_2^7]$ | $E[x_3^7]$ | $E_2^7 = 0$ | $E_3^7 = 1$ |
| 8 | $w_2^8 = 0$ | $w_3^8 = 1$ | $l_2^8 = 0$ | $l_3^8 = \frac{1}{2}$ | $p_2^8 = 0$ | $p_3^8 = 1$ | $E[x_2^8]$ | $E[x_3^8]$ | $E_2^8 = 0$ | $E_3^8 = 1$ |
| 9 | $w_2^9 = 0$ | $w_4^9 = 0$ | $l_2^9 = 0$ | $l_4^9 = 0$ | $p_2^9 = p_2^1$ | $p_4^9 = p_3^1$ | $E[x_2^9] = E[x_2^1]$ | $E[x_4^9] = E[x_3^1]$ | $E_2^9 = E_2^1$ | $E_4^9 = E_3^1$ |
| 10 | $w_2^{10} = \frac{1}{2}$ | $w_4^{10} = \frac{1}{2}$ | $l_2^{10} = 0$ | $l_4^{10} = 0$ | $p_2^{10} = p_2^2$ | $p_4^{10} = p_3^2$ | $E[x_2^{10}] = E[x_2^2]$ | $E[x_4^{10}] = E[x_3^2]$ | $E_2^{10} = E_2^2$ | $E_4^{10} = E_3^2$ |
| 11 | $w_2^{11} = \frac{1}{3}$ | $w_4^{11} = 1$ | $l_2^{11} = 0$ | $l_4^{11} = \frac{1}{3}$ | $p_2^{11} = p_2^3$ | $p_4^{11} = p_3^3$ | $E[x_2^{11}] = E[x_2^3]$ | $E[x_4^{11}] = E[x_3^3]$ | $E_2^{11} = E_2^3$ | $E_4^{11} = E_3^3$ |
| 12 | $w_2^{12} = \frac{1}{2}$ | $w_4^{12} = 1$ | $l_2^{12} = 0$ | $l_4^{12} = \frac{1}{2}$ | $p_2^{12} = p_2^4$ | $p_4^{12} = p_3^4$ | $E[x_2^{12}] = E[x_2^4]$ | $E[x_4^{12}] = E[x_3^4]$ | $E_2^{12} = E_2^4$ | $E_4^{12} = E_3^4$ |
| 13 | $w_2^{13} = 0$ | $w_4^{13} = 0$ | $l_2^{13} = 0$ | $l_4^{13} = 0$ | $p_2^{13} = p_2^5$ | $p_4^{13} = p_3^5$ | $E[x_2^{13}] = E[x_2^5]$ | $E[x_4^{13}] = E[x_3^5]$ | $E_2^{13} = E_2^5$ | $E_4^{13} = E_3^5$ |
| 14 | $w_2^{14} = 0$ | $w_4^{14} = \frac{1}{3}$ | $l_2^{14} = 0$ | $l_4^{14} = 0$ | $p_2^{14} = p_2^6$ | $p_4^{14} = p_3^6$ | $E[x_2^{14}] = E[x_2^6]$ | $E[x_4^{14}] = E[x_3^6]$ | $E_2^{14} = E_2^6$ | $E_4^{14} = E_3^6$ |
| 15 | $w_2^{15} = 0$ | $w_4^{15} = 1$ | $l_2^{15} = 0$ | $l_4^{15} = \frac{1}{2}$ | $p_2^{15} = p_2^7$ | $p_4^{15} = p_3^7$ | $E[x_2^{15}] = E[x_2^7]$ | $E[x_4^{15}] = E[x_3^7]$ | $E_2^{15} = E_2^7$ | $E_4^{15} = E_3^7$ |
| 16 | $w_2^{16} = 0$ | $w_4^{16} = 1$ | $l_2^{16} = 0$ | $l_4^{16} = \frac{1}{2}$ | $p_2^{16} = p_2^8$ | $p_4^{16} = p_3^8$ | $E[x_2^{16}] = E[x_2^8]$ | $E[x_4^{16}] = E[x_3^8]$ | $E_2^{16} = E_2^8$ | $E_4^{16} = E_3^8$ |

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 17 | $w_1^{17}=0$ | $w_3^{17}=0$ | $l_2^{17}=0$ | $l_3^{17}=0$ | $p_1^{17}=p_2^1$ | $p_3^{17}=p_3^1$ | $E[x_1^{17}]=E[x_2^1]$ | $E[x_3^{17}]=E[x_3^1]$ | $E_1^{17}=E_2^1$ | $E_3^{17}=E_3^1$ |
| 18 | $w_1^{18}=\frac{1}{2}$ | $w_3^{18}=\frac{1}{2}$ | $l_2^{18}=0$ | $l_3^{18}=0$ | $p_1^{18}=p_2^2$ | $p_3^{18}=p_3^2$ | $E[x_1^{18}]=E[x_2^2]$ | $E[x_3^{18}]=E[x_3^2]$ | $E_1^{18}=E_2^2$ | $E_3^{18}=E_3^2$ |
| 19 | $w_1^{19}=\frac{1}{3}$ | $w_3^{19}=1$ | $l_2^{19}=0$ | $l_3^{19}=\frac{1}{3}$ | $p_1^{19}=p_2^3$ | $p_3^{19}=p_3^3$ | $E[x_1^{19}]=E[x_2^3]$ | $E[x_3^{19}]=E[x_3^3]$ | $E_1^{19}=E_2^3$ | $E_3^{19}=E_3^3$ |
| 20 | $w_1^{20}=\frac{1}{2}$ | $w_3^{20}=1$ | $l_2^{20}=0$ | $l_3^{20}=\frac{1}{2}$ | $p_1^{20}=p_2^4$ | $p_3^{20}=p_3^4$ | $E[x_1^{20}]=E[x_2^4]$ | $E[x_3^{20}]=E[x_3^4]$ | $E_1^{20}=E_2^4$ | $E_3^{20}=E_3^4$ |
| 21 | $w_1^{21}=0$ | $w_3^{21}=0$ | $l_1^{21}=0$ | $l_3^{21}=0$ | $p_1^{21}=p_2^5$ | $p_3^{21}=p_3^5$ | $E[x_1^{21}]=E[x_2^5]$ | $E[x_3^{21}]=E[x_3^5]$ | $E_1^{21}=E_2^5$ | $E_3^{21}=E_3^5$ |
| 22 | $w_1^{22}=0$ | $w_3^{22}=\frac{1}{3}$ | $l_1^{22}=0$ | $l_3^{22}=0$ | $p_1^{22}=p_2^6$ | $p_3^{22}=p_3^6$ | $E[x_1^{22}]=E[x_2^6]$ | $E[x_3^{22}]=E[x_3^6]$ | $E_1^{22}=E_2^6$ | $E_3^{22}=E_3^6$ |
| 23 | $w_1^{23}=0$ | $w_3^{23}=1$ | $l_1^{23}=0$ | $l_3^{23}=\frac{1}{2}$ | $p_1^{23}=p_2^7$ | $p_3^{23}=p_3^7$ | $E[x_1^{23}]=E[x_2^7]$ | $E[x_3^{23}]=E[x_3^7]$ | $E_1^{23}=E_2^7$ | $E_3^{23}=E_3^7$ |
| 24 | $w_1^{24}=0$ | $w_3^{24}=1$ | $l_1^{24}=0$ | $l_3^{24}=\frac{1}{2}$ | $p_1^{24}=p_2^8$ | $p_3^{24}=p_3^8$ | $E[x_1^{24}]=E[x_2^8]$ | $E[x_3^{24}]=E[x_3^8]$ | $E_1^{24}=E_2^8$ | $E_3^{24}=E_3^8$ |
| 25 | $w_1^{25}=0$ | $w_4^{25}=0$ | $l_1^{25}=0$ | $l_4^{25}=0$ | $p_1^{25}=p_2^1$ | $p_4^{25}=p_3^1$ | $E[x_1^{25}]=E[x_2^1]$ | $E[x_4^{25}]=E[x_3^1]$ | $E_1^{25}=E_2^1$ | $E_4^{25}=E_3^1$ |
| 26 | $w_1^{26}=\frac{1}{2}$ | $w_4^{26}=\frac{1}{2}$ | $l_1^{26}=0$ | $l_4^{26}=0$ | $p_1^{26}=p_2^2$ | $p_4^{26}=p_3^2$ | $E[x_1^{26}]=E[x_2^2]$ | $E[x_4^{26}]=E[x_3^2]$ | $E_1^{26}=E_2^2$ | $E_4^{26}=E_3^2$ |
| 27 | $w_1^{27}=\frac{1}{3}$ | $w_4^{27}=1$ | $l_1^{27}=0$ | $l_4^{27}=\frac{1}{3}$ | $p_1^{27}=p_2^3$ | $p_4^{27}=p_3^3$ | $E[x_1^{27}]=E[x_2^3]$ | $E[x_4^{27}]=E[x_3^3]$ | $E_1^{27}=E_2^3$ | $E_4^{27}=E_3^3$ |
| 28 | $w_1^{28}=\frac{1}{2}$ | $w_4^{28}=1$ | $l_1^{28}=0$ | $l_4^{28}=\frac{1}{2}$ | $p_1^{28}=p_2^4$ | $p_4^{28}=p_3^4$ | $E[x_1^{28}]=E[x_2^4]$ | $E[x_4^{28}]=E[x_3^4]$ | $E_1^{28}=E_2^4$ | $E_4^{28}=E_3^4$ |
| 29 | $w_1^{29}=0$ | $w_4^{29}=0$ | $l_1^{29}=0$ | $l_4^{29}=0$ | $p_1^{29}=p_2^5$ | $p_4^{29}=p_3^5$ | $E[x_1^{29}]=E[x_2^5]$ | $E[x_4^{29}]=E[x_3^5]$ | $E_1^{29}=E_2^5$ | $E_4^{29}=E_3^5$ |
| 30 | $w_1^{30}=0$ | $w_4^{30}=\frac{1}{3}$ | $l_1^{30}=0$ | $l_4^{30}=0$ | $p_1^{30}=p_2^6$ | $p_4^{30}=p_3^6$ | $E[x_1^{30}]=E[x_2^6]$ | $E[x_4^{30}]=E[x_3^6]$ | $E_1^{30}=E_2^6$ | $E_4^{30}=E_3^6$ |
| 31 | $w_1^{31}=0$ | $w_4^{31}=1$ | $l_1^{31}=0$ | $l_4^{31}=\frac{1}{2}$ | $p_1^{31}=p_2^7$ | $p_4^{31}=p_3^7$ | $E[x_1^{31}]=E[x_2^7]$ | $E[x_4^{31}]=E[x_3^7]$ | $E_1^{31}=E_2^7$ | $E_4^{31}=E_3^7$ |
| 32 | $w_1^{32}=0$ | $w_4^{32}=1$ | $l_1^{32}=0$ | $l_4^{32}=\frac{1}{2}$ | $p_1^{32}=p_2^8$ | $p_4^{32}=p_3^8$ | $E[x_1^{32}]=E[x_2^8]$ | $E[x_4^{32}]=E[x_3^8]$ | $E_1^{32}=E_2^8$ | $E_4^{32}=E_3^8$ |

Similarly, continuation payoffs of the respective players in nodes 33 - 48 can be derived:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 33 | $w_1^{33} = p_2^1 1 + p_3^1 1 = 1$ | $w_4^{33} = p_2^2 0 + p_3^2 0 = 0$ | $l_1^{33} = p_2^2 \frac{1}{2} + p_3^2 \frac{1}{2} = \frac{1}{2}$ | $l_4^{33} = p_1^1 0 + p_3^1 0 = 0$ |
| 34 | $w_1^{34} = p_2^3 \frac{1}{3} + p_3^3 0 > 0$ | $w_4^{34} = p_2^4 0 + p_3^4 0 = 0$ | $l_1^{34} = p_2^4 0 + p_3^4 0 = 0$ | $l_4^{34} = p_2^3 0 + p_3^3 0 = 0$ |
| 35 | $w_1^{35} = p_2^5 1 + p_3^5 1 = 1$ | $w_4^{35} = p_2^6 \frac{1}{2} + p_3^6 \frac{1}{3} = \frac{1}{3}$ | $l_1^{35} = p_2^6 \frac{1}{2} + p_3^6 \frac{1}{3} = \frac{1}{3}$ | $l_4^{35} = p_2^5 0 + p_3^5 0 = 0$ |
| 36 | $w_1^{36} = p_2^7 \frac{1}{2} + p_3^7 0 = 0$ | $w_4^{36} = p_2^8 \frac{1}{2} + p_3^8 0 = 0$ | $l_1^{36} = p_2^8 0 + p_3^8 0 = 0$ | $l_4^{36} = p_2^7 0 + p_3^7 0 = 0$ |
| 37 | $w_1^{37} = p_2^9 1 + p_4^9 1 = 1 = w_1^{33}$ | $w_3^{37} = p_2^{10} 0 + p_4^{10} 0 = 0 = w_4^{33}$ | $l_1^{37} = p_2^{10} \frac{1}{2} + p_4^{10} \frac{1}{2} = \frac{1}{2} = l_2^{33}$ | $l_3^{37} = p_2^9 0 + p_4^9 0 = 0 = l_4^{33}$ |
| 38 | $w_1^{38} = p_2^{11} \frac{1}{3} + p_4^{11} 0 = p_2^3 \frac{1}{3} + p_3^3 0 = w_1^{34}$ | $w_3^{38} = p_2^{12} 0 + p_4^{12} 0 = 0 = w_4^{34}$ | $l_1^{38} = p_2^{12} 0 + p_4^{12} 0 = 0 = l_2^{34}$ | $l_3^{38} = p_2^{11} 0 + p_4^{11} 0 = 0 = l_4^{34}$ |
| 39 | $w_1^{39} = p_2^{13} 1 + p_4^{13} 1 = 1 = w_1^{35}$ | $w_3^{39} = p_2^{14} \frac{1}{2} + p_4^{14} \frac{1}{3} = p_2^6 \frac{1}{2} + p_3^6 \frac{1}{3} = w_4^{35}$ | $l_1^{39} = p_2^{14} \frac{1}{2} + p_4^{14} \frac{1}{3} = p_2^6 \frac{1}{2} + p_3^6 \frac{1}{3} = l_1^{35}$ | $l_3^{39} = p_2^{13} 0 + p_4^{13} 0 = 0 = l_4^{35}$ |
| 40 | $w_1^{40} = p_2^{15} \frac{1}{2} + p_4^{15} 0 = p_2^7 \frac{1}{2} + p_3^7 0 = w_1^{36}$ | $w_3^{40} = p_2^{16} \frac{1}{2} + p_4^{16} 0 = p_2^8 \frac{1}{2} + p_3^8 0 = w_4^{36}$ | $l_1^{40} = p_2^{16} 0 + p_4^{16} 0 = 0 = l_1^{36}$ | $l_3^{40} = p_2^{15} 0 + p_4^{15} 0 = 0 = l_4^{36}$ |

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 41 | $w_2^{41} = p_1^{17}1 + p_3^{17}1 = 1 = w_1^{33}$ | $w_4^{41} = p_1^{18}0 + p_3^{18}0 = 0 = w_4^{33}$ | $l_2^{41} = p_1^{18}\frac{1}{2} + p_3^{18}\frac{1}{2} = \frac{1}{2} = l_2^{33}$ | $l_4^{41} = p_1^{17}0 + p_3^{17}0 = 0 = l_4^{33}$ |
| 42 | $w_2^{42} = p_1^{19}\frac{1}{3} + p_3^{19}0 = p_2^{3}\frac{1}{3} + p_3^{3}0 = w_1^{34}$ | $w_4^{42} = p_1^{20}0 + p_3^{20}0 = 0 = w_4^{34}$ | $l_2^{42} = p_1^{20}0 + p_3^{20}0 = 0 = l_2^{34}$ | $l_4^{42} = p_1^{19}0 + p_3^{19}0 = 0 = l_4^{34}$ |
| 43 | $w_2^{43} = p_1^{21}1 + p_3^{21}1 = 1 = w_1^{35}$ | $w_4^{43} = p_1^{22}\frac{1}{2} + p_3^{22}\frac{1}{3} = p_2^{6}\frac{1}{2} + p_3^{6}\frac{1}{3} = w_4^{35}$ | $l_2^{43} = p_1^{22}\frac{1}{2} + p_3^{22}\frac{1}{3} = p_2^{6}\frac{1}{2} + p_3^{6}\frac{1}{3} = l_1^{35}$ | $l_4^{43} = p_1^{21}0 + p_3^{21}0 = 0 = l_4^{35}$ |
| 44 | $w_2^{44} = p_1^{23}\frac{1}{2} + p_3^{23}0 = p_2^{7}\frac{1}{2} + p_3^{7}0 = w_1^{36}$ | $w_4^{44} = p_1^{24}\frac{1}{2} + p_3^{24}0 = p_2^{8}\frac{1}{2} + p_3^{8}0 = w_4^{36}$ | $l_2^{44} = p_1^{24}0 + p_3^{24}0 = 0 = l_1^{36}$ | $l_4^{44} = p_1^{23}0 + p_3^{23}0 = 0 = l_4^{36}$ |
| 45 | $w_2^{45} = p_1^{25}1 + p_4^{25}1 = 1 = w_1^{33}$ | $w_3^{45} = p_1^{26}0 + p_4^{26}0 = 0 = w_4^{33}$ | $l_2^{45} = p_1^{26}\frac{1}{2} + p_4^{26}\frac{1}{2} = \frac{1}{2} = l_2^{33}$ | $l_3^{45} = p_1^{25}0 + p_4^{25}0 = 0 = l_4^{33}$ |
| 46 | $w_2^{46} = p_1^{27}\frac{1}{3} + p_4^{27}0 = p_2^{3}\frac{1}{3} + p_3^{3}0 = w_1^{34}$ | $w_3^{46} = p_1^{28}0 + p_4^{28}0 = 0 = w_4^{34}$ | $l_2^{46} = p_1^{28}0 + p_4^{28}0 = 0 = l_2^{34}$ | $l_3^{27} = p_1^{11}0 + p_4^{27}0 = 0 = l_4^{34}$ |
| 47 | $w_2^{47} = p_1^{29}1 + p_4^{29}1 = 1 = w_1^{35}$ | $w_3^{47} = p_1^{30}\frac{1}{2} + p_4^{30}\frac{1}{3} = p_2^{6}\frac{1}{2} + p_3^{6}\frac{1}{3} = w_4^{35}$ | $l_2^{47} = p_1^{30}\frac{1}{2} + p_4^{30}\frac{1}{3} = p_2^{6}\frac{1}{2} + p_3^{6}\frac{1}{3} = l_1^{35}$ | $l_3^{47} = p_1^{29}0 + p_4^{29}0 = 0 = l_4^{35}$ |
| 48 | $w_2^{48} = p_1^{31}\frac{1}{2} + p_4^{31}0 = p_2^{7}\frac{1}{2} + p_3^{7}0 = w_1^{36}$ | $w_3^{48} = p_1^{32}\frac{1}{2} + p_4^{32}0 = p_2^{8}\frac{1}{2} + p_3^{8}0 = w_4^{36}$ | $l_2^{48} = p_1^{32}0 + p_4^{32}0 = 0 = l_1^{36}$ | $l_3^{48} = p_1^{31}0 + p_4^{31}0 = 0 = l_4^{36}$ |

These continuation payoffs imply the following winning probabilities, expected payoffs and expected efforts of the respective players in nodes 33 - 48:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 33 | $p_1^{33} = 1$ | $p_4^{33} = 0$ | $E[x_1^{33}]$ | $E[x_4^{33}]$ | $E_1^{33} = 1$ | $E_4^{33} = 0$ |
| 34 | $p_1^{34} = 1$ | $p_4^{34} = 0$ | $E[x_1^{34}]$ | $E[x_4^{34}]$ | $E_1^{34} = w_1^{34} > 0$ | $E_4^{34} = 0$ |
| 35 | $p_1^{35} = p_3^3$ | $p_4^{35} = p_2^3$ | $E[x_1^{35}] = E[x_3^3]$ | $E[x_4^{35}] = E[x_2^3]$ | $E_1^{35} = E_3^3$ | $E_3^{35} = p_2^3$ |
| 36 | $p_1^{36} = \frac{1}{2}$ | $p_4^{36} = \frac{1}{2}$ | $E[x_1^{36}] = 0$ | $E[x_4^{36}] = 0$ | $E_1^{36} = 0$ | $E_4^{36} = 0$ |
| 37 | $p_1^{37} = p_1^{33}$ | $p_3^{37} = p_4^{33}$ | $E[x_1^{37}] = E[x_1^{33}]$ | $E[x_3^{37}] = E[x_4^{33}]$ | $E_1^{37} = E_1^{33}$ | $E_3^{37} = E_4^{33}$ |
| 38 | $p_1^{38} = p_1^{34}$ | $p_3^{38} = p_4^{34}$ | $E[x_1^{38}] = E[x_1^{34}]$ | $E[x_3^{38}] = E[x_4^{34}]$ | $E_1^{38} = E_1^{34}$ | $E_3^{38} = E_4^{34}$ |
| 39 | $p_1^{39} = p_1^{35}$ | $p_3^{39} = p_4^{35}$ | $E[x_1^{39}] = E[x_1^{35}]$ | $E[x_3^{39}] = E[x_4^{35}]$ | $E_1^{39} = E_1^{35}$ | $E_3^{39} = E_4^{35}$ |
| 40 | $p_1^{40} = p_1^{36}$ | $p_3^{40} = p_4^{36}$ | $E[x_1^{40}] = E[x_1^{36}]$ | $E[x_3^{40}] = E[x_4^{36}]$ | $E_1^{40} = E_1^{36}$ | $E_3^{40} = E_4^{36}$ |
| 41 | $p_2^{41} = p_1^{33}$ | $p_4^{41} = p_4^{33}$ | $E[x_2^{41}] = E[x_1^{33}]$ | $E[x_4^{41}] = E[x_4^{33}]$ | $E_2^{41} = E_1^{33}$ | $E_4^{41} = E_4^{33}$ |
| 42 | $p_2^{42} = p_1^{34}$ | $p_4^{42} = p_4^{34}$ | $E[x_2^{42}] = E[x_1^{34}]$ | $E[x_4^{42}] = E[x_4^{34}]$ | $E_2^{42} = E_1^{34}$ | $E_4^{42} = E_4^{34}$ |
| 43 | $p_2^{43} = p_1^{35}$ | $p_4^{43} = p_4^{35}$ | $E[x_2^{43}] = E[x_1^{35}]$ | $E[x_4^{43}] = E[x_4^{35}]$ | $E_2^{43} = E_1^{35}$ | $E_4^{43} = E_4^{35}$ |
| 44 | $p_2^{44} = p_1^{36}$ | $p_4^{44} = p_4^{36}$ | $E[x_2^{44}] = E[x_1^{36}]$ | $E[x_4^{44}] = E[x_4^{36}]$ | $E_2^{44} = E_1^{36}$ | $E_4^{44} = E_4^{36}$ |
| 45 | $p_2^{45} = p_1^{33}$ | $p_3^{45} = p_4^{33}$ | $E[x_2^{45}] = E[x_1^{33}]$ | $E[x_3^{45}] = E[x_4^{33}]$ | $E_2^{45} = E_1^{33}$ | $E_3^{45} = E_4^{33}$ |
| 46 | $p_2^{46} = p_1^{34}$ | $p_3^{46} = p_4^{34}$ | $E[x_2^{46}] = E[x_1^{34}]$ | $E[x_3^{46}] = E[x_4^{34}]$ | $E_2^{46} = E_1^{34}$ | $E_3^{46} = E_4^{34}$ |
| 47 | $p_2^{47} = p_1^{35}$ | $p_3^{47} = p_4^{35}$ | $E[x_2^{47}] = E[x_1^{35}]$ | $E[x_3^{47}] = E[x_4^{35}]$ | $E_2^{47} = E_1^{35}$ | $E_3^{47} = E_4^{35}$ |
| 48 | $p_2^{48} = p_1^{36}$ | $p_3^{48} = p_4^{36}$ | $E[x_2^{48}] = E[x_1^{36}]$ | $E[x_3^{48}] = E[x_4^{36}]$ | $E_2^{48} = E_1^{36}$ | $E_3^{48} = E_4^{36}$ |

In nodes 49 - 56 representing match 4, the continuation payoffs of the respective players are thus:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 49 | $w_1^{49}=E_1^{33}=1$ | $w_3^{49}=p_1^{34}E_3^3+p_4^{34}E_3^4=E_3^3$ | $l_1^{49}=E_1^{34}=w_1^{34}=p_2^3\frac{1}{3}=l_1^{49}$ | $l_3^{49}=p_1^{33}E_3^1+p_4^{33}E_3^2=0$ |
| 50 | $w_1^{50}=E_1^{35}=E_3^3=w_3^{49}$ | $w_3^{50}=p_1^{36}E_3^7+p_4^{36}E_3^8=1=w_1^{49}$ | $l_1^{50}=E_1^{36}=0=l_3^{49}$ | $l_3^{50}=p_1^{35}E_3^5+p_4^{35}E_3^6=p_2^3\frac{1}{3}=l_1^{49}$ |
| 51 | $w_1^{51}=E_1^{37}=E_1^{33}=w_1^{49}$ | $w_4^{51}=p_1^{38}E_4^{11}+p_3^{38}E_4^{12}=p_1^{34}E_3^3+p_4^{34}E_3^4=w_3^{49}$ | $l_1^{51}=E_1^{38}=E_1^{34}=l_1^{49}$ | $l_4^{51}=p_1^{37}E_4^9+p_3^{37}E_4^{10}=p_1^{33}E_3^1+p_4^{33}E_3^2=l_3^{49}$ |
| 52 | $w_1^{52}=E_1^{39}=E_1^{35}=w_1^{50}=w_3^{49}$ | $w_4^{52}=p_1^{40}E_4^{15}+p_3^{40}E_4^{16}=p_1^{36}E_3^7+p_4^{36}E_3^8=w_3^{50}=w_1^{49}$ | $l_1^{52}=E_1^{40}=E_1^{36}=l_1^{50}=l_3^{49}$ | $l_4^{52}=p_1^{39}E_4^{13}+p_3^{39}E_4^{14}=p_1^{35}E_3^5+p_4^{35}E_3^6=l_3^{50}=l_1^{49}$ |
| 53 | $w_2^{53}=E_2^{41}=E_1^{33}=w_1^{49}$ | $w_3^{53}=p_2^{42}E_3^{19}+p_4^{42}E_3^{20}=p_1^{34}E_3^3+p_4^{34}E_3^4=w_3^{49}$ | $l_2^{53}=E_2^{42}=E_1^{34}=l_1^{49}$ | $l_3^{53}=p_2^{41}E_3^{17}+p_4^{41}E_3^{18}=p_1^{33}E_3^1+p_4^{33}E_3^2=l_3^{49}$ |
| 54 | $w_2^{54}=E_2^{43}=E_1^{35}=w_1^{50}=w_3^{49}$ | $w_3^{54}=p_2^{44}E_3^{23}+p_4^{44}E_3^{24}=p_1^{36}E_3^7+p_4^{36}E_3^8=w_3^{50}=w_1^{49}$ | $l_2^{54}=E_2^{44}=E_1^{36}=l_1^{50}=l_3^{49}$ | $l_3^{54}=p_2^{43}E_3^{21}+p_4^{43}E_3^{22}=p_1^{35}E_3^5+p_4^{35}E_3^6=l_3^{50}=l_1^{49}$ |
| 55 | $w_2^{55}=E_2^{45}=E_1^{33}=w_1^{49}$ | $w_4^{55}=p_2^{46}E_4^{27}+p_3^{46}E_4^{28}=p_1^{34}E_3^3+p_4^{34}E_3^4=w_3^{49}$ | $l_2^{55}=E_2^{46}=E_1^{34}=l_1^{49}$ | $l_4^{55}=p_2^{45}E_4^{25}+p_3^{45}E_4^{26}=p_1^{33}E_3^1+p_4^{33}E_3^2=l_3^{49}$ |
| 56 | $w_2^{56}=E_2^{47}=E_1^{35}=w_1^{50}=w_3^{49}$ | $w_4^{56}=p_2^{48}E_4^{31}+p_3^{48}E_4^{32}=p_1^{36}E_3^7+p_4^{36}E_3^8=w_3^{50}==w_1^{49}$ | $l_2^{56}=E_2^{48}=E_1^{36}=l_1^{50}=l_3^{49}$ | $l_4^{56}=p_2^{47}E_4^{29}+p_3^{47}E_4^{30}=p_1^{35}E_3^5+p_4^{35}E_3^6=l_3^{50}=l_1^{49}$ |

These continuation payoffs imply the following winning probabilities, expected payoffs and expected efforts for players in match 4:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 49 | $p_1^{49}$ | $p_3^{49}$ | $E[x_1^{49}]$ | $E[x_3^{49}]$ | $E_1^{49}$ | $E_3^{49}$ |
| 50 | $p_1^{50} = p_3^{49}$ | $p_3^{50} = p_1^{49}$ | $E[x_1^{50}] = E[x_3^{49}]$ | $E[x_3^{50}] = E[x_1^{49}]$ | $E_1^{50} = E_3^{49}$ | $E_3^{50} = E_1^{49}$ |
| 51 | $p_1^{51} = p_1^{49}$ | $p_4^{51} = p_3^{49}$ | $E[x_1^{51}] = E[x_1^{49}]$ | $E[x_4^{51}] = E[x_3^{49}]$ | $E_1^{51} = E_1^{49}$ | $E_4^{51} = E_3^{49}$ |
| 52 | $p_1^{52} = p_3^{49}$ | $p_4^{52} = p_1^{49}$ | $E[x_1^{52}] = E[x_3^{49}]$ | $E[x_4^{52}] = E[x_1^{49}]$ | $E_1^{52} = E_3^{49}$ | $E_4^{52} = E_1^{49}$ |
| 53 | $p_2^{53} = p_1^{49}$ | $p_3^{53} = p_3^{49}$ | $E[x_2^{53}] = E[x_1^{49}]$ | $E[x_3^{53}] = E[x_3^{49}]$ | $E_2^{53} = E_1^{49}$ | $E_3^{53} = E_3^{49}$ |
| 54 | $p_2^{54} = p_3^{49}$ | $p_3^{54} = p_1^{49}$ | $E[x_2^{54}] = E[x_3^{49}]$ | $E[x_3^{54}] = E[x_1^{49}]$ | $E_2^{54} = E_3^{49}$ | $E_3^{54} = E_1^{49}$ |
| 55 | $p_2^{55} = p_1^{49}$ | $p_4^{55} = p_3^{49}$ | $E[x_2^{55}] = E[x_1^{49}]$ | $E[x_4^{55}] = E[x_3^{49}]$ | $E_2^{55} = E_1^{49}$ | $E_4^{55} = E_3^{49}$ |
| 56 | $p_2^{56} = p_3^{49}$ | $p_4^{56} = p_1^{49}$ | $E[x_2^{56}] = E[x_3^{49}]$ | $E[x_4^{56}] = E[x_1^{49}]$ | $E_2^{56} = E_3^{49}$ | $E_4^{56} = E_1^{49}$ |

In nodes 57 - 60 representing all possible versions of match 3, the continuation payoffs are hence given by:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 57 | $w_2^{57} = p_1^{49}(p_1^{33}E_2^1 + p_4^{33}E_2^2) + p_3^{49}(p_1^{34}E_2^3 + p_4^{34}E_2^4) = p_3^{49}E_2^3$ | $w_4^{57} = p_1^{50}E_4^{35} + p_3^{50}E_4^{36} = p_3^{49}E_2^3 = w_2^{57}$ | $l_2^{57} = p_1^{50}(p_1^{35}E_2^5 + p_4^{35}E_2^6) + p_3^{50}(p_1^{36}E_2^7 + p_4^{36}E_2^8) = 0$ | $l_4^{57} = p_1^{49}E_4^{33} + p_3^{49}E_4^{34} = 0 = l_2^{57}$ |
| 58 | $w_2^{58} = p_1^{51}(p_1^{37}E_2^9 + p_3^{37}E_2^{10}) + p_4^{51}(p_1^{38}E_2^{11} + p_3^{38}E_2^{12}) = p_1^{49}(p_1^{33}E_2^1 + p_4^{33}E_2^2) + p_3^{49}(p_1^{34}E_2^3 + p_4^{34}E_2^4) = w_2^{57}$ | $w_3^{58} = p_1^{52}E_3^{39} + p_4^{52}E_3^{40} = p_1^{50}E_4^{35} + p_3^{50}E_4^{36} = w_4^{57}$ | $l_2^{58} = p_1^{52}(p_1^{39}E_2^{13} + p_3^{39}E_2^{14}) + p_4^{52}(p_1^{40}E_2^{15} + p_3^{40}E_2^{16}) = p_1^{50}(p_1^{35}E_2^5 + p_4^{35}E_2^6) + p_3^{50}(p_1^{36}E_2^7 + p_4^{36}E_2^8) = l_2^{57}$ | $l_3^{58} = p_1^{51}E_3^{37} + p_4^{51}E_3^{38} = p_1^{49}E_4^{33} + p_3^{49}E_4^{34} = l_4^{57}$ |
| 59 | $w_2^{59} = p_2^{53}(p_2^{41}E_2^{17} + p_4^{41}E_2^{18}) + p_3^{53}(p_2^{42}E_2^{19} + p_4^{42}E_2^{20}) = p_1^{49}(p_1^{33}E_2^1 + p_4^{33}E_2^2) + p_3^{49}(p_1^{34}E_2^3 + p_4^{34}E_2^4) = w_2^{57}$ | $w_4^{59} = p_2^{54}E_4^{43} + p_3^{54}E_4^{44} = p_1^{52}E_3^{39} + p_4^{52}E_3^{40} = p_1^{50}E_4^{35} + p_3^{50}E_4^{36} = w_4^{57}$ | $l_2^{59} = p_2^{54}(p_2^{43}E_2^{21} + p_4^{43}E_2^{22}) + p_3^{54}(p_2^{44}E_2^{23} + p_4^{44}E_2^{24}) = p_1^{50}(p_1^{35}E_2^5 + p_4^{35}E_2^6) + p_3^{50}(p_1^{36}E_2^7 + p_4^{36}E_2^8) = l_2^{57}$ | $l_4^{59} = p_2^{53}E_4^{41} + p_3^{53}E_4^{42} = p_1^{49}E_4^{33} + p_3^{49}E_4^{34} = l_4^{57}$ |
| 60 | $w_1^{60} = p_2^{55}(p_2^{45}E_1^{25} + p_3^{45}E_1^{26}) + p_4^{55}(p_2^{46}E_1^{27} + p_3^{46}E_1^{28}) = p_1^{49}(p_1^{33}E_2^1 + p_4^{33}E_2^2) + p_3^{49}(p_1^{34}E_2^3 + p_4^{34}E_2^4) = w_2^{57}$ | $w_3^{60} = p_2^{56}E_3^{47} + p_4^{56}E_3^{48} = p_1^{52}E_3^{39} + p_4^{52}E_3^{40} = p_1^{50}E_4^{35} + p_3^{50}E_4^{36} = w_4^{57}$ | $l_1^{60} = p_2^{56}(p_2^{47}E_1^{29} + p_3^{47}E_1^{30}) + p_4^{56}(p_2^{48}E_1^{31} + p_3^{48}E_1^{32}) = p_1^{50}(p_1^{35}E_2^5 + p_4^{35}E_2^6) + p_3^{50}(p_1^{36}E_2^7 + p_4^{36}E_2^8) = l_2^{57}$ | $l_3^{60} = p_2^{55}E_3^{45} + p_4^{55}E_3^{46} = p_1^{49}E_4^{33} + p_3^{49}E_4^{34} = l_4^{57}$ |

Winning probabilities, expected payoffs and expected efforts of all variations of match 3 are hence given by:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 57 | $p_2^{57} = \frac{1}{2}$ | $p_4^{57} = \frac{1}{2}$ | $E[x_2^{57}]$ | $E[x_4^{57}] = E[x_2^{57}]$ | $E_2^{57}$ | $E_4^{57} = E_2^{57}$ |
| 58 | $p_2^{58} = p_2^{57} = \frac{1}{2}$ | $p_3^{58} = p_4^{57} = \frac{1}{2}$ | $E[x_2^{58}] = E[x_2^{57}]$ | $E[x_3^{58}] = E[x_2^{57}]$ | $E_2^{58} = E_2^{57}$ | $E_3^{58} = E_2^{57}$ |
| 59 | $p_1^{59} = p_2^{57} = \frac{1}{2}$ | $p_4^{59} = p_4^{57} = \frac{1}{2}$ | $E[x_1^{59}] = E[x_2^{57}]$ | $E[x_4^{59}] = E[x_2^{57}]$ | $E_1^{59} = E_2^{57}$ | $E_4^{59} = E_2^{57}$ |
| 60 | $p_1^{60} = p_2^{57} = \frac{1}{2}$ | $p_3^{60} = p_4^{57} = \frac{1}{2}$ | $E[x_1^{60}] = E[x_2^{57}]$ | $E[x_3^{60}] = E[x_2^{57}]$ | $E_1^{60} = E_2^{57}$ | $E_3^{60} = E_2^{57}$ |

In nodes 61 and 62 representing all versions of match 2, continuation payoffs are derived as follows:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 61 | $w_3^{61} = p_2^{57}E_3^{49} + p_4^{57}E_3^{50} = \frac{1}{2}E_3^{49} + \frac{1}{2}E_1^{49}$ | $w_4^{61} = p_2^{58}E_4^{51} + p_3^{58}E_4^{52} = p_2^{57}E_3^{49} + p_4^{57}E_3^{50} = w_3^{61}$ | $l_3^{61} = E_3^{58} = E_4^{57}$ | $l_4^{61} = E_4^{57} = l_3^{61}$ |
| 62 | $w_3^{62} = p_1^{59}E_3^{53} + p_4^{59}E_3^{54} = p_2^{57}E_3^{49} + p_4^{57}E_3^{50} = w_3^{61}$ | $w_4^{62} = p_1^{60}E_4^{55} + p_3^{60}E_4^{56} = p_2^{57}E_3^{49} + p_4^{57}E_3^{50} = w_3^{61}$ | $l_3^{62} = E_3^{60} = E_2^{57} = l_3^{61}$ | $l_4^{62} = E_4^{59} = E_2^{57} = l_3^{61}$ |

Therefore, winning probabilities, expected payoffs and expected efforts in nodes 61 and 62 are given by:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 61 | $p_3^{61} = \frac{1}{2}$ | $p_4^{61} = p_3^{61} = \frac{1}{2}$ | $E[x_3^{61}]$ | $E[x_4^{61}] = E[x_3^{61}]$ | $E_3^{61}$ | $E_4^{61} = E_3^{61}$ |
| 62 | $p_3^{62} = p_3^{61} = \frac{1}{2}$ | $p_4^{62} = p_3^{61} = \frac{1}{2}$ | $E[x_3^{62}] = E[x_3^{61}]$ | $E[x_4^{62}] = E[x_3^{61}]$ | $E_3^{62} = E_3^{61}$ | $E_4^{62} = E_3^{61}$ |

Finally, the continuation payoffs for the respective players in node 63 (i.e., match 1) can be rewritten as follows:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 63 | $w_1^{63} = p_3^{61}(p_2^{57}E_1^{49} + p_4^{57}E_1^{50}) + p_4^{61}(p_2^{58}E_1^{51} + p_3^{58}E_1^{52}) = \frac{1}{2}E_1^{49} + \frac{1}{2}E_3^{49}$ | $w_2^{63} = p_3^{62}(p_1^{59}E_2^{53} + p_4^{59}E_2^{54}) + p_4^{62}(p_1^{60}E_2^{55} + p_3^{60}E_2^{56}) = \frac{1}{2}E_1^{49} + \frac{1}{2}E_3^{49}$ | $l_1^{63} = p_3^{62}E_1^{59} + p_4^{62}E_1^{60} = E_2^{57}$ | $l_2^{63} = p_3^{61}E_2^{57} + p_4^{61}E_2^{58} = E_2^{57}$ |

Therefore, the winning probabilities, expected payoffs and expected efforts are thus given by:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 63 | $p_1^{63} = \frac{1}{2}$ | $p_2^{63} = \frac{1}{2}$ | $E[x_1^{63}]$ | $E[x_2^{63}] = E[x_1^{63}]$ | $E_1^{63}$ | $E_2^{63} = E_1^{63}$ |

Notice that $E_1^{63} = E_2^{63}$. Also, $E_3^{63} = \frac{1}{2}E_3^{61} + \frac{1}{2}E_3^{62} = \frac{1}{2}E_3^{61} + \frac{1}{2}E_3^{61} = E_3^{61}$ and $E_4^{63} = \frac{1}{2}E_4^{61} + \frac{1}{2}E_4^{62} = \frac{1}{2}E_3^{61} + \frac{1}{2}E_3^{61} = E_3^{61} = E_3^{61}$ implying $E_3^{63} = E_4^{63}$. Straightforwardly, $w_3^{61} = \frac{1}{2}E_3^{49} + \frac{1}{2}E_1^{49} = w_4^{61} = w_1^{63} = w_2^{63}$ and $l_3^{61} = E_2^{57} = l_4^{61} = l_1^{63} = l_2^{63}$ such that $E_1^{63} = E_2^{63} = E_3^{63} = E_4^{63}$.[17]

[17] Straightforward computations show that the players' ex-ante expected winnings coincide as well.

## A.4 Proof of Proposition 3

To prove the only-if-part of Proposition 3, we show that there is a round-robin tournament with four symmetric players and multiple prizes in which at least two players will have different ex-ante expected payoffs if an endogenous sequence, which matches the two winners (losers) of the first round in the second round, is not advanced.[18] To this end, we consider the example of Table 4 for a round-robin tournament with matches organized as all-pay auctions and $(a, b) = (.6, 0)$. Applying the results of Appendix A.1 repeatedly, we solve the games resulting under the different endogenous sequences by backward induction for their subgame perfect equilibria.[19] We collect the players' respective equilibrium ex-ante expected payoffs in Table 4. They show that there are, indeed, always two players with different ex-ante expected payoffs if the endogenous sequence, which matches the two winners (losers) of the first round in the second round, is not advanced.

To prove the if-part, we demonstrate the arguments for the sequence $LLL_4$; analog reasoning applies to all other advanced sequences that match the two winners (losers) of the first round in the second round.

Suppose that players are symmetric ($v_1 = v_2 = v_3 = v_4$) and there is a general prize structure $1 \geq a \geq b \geq 0$. Assume, moreover, that the CSF that shapes competition on the match level satisfies Assumption 1. Then, continuation payoffs, winning probabilities, expected payoffs and expected efforts of the respective players in nodes 1 - 32 representing all possible alternatives of match 6 can be rewritten as follows.

[18]Proposition 2 implies that only endogenous sequences, which match the two winners (losers) of the first round in the second round, have to be considered any further.

[19]The details of this procedure are outlined, e.g., in Krumer et al. (2017a); Laica et al. (2021).

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $w_1^1 = 1$ | $w_4^1 = \frac{b}{2}$ | $l_1^1 = \Delta$ | $l_4^1 = 0$ | $p_1^1$ | $p_4^1$ | $E[x_1^1]$ | $E[x_4^1]$ | $E_1^1$ | $E_4^1$ |
| 2 | $w_1^2 = 1$ | $w_4^2 = \frac{b}{2}$ | $l_1^2 = \Delta$ | $l_4^2 = 0$ | $p_1^2$ | $p_4^2$ | $E[x_1^2]$ | $E[x_4^2]$ | $E_1^2$ | $E_4^2$ |
| 3 | $w_2^3 = \Omega$ | $w_3^3 = 1$ | $l_2^3 = b$ | $l_3^3 = \Omega$ | $p_2^3$ | $p_3^3$ | $E[x_2^3]$ | $E[x_3^3]$ | $E_2^3$ | $E_3^3$ |
| 4 | $w_2^4 = \Delta$ | $w_3^4 = 1$ | $l_2^4 = \Theta$ | $l_3^4 = \Delta$ | $p_2^4$ | $p_3^4$ | $E[x_2^4]$ | $E[x_3^4]$ | $E_2^4$ | $E_3^4$ |
| 5 | $w_1^5 = 1$ | $w_4^5 = \Delta$ | $l_1^5 = \Delta$ | $l_4^5 = \Theta$ | $p_1^5 = p_3^4$ | $p_4^5 = p_2^4$ | $E[x_1^5] = E[x_3^4]$ | $E[x_4^5] = E[x_2^4]$ | $E_1^5 = E_3^4$ | $E_4^5 = E_2^4$ |
| 6 | $w_1^6 = 1$ | $w_4^6 = \Omega$ | $l_1^6 = \Omega$ | $l_4^6 = b$ | $p_1^6 = p_3^3$ | $p_4^6 = p_2^3$ | $E[x_1^6] = E[x_3^3]$ | $E[x_4^6] = E[x_2^3]$ | $E_1^6 = E_3^3$ | $E_4^6 = E_2^3$ |
| 7 | $w_2^7 = \frac{b}{2}$ | $w_3^7 = 1$ | $l_2^7 = 0$ | $l_3^7 = \Delta$ | $p_2^7 = p_4^2$ | $p_3^7 = p_1^2$ | $E[x_2^7] = E[x_4^2]$ | $E[x_3^7] = E[x_1^2]$ | $E_2^7 = E_4^2$ | $E_3^7 = E_1^2$ |
| 8 | $w_2^8 = \frac{b}{2}$ | $w_3^8 = 1$ | $l_2^8 = 0$ | $l_3^8 = \Delta$ | $p_2^8 = p_4^1$ | $p_3^8 = p_1^1$ | $E[x_2^8] = E[x_4^1]$ | $E[x_3^8] = E[x_1^1]$ | $E_2^8 = E_4^1$ | $E_3^8 = E_1^1$ |
| 9 | $w_1^9 = 1$ | $w_3^9 = \frac{b}{2}$ | $l_1^9 = \Delta$ | $l_3^9 = 0$ | $p_1^9 = p_1^1$ | $p_3^9 = p_4^1$ | $E[x_1^9] = E[x_1^1]$ | $E[x_3^9] = E[x_4^1]$ | $E_1^9 = E_1^1$ | $E_3^9 = E_4^1$ |
| 10 | $w_1^{10} = 1$ | $w_3^{10} = \frac{b}{2}$ | $l_1^{10} = \Delta$ | $l_3^{10} = 0$ | $p_1^{10} = p_1^2$ | $p_3^{10} = p_4^2$ | $E[x_1^{10}] = E[x_1^2]$ | $E[x_3^{10}] = E[x_4^2]$ | $E_1^{10} = E_1^2$ | $E_3^{10} = E_4^2$ |
| 11 | $w_2^{11} = \Omega$ | $w_4^{11} = 1$ | $l_2^{11} = b$ | $l_4^{11} = \Omega$ | $p_2^{11} = p_2^3$ | $p_4^{11} = p_3^3$ | $E[x_2^{11}] = E[x_2^3]$ | $E[x_4^{11}] = E[x_3^3]$ | $E_2^{11} = E_2^3$ | $E_4^{11} = E_3^3$ |
| 12 | $w_2^{12} = \Delta$ | $w_4^{12} = 1$ | $l_2^{12} = \Theta$ | $l_4^{12} = \Delta$ | $p_2^{12} = p_2^4$ | $p_4^{12} = p_3^4$ | $E[x_2^{12}] = E[x_2^4]$ | $E[x_4^{12}] = E[x_3^4]$ | $E_2^{12} = E_2^4$ | $E_4^{12} = E_3^4$ |
| 13 | $w_1^{13} = 1$ | $w_3^{13} = \Delta$ | $l_1^{13} = \Delta$ | $l_3^{13} = \Theta$ | $p_1^{13} = p_3^4$ | $p_3^{13} = p_2^4$ | $E[x_1^{13}] = E[x_3^4]$ | $E[x_3^{13}] = E[x_2^4]$ | $E_1^{13} = E_3^4$ | $E_3^{13} = E_2^4$ |
| 14 | $w_1^{14} = 1$ | $w_3^{14} = \Omega$ | $l_1^{14} = \Omega$ | $l_3^{14} = b$ | $p_1^{14} = p_3^3$ | $p_3^{14} = p_2^3$ | $E[x_1^{14}] = E[x_3^3]$ | $E[x_3^{14}] = E[x_2^3]$ | $E_1^{14} = E_3^3$ | $E_3^{14} = E_2^3$ |
| 15 | $w_2^{15} = \frac{b}{2}$ | $w_4^{15} = 1$ | $l_2^{15} = 0$ | $l_4^{15} = \Delta$ | $p_2^{15} = p_4^2$ | $p_4^{15} = p_1^2$ | $E[x_2^{15}] = E[x_4^2]$ | $E[x_4^{15}] = E[x_1^2]$ | $E_2^{15} = E_4^2$ | $E_4^{15} = E_1^2$ |
| 16 | $w_2^{16} = \frac{b}{2}$ | $w_4^{16} = 1$ | $l_2^{16} = 0$ | $l_4^{16} = \Delta$ | $p_2^{16} = p_4^1$ | $p_4^{16} = p_1^1$ | $E[x_2^{16}] = E[x_4^1]$ | $E[x_4^{16}] = E[x_1^1]$ | $E_2^{16} = E_4^1$ | $E_4^{16} = E_1^1$ |

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 17 | $w_2^{17} = 1$ | $w_4^{17} = \frac{b}{2}$ | $l_2^{17} = \Delta$ | $l_4^{17} = 0$ | $p_2^{17} = p_1^1$ | $p_4^{17} = p_4^1$ | $E[x_2^{17}] = E[x_1^1]$ | $E[x_4^{17}] = E[x_4^1]$ | $E_2^{17} = E_1^1$ | $E_4^{17} = E_4^1$ |
| 18 | $w_2^{18} = 1$ | $w_4^{18} = \frac{b}{2}$ | $l_2^{18} = \Delta$ | $l_4^{18} = 0$ | $p_2^{18} = p_1^2$ | $p_4^{18} = p_4^2$ | $E[x_2^{18}] = E[x_1^2]$ | $E[x_4^{18}] = E[x_4^2]$ | $E_2^{18} = E_1^2$ | $E_4^{18} = E_4^2$ |
| 19 | $w_1^{19} = \Omega$ | $w_3^{19} = 1$ | $l_1^{19} = b$ | $l_3^{19} = \Omega$ | $p_1^{19} = p_2^3$ | $p_3^{19} = p_3^3$ | $E[x_1^{19}] = E[x_2^3]$ | $E[x_3^{19}] = E[x_3^3]$ | $E_1^{19} = E_2^3$ | $E_3^{19} = E_3^3$ |
| 20 | $w_1^{20} = \Delta$ | $w_3^{20} = 1$ | $l_1^{20} = \Theta$ | $l_3^{20} = \Delta$ | $p_1^{20} = p_2^4$ | $p_3^{20} = p_3^4$ | $E[x_1^{20}] = E[x_2^4]$ | $E[x_3^{20}] = E[x_3^4]$ | $E_1^{20} = E_2^4$ | $E_3^{20} = E_3^4$ |
| 21 | $w_2^{21} = 1$ | $w_4^{21} = \Delta$ | $l_2^{21} = \Delta$ | $l_4^{21} = \Theta$ | $p_2^{21} = p_3^4$ | $p_4^{21} = p_2^4$ | $E[x_2^{21}] = E[x_3^4]$ | $E[x_4^{21}] = E[x_2^4]$ | $E_2^{21} = E_3^4$ | $E_4^{21} = E_2^4$ |
| 22 | $w_2^{22} = 1$ | $w_4^{22} = \Omega$ | $l_2^{22} = \Omega$ | $l_4^{22} = b$ | $p_2^{22} = p_3^3$ | $p_4^{22} = p_2^3$ | $E[x_2^{22}] = E[x_3^3]$ | $E[x_4^{22}] = E[x_2^3]$ | $E_2^{22} = E_3^3$ | $E_4^{22} = E_2^3$ |
| 23 | $w_1^{23} = \frac{b}{2}$ | $w_3^{23} = 1$ | $l_1^{23} = 0$ | $l_3^{23} = \Delta$ | $p_1^{23} = p_4^2$ | $p_3^{23} = p_1^2$ | $E[x_1^{23}] = E[x_4^2]$ | $E[x_3^{23}] = E[x_1^2]$ | $E_1^{23} = E_4^2$ | $E_3^{23} = E_1^2$ |
| 24 | $w_1^{24} = \frac{b}{2}$ | $w_3^{24} = 1$ | $l_1^{24} = 0$ | $l_3^{24} = \Delta$ | $p_1^{24} = p_4^1$ | $p_3^{24} = p_1^1$ | $E[x_1^{24}] = E[x_4^1]$ | $E[x_3^{24}] = E[x_1^1]$ | $E_1^{24} = E_4^1$ | $E_3^{24} = E_1^1$ |
| 25 | $w_2^{25} = 1$ | $w_3^{25} = \frac{b}{2}$ | $l_2^{25} = \Delta$ | $l_3^{25} = 0$ | $p_2^{25} = p_1^1$ | $p_3^{25} = p_4^1$ | $E[x_2^{25}] = E[x_1^1]$ | $E[x_3^{25}] = E[x_4^1]$ | $E_2^{25} = E_1^1$ | $E_3^{25} = E_4^1$ |
| 26 | $w_2^{26} = 1$ | $w_3^{26} = \frac{b}{2}$ | $l_2^{26} = \Delta$ | $l_3^{26} = 0$ | $p_2^{26} = p_1^2$ | $p_3^{26} = p_4^2$ | $E[x_2^{26}] = E[x_1^2]$ | $E[x_3^{26}] = E[x_4^2]$ | $E_2^{26} = E_1^2$ | $E_3^{26} = E_4^2$ |
| 27 | $w_1^{27} = \Omega$ | $w_4^{27} = 1$ | $l_1^{27} = b$ | $l_4^{27} = \Omega$ | $p_1^{27} = p_2^3$ | $p_4^{27} = p_3^3$ | $E[x_1^{27}] = E[x_2^3]$ | $E[x_4^{27}] = E[x_3^3]$ | $E_1^{27} = E_2^3$ | $E_4^{27} = E_3^3$ |
| 28 | $w_1^{28} = \Delta$ | $w_4^{28} = 1$ | $l_1^{28} = \Theta$ | $l_4^{28} = \Delta$ | $p_1^{28} = p_2^4$ | $p_4^{28} = p_3^4$ | $E[x_1^{28}] = E[x_2^4]$ | $E[x_4^{28}] = E[x_3^4]$ | $E_1^{28} = E_2^4$ | $E_4^{28} = E_3^4$ |
| 29 | $w_2^{29} = 1$ | $w_3^{29} = \Delta$ | $l_2^{29} = \Delta$ | $l_3^{29} = \Theta$ | $p_2^{29} = p_3^4$ | $p_3^{29} = p_2^4$ | $E[x_2^{29}] = E[x_3^4]$ | $E[x_3^{29}] = E[x_2^4]$ | $E_2^{29} = E_3^4$ | $E_3^{29} = E_2^4$ |
| 30 | $w_2^{30} = 1$ | $w_3^{30} = \Omega$ | $l_2^{30} = \Omega$ | $l_3^{30} = b$ | $p_2^{30} = p_3^3$ | $p_3^{30} = p_2^3$ | $E[x_2^{30}] = E[x_3^3]$ | $E[x_3^{30}] = E[x_2^3]$ | $E_2^{30} = E_3^3$ | $E_3^{30} = E_2^3$ |
| 31 | $w_1^{31} = \frac{b}{2}$ | $w_4^{31} = 1$ | $l_1^{31} = 0$ | $l_4^{31} = \Delta$ | $p_1^{31} = p_4^2$ | $p_4^{31} = p_1^2$ | $E[x_1^{31}] = E[x_4^2]$ | $E[x_4^{31}] = E[x_1^2]$ | $E_1^{31} = E_4^2$ | $E_4^{31} = E_1^2$ |
| 32 | $w_1^{32} = \frac{b}{2}$ | $w_4^{32} = 1$ | $l_1^{32} = 0$ | $l_4^{32} = \Delta$ | $p_1^{32} = p_4^1$ | $p_4^{32} = p_1^1$ | $E[x_1^{32}] = E[x_4^1]$ | $E[x_4^{32}] = E[x_1^1]$ | $E_1^{32} = E_4^1$ | $E_4^{32} = E_1^1$ |

Similarly, continuation payoffs of the respective players in nodes 33 - 48 can be derived.

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 33 | $w_2^{33} = p_1^1 a + p_4^1\Delta$ | $w_3^{33} = p_1^2 a + p_4^2\Delta$<br>$= p_1^1 a + p_4^1\Delta = w_2^{33}$ | $l_2^{33} = p_1^2 b + p_4^2\frac{b}{2}$<br>$= p_1^1 b + p_4^1\frac{b}{2}$ | $l_3^{33} = p_1^1 b + p_4^1\frac{b}{2}$<br>$= l_2^{33}$ |
| 34 | $w_1^{34} = p_2^3\Omega + p_3^3 a$ | $w_4^{34} = p_2^4\frac{b}{2} + p_3^4\Theta$ | $l_1^{34} = p_2^4\frac{b}{2} + p_3^4\Theta$<br>$= w_4^{34}$ | $l_4^{34} = p_2^3 0 + p_3^3 0 = 0$ |
| 35 | $w_2^{35} = p_1^5\Theta + p_4^5\frac{b}{2}$<br>$= p_3^4\Theta + p_2^4\frac{b}{2} = w_4^{34}$ | $w_3^{35} = p_1^6 a + p_4^6\Omega$<br>$= p_3^3 a + p_2^3\Omega = w_1^{34}$ | $l_2^{35} = p_1^6 0 + p_4^6 0$<br>$= l_4^{34}$ | $l_3^{35} = p_1^5\Theta + p_4^5\frac{b}{2}$<br>$= p_3^4\Theta + p_2^4\frac{b}{2} = l_1^{34}$ |
| 36 | $w_1^{36} = p_2^7\Delta + p_3^7 a$<br>$= p_4^1\Delta + p_1^1 a = w_3^{33}$ | $w_4^{36} = p_2^8\Delta + p_3^8 a$<br>$= p_4^1\Delta + p_1^1 a = w_2^{33}$ | $l_1^{36} = p_2^8\frac{b}{2} + p_3^8 b$<br>$= p_4^1\frac{b}{2} + p_1^1 b = l_3^{33}$ | $l_4^{36} = p_2^7\frac{b}{2} + p_3^7 b$<br>$= p_4^1\frac{b}{2} + p_1^1 b = l_2^{33}$ |
| 37 | $w_2^{37} = p_1^9 a + p_3^9\Delta$<br>$= p_1^1 a + p_4^1\Delta = w_2^{33}$ | $w_4^{37} = p_1^{10} a + p_3^{10}\Delta$<br>$= p_1^a + p_4^1\Delta = w_3^{33}$ | $l_2^{37} = p_1^{10} b + p_3^{10}\frac{b}{2}$<br>$= p_1^1 b + p_4^1\frac{b}{2} = l_2^{33}$ | $l_4^{37} = p_1^9 b + p_3^9\frac{b}{2}$<br>$= p_1^1 b + p_4^1\frac{b}{2} = l_2^{33}$ |
| 38 | $w_1^{38} = p_2^{11}\Omega + p_4^{11} a$<br>$= p_2^3\Omega + p_3^3 a = w_1^{34}$ | $w_3^{38} = p_2^{12}\frac{b}{2} + p_4^{12}\Theta$<br>$= p_2^4\frac{b}{2} + p_3^4\Theta = w_4^{34}$ | $l_1^{38} = p_2^{12}\frac{b}{2} + p_4^{12}\Theta$<br>$= p_2^4\frac{b}{2} + p_3^4\Theta = l_1^{34}$ | $l_3^{38}$<br>$= p_2^{11}0 + p_4^{11}0$<br>$= 0 = l_4^{34}$ |
| 39 | $w_2^{39} = p_1^{13}\Theta + p_3^{13}\frac{b}{2}$<br>$= p_3^4\Theta + p_2^4\frac{b}{2} = w_4^{34}$ | $w_4^{39} = p_1^{14} a + p_3^{14}\Omega$<br>$= p_3^3 a + p_2^3\Omega = w_1^{34}$ | $l_2^{39} = p_1^{14}0 + p_3^{14}0$<br>$= 0 = l_4^{34}$ | $l_4^{39} = p_1^{13}\Theta + p_3^{13}\frac{b}{2}$<br>$= p_3^4\Theta + p_2^4\frac{b}{2} = l_1^{34}$ |
| 40 | $w_1^{40} = p_2^{15}\Delta + p_4^{15} a$<br>$= p_4^1\Delta + p_1^1 a = w_3^{33}$ | $w_3^{40} = p_2^{16}\Delta + p_4^{16} a$<br>$= p_4^1\Delta + p_1^1 a = w_2^{33}$ | $l_1^{40} = p_2^{16}\frac{b}{2} + p_4^{16} b$<br>$= p_4^1\frac{b}{2} + p_1^1 b = l_3^{33}$ | $l_3^{40} = p_2^{15}\frac{b}{2} + p_4^{15} b$<br>$= p_4^1\frac{b}{2} + p_1^1 b = l_2^{33}$ |

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 41 | $w_1^{41} = p_2^{17}a + p_4^{17}\Delta$<br>$= p_1^1 a + p_4^1\Delta = w_2^{33}$ | $w_3^{41} = p_2^{18}a + p_4^{18}\Delta$<br>$= p_1^a + p_4^1\Delta = w_3^{33}$ | $l_1^{41} = p_2^{18}b + p_4^{18}\frac{b}{2}$<br>$= p_1^1 b + p_4^1\frac{b}{2} = l_2^{33}$ | $l_3^{41} = p_2^{17}b + p_4^{17}\frac{b}{2}$<br>$= p_1^1 b + p_4^1\frac{b}{2} = l_2^{33}$ |
| 42 | $w_2^{42} = p_1^{19}\Omega + p_3^{19}a$<br>$= p_2^3\Omega + p_3^3 a = w_1^{34}$ | $w_4^{42} = p_1^{20}\frac{b}{2} + p_3^{20}\Theta$<br>$= p_2^4\frac{b}{2} + p_3^4\Theta = w_4^{34}$ | $l_2^{42} = p_1^{20}\frac{b}{2} + p_3^{20}\Theta$<br>$= p_2^4\frac{b}{2} + p_3^4\Theta = l_1^{34}$ | $l_4^{42}$<br>$= p_1^{19}0 + p_3^{19}0$<br>$= 0 = l_4^{34}$ |
| 43 | $w_1^{43} = p_2^{21}\Theta + p_4^{21}\frac{b}{2}$<br>$= p_3^4\Theta + p_2^4\frac{b}{2} = w_4^{34}$ | $w_3^{43} = p_2^{22}a + p_4^{22}\Omega$<br>$= p_3^3 a + p_2^3\Omega = w_1^{34}$ | $l_1^{43} = p_2^{22}0 + p_4^{22}0$<br>$= 0 = l_4^{34}$ | $l_3^{43} = p_2^{21}\Theta + p_4^{21}\frac{b}{2}$<br>$= p_3^4\Theta + p_2^4\frac{b}{2} = l_1^{34}$ |
| 44 | $w_2^{44} = p_1^{23}\Delta + p_3^{23}a$<br>$= p_4^1\Delta + p_1^1 a = w_3^{33}$ | $w_4^{44} = p_1^{24}\Delta + p_3^{24}a$<br>$= p_4^1\Delta + p_1^1 a = w_2^{33}$ | $l_2^{44} = p_1^{24}\frac{b}{2} + p_3^{24}b$<br>$= p_4^1\frac{b}{2} + p_1^1 b = l_3^{33}$ | $l_4^{44} = p_1^{23}\frac{b}{2} + p_3^{23}b$<br>$= p_4^1\frac{b}{2} + p_1^1 b = l_2^{33}$ |
| 45 | $w_1^{45} = p_2^{25}a + p_3^{25}\Delta$<br>$= p_1^1 a + p_4^1\Delta = w_2^{33}$ | $w_4^{45} = p_2^{26}a + p_3^{26}\Delta$<br>$= p_1^a + p_4^1\Delta = w_3^{33}$ | $l_1^{45} = p_2^{26}b + p_3^{26}\frac{b}{2}$<br>$= p_1^1 b + p_4^1\frac{b}{2} = l_2^{33}$ | $l_4^{45} = p_2^{25}b + p_3^{25}\frac{b}{2}$<br>$= p_1^1 b + p_4^1\frac{b}{2} = l_2^{33}$ |
| 46 | $w_2^{46} = p_1^{27}\Omega + p_4^{27}a$<br>$= p_2^3\Omega + p_3^3 a = w_1^{34}$ | $w_3^{46} = p_1^{28}\frac{b}{2} + p_4^{28}\Theta$<br>$= p_2^4\frac{b}{2} + p_3^4\Theta = w_4^{34}$ | $l_2^{46} = p_1^{28}\frac{b}{2} + p_4^{28}\Theta$<br>$= p_2^4\frac{b}{2} + p_3^4\Theta = l_1^{34}$ | $l_3 46$<br>$= p_1^{27}0 + p_4^{27}0$<br>$= 0 = l_4^{34}$ |
| 47 | $w_1^{47} = p_2^{29}\Theta + p_3^{29}\frac{b}{2}$<br>$= p_3^4\Theta + p_2^4\frac{b}{2} = w_4^{34}$ | $w_4^{47} = p_2^{30}a + p_3^{30}\Omega$<br>$= p_3^3 a + p_2^3\Omega = w_1^{34}$ | $l_1^{47} = p_2^{30}0 + p_3^{30}0$<br>$= 0 = l_4^{34}$ | $l_4^{47} = p_2^{29}\Theta + p_3^{29}\frac{b}{2}$<br>$= p_3^4\Theta + p_2^4\frac{b}{2} = l_1^{34}$ |
| 48 | $w_2^{48} = p_1^{31}\Delta + p_4^{31}a$<br>$= p_4^1\Delta + p_1^1 a = w_3^{33}$ | $w_3^{48} = p_1^{32}\Delta + p_4^{32}a$<br>$= p_4^1\Delta + p_1^1 a = w_2^{33}$ | $l_2^{48} = p_1^{32}\frac{b}{2} + p_4^{32}b$<br>$= p_4^1\frac{b}{2} + p_1^1 b = l_3^{33}$ | $l_3^{48} = p_1^{31}\frac{b}{2} + p_4^{31}b$<br>$= p_4^1\frac{b}{2} + p_1^1 b = l_2^{33}$ |

These continuation payoffs imply the following winning probabilities, expected payoffs and expected efforts of the respective players in nodes 33 - 48.

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 33 | $p_2^{33}$ | $p_3^{33}$ | $E[x_2^{33}]$ | $E[x_3^{33}]$ | $E_2^{33}$ | $E_3^{33}$ |
| 34 | $p_1^{34}$ | $p_4^{34}$ | $E[x_1^{34}]$ | $E[x_4^{34}]$ | $E_1^{34}$ | $E_4^{34}$ |
| 35 | $p_2^{35} = p_4^{34}$ | $p_3^{35} = p_1^{34}$ | $E[x_2^{35}] = E[x_4^{34}]$ | $E[x_3^{35}] = E[x_1^{34}]$ | $E_2^{35} = E_4^{34}$ | $E_3^{35} = E_1^{34}$ |
| 36 | $p_1^{36} = p_3^{33}$ | $p_4^{36} = p_2^{33}$ | $E[x_1^{36}] = E[x_3^{33}]$ | $E[x_4^{36}] = E[x_2^{33}]$ | $E_1^{36} = E_3^{33}$ | $E_4^{36} = E_2^{33}$ |
| 37 | $p_2^{37} = p_2^{33}$ | $p_4^{37} = p_3^{33}$ | $E[x_2^{37}] = E[x_2^{33}]$ | $E[x_4^{37}] = E[x_3^{33}]$ | $E_2^{37} = E_2^{33}$ | $E_4^{37} = E_3^{33}$ |
| 38 | $p_1^{38} = p_1^{34}$ | $p_3^{38} = p_4^{34}$ | $E[x_1^{38}] = E[x_1^{34}]$ | $E[x_3^{38}] = E[x_4^{34}]$ | $E_1^{38} = E_1^{34}$ | $E_3^{38} = E_4^{34}$ |
| 39 | $p_2^{39} = p_4^{34}$ | $p_4^{39} = p_1^{34}$ | $E[x_2^{39}] = E[x_4^{34}]$ | $E[x_4^{39}] = E[x_1^{34}]$ | $E_2^{39} = E_4^{34}$ | $E_4^{39} = E_1^{34}$ |
| 40 | $p_1^{40} = p_3^{33}$ | $p_3^{40} = p_2^{33}$ | $E[x_1^{40}] = E[x_3^{33}]$ | $E[x_3^{40}] = E[x_2^{33}]$ | $E_1^{40} = E_3^{33}$ | $E_3^{40} = E_2^{33}$ |
| 41 | $p_1^{41} = p_2^{33}$ | $p_3^{41} = p_3^{33}$ | $E[x_1^{41}] = E[x_2^{33}]$ | $E[x_3^{41}] = E[x_3^{33}]$ | $E_1^{41} = E_2^{33}$ | $E_3^{41} = E_3^{33}$ |
| 42 | $p_2^{42} = p_1^{34}$ | $p_4^{42} = p_4^{34}$ | $E[x_2^{42}] = E[x_1^{34}]$ | $E[x_4^{42}] = E[x_4^{34}]$ | $E_2^{42} = E_1^{34}$ | $E_4^{42} = E_4^{34}$ |
| 43 | $p_1^{43} = p_4^{34}$ | $p_3^{43} = p_1^{34}$ | $E[x_1^{43}] = E[x_4^{34}]$ | $E[x_3^{43}] = E[x_1^{34}]$ | $E_1^{43} = E_4^{34}$ | $E_3^{43} = E_1^{34}$ |
| 44 | $p_2^{44} = p_3^{33}$ | $p_4^{44} = p_2^{33}$ | $E[x_2^{44}] = E[x_3^{33}]$ | $E[x_4^{44}] = E[x_2^{33}]$ | $E_2^{44} = E_3^{33}$ | $E_4^{44} = E_2^{33}$ |
| 45 | $p_1^{45} = p_2^{33}$ | $p_4^{45} = p_3^{33}$ | $E[x_1^{45}] = E[x_2^{33}]$ | $E[x_4^{45}] = E[x_3^{33}]$ | $E_1^{45} = E_2^{33}$ | $E_4^{45} = E_3^{33}$ |
| 46 | $p_2^{46} = p_1^{34}$ | $p_3^{46} = p_4^{34}$ | $E[x_2^{46}] = E[x_1^{34}]$ | $E[x_3^{46}] = E[x_4^{34}]$ | $E_2^{46} = E_1^{34}$ | $E_3^{46} = E_4^{34}$ |
| 47 | $p_1^{47} = p_4^{34}$ | $p_4^{47} = p_1^{34}$ | $E[x_1^{47}] = E[x_4^{34}]$ | $E[x_4^{47}] = E[x_1^{34}]$ | $E_1^{47} = E_4^{34}$ | $E_4^{47} = E_1^{34}$ |
| 48 | $p_2^{48} = p_3^{33}$ | $p_3^{48} = p_2^{33}$ | $E[x_2^{48}] = E[x_3^{33}]$ | $E[x_3^{48}] = E[x_2^{33}]$ | $E_2^{48} = E_3^{33}$ | $E_3^{48} = E_2^{33}$ |

In nodes 49 - 56 representing match 4, the continuation payoffs of the respective players are thus:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 49 | $w_1^{49} = p_2^{33}E_1^1 + p_3^{33}E_1^2$ | $w_3^{49} = p_1^{34}E_3^3 + p_4^{34}E_3^4$ | $l_1^{49} = E_1^{34}$ | $l_3^{49} = E_3^{33}$ |
| 50 | $w_1^{50} = p_2^{35}E_1^5 + p_3^{35}E_1^6$<br>$= p_4^{34}E_3^4 + p_1^{34}E_3^3 = w_3^{49}$ | $w_3^{50} = p_1^{36}E_3^7 + p_4^{36}E_3^8$<br>$= p_3^{33}E_1^2 + p_2^{35}E_1^1 = w_1^{49}$ | $l_1^{50} = E_1^{36}$<br>$= E_3^{33} = l_3^{49}$ | $l_3^{50} = E_3^{35}$<br>$= E_1^{34} = l_1^{49}$ |
| 51 | $w_1^{51} = p_2^{37}E_1^9 + p_4^{37}E_1^{10}$<br>$= p_2^{33}E_1^1 + p_3^{33}E_1^2 = w_1^{49}$ | $w_4^{51} = p_1^{38}E_4^{11} + p_3^{38}E_4^{12}$<br>$= p_1^{34}E_3^3 + p_4^{34}E_3^4 = w_3^{49}$ | $l_1^{51} = E_1^{38}$<br>$= E_1^{34} = l_1^{49}$ | $l_4^{51} = E_4^{37}$<br>$= E_3^{33} = l_3^{49}$ |
| 52 | $w_1^{52} = p_2^{39}E_1^{13} + p_3^{41}E_2^{14}$<br>$= p_4^{34}E_3^4 + p_1^{34}E_3^3 = w_3^{49}$ | $w_4^{52} = p_1^{40}E_4^{15} + p_3^{40}E_4^{16}$<br>$= p_3^{33}E_1^2 + p_2^{35}E_1^1 = w_1^{49}$ | $l_1^{52} = E_1^{40}$<br>$= E_3^{33} = l_3^{49}$ | $l_4^{52} = E_4^{39}$<br>$= E_1^{34} = l_1^{49}$ |
| 53 | $w_2^{53} = p_1^{41}E_2^{17} + p_3^{41}E_2^{18}$<br>$= p_2^{33}E_1^1 + p_3^{33}E_1^2 = w_1^{49}$ | $w_3^{53} = p_2^{42}E_3^{19} + p_4^{42}E_3^{20}$<br>$= p_1^{34}E_3^3 + p_4^{34}E_3^4 = w_3^{49}$ | $l_2^{53} = E_2^{42}$<br>$= E_1^{34} = l_1^{49}$ | $l_3^{53} = E_3^{41}$<br>$= E_3^{33} = l_3^{49}$ |
| 54 | $w_2^{54} = p_1^{43}E_2^{21} + p_3^{43}E_2^{22}$<br>$= p_4^{34}E_3^4 + p_1^{34}E_3^3 = w_3^{49}$ | $w_3^{54} = p_2^{44}E_3^{23} + p_4^{44}E_3^{24}$<br>$= p_3^{33}E_1^2 + p_2^{35}E_1^1 = w_1^{49}$ | $l_2^{54} = E_2^{44}$<br>$= E_3^{33} = l_3^{49}$ | $l_3^{54} = E_3^{43}$<br>$= E_1^{34} = l_1^{49}$ |
| 55 | $w_2^{55} = p_1^{45}E_2^{25} + p_4^{45}E_2^{26}$<br>$= p_2^{33}E_1^1 + p_3^{33}E_1^2 = w_1^{49}$ | $w_4^{55} = p_2^{46}E_4^{27} + p_3^{46}E_4^{28}$<br>$= p_1^{34}E_3^3 + p_4^{34}E_3^4 = w_3^{49}$ | $l_2^{55} = E_2^{46}$<br>$= E_1^{34} = l_1^{49}$ | $l_4^{55} = E_4^{45}$<br>$= E_3^{33} = l_3^{49}$ |
| 56 | $w_2^{56} = p_1^{47}E_2^{29} + p_4^{47}E_2^{30}$<br>$= p_4^{34}E_3^4 + p_1^{34}E_3^3 = w_3^{49}$ | $w_4^{56} = p_2^{48}E_4^{31} + p_3^{48}E_4^{32}$<br>$= p_3^{33}E_1^2 + p_2^{35}E_1^1 = w_1^{49}$ | $l_2^{56} = E_2^{48}$<br>$= E_3^{33} = l_3^{49}$ | $l_4^{56} = E_4^{47}$<br>$= E_1^{34} = l_1^{49}$ |

These continuation payoffs imply the following winning probabilities, expected payoffs and expected efforts for players in game 4:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 49 | $p_1^{49}$ | $p_3^{49}$ | $E[x_1^{49}]$ | $E[x_3^{49}]$ | $E_1^{49}$ | $E_3^{49}$ |
| 50 | $p_1^{50} = p_3^{49}$ | $p_3^{50} = p_1^{49}$ | $E[x_1^{50}] = E[x_3^{49}]$ | $E[x_3^{50}] = E[x_1^{49}]$ | $E_1^{50} = E_3^{49}$ | $E_3^{50} = E_1^{49}$ |
| 51 | $p_1^{51} = p_1^{49}$ | $p_4^{51} = p_3^{49}$ | $E[x_1^{51}] = E[x_1^{49}]$ | $E[x_4^{51}] = E[x_3^{49}]$ | $E_1^{51} = E_1^{49}$ | $E_4^{51} = E_3^{49}$ |
| 52 | $p_1^{52} = p_3^{49}$ | $p_4^{52} = p_1^{49}$ | $E[x_1^{52}] = E[x_3^{49}]$ | $E[x_4^{52}] = E[x_1^{49}]$ | $E_1^{52} = E_3^{49}$ | $E_4^{52} = E_1^{49}$ |
| 53 | $p_2^{53} = p_1^{49}$ | $p_3^{53} = p_3^{49}$ | $E[x_2^{53}] = E[x_1^{49}]$ | $E[x_3^{53}] = E[x_3^{49}]$ | $E_2^{53} = E_1^{49}$ | $E_3^{53} = E_3^{49}$ |
| 54 | $p_2^{54} = p_3^{49}$ | $p_3^{54} = p_1^{49}$ | $E[x_2^{54}] = E[x_3^{49}]$ | $E[x_3^{54}] = E[x_1^{49}]$ | $E_2^{54} = E_3^{49}$ | $E_3^{54} = E_1^{49}$ |
| 55 | $p_2^{55} = p_1^{49}$ | $p_4^{55} = p_3^{49}$ | $E[x_2^{55}] = E[x_1^{49}]$ | $E[x_4^{55}] = E[x_3^{49}]$ | $E_2^{55} = E_1^{49}$ | $E_4^{55} = E_3^{49}$ |
| 56 | $p_2^{56} = p_3^{49}$ | $p_4^{56} = p_1^{49}$ | $E[x_2^{56}] = E[x_3^{49}]$ | $E[x_4^{56}] = E[x_1^{49}]$ | $E_2^{56} = E_3^{49}$ | $E_4^{56} = E_1^{49}$ |

In nodes 57 - 60 representing all possible versions of game 3, the continuation payoffs are hence given by:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 57 | $w_2^{57} = p_1^{49}E_2^{33} + p_3^{49}(p_1^{34}E_2^3 + p_4^{34}E_2^4)$ | $w_4^{57} = p_1^{50}(p_2^{35}E_4^5 + p_3^{35}E_4^6) + p_3^{50}E_4^{36} = p_3^{49}(p_4^{34}E_2^4 + p_1^{34}E_2^3) + p_1^{49}E_2^{33} = w_2^{57}$ | $l_2^{57} = p_1^{50}E_2^{35} + p_3^{50}(p_1^{36}E_2^7 + p_4^{36}E_2^8) = p_3^{49}E_4^{34} + p_1^{49}(p_3^{33}E_4^2 + p_2^{33}E_4^1)$ | $l_4^{57} = p_1^{49}(p_2^{33}E_4^1 + p_3^{33}E_4^2) + p_3^{49}E_4^{34} = l_2^{57}$ |
| 58 | $w_2^{58} = p_1^{51}E_2^{37} + p_4^{51}(p_1^{38}E_2^{11} + p_3^{38}E_2^{12}) = p_1^{49}E_2^{33} + p_3^{49}(p_1^{34}E_2^3 + p_4^{34}E_2^4) = w_2^{57}$ | $w_3^{58} = p_1^{52}(p_2^{39}E_3^{13} + p_4^{39}E_3^{14}) + p_4^{52}E_3^{40} = p_3^{49}(p_4^{34}E_2^4 + p_1^{34}E_2^3) + p_1^{49}E_2^{33} = w_2^{57}$ | $l_2^{58} = p_1^{52}E_2^{39} + p_4^{52}(p_1^{40}E_2^{15} + p_3^{40}E_2^{16}) = p_3^{49}E_4^{34} + p_1^{49}(p_3^{33}E_4^2 + p_2^{33}E_4^1) = l_2^{57}$ | $l_3^{58} = p_1^{51}(p_2^{37}E_3^9 + p_4^{37}E_3^{10}) + p_4^{51}E_3^{38} = p_1^{49}(p_2^{33}E_4^1 + p_3^{33}E_4^2) + p_3^{49}E_4^{34} = l_2^{57}$ |
| 59 | $w_1^{59} = p_2^{53}E_1^{41} + p_3^{53}(p_2^{42}E_1^{19} + p_4^{42}E_1^{20}) = p_1^{49}E_2^{33} + p_3^{49}(p_1^{34}E_2^3 + p_4^{34}E_2^4) = w_2^{57}$ | $w_4^{59} = p_2^{54}(p_1^{43}E_4^{21} + p_3^{43}E_4^{22}) + p_3^{54}E_4^{44} = p_3^{49}(p_4^{34}E_2^4 + p_1^{34}E_2^3) + p_1^{49}E_2^{33} = w_2^{57}$ | $l_1^{59} = p_2^{54}E_1^{43} + p_4^{56}(p_2^{48}E_1^{31} + p_3^{48}E_1^{32}) = p_3^{49}E_4^{34} + p_1^{49}(p_3^{33}E_4^2 + p_2^{33}E_4^1) = l_2^{57}$ | $l_4^{59} = p_2^{53}(p_1^{41}E_4^{17} + p_3^{41}E_4^{18}) + p_3^{53}E_4^{42} = p_1^{49}(p_2^{33}E_4^1 + p_3^{33}E_4^2) + p_3^{49}E_4^{34} = l_2^{57}$ |
| 60 | $w_1^{60} = p_2^{55}E_1^{45} + p_4^{55}(p_2^{46}E_1^{27} + p_3^{46}E_1^{28}) = p_1^{49}E_2^{33} + p_3^{49}(p_1^{34}E_2^3 + p_4^{34}E_2^4) = w_2^{57}$ | $w_3^{60} = p_2^{56}(p_1^{47}E_3^{29} + p_4^{47}E_3^{30}) + p_4^{56}E_3^{44} = p_3^{49}(p_4^{34}E_2^4 + p_1^{34}E_2^3) + p_1^{49}E_2^{33} = w_2^{57}$ | $l_1^{60} = p_2^{56}E_1^{47} + p_4^{56}(p_2^{48}E_1^{31} + p_3^{48}E_1^{32}) = p_3^{49}E_4^{34} + p_1^{49}(p_3^{33}E_4^2 + p_2^{33}E_4^1) = l_2^{57}$ | $l_3^{60} = p_2^{55}(p_1^{45}E_3^{25} + p_4^{45}E_3^{26}) + p_4^{55}E_3^{46} = p_1^{49}(p_2^{33}E_4^1 + p_3^{33}E_4^2) + p_3^{49}E_4^{34} = l_2^{57}$ |

Hence, winning probabilities, expected payoffs and expected efforts are given by:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 57 | $p_2^{57}$ | $p_4^{57} = p_2^{57}$ | $E[x_2^{57}]$ | $E[x_4^{57}] = E[x_2^{57}]$ | $E_2^{57}$ | $E_4^{57} = E_2^{57}$ |
| 58 | $p_2^{58} = p_2^{57}$ | $p_3^{58} = p_2^{57}$ | $E[x_2^{58}] = E[x_2^{57}]$ | $E[x_3^{58}] = E[x_2^{57}]$ | $E_2^{58} = E_2^{57}$ | $E_3^{58} = E_2^{57}$ |
| 59 | $p_1^{59} = p_2^{57}$ | $p_4^{59} = p_2^{57}$ | $E[x_1^{59}] = E[x_2^{57}]$ | $E[x_4^{59}] = E[x_2^{57}]$ | $E_1^{59} = E_2^{57}$ | $E_4^{59} = E_2^{57}$ |
| 60 | $p_1^{60} = p_2^{57}$ | $p_3^{60} = p_2^{57}$ | $E[x_1^{60}] = E[x_2^{57}]$ | $E[x_3^{60}] = E[x_2^{57}]$ | $E_1^{60} = E_2^{57}$ | $E_3^{60} = E_2^{57}$ |

In nodes 61 and 62 representing all versions of match 2, continuation payoffs are derived as follows:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 61 | $w_3^{61} = p_2^{57}E_3^{49} + p_4^{57}E_3^{50} =$<br>$p_2^{57}E_3^{49} + p_2^{57}E_1^{49} =$<br>$0.5E_3^{49} + 0.5E_1^{49}$ | $w_4^{61} = p_2^{58}E_4^{51} + p_3^{58}E_4^{52} =$<br>$p_2^{57}E_3^{49} + p_2^{57}E_1^{49} =$<br>$0.5E_3^{49} + 0.5E_1^{49} = w_3^{61}$ | $l_3^{61} = E_3^{58} = E_2^{57}$ | $l_4^{61} = E_4^{57} = E_2^{57} = l_3^{61}$ |
| 62 | $w_3^{62} = p_1^{59}E_3^{53} + p_4^{59}E_3^{54} =$<br>$p_2^{57}E_3^{49} + p_2^{57}E_1^{49} = w_3^{61}$ | $w_4^{62} = p_1^{60}E_4^{55} + p_3^{60}E_4^{56} =$<br>$p_2^{57}E_3^{49} + p_2^{57}E_1^{49} = w_3^{61}$ | $l_3^{62} = E_3^{60} = E_2^{57} = l_3^{61}$ | $l_4^{62} = E_4^{59} = E_2^{57} = l_3^{61}$ |

Winning probabilities, expected payoffs and expected efforts in nodes 61 and 62 are given by:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 61 | $p_3^{61}$ | $p_4^{61} = p_3^{61}$ | $E[x_3^{61}]$ | $E[x_4^{61}] = E[x_3^{61}]$ | $E_3^{61}$ | $E_4^{61} = E_3^{61}$ |
| 62 | $p_3^{62} = p_3^{61}$ | $p_4^{62} = p_3^{61}$ | $E[x_3^{62}] = E[x_3^{61}]$ | $E[x_4^{62}] = E[x_4^{61}]$ | $E_3^{62} = E_3^{61}$ | $E_4^{62} = E_3^{61}$ |

Finally, the continuation payoffs for the respective players in node 63 (i.e. match 1) can be rewritten as follows:

| Node | $w_A^k$ | $w_B^k$ | $l_A^k$ | $l_B^k$ |
|---|---|---|---|---|
| 63 | $w_1^{63} = p_3^{61}(p_2^{57}E_1^{49} + p_4^{57}E_1^{50}) +$<br>$p_4^{61}(p_2^{58}E_1^{51} + p_3^{58}E_1^{52}) =$<br>$0.5(0.5E_1^{49} + 0.5E_3^{49}) +$<br>$0.5(0.5E_1^{49} + 0.5E_3^{49}) =$<br>$0.5E_1^{49} + 0.5E_3^{49} = w_3^{61}$ | $w_2^{63} = p_3^{62}(p_1^{59}E_2^{53} + p_4^{59}E_2^{54}) +$<br>$p_4^{62}(p_1^{60}E_2^{55} + p_3^{60}E_2^{56}) =$<br>$0.5(0.5E_1^{49} + 0.5E_3^{49}) +$<br>$0.5(0.5E_1^{49} + 0.5E_3^{49}) =$<br>$0.5E_1^{49} + 0.5E_3^{49} = w_3^{61}$ | $l_1^{63} = p_3^{62}E_1^{59} + p_4^{62}E_1^{60} =$<br>$0.5E_2^{57} + 0.5E_2^{57} = E_2^{57} = l_3^{61}$ | $l_2^{63} = p_3^{61}E_2^{57} + p_4^{61}E_2^{58} =$<br>$0.5E_2^{57} + 0.5E_2^{57} = E_2^{57} = l_3^{61}$ |

The winning probabilities, expected payoffs and expected efforts of match 1 are thus given by:

| Node | $p_A^k$ | $p_B^k$ | $E[x_A^k]$ | $E[x_B^k]$ | $E_A^k$ | $E_B^k$ |
|---|---|---|---|---|---|---|
| 63 | $p_1^{63} = p_3^{61}$ | $p_2^{63} = p_3^{61}$ | $E[x_1^{63}] = E[x_3^{61}]$ | $E[x_2^{63}] = E[x_3^{61}]$ | $E_1^{63} = E_3^{61}$ | $E_2^{63} = E_3^{61}$ |

Notice that $E_1^{63} = E_3^{61}$ and $E_2^{63} = E_3^{61}$. Moreover, $E_3^{63} = p_1^{63} E_3^{61} + p_2^{63} E_3^{62} = 0.5 E_3^{61} + 0.5 E_3^{61} = E_3^{61}$ and $E_4^{63} = p_1^{63} E_4^{61} + p_2^{63} E_4^{62} = 0.5 E_3^{61} + 0.5 E_3^{61} = E_3^{61}$. Therefore, $E_1^{63} = E_2^{63} = E_3^{63} = E_4^{63}$.[20]

[20]Straightforward computations show that the players' ex-ante expected winnings coincide as well.

# B Game trees

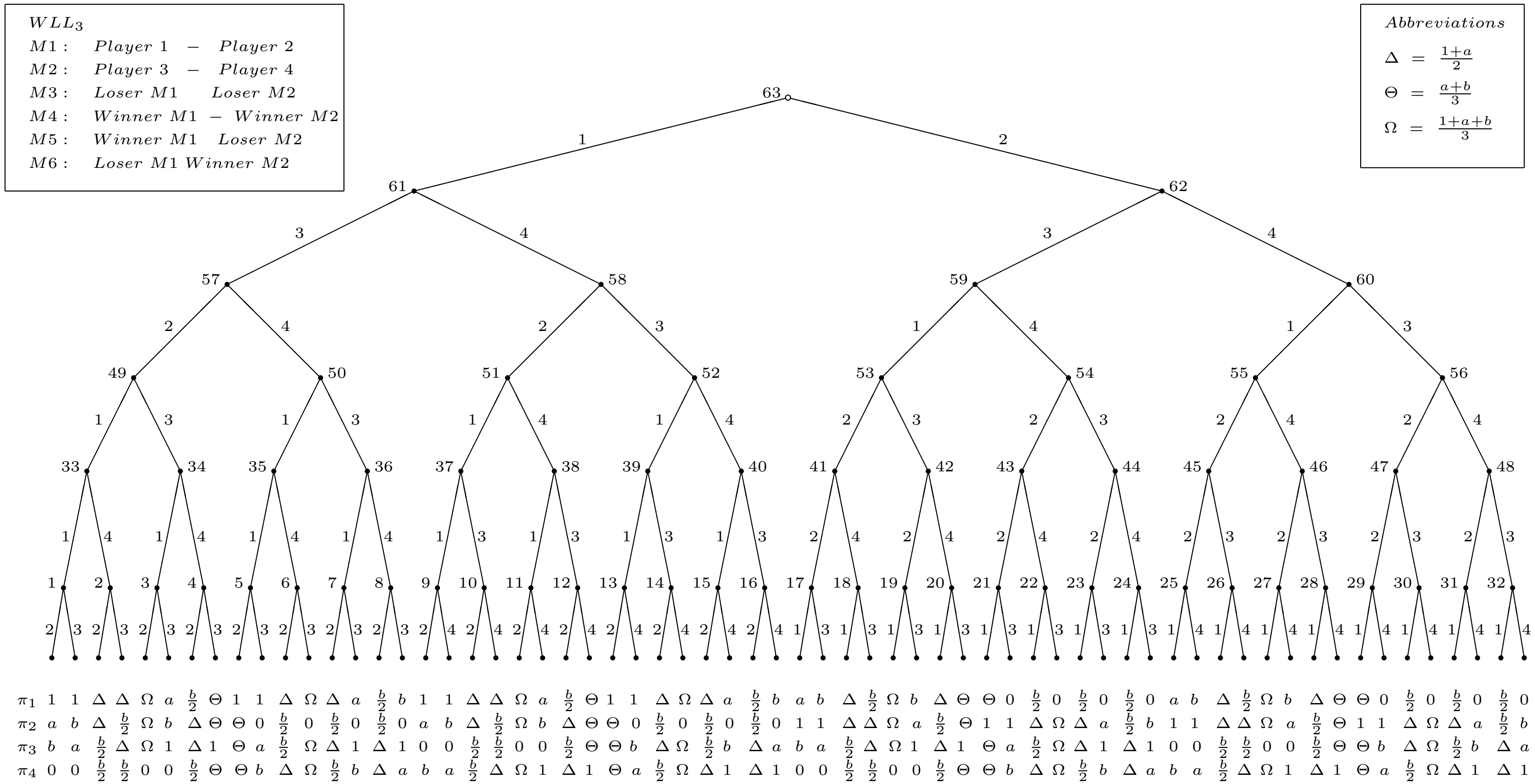



Figure 1: Game tree for $LLW_1$

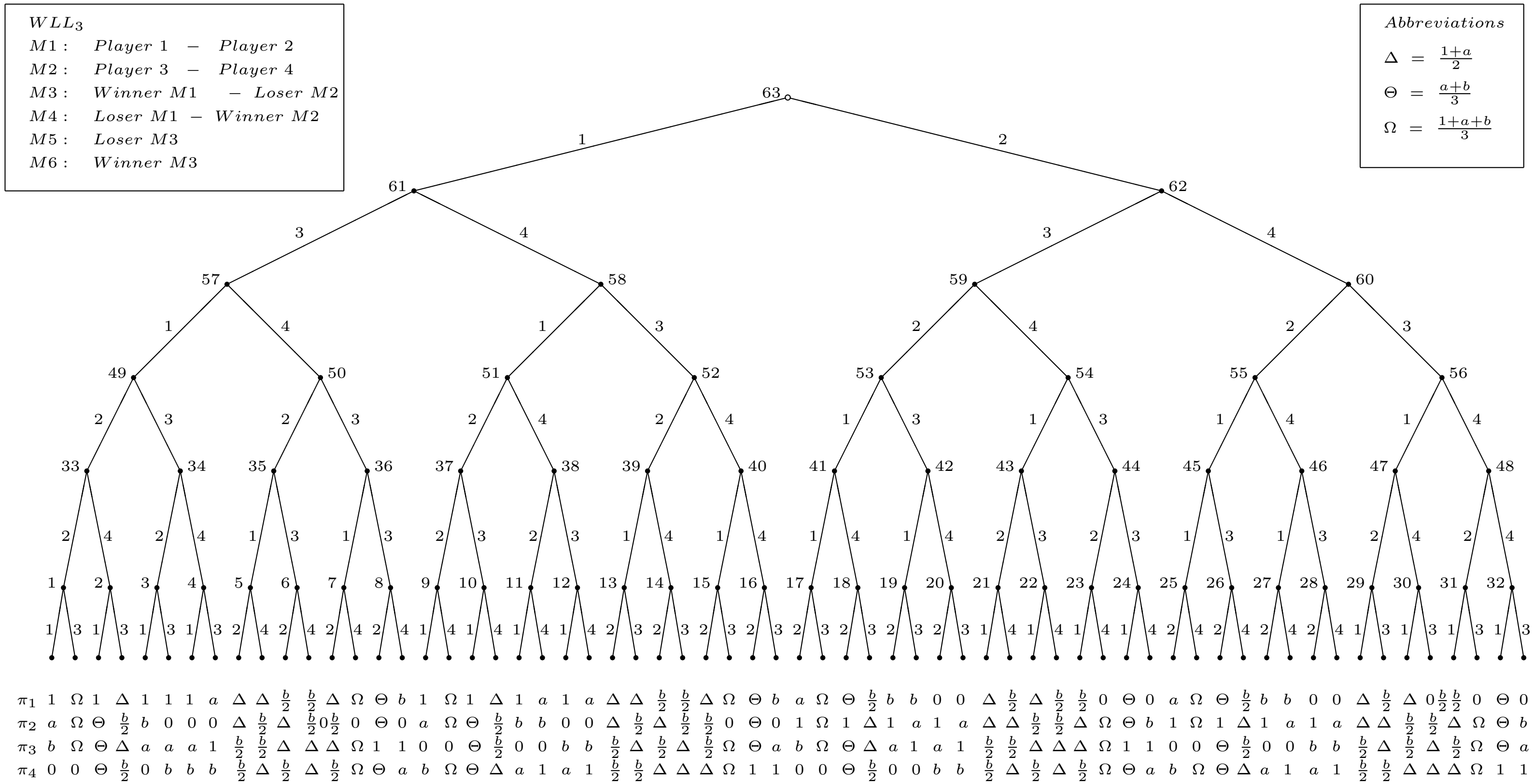



Figure 2: Game tree for $WLL_3$

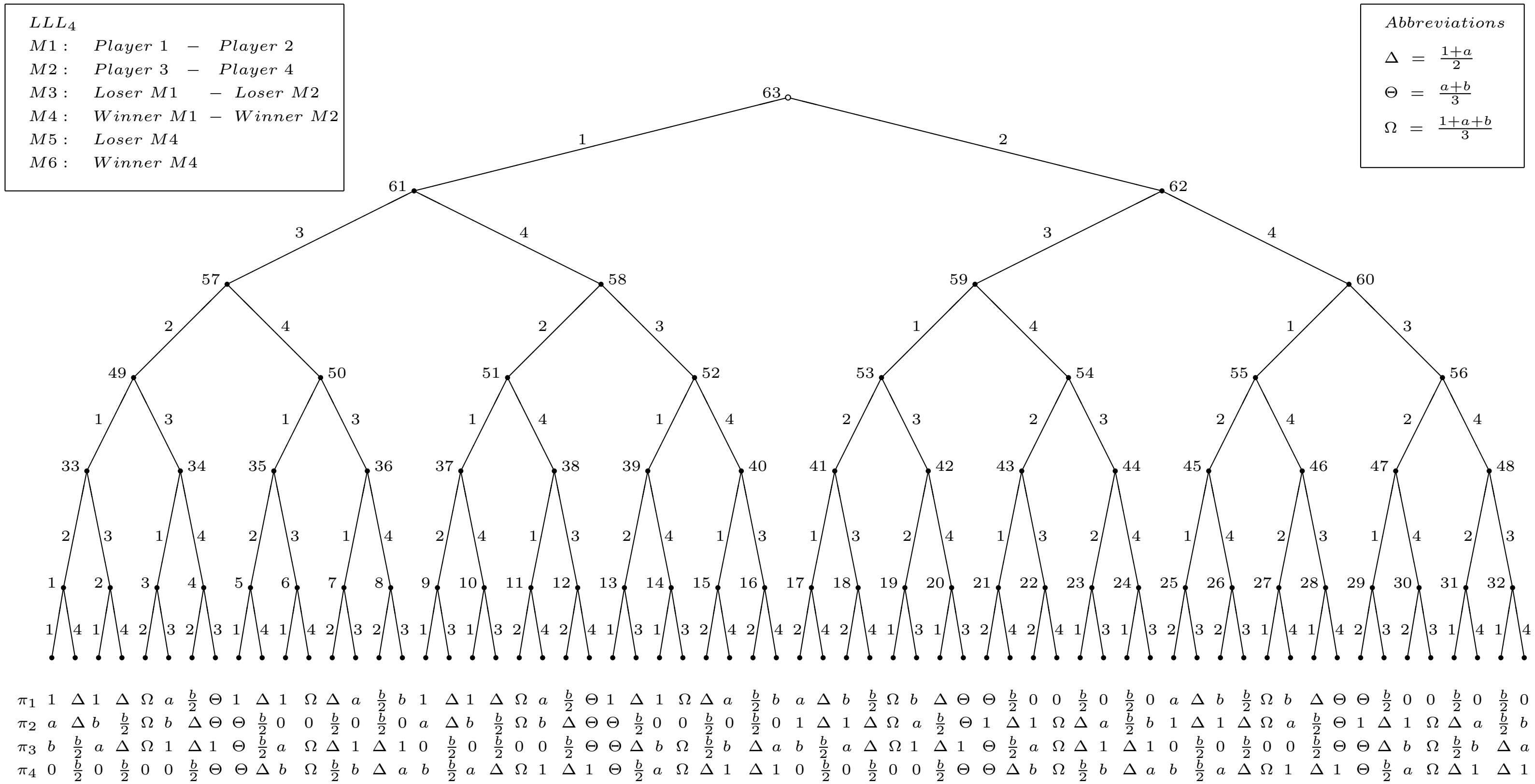



Figure 3: Game tree for $LLL_4$

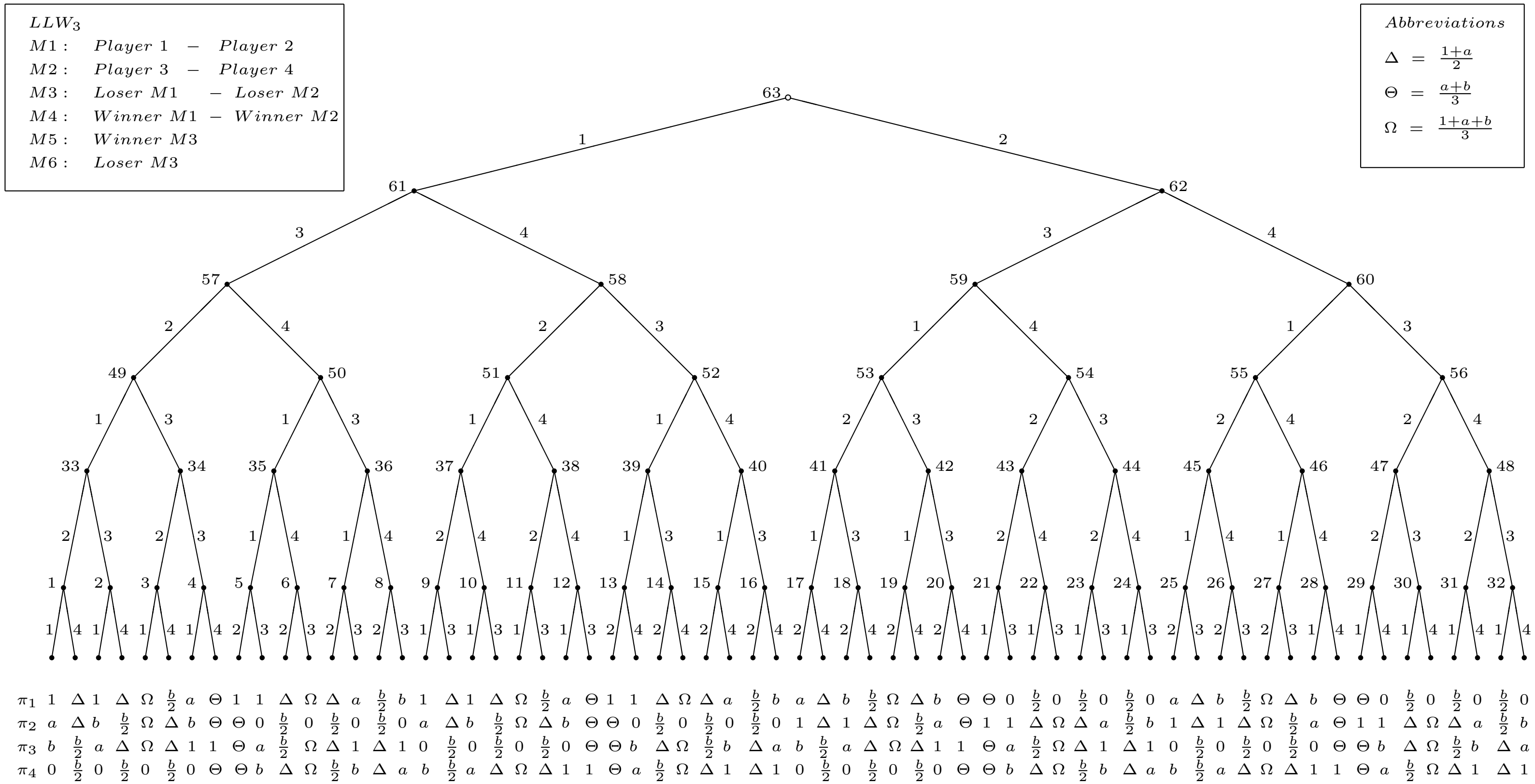



Figure 4: Game tree for $LLW_3$

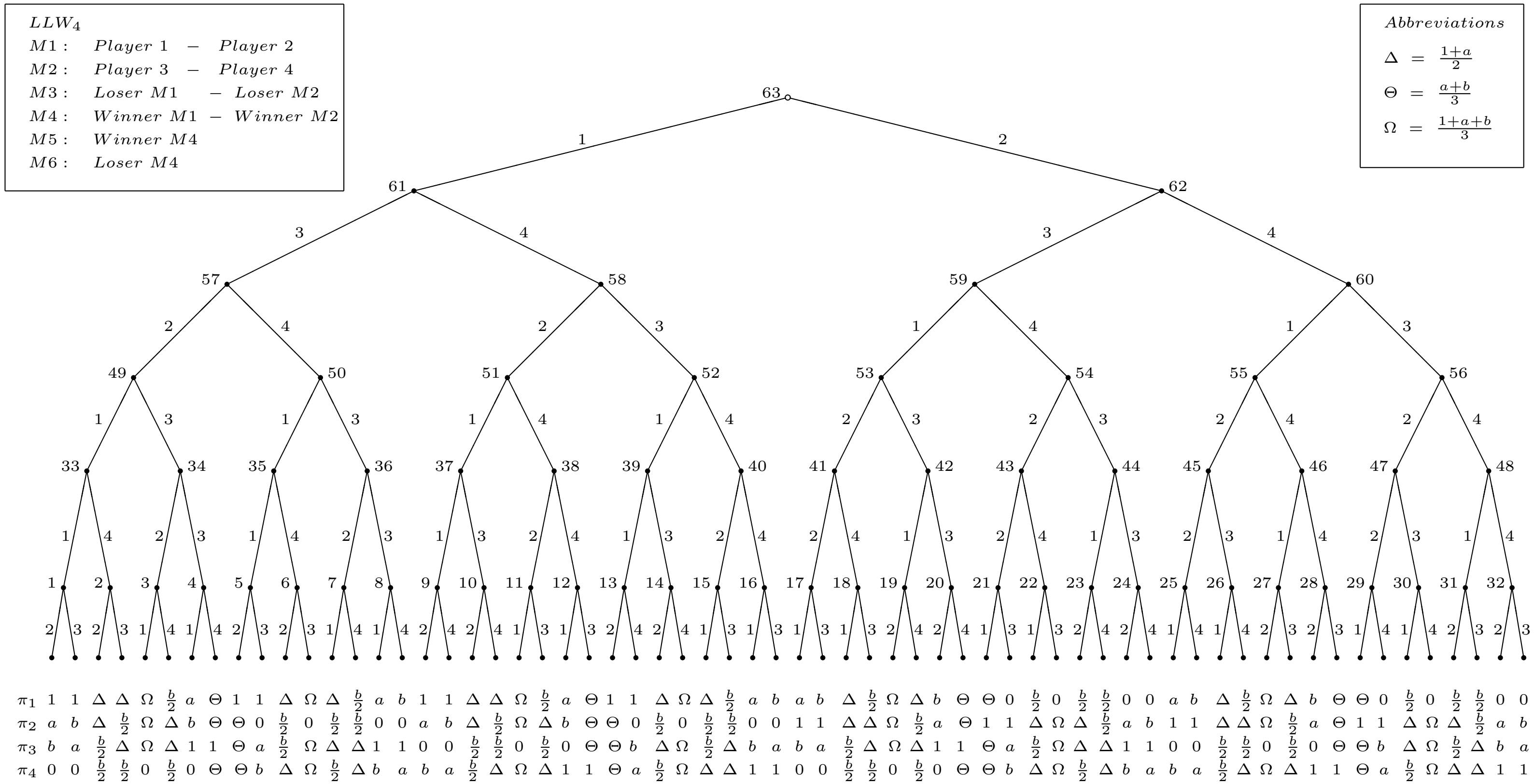



Figure 5: Game tree for $LLW_4$

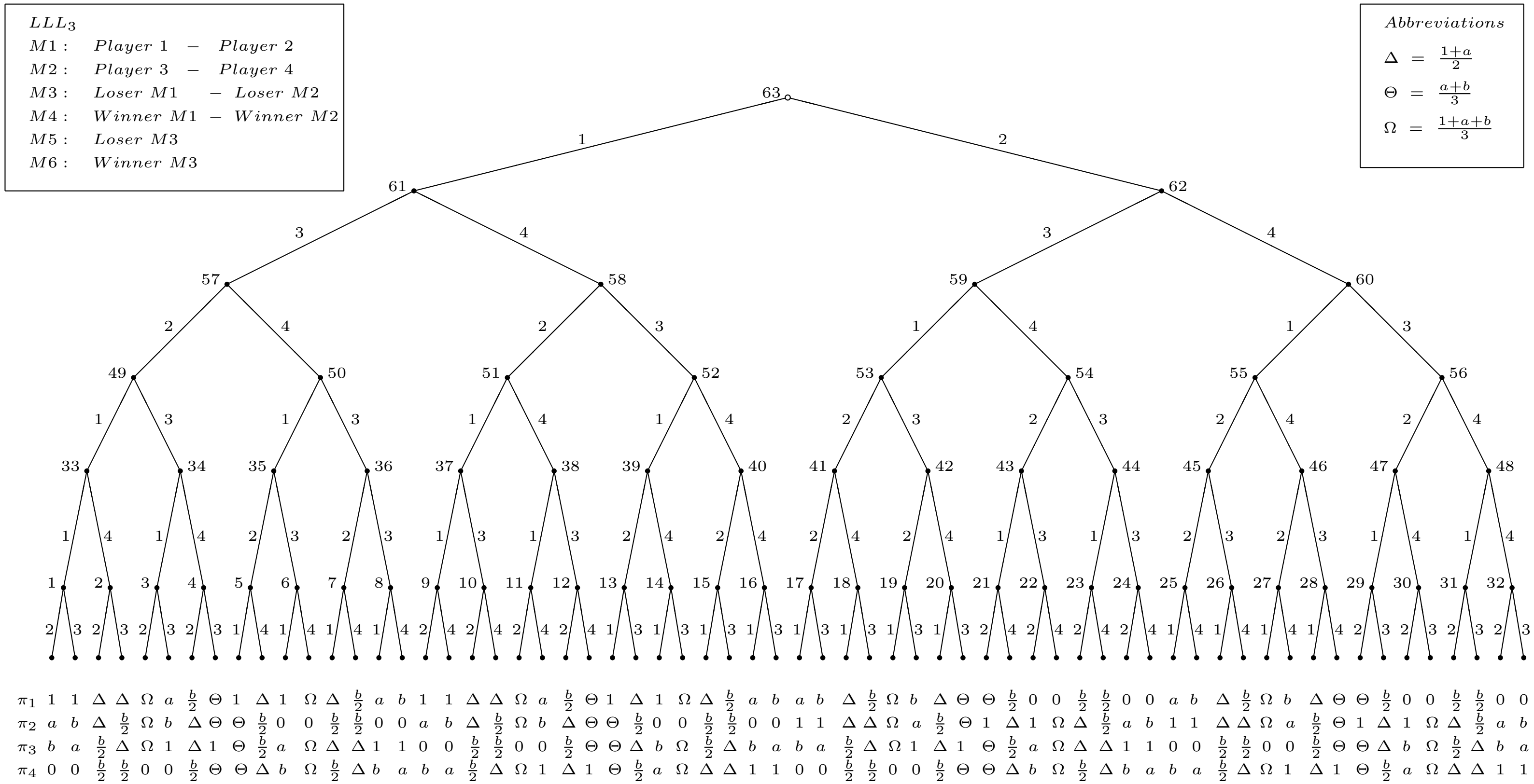



Figure 6: Game tree for $LLL_3$

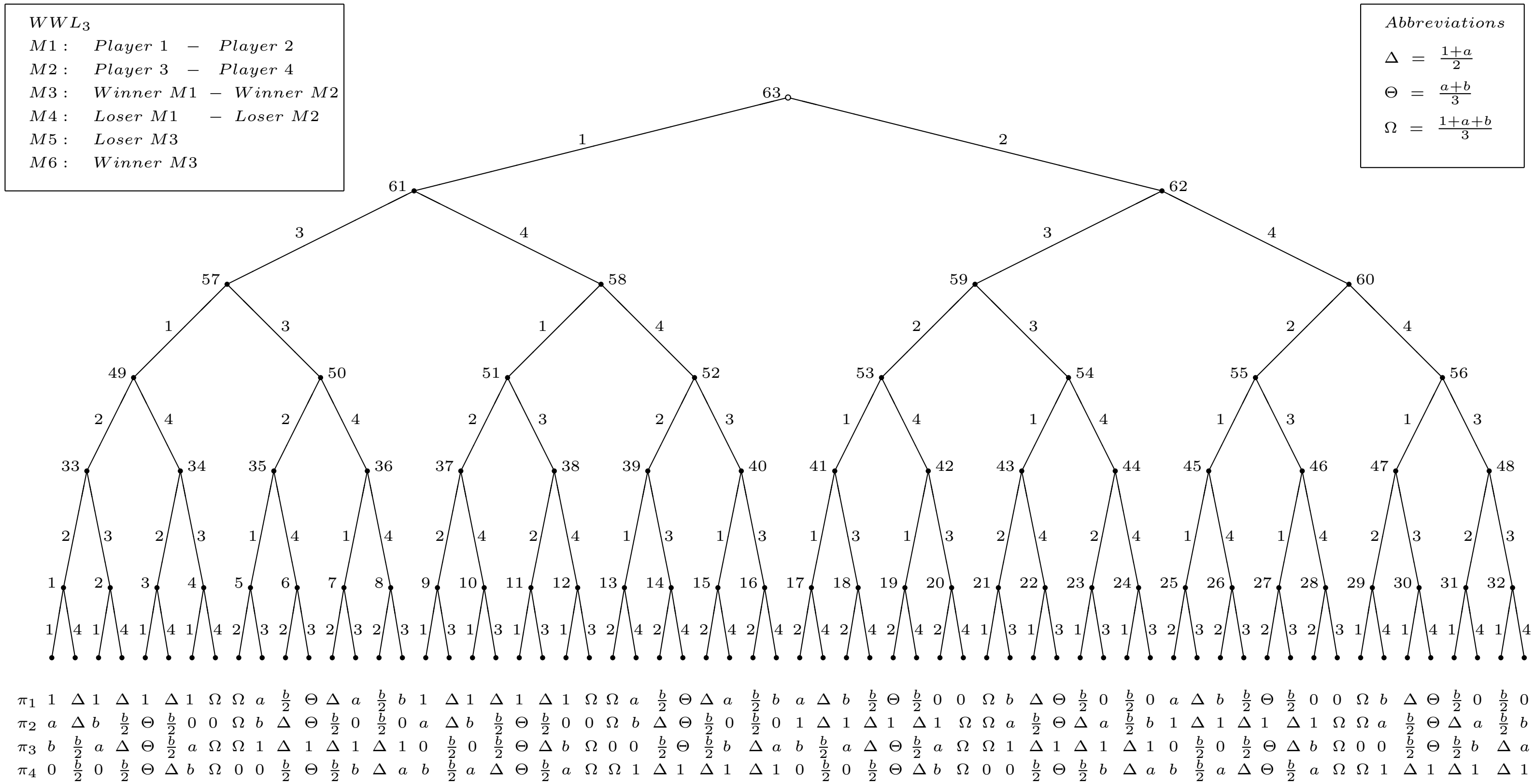



Figure 7: Game tree for $WWL_3$

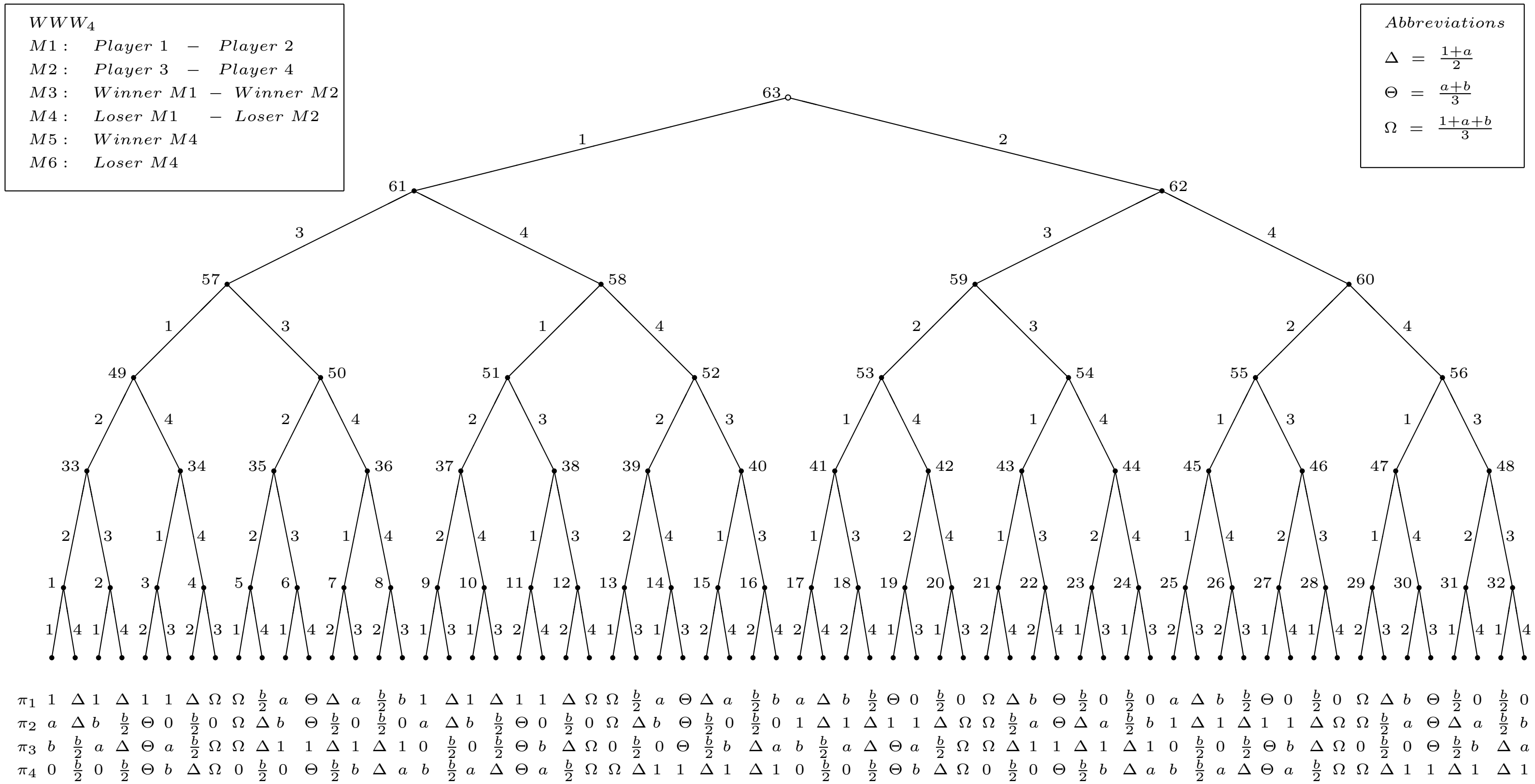



Figure 8: Game tree for $WWW_4$

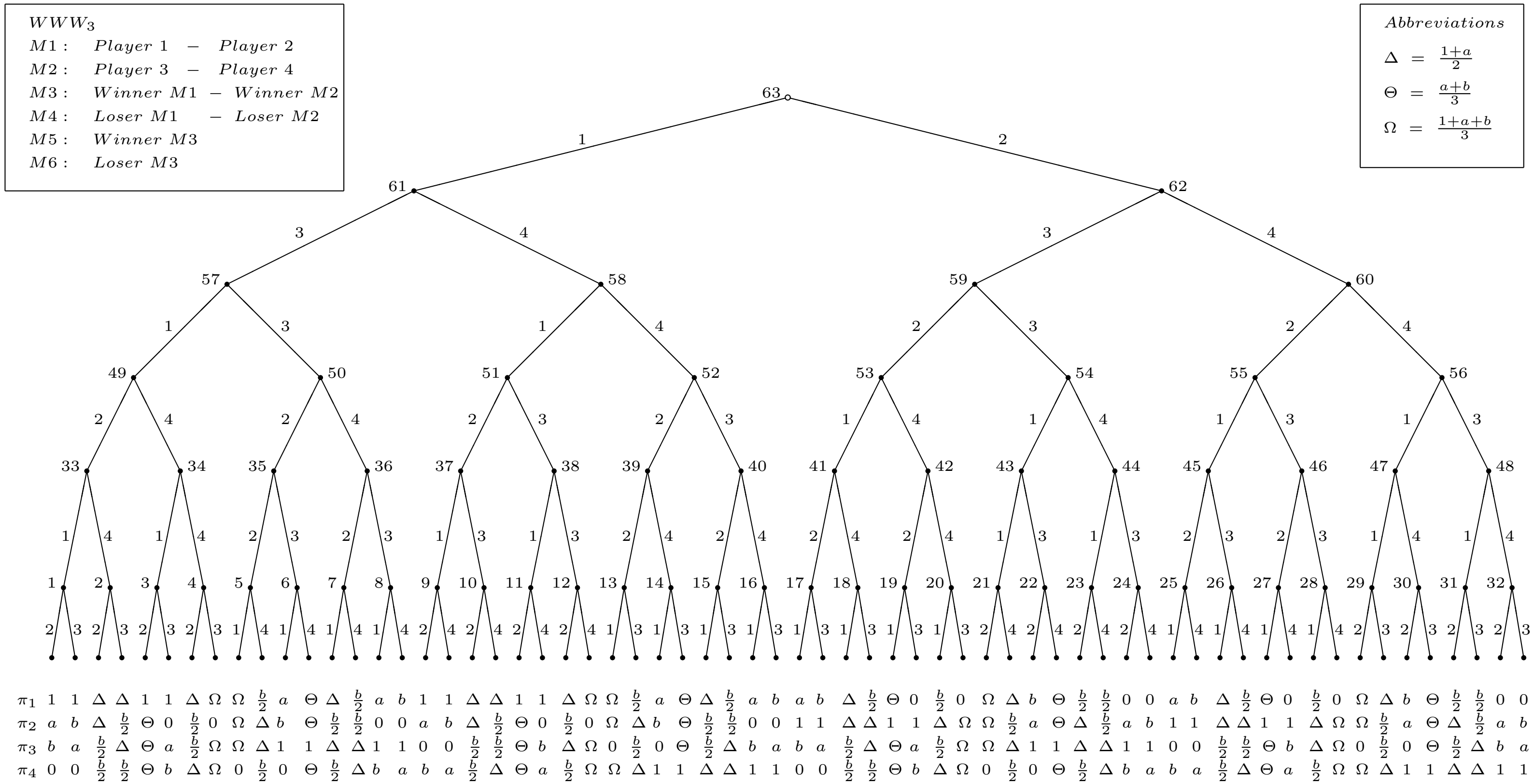



Figure 9: Game tree for $WWW_3$

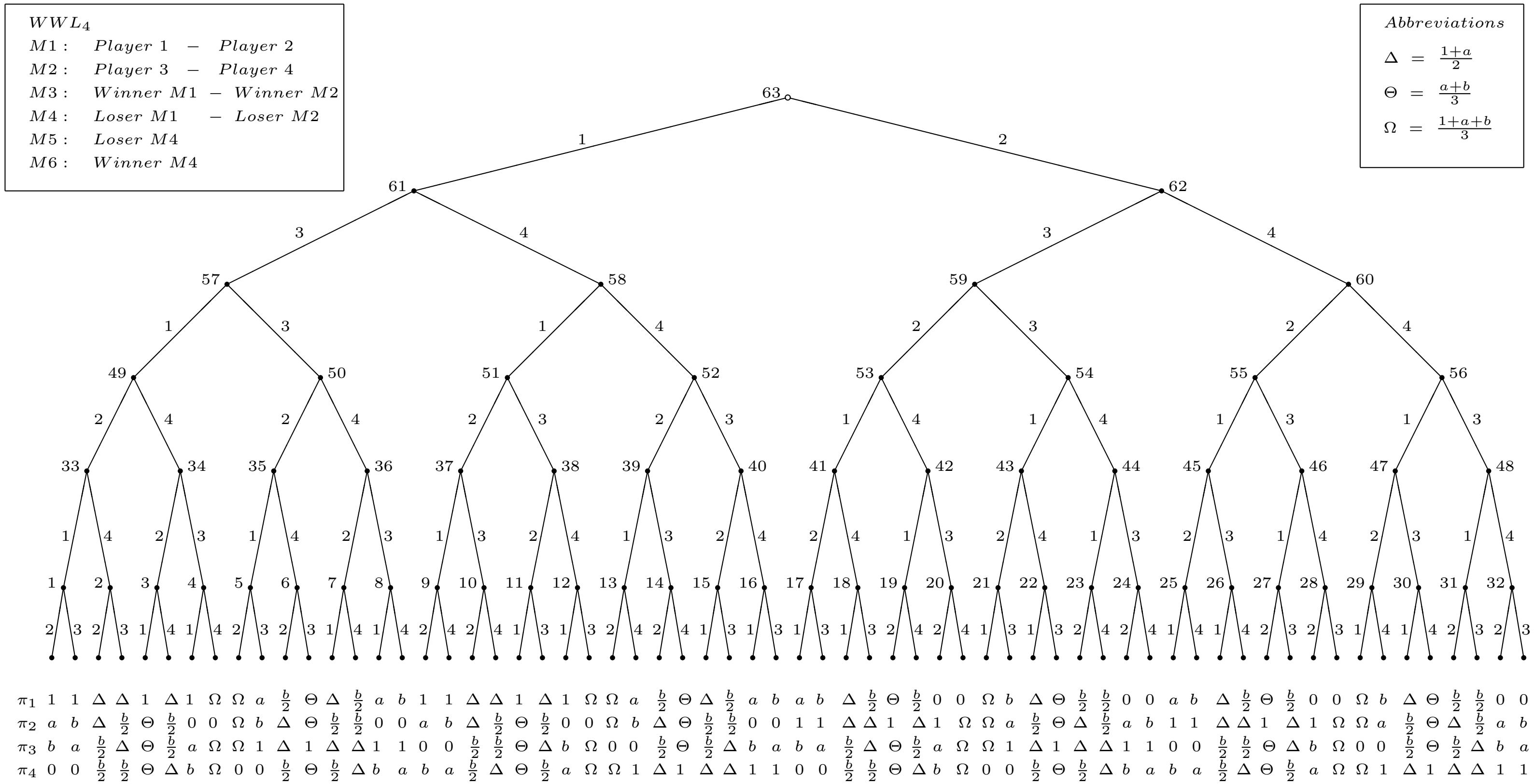



Figure 10: Game tree for $WWL_4$

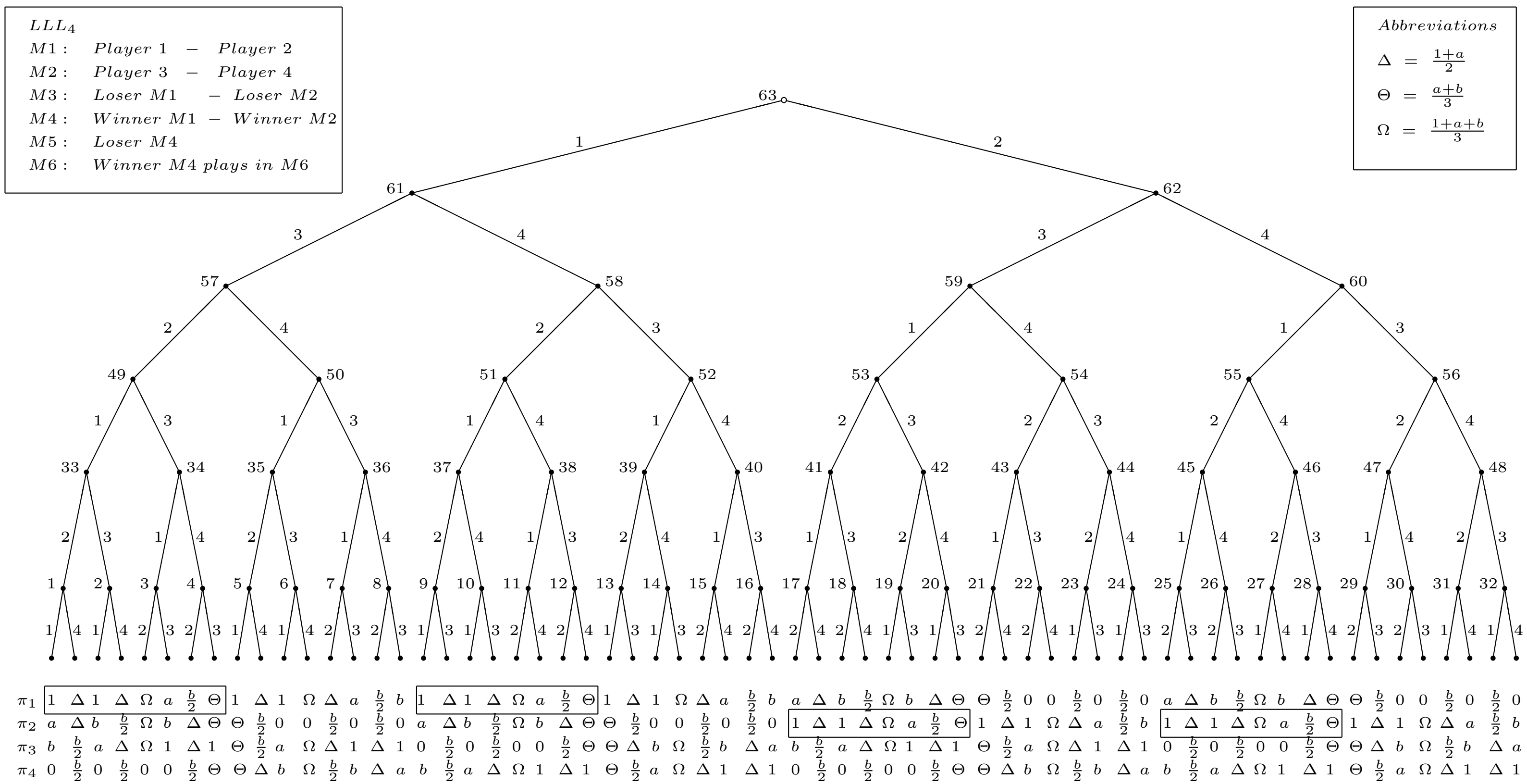
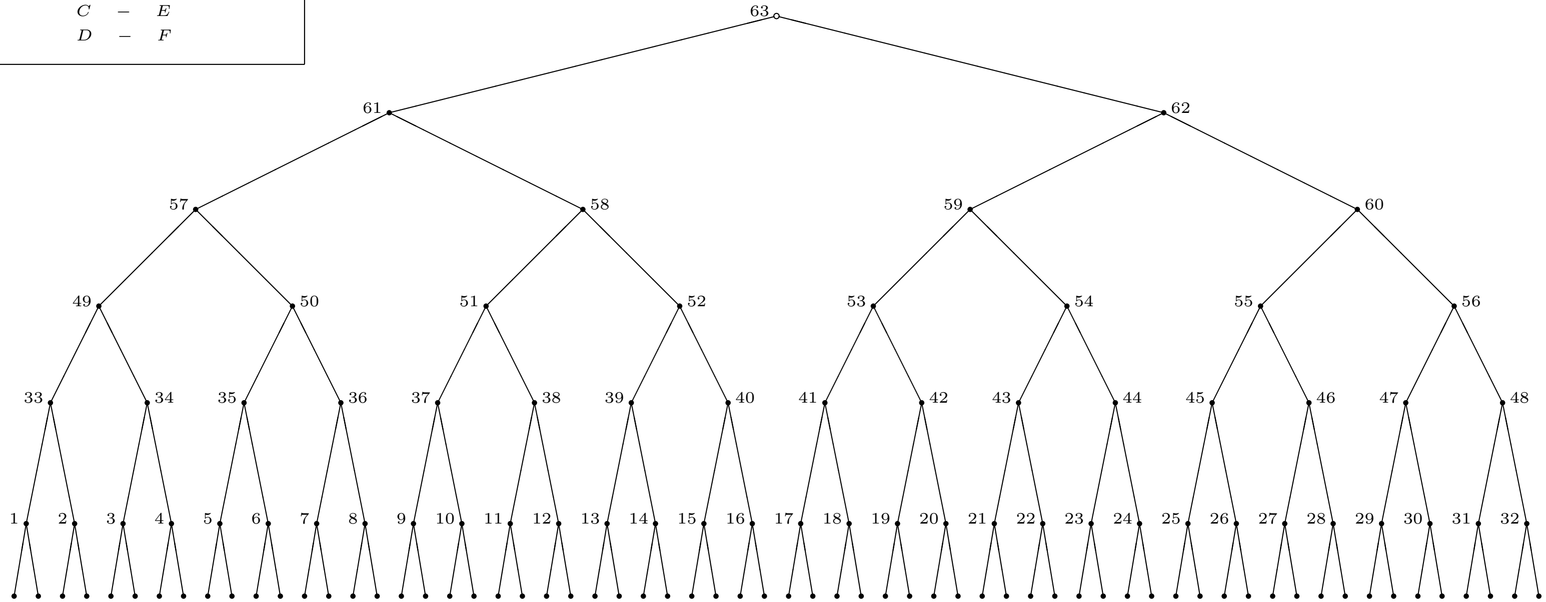



Figure 11: Game tree for $LLL_4$: in boxes find payoff sequence A

*Mirror − inverse counterparts*

| | | |
|---|---|---|
| $A$ | – | $G$ |
| $B$ | – | $H$ |
| $C$ | – | $E$ |
| $D$ | – | $F$ |




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $A$ | $E$ | $A$ | $E$ | $B$ | $F$ | $B$ | $F$ |
| $\pi_2$ | $B$ | $F$ | $B$ | $F$ | $A$ | $E$ | $A$ | $E$ |
| $\pi_3$ | $C$ | $G$ | $D$ | $H$ | $C$ | $G$ | $D$ | $H$ |
| $\pi_4$ | $D$ | $H$ | $C$ | $G$ | $D$ | $H$ | $C$ | $G$ |

Figure 12: Game tree illustrating the symmetry argument

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