OPTIMAL TAXATION AND NORMALISATIONS

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Abstract

There still seems to be some confusion about the consequences of normalisations in the optimal taxation literature. We claim that:
1) Normalisations do not matter for the real solution of optimal taxation problem.
2) Normalisations do matter for good characterisations of the solutions to optimal taxation problems.
Whereas the first point is uncontroversial, the second one is less well understood. There is also a need to distinguish between the following senses of taxation of endowment:
1) The taxation of own consumption of initial endowments (e.g. leisure).
2) The taxation of the sale of initial endowments (e.g. labour).
By postponing the normalisation of consumer prices, we detail how normalisations of consumer prices affect the characterisation of optimal commodity taxes, derive the preferred characterisation, and show how it depends on the normalisation. On the way, we discuss the effect of normalisations on measures of the marginal efficiency loss of taxation.

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1 Introduction

There still seems to be some confusion about the the role of normalisations in the optimal taxation literature, even with perfect competition, complete markets and full information. We set out to clarify this matter, and claim that, with all income generated by endowments:

1. Normalisations do not matter for the real solution of optimal taxation problem.[1]

2. Normalisations do matter for (good) characterisations of the solutions to optimal taxation problems.

The first point is uncontroversial. I think the second one is also fairly obvious. The idea is clarified by an analogy with the view (analogous to the normalisation) of a (3-dimensional) object (analogous to the solution). For such objects, it is clear that: 1) The view does not change the object. 2) The view is important for seeing the essential properties of the object. In our case the homogenity of demand in consumer prices introduces one degree of freedom in choosing the "view" on the solution.

We support this distinction by showing in detail how normalisations affect the characterisation, and arrive at a canonical (preferred) normalisation.

One also needs to distinguish between two kinds of taxation of leisure:

1. The taxation of own consumption of initial endowments (e.g. leisure).

2. The taxation of the sale of initial endowments (e.g. labour).

In standard optimal taxations models, untaxed own consumption of initial endowments is usually what makes first-best unattainable, and thus a basic trait of the models. On the other hand, untaxed sales of initial endowments is only a normalisation of consumer prices, though, as we will argue, the canonical one. More specifically, we claim that, as long as there are no profit in the economy, consumer prices should be normalised so that the *tax income from initial endowments is zero* at the optimum. This is of course a straightforward generalisation of the standard case, where leisure is the only endowment and labour is untaxed. Using this normalisation, we arrive at a generalised inverse elastisity rule, generalising slightly the characterisation in Deaton (1979). The rule says that one should tax complements to the initial endowments harder. The intuitive explanation is that doing this is an indirect way of introducing a tax on the own consumption of endowments.

The assumption of perfect competition is important, for, as noted by Gabszewicz and Vial (1972), with imperfect competition the price normalization also matters for the real solution of the taxation problem. Additionally, one are usually not able to solve such general equilibrium models without making a normalization.

In the following, after discussing some of the literature, we first set up the commodity tax model, then makes our simple points concerning the second

[1] This presupposes demand and supply functions which are homogenous of degree 0 in consumer and producer prices. If not, Dixit and Munk (1977) show that normalisations also matter for the real solution.

distinction using the homogeneity of demand. Finally, we detail how the normalisation of consumer prices affect the characterisation of optimal commodity taxes, and arrive at our canonical normalisation.

1.1 Some examples from the literature

Are not these two distinctions well-known? In one sense they are, as most people use the appropriate normalisations and the resulting intuitive interpretations. People explicitly discussing normalisations in an optimal taxation context, however, tend to miss at least the first distinction, as I will try to show. Otherwise, however, this article build on the work of the people we criticise here, as the model presentation is only a slight variation of that of Deaton (1979) and Auerbach (1987).

Myles Myles in his textbook (1995, p. 122-123) seems to claim that interpretations should be independent of normalisations:

> It has been shown that in an economy with constant returns to scale, consumer and producer prices can be normalised separately and that the standard procedure is to make one good the numeraire and set its consumer and producer prices equal. This normalisation also has the effect of setting the tax on that good to zero. The latter fact is clearly seen to be of no consequence whatsoever since the zero tax is just a result of the normalisation rule. In particular, the zero tax carries no implications about the nature of the good nor about the ability to tax that good. This follows since the good with zero tax can be chosen arbitrarily from the set of available goods.

The "no consequences whatsoever" might indicate that he misses the first distinction above, but this is not clear. He continues:

> Unfortunately, this reasoning has not been as clearly appreciated in some of the literature as it should have been. The reason for this has been the convention, as adopted in sections 3 and 4 of this chapter, of taking labour as the untaxed commodity. Since labour is often viewed as the negative of leisure, it has been inferred from this that, since leisure cannot be measured in the same way that purchases of other commodities can, the zero tax on leisure is a restriction on the permissible tax system brought about by an inability to tax leisure. In addition, the further inference is usually made that the optimal tax system aims to overcome the missing tax on leisure by taxing goods complementary to leisure.

Here the second distinction between the two types of taxation of leisure seems to be lost. He continues:

> Particular examples of this are found in Corlett and Hague (1953) 'By taxing those goods complementary with leisure, one is to some extent taxing leisure itself' (p. 26) and Layard and Walters (1978) 'The theory of second best tells us that it we cannot tax leisure, we can do better than by taxing all other goods, equiproportionately'

> (p. 184). Many other instances of similar statements could easily be given. This, of course, is a false interpretation. When real restrictions upon the permissible range of tax instruments are introduced the results obtained are affected. A number of such restrictions are considered in Munk (1980) where it is shown that the resulting optimal tax structure is sensitive to the precise restrictions imposed.

Here he seems to infer that interpretations based on a specific normalisation is illegitimate, and thus our first distinction is lost. Further down the same page he continues mixing up the difference between the two types of taxation of leisure:

> A further mistake that has arisen in this context can be found in Dixit (1970) and Lerner (1970). In a single-household economy, any required revenue can be raised most efficiently by a lump-sum tax on the household equal to the value of the revenue. Noticing this, it has been suggested that a set of commodity taxes which raise the price of all goods by the same proportion will have the same effect as the lump-sum tax and therefore that when all goods can be taxed, the optimal system has the same proportional tax on all goods. This conclusion is clearly in contrast to that of the Ramsey rule. The mistake in the reasoning was pointed out by Sandmo (1974) who demonstrated that such a proportional tax system would raise no revenue. This follows since households both demand goods and supply labour. A proportional tax then taxes demands but subsidises supplies and, since the value of household demand equals the value of supply, the proportional tax is just offset by the proportional subsidy. Effectively, the proportiona1 tax on all commodities is just a rescaling of the consumer price vector which does not affect household choices.

Here, the first claim, attributed to Dixit and Lerner, is true for an equal (value) tax on all consumption, including *own consumption* of initial endowments (e.g. leisure), whereas the second claim, attributed to Sandmo, is valid for an equal value tax on excess demand. With leisure as the only initial endowment, this means an equal value on demand of goods and *sales of leisure* (labour). Thus in our interpretation, there is essentially no conflict between the two claims, though we are back to first-best with taxation of own consumption of endowments.

Auerbach Auerbach in his handbook article (1985, p. 89) clearly notes the difference between the two types of taxation of leisure. At another place, however, he seems more sloppy. Thus in the case with an endowment of leisure only, while we argue that one should use leisure as numeraire, as this gives the most intuitive interpretation, he warns (on p. 90) that making leisure untaxed leads to a "loss of distinction between untaxable and untaxed goods." As mentioned above, this only happens only if one does not discriminate between own consumption of leisure (untaxable by model setup) and sale of leisure (untaxed due to the normalisation). Then (on p. 92) in discussing the Corlett and Hague (1953-54) rule, he states:

> Expression (5.21) [the Corlett-Hague rule] calls for a higher tax on on the taxed good that is a complement to the numeraire. This generated the somewhat misleading explanation that we "cannot" tax good zero, so we minimize distortions by taxing more heavily its relative complements. Recall that the choice of untaxed good is arbitrary, and that (5.21] applies for any numbering of the three goods.

This is in a way correct, but suggests that there is nothing special by making leisure untaxed. Our point is that the *interpretation* of the Corlett-Hague rule only makes sense if leisure is untaxed. Taxing more heavily relative complements with leisure makes sense, as the untaxed own consumption of leisure is the reason for not obtaining the first-best in this model. To give an intuitive explanation of why one should tax relative complements with an arbitrary consumer good, taken as numeraire, on the other hand, is close to impossible. As we will see, it is only with leisure untaxed, that we can generalize the Corlett-Hague rule, in the sense that we get a general characterisation with essentially the same interpretation.

Deaton Even Deaton (1981, p. 1256) seems confused about the role of normalisations. Discussing the characterisation of optimal taxes (equation (14) below) he states:

> Note the special place occupied by good 0, leisure. ... the asymmetry is due to the numeraire role of labor (or leisure). Since leisure is untaxed, government revenue is implicitly measured in labor units so that by taxing complements with the revenue good, taxation is rendered easier. In general, the government will presumably wish to purchase goods other than labor and this would lead to a different tax rule. For example, a king who must pay a tribute of oxen to a neighboring conqueror would do well to levy relatively high taxes on goods complementary with oxen.

Stern Stern (1986, p. 298) claims that "there have been a temptation "to elevate the innocent normalization ... into something of of real substance". He then points out the error in Deatons above claim. He is, however, not quite precise about what is going on as he states that

> The crucial reason for the central role of complementarity with leisure in the results concerning the optimum proportion of tax in price is is that there is an endowment of leisure which cannot be taxed in this second-best problem.

The root of the problem is as we have said that own consumption of leisure is untaxed. That labor (sales of leisure) is untaxed is an innocent normalisation. Furthermore, the fact that normalisations are irrelevant for the solution of the optimal taxation problem does not imply that they are irrelevant for the characterisation of the problem.

It looks like Stern does not discriminate between the task of *understanding* the principles of taxation and the task of *solving* a taxation problem. For the first task we need an intuitive characterisation, and for this normalisations matter, whereas for the second task normalisations are irellevant – but so are normally the characterisations. This is different from the case with explicit analytic solutions, where one wants to arrange the solution in an easily interpretable way.

Stern also lists some other questionable reservations against Deaton's approach. One is that the symmetry between goods and factors are lost on Deatons approach. This is true, but that symmetry is replaced by the more fundamental one between positive and negative net trades. Another is, if I understand him correctly, is that the duality results underlying Deaton's approach presuppose positive lump-sum income. This is wrong, as the duality results only presupposes positive (full) income, and this is for all practical purposes fulfilled without lump-som income. Finally, he claims that it is hard to measure or even define the endowment of leisure. But the measurement of the endowment of goods is more a problem when it comes *solving* an optimal taxation problem than for *understanding* the principles of taxation. For the last task, I think Deatons's approach is the preferred one.

2 The optimal commodity taxation model

We use the standard optimal commodity taxation model with one price-taking individual, except that we allow for initial endowments, $\overline{\mathbf{x}} \geq \mathbf{0}$, of all goods.[2] Assume fixed producer prices, $\overline{\mathbf{p}}$.[3] With quantity taxes, $\mathbf{t}$, on market transactions, $\mathbf{x} - \overline{\mathbf{x}}$, and no lump sum tax,[4] consumer prices are $\mathbf{p} = \overline{\mathbf{p}} + \mathbf{t}$, and the income of the individual, $m = \overline{m} + \mathbf{p}'\overline{\mathbf{x}}$, where $\overline{m} = 0$ is exogenously given income. The problem for efficiency is as usual that own consumption of initial endowments is untaxed.

Assume that the public sector maximises the utility of the individual given an exogenously given tax requirement, $\overline{T}$. Normalise producer prices by setting income at producer prices, $\overline{\mathbf{p}}'\overline{\mathbf{x}} = 1$. Then $\overline{T}$ is the tax requirement as a share of the income at producer prices. Also let $v(\mathbf{p}, m)$ be the indirect utility function and $\mathbf{x}(\mathbf{p}, m)$ the demand of the individual. The governments problem is then:

$$V(\overline{\mathbf{p}}, \overline{m}, \overline{T}) = \max_{\mathbf{t}} v(\mathbf{p}, m) \quad \text{subject to} \quad \mathbf{t}'\left(\mathbf{x}(\mathbf{p}, m) - \overline{\mathbf{x}}\right) - \overline{T} \geq 0 \quad (\mu). \tag{1}$$

The marginal social utility of (exogenously) income to the individual**,** $V_{\overline{m}} = v_m + \mu \mathbf{t}'\mathbf{x}_m$, and the marginal social utility of increasing the tax income requirement, $V_{\overline{T}} = -\mu$. Thus μ is the marginal social utility of reducing the public tax requirement, and $\theta = (\mu - V_{\overline{m}})/\mu$ is the value of transferring one pound (exogenously) from the individual to the public. Note that all these concepts are evaluated at consumer prices.

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[2] We also assume a complete set of markets, price-taking actors and no profit. Without these assumptions, even the real solution to the problem might depend on the normalisation.

[3] This corresponds to a (constant scale) Leontief technology. The assumption of fixed producer prices can be replaced with a linear technology without any consequences for the results. The only difference is that then we also have to make the producer price normalisation at the prices in optimum, as we do for the consumer price normalisation below.

[4] This is of course unrealistic. It is usually justified by saying that one are interested in efficiency results in an economy with many individuals, but this is not efficiency in the Pareto sense.

Before characterising the solution, we discuss different normalisations of consumer prices.

3 Normalisation of consumer prices

Individual demand, $\mathbf{x}(\mathbf{p}, m)$, is homogenous of degree 0 in prices and income. With no exogenously income, $\overline{m} = 0$, demand $\mathbf{x}(\mathbf{p}, \mathbf{p}'\overline{\mathbf{x}})$ is homogeneous of degree 0 in consumer prices, $\mathbf{p}$. Thus we can scale (normalise) the consumer prices, $\mathbf{p}$, without affecting demand. This homogeneity is often used to set the consumer price on one good equal to its producer price, so that the tax on this good is 0. We illustrate the consequences of different choices with an example.

Example 1 *With a (constant scale) Leontief technology, choose units so that all goods have producer (firm) price* 1*. Assume two goods,* $i = 1, 2$ *in addition to leisure (good* 0*). Also assume that with leisure untaxed (the canonical normalisation, as we shall see), the tax on good* 2 *is larger than that on good* 1*, i.e.* $t_2 > t_1$*. The consumer prices are then* $\mathbf{p}^0 = (1, 1 + t_1, 1 + t_2)^{'}$*.*
What happens if we instead choose good 2 *as untaxed, by dividing consumer prices by* $1 + t_2$*? We get new consumer prices,* $\mathbf{p}^2 = (\frac{1}{1+t_2}, \frac{1+t_1}{1+t_2}, 1)'$*. But then* $p_0^2, p_1^2 < 1$*, thus we have a subsidy on good* 1 *and tax on good* 0*. The latter since the price of leisure is smaller to the consumer than the firms.*

The result in the case with good 2 as untaxed is hard to interpret: To obtain our required public revenue, it looks as if we should subsidise good 1. In contrast, the case with leisure untaxed is more comprehensible, with two taxed goods. I think this example already points to the importance of our first distinction. We get briefly back to this example at the end of the paper.

3.1 Equivalence results

We state three trivial and well-known equivalence results on taxation of endowments (leisure). They all follow directly from the homogeneity of demand in consumer prices, and show the importance of discriminating between the two forms of taxation of endowments (leisure).

Proposition 2

1. *An equal value tax on all consumption,* $\mathbf{x}$*, including own consumption of initial endowments, is equivalent to a lump sum tax.*

2. *An equal value tax on net trade,* $\mathbf{x} - \overline{\mathbf{x}}$*, have no effect at all.*

3. *A uniform tax on goods without endowments is equivalent to an uniform tax on the sales of endowments.*

In the sense of 1), the claim attributed to Dixit and Learner above is true, whereas 2) is attributed to Sandmo above. From 3) the assumption of no tax on labour is only seemingly a restriction on the model - at least for linear taxes.

Proof. 1) It is easily seen from the individual's budget condition which in this case is $(1+t)\overline{\mathbf{p}}'\mathbf{x} = \overline{\mathbf{p}}'\overline{\mathbf{x}}$. This budget condition can easily be rewritten as a tax on the exogenously income, $\overline{\mathbf{p}}'\overline{\mathbf{x}}$:[5]

$$\overline{\mathbf{p}}'\mathbf{x} = \frac{\overline{\mathbf{p}}'\overline{\mathbf{x}}}{(1+t)} = (1-t')\overline{\mathbf{p}}'\overline{\mathbf{x}} \quad \text{where} \quad t' = \frac{t}{1+t}.$$

2) This is immediate from the budget condition in this case, $(1+t)\mathbf{p}'(\mathbf{x}-\overline{\mathbf{x}}) = 0$, by shorting the factor $1+t$. The reason is that whereas t is a tax for goods without initial endowments, it is a subsidy on sales of endowments, as in this case, the price to the individual is larger than that to a firm.
3) To show this, let $\mathbf{x} = (\mathbf{x}^e, \mathbf{x}^0)$, where $\mathbf{x}^e$ are the goods for which the individual holds endowments and $\mathbf{x}^0$ the ones without, and let $\mathbf{p} = (\mathbf{p}^e, \mathbf{p}^0)$ be the corresponding prices. With a uniform tax t on non-endowed goods the individuals budget is $(1+t)\left(\mathbf{p}^0\right)'\mathbf{x}^0 = \mathbf{p}^e(\overline{\mathbf{x}}^e - \mathbf{x}^e)$. Dividing by $1+t$, letting the uniform tax on labour, τ, be defined by $1-\tau = 1/(1+t)$, i.e. $\tau = t/(1+t)$, the above equation is equivalent to $\left(\mathbf{p}^0\right)'\mathbf{x}^0 = (1-\tau)\mathbf{p}^e(\overline{\mathbf{x}}^e - \mathbf{x}^e)$, thus the uniform tax t on non-endowed goods is equivalent to the uniform tax, τ, on the endowments.

It is also easy to verify that the tax income is the same in each case with the two tax forms in the three cases. ∎

4 Characterising optimal commodity taxes

As mentioned, we postpone the consumer price normalisation, to get a clearer view of it's effect.

The first order condition of our taxation problem, (1), with respect to the tax rates, $\mathbf{t}$, is

$$v_{\mathbf{p}} + \mu(\mathbf{x} - \overline{\mathbf{x}} + \mathbf{t}'\mathbf{x}_{\mathbf{p}}) = \mathbf{0}.$$

With initial endowments, Roy's theorem and the Slutsky equation are $v_{\mathbf{p}} = -v_m(\mathbf{x}-\overline{\mathbf{x}})$ and $\mathbf{x}_{\mathbf{p}} = \mathbf{x}^H_{\mathbf{p}} - \mathbf{x}_m(\mathbf{x}-\overline{\mathbf{x}})'$, where $\mathbf{x}^H$ is the compensated demand. Inserting from this into the first-order condition gives

$$-v_m(\mathbf{x}-\overline{\mathbf{x}}) + \mu\left(\mathbf{x} - \overline{\mathbf{x}} + \mathbf{t}'\left(\mathbf{x}^H_{\mathbf{p}} - \mathbf{x}_m(\mathbf{x}-\overline{\mathbf{x}})\right)\right) = \mathbf{0}.$$

Collecting the terms with $\mathbf{x}-\overline{\mathbf{x}}$, dividing by μ and inserting for $V_{\overline{m}}$ and θ, we get the *Ramsey rule*:

$$\theta(\mathbf{x}-\overline{\mathbf{x}})' + \mathbf{t}'\mathbf{x}^H_{\mathbf{p}} = \mathbf{0}. \tag{2}$$

Compared to the standard case with leisure as the only endowment, and where consumer prices is normalised by setting the tax on labour to zero, we additionally gets the term $-\theta\overline{\mathbf{x}}$.[6]

A direct interpretation of the Ramsey rule is that an equal relative increase in all tax rates (at optimum) should give an equal relative reduction in the compensated quantity of all non-endowed goods.[7] This interpretation, however, does not convey much information about the implied taxes rates as these are quite implicit in the characterisation. But some stronger assumptions gives simpler interpretations.

[5] With our normalisation, income at producer prices, $\overline{\mathbf{p}}'\overline{\mathbf{x}} = 1$.
[6] Replacing compensated demand, $\mathbf{x}^H$, by compensated excess demand, $\mathbf{z}^H = \mathbf{x}^H - \overline{\mathbf{x}}$, we get exactly the same condition as in the standard case.
[7] The characterisation gets somewhat more complex for endowed goods.

The (compensated) inverse elasticity rule Assume that leisure (good 0) is the only endowment, and no compensated cross price elasticities between goods, except towards leisure. Then Ramsey rule, (2), gives the *inverse elasticity rule* (for $k \neq 0$):[8]

$$\frac{t_k}{p_k} = -\frac{\theta}{El_{p_k} x_k^H} = \frac{\theta}{El_{p_0} x_k^H}. \tag{3}$$

The first form of this rule is mostly used, but the second gives the best interpretation. It says that one should *tax relative complements to leisure harder.* We come back to this important interpretation in section 4.2.

The Corlett-Hague rule Assume that leisure (good 0) is the only endowment and that there are only two other goods. In this case, the Ramsey rule, (2), gives the *Corlett-Hague rule*:[9]

$$\frac{\frac{t_1}{p_1}}{\frac{t_2}{p_2}} = \frac{El_{p_2} x_1^H + El_{p_0} x_2^H + El_{p_1} x_2^H}{El_{p_2} x_1^H + El_{p_0} x_1^H + El_{p_1} x_2^H}. \tag{4}$$

On the right hand side, only the middle terms are different. Thus the interpretation is again that we should tax relative complements to leisure harder.[10]

The importance of the common interpretation of the second form of the inverse elasticity rule and the Corlett-Hague rule is argued by Sandmo (1987). The argument below, essentially due to Deaton (1979), gives a general characterisation of optimal commodity taxes with the same intuitive interpretation. Thus the argument outdates Sandmo's (1976, p. 46) remark: "Elasticity formulae become very complicated in the general case and provide little intuitive insight into the structure of taxation."

4.1 Inverting the Ramsey rule

We use the Antonelli matrix, which is the derivative of the (compensated) marginal willingness to pay function, to invert the Ramsey rule. First, however, we recall these concepts.

4.1.1 The (compensated) marginal willingness to pay

First introduce prices, $\mathbf{q} = \mathbf{p}/\mathbf{p}'\overline{\mathbf{x}}$, scaled so that income is one. With these prices, $\mathbf{q}$, the indirect utility function can be written $U^*(\mathbf{q}) = v(\mathbf{p}/\mathbf{p}'\overline{\mathbf{x}}, 1)$.

The (compensated) *willingness to pay function* (or *indirect expenditure function*), e^*, is the least one is willing to pay for quantities $\mathbf{x}$, to keep utility at or below u, i.e.:[11]

$$e^*(\mathbf{x}, u) = \min_{\mathbf{q}} \mathbf{q}'\mathbf{x} \quad \text{subject to} \quad U^*(\mathbf{q}) - u \leq 0.$$

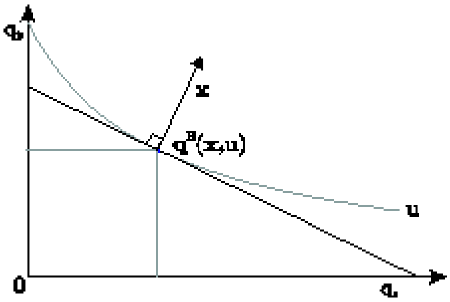
[8] Here $El_b a = (b/a)(da/db)$ is the elasticity of a with respect to b.

[9] Dalton and Sadka (1979) gives a more direct generalisation of the Corlett-Hague rule than the one by Deaton which we advocate, but in their case there are still only two taxed goods.

[10] In contrast to the assumptions of the inverse elasticity rule, the assumptions of the Corlett-Hague rule allows complements to leisure.

[11] Deaton actually use the equivalent distance function instead, but the willingness to pay function is more intuitive.

It is easy to show that this function has the same properties (in $\mathbf{x}$) as the exprenditure function (in $\mathbf{p}$). The solution to this problem, $\mathbf{q}^H(\mathbf{x}, u)$, is the (compensated) *marginal willingness to pay* for $\mathbf{x}$ (given utility level u) or the (compensated) *inverse demand for* $\mathbf{x}$. By the envelope property, $e^*_{\mathbf{x}}(\mathbf{x}, u) = \mathbf{q}^H(\mathbf{x}, u)$. We draw a picture of the problem with the given level curve $U^*(\mathbf{q}) = u$ in grey. With prices on the axes, utility is increasing towards origin in the diagram:



The (compensated) willingness to pay function is concave, homogenous of degree 1, and nondecreasing in $\mathbf{x}$, and additionally continuous for $\mathbf{x} \gg \mathbf{0}$.

Compensated demand, $\mathbf{x}^H(\mathbf{p}, u)$, and compensated inverse demand, $\mathbf{q}^H(\mathbf{x},u)$, are generalised inverse functions of prices for a given utility level in the following sense:[12]

$$\mathbf{q}^H(\mathbf{x}^H(\mathbf{p}, u), u) = \frac{\mathbf{p}}{e(\mathbf{p}, u)}. \tag{5}$$

This relation says that given prices $\mathbf{p}$, with expenditure 1 to reach utility level u, we get back to $\mathbf{p}$ by first taking the compensated demand at these prices, and then the compensated marginal willingness to pay, given this demand. If expenditure to reach utility level u is different from 1, we only get back to original prices, scaled to expenditure 1.

Taking the derivative of the above expression wrt. prices $\mathbf{p}$, we get that the *Slutsky matrix*, $\mathbf{x}^H_{\mathbf{p}}$, and the *Antonelli matrix*, $\mathbf{q}^H_{\mathbf{x}}$, are *generalised inverses*, i.e.

$$(\mathbf{p}'\overline{\mathbf{x}})\,\mathbf{q}^H_{\mathbf{x}}\mathbf{x}^H_{\mathbf{p}} = \mathbf{I} - \mathbf{x}\mathbf{q}', \tag{6}$$

where $\mathbf{I}$ is the identity matrix.

4.1.2 The inverse Ramsey rule

Let $\overline{\theta} = \theta\mathbf{p}'\overline{\mathbf{x}}$.[13] Right multiplying the Ramsey rule, (2), with $\mathbf{p}'\overline{\mathbf{x}}$ times the Antonelli matrix, $\mathbf{q}^H_{\mathbf{x}}$, using the generalized inverse property, (6), and the definition of $\overline{\theta}$, we get:

$$\overline{\theta}(\mathbf{x}-\overline{\mathbf{x}})'\mathbf{q}^H_{\mathbf{x}} = (\mathbf{p}'\overline{\mathbf{x}})\,\mathbf{t}'\mathbf{x}^H_{\mathbf{p}}\mathbf{q}^H_{\mathbf{x}} = \mathbf{t}'(\mathbf{I} - \mathbf{x}\mathbf{q}').$$

As $\mathbf{q}^H(\mathbf{x}, u)$ is homogenous of degree 0 in quantities $\mathbf{x}$, $\mathbf{q}^H_{\mathbf{x}}\mathbf{x} = \mathbf{0}$. Hence, $\mathbf{x}'\mathbf{q}^H_{\mathbf{x}} = \mathbf{0}$, since $\mathbf{q}^H_{\mathbf{x}}$ is symmetric. Inserting from the public budget constraint, $\mathbf{t}'(\mathbf{x}-\overline{\mathbf{x}}) = \overline{T}$, we get the (unnormalised) *inverse Ramsey rule*:[14]

$$\mathbf{t}' = (\overline{T} + \mathbf{t}'\overline{\mathbf{x}})\mathbf{q}' + \overline{\theta}\overline{\mathbf{x}}'\mathbf{q}^H_{\mathbf{x}}. \tag{7}$$

[12] A simple proof is given in an appendix. Another proof of (6) below is given in Salvas-Bronsard, Leblanc and Bronsard (1977).

[13] We get back to the interpretation of $\overline{\theta}$.

[14] This is essentially Auerbach's (1987) formula (6.6). Thus, so far there is nothing new.

Before proceeding, we introduce some definitions.

4.1.3 Endowment elasticities

The (compensated) *endowment elasticity of good k*,

$$\iota_k \overset{def}{=} -\frac{\tau}{q_k}\frac{\partial q_k^H(\tau\mathbf{x}+(1-\tau)\overline{\mathbf{x}},u)}{\partial\tau}\,|_{\tau=1}\ .$$

This is the elasticity of the (compensated) marginal willingness to pay for good k with respect to an adjustment of consumption (from the optimum $\mathbf{x}$) in the direction of the endowments, $\overline{\mathbf{x}}$.[15]

Expanding this definition, using homogeneity and symmetry, we get:

$$q_k\iota_k = \sum_j \frac{\partial q_k^H}{\partial x_j}(\overline{x}_j - x_j) = \sum_j \frac{\partial q_k^H}{\partial x_j}\overline{x}_j = \sum_j \overline{x}_j \frac{\partial q_j^H}{\partial x_k}. \tag{8}$$

Let $\beta_k \overset{def}{=} q_k\overline{x}_k$ be the *income share* of good k, evaluated at the marginal willingness to pay at optimum, and $\overline{\iota}^\beta = \sum_j \beta_j\iota_j$ the *average* (compensated) *endowment elasticity*, with the income share weights.

4.1.4 More on $\overline{\theta}$

Right multiplying (7) by $\overline{\mathbf{x}}$, using that $\mathbf{q}'\overline{\mathbf{x}} = 1$, we get an expression for the marginal value of transferring one pound (exogenously) from the individual to the public (in units of the public budget):[16]

$$\overline{\theta} = \frac{\overline{T}}{-\overline{\mathbf{x}}'\mathbf{q}_{\mathbf{x}}^H\overline{\mathbf{x}}} \geq 0. \tag{9}$$

The quadratic form in the denominator of (9), however, equals the average marginal willingness to pay elasticity, as by the definition of β_k, (8), and the definition of $\overline{\iota}^\beta$:

$$\begin{aligned}\overline{\mathbf{x}}'\mathbf{q}_{\mathbf{x}}^H\overline{\mathbf{x}} &= \sum_{k,j}\overline{x}_k\frac{\partial q_j^H}{\partial x_k}\overline{x}_j = \sum_{k,j} q_k\overline{x}_k\frac{\overline{x}_j}{q_k}\frac{\partial q_j^H}{\partial x_k}\\ &= \sum_k \beta_k \sum_j \frac{\overline{x}_j}{q_k}\frac{\partial q_j^H}{\partial x_k} = \sum_k \beta_k\iota_k = \overline{\iota}^\beta.\end{aligned}$$

Thus from (9),

$$\overline{\theta} = \frac{\overline{T}}{-\overline{\iota}^\beta}. \tag{10}$$

[15] In the standard case with an endowment only of leisure, the endowment elasticity of a good is simply the elastisity of the (compensated) marginal willingness to pay for that good wrt. the quantity of leisure.

[16] The inequality follows as $\mathbf{e}^*(\mathbf{x},u)$ is concave i $\mathbf{x}$, thus $\mathbf{q}_{\mathbf{x}}^H$ is negative semidefinit.

The marginal efficiency loss of taxation We saw above that θ is the value of transferring one pound exogenously, *at consumer prices*, from the individual to the government. Then $\overline{\theta} = \theta \mathbf{p}'\overline{\mathbf{x}}$ is the value of transferring one pound exogenously, *at producer prices*, from the individual to the government. This is so, since with our producer price normalisation, the income at producer prices, $\overline{m} = \overline{\mathbf{p}}'\overline{\mathbf{x}} = 1$, while income at consumer prices, $m = \mathbf{p}'\overline{\mathbf{x}}$. Thus $m = \mathbf{p}'\overline{\mathbf{x}}\,\overline{m}$, so $dm/d\overline{m} = \mathbf{p}'\overline{\mathbf{x}}$.

By its interpretation, θ looks like a natural measure of the marginal efficiency loss of taxation. As pointed out by Håkonsen (1998), however, this is problematic, as one would like such a measure to be independent of the consumer price normalisation, and θ is not.[17] But, as seen from (9), $\overline{\theta}$ is independent of the consumer price normalisation. Therefore, $\overline{\theta}$ is a measure of the marginal efficiency loss of taxation. Again, with our (canonical) normalisation below, $\theta = \overline{\theta}$, so with this normalisation, θ also is a measure of the marginal efficiency loss of taxation.

4.2 The normalisation and inverse characterisation

We are now ready for our (canonical) normalisation, being a straightforward generalisation of the standard practice with leisure as the only endowment, and untaxed labour.

Normalise consumer prices by setting income at the *optimal* consumer prices equal to one, $\mathbf{p}'\overline{\mathbf{x}} = 1$. Then, at optimum, the initial endowments are a nontaxed (composite) good, i.e. $\mathbf{t}'\overline{\mathbf{x}} = 0$. Then also $\mathbf{p} = \mathbf{q}$, and (7) simplifies to the *inverse Ramsey rule*:

$$\mathbf{t}' = \overline{T}\mathbf{p}' + \overline{\theta}\overline{\mathbf{x}}'\mathbf{q}^H_{\mathbf{x}}. \tag{11}$$

On component form, (11) is:

$$t_k = \overline{T}p_k + \overline{\theta}\sum_j \overline{x}_j \frac{\partial q_j^H}{\partial x_k}.$$

Dividing by p_k, inserting for ι_k from (8) and $\overline{\theta}$ from (10), gives the *generalised inverse elasticity rule*:

$$\frac{t_k}{p_k} = \overline{T}\left(1 + \frac{\iota_k}{-\,\overline{\iota}^\beta}\right). \tag{12}$$

Thus the *public tax requirement* (as a share of income at producer prices) is a *basic tax rate.* Deviations from this base tax rate is for each good proportional to its endowment elasticity.

Call a good k an *endowment substitute* if $\iota_k < 0$, and an *endowment complement* if $\iota_k > 0$. The basis tax rate should then be raised for endowment complements and lowered for endowment substitutes.[18] The intuition is clear: A tax on own consumption of endowments removes the efficiency loss. Thus a tax on endowment complements is an indirect way of taxing own consumption of endowments, thereby reducing the efficiency loss. Taxing endowment substitutes, on the other hand, only increases the own use of untaxed endowments.

[17] Håkonsen then proceeds to define the (total) efficiency loss by means of a pair of dual optimal value functions for the optimization problem.

[18] With only endowments of one good, this is essentially Hicks' (1956) notions of q-compements and q-substitutes with respect to this good.

This is of course the same intuition as one get both from the second form of the inverse elasticity rule, (3), and the Corlett and Hague rule, (4), above. It does, however, avoid the restrictive assumptions of both these characterisations.

Without this canonical normalisation, from (7) we get the characterisation:

$$\frac{t_k}{p_k} = \frac{1}{\mathbf{p}'\overline{\mathbf{x}}}\left(\mathbf{t}'\overline{\mathbf{x}} + \overline{T}\left(1 + \frac{\iota_k}{-\overline{\iota}^{\beta}}\right)\right). \tag{13}$$

In this characterisation especially the term for the tax value of endowments, $\mathbf{t}'\overline{\mathbf{x}}$, is hard to interpret, as the other new term, $(\mathbf{p}'\overline{\mathbf{x}})^{-1}$, is only a proportionality factor. In example 1, it is this term which explains why we get negative tax rate under the last normalisation, as it is negative.

Our generalised inverse elasticity rule, (12), is simpler and easier to interpret than the similar formula (6.7) in Auerbach (1985), which is essentially of the form (13) (although he does not introduce the endowment elasticities), obtained by setting an arbitrary tax rate equal to 0. Our (canonical) normalisation, however, looks like the obvious choice at this point. Thus I suspect that the reason Auerbach does not use the normalisation is that he thinks it is illegitimate.

Remark 3 *Auerbach gets around the problems caused by the tax value of endowments, by looking at the differences between the relative tax rates in his formula (6.8), before interpreting the results. Thus he looks at*

$$\frac{t_k}{p_k} - \frac{t_j}{pj} = \frac{\overline{T}}{\mathbf{p}'\overline{\mathbf{x}}\left(-\overline{\iota}^{\beta}\right)}\left(\iota_k - \iota_j\right) \backsim \iota_k - \iota_j .$$

Then, however, i.e. the interpretation of $\overline{T}$ *as the base tax rate is lost.*

Remark 4 *Normally, one normalise before solving the problem. This is not possible in our case, as we normalise income by setting income at the* optimal *consumer prices to* 1*. But this is unproblematic as the point of the normalisation is to characterise the solution, i.e. helping us understand it, and not to solve the problem.*

4.2.1 Only one endowment

With endowments of only one good, 0 (say leisure), from (8) using symmetry, the (compensated) endowment elasticity of good is the elasticity of (compensated) marginal willingness to pay wrt. leisure times the endowment share of consumption:

$$\iota_k = \frac{\overline{x}_0}{q_k}\frac{\partial q_k^H}{\partial x_0} = \frac{\overline{x}_0}{x_0} El_{x_0} q_k^H .$$

In this case $\beta_0 = 1$. Thus from the definition:

$$\overline{\iota}^{\beta} = \iota_0 = \frac{\overline{x}_0}{x_0} El_{x_0} q_0^H .$$

Hence the generalized inverse elasticity rule (12) simplifies to:[19]

$$\frac{t_k}{p_k} = \overline{T}\left(1 + \frac{El_{x_0} q_k^H}{-El_{x_0} q_0^H}\right). \tag{14}$$

[19] This is Deaton's (1979) formula (51).

The departure of the tax rate on good k from the base tax rate is for each good proportional to the (compensated) elasticity of the marginal willingness to pay wrt. the price of leisure.

A comparison The generalized inverse elasticity rule in this case, (14) is similar to the second form of the inverse elasticity rule, (3), except that it involves the elasticity of the (compensated) marginal willingness to pay (i.e. inverse demand) wrt. leisure, whereas the standard inverse elasticity rule involves the inverse of the elasticity of (compensated) demand with respect to the price of leisure.

A The generalized inverse property

Here, we verify the generalized inverse property used in the main text, using the duality between the compensated direct and compensated indirect demand. We state the basic result for demand correspondences, with respect to income normalized prices and quantities.

Given a utility level u, define the *compensated demand correspondence*, c^u, by $\mathbf{x} \in c^u(\mathbf{q})$ if $\mathbf{qx} \leq 1$, $U(\mathbf{x}) \geq u$ and for all $\mathbf{x}'$ such that $U(\mathbf{x}') \geq u$, $\mathbf{qx}' \geq 1$, and the *compensated inverse demand correspondence*, c^{*u}, by $\mathbf{q} \in c^{*u}(\mathbf{x})$ if $\mathbf{qx} \leq 1$, $U^*(\mathbf{q}) \leq u$ and and for all $\mathbf{q}'$ such that $U^*(\mathbf{q}) \leq u$, $\mathbf{q}'\mathbf{x} \geq 1$. The following straightforward proposition states that the two concepts are dual in a simple way.

Proposition 5 *Assume monotone preferences. Then* $\mathbf{x} \in c^u(\mathbf{q})$ *if and only if* $\mathbf{q} \in c^{*u}(\mathbf{x})$.

Proof. $\Rightarrow$: Assume $\mathbf{x} \in c^u(\mathbf{q})$. To show that $\mathbf{q} \in c^{*u}(\mathbf{x})$. Trivially $\mathbf{qx} \leq 1$, so we need to show first that $U^*(\mathbf{q}) \leq u$ and secondly that if $U(\mathbf{q}') \leq u$, then $\mathbf{q}'\mathbf{x} \leq 0$.

1. Assume $U^*(\mathbf{q}) > u$. Then since $U^*(\mathbf{q}) = \sup_{\mathbf{x}} \{U(\mathbf{x}) | \mathbf{qx} \leq 1\}$, there is $\mathbf{x}'$ such that $U(\mathbf{x}') > u$ and $\mathbf{qx}' \leq 1$. Hence by monotonicity there is $\mathbf{x}''$ such that $U(\mathbf{x}'') > u$ and $\mathbf{qx}'' < 1$, contradicting $\mathbf{x} \in c^u(\mathbf{q})$.

2. Let $U(\mathbf{q}') \leq u$, and assume that $\mathbf{q}'\mathbf{x} > 1$. Then by monotonicity, there is $\mathbf{q}''$ such that $U(\mathbf{q}'') < u$ and $\mathbf{q}''\mathbf{x} > 1$. Since $U(\mathbf{x}) = \inf_{\mathbf{q}} \{U^*(\mathbf{q}) | \mathbf{qx} \geq 1\}$, then $U(\mathbf{x}) \leq U(\mathbf{q}'') < U(\mathbf{q}) = u$, contradicting $\mathbf{x} \in c^u(\mathbf{q})$.

$\Leftarrow$: This is essentially the same argument. ■

If both the above correspondences are single-valued, we essentially have the compensated demand and inverse demand *functions*, $\mathbf{x}^H(\mathbf{q}, u)$ and $\mathbf{q}^H(\mathbf{x}, u)$, ie. $\mathbf{x} \in c^u(\mathbf{q})$ can be written $\mathbf{x} = \mathbf{x}^H(\mathbf{q}, u)$ and $\mathbf{q} \in c^{*u}(\mathbf{x})$ can be written $\mathbf{q} = \mathbf{q}^H(\mathbf{x}, u)$. In this case, a consequence of the proposition is the generalised inverse property (5), as

$$\mathbf{q}^H\left(\mathbf{x}^H(\mathbf{q}, u), u\right) = \mathbf{q} = \frac{\mathbf{p}}{\mathbf{p}'\mathbf{x}(\mathbf{p}, u)} = \frac{\mathbf{p}}{e(\mathbf{p}, u)}.$$

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B Other literature on normalizations in optimal taxation

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