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# REDISTRIBUTION AND THE MARGINAL COST OF PUBLIC FUNDS

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## Abstract

Discussions of the marginal cost of public funds with distortionary taxation are often cast in the framework of a one-consumer economy, while the main justification of distortionary taxes is that they are needed for redistribution. This paper analyzes the issue in a model with heterogeneous consumers and a linear income tax, focusing on the tradeoff between labour market distortions and the redistribution from high-wage to low-wage workers. In an optimal tax system the MCF will be the same for all sources of funds and under certain assumptions less than one. Without optimality the MCF will in general differ between different sources of finance.

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# REDISTRIBUTION AND THE MARGINAL COST OF PUBLIC FUNDS.

## 1. Introduction.

Resource use in the public sector, whether on public goods or the public provision of private goods, involves benefits and costs, and in order to do systematic cost-benefit studies one has to estimate both of these. The traditional view has been that it is the assessment of benefits that presents the main problem for applied work, and that the computation of costs basically involves problems that are no different from those encountered in the private sector. This might indeed be true if public expenditure could be financed through non-distortionary lump sum taxes, but in a second best world of distortionary taxation the situation is more complicated. It was pointed out by Pigou (1947) that the financing of public expenditure by distortionary taxes imposes an efficiency loss on the economy, and this efficiency cost should be added to the direct resource cost of public expenditure. This raises problems of a level of difficulty comparable to that of the estimation of benefits, since the efficiency costs of taxes are not directly observable from market transactions.

According to Pigou, if the private sector is otherwise perfectly competitive, the marginal cost to be used in public cost-benefit analyses should exceed that of private firms using comparable inputs. Pigou's idea was taken up in a famous paper by Atkinson and Stern (1974), which set the problem in the context of the theory of optimum commodity taxation; their formulation was later extended by Wildasin (1984) and King (1986). A second line of research, which took its point of departure from the theory of optimal non-linear income taxation, beginning with Hylland and Zeckhauser (1979) and continuing with Christiansen (1981), focused on the conditions that would have to be satisfied for the conventional cost-benefit criterion to be valid even in the absence of lump sum taxes.

In more recent work in this area the central theoretical concept has become that of the marginal cost of public funds (MCF), which has been defined as the multiplier to be applied to the direct resource cost in order to arrive at the socially relevant shadow price of resources to be used in the public sector. Under first best conditions, the marginal social benefit should obviously be equal to the marginal resource cost, so that the MCF in this case must equal one. The basic economic intuition behind the concept would seem to imply that with distortionary taxes one must necessarily have MCF>1, but there are several reasons why one should be careful about drawing this conclusion. One reason, which was stressed by Atkinson and Stern, has to do with demand complementarities between private and public goods, another, which will be emphasized here, concerns the effects of taxes on the distribution of income and welfare. Presumably the main reason why we have distortionary taxes is precisely the distributional problem; if issues of equity and justice could be disregarded altogether, the design of an efficient tax system would be a fairly easy task. Moreover, it is important to keep in mind that in a comparison of the first and the second best, the first best is a well-defined alternative, whereas the second best can be defined in a number of ways, depending on the precise set of tax instruments which is at the disposal of the government.

Of the previous contributions to the area, the present paper is related to those of King (1986), Wildasin (1984) and particularly to Wilson (1991) in assuming a population of heterogeneous consumers. Unlike them, however, I assume that redistribution occurs via a negative income tax, which on the one hand makes the equity-efficiency tradeoff more transparent and on the other hand focuses attention more directly on the labour market distortion which has figured so prominently in applied work. As in Wilson (1991) this introduces a lump sum element in the tax system, although one that is constrained to be uniform across consumers. The linearity assumption is in contrast to e.g. Christiansen (1981), who analyzes a fully optimal non-linear tax. The restriction to linear tax schemes tries to capture the idea that tax systems, for administrative and perhaps political reasons, must have a structure which is relatively simple.[2] I also emphasize that the magnitude of the MCF may depend critically on one's assumptions about the kind of tax instruments that the government can use, and I try to distinguish clearly between, on the one hand, the MCF that results from a process of social welfare optimization, and, on the other hand, from a partial change of policy that starts from an arbitrary situation in terms of tax and expenditure policies.

The MCF has recently attracted a great deal of interest from policy makers who have begun to think of it as a practical tool for applied cost-benefit analysis. This would involve instructing public sector organizations to apply the MCF to the estimated cost of a project as a part of the routine of carrying out a cost-benefit analysis. I discuss briefly how such a procedure should affect the definition of the MCF and how it could be expected to affect the interplay between the central government and individual public agencies.

## 2. The behaviour of individual agents.

I assume that there are $n$ consumers in the economy. The preferences of consumers are defined on their consumption of a private good, $c^i$, on the amount of leisure, $l^i$, and on the supply of a public good provided by the goverment in amount $z$. All consumers have the same preference map which is represented by the common utility function $u$:

$$(1) \qquad u^i=u(c^i, l^i, z). \qquad (i=1,...,n)$$

Consumption is equal to labour income net of taxes. The income tax system has two parameters, viz. a constant marginal tax rate, $t$, and a lump sum transfer, $a$. In other words, we have a negative income tax. When $t$ and $a$ are both positive, we have a progressive tax system in the sense that the average tax rate is increasing with gross income. The budget constraint becomes

$$(2) \qquad c^i=(1-t)w^ih^i+a. \qquad (i=1,...,n)$$

Here $h^i$ is labour supply, and $h^i+l^i=T$, which is the time endowment. $w^i$ is the wage rate of consumer $i$. In the tradition of Mirrlees (1971) wage rates are assumed to reflect individuals'

[2] Real income tax systems are usually far from simple in that they include a number of special provision which may include such factors as family size, age, health status, cost of travel to work etc. - not to mention the many special provisions for the taxation of income from capital. But none of these factors enters the present model. The point here is that for individuals whose characteristics in terms if these factors are the same, the basic «mathematical» structure of the tax schedule related to labour income is fairly simple.

productivities or «abilities» via the operation of competitive labour markets and to be the source of inequality in the model. Abilities are assumed to be exogeneously given, and so, therefore, are wages. This particular assumption about the generation of inequality is not important for the conclusions of the present paper and has been adopted mainly because of its familiarity.[3]

Maximization of (1) subject to (2) yields the familiar equality between the marginal rate of substitution between leisure and consumption and the wage rate after tax:

$$(3) \qquad u^i_l / u^i_c = (1-t)w^i. \qquad\qquad (i=1,...,n)$$

From the solution to the maximization problem we can also derive the indirect utility function

$$(4) \qquad v^i = v((1-t)w^i, a, z) \qquad\qquad (i=1,...,n)$$

and its partial derivatives with respect to the public policy parameters

$$(5) \qquad v^i_t = -\lambda^i w^i h^i, \qquad v^i_a = \lambda^i, \qquad v^i_z = u^i_z. \qquad (i=1,...,n)$$

The individual consumer's marginal willingness to pay for the public good, i.e. his marginal rate of substitution between the public good and private consumption, can be defined both in terms of the direct and indirect utility functions, i.e.

$$(6) \qquad m^i = u^i_z / u^i_c = v^i_z / \lambda^i. \qquad\qquad (i=1,...,n)$$

We can also write down the labour supply function as

$$(7) \qquad h^i = h((1-t)w^i, a, z). \qquad\qquad (i=1,...,n)$$

I make the standard assumption that leisure is a normal good, so that labour supply is decreasing in $a$. The net wage rate then has substitution and income effects that are of opposite signs. Obviously, labour supply depends on the supply of public goods as well as on the tax system, but utility maximization does not imply any restrictions on the sign of the public goods effect on labour supply.

[3] An alternative assumption would be that inequality is generated through differences in the consumption-leisure preferences of individuals. For a discussion of the implications of this for redistribution policy see Sandmo (1993).

### 3. The government's optimization problem.

A model of optimal government policy is the natural framework for studying the marginal cost of public funds. One may have legitimate objections to a view of government that involves the maximization of a social welfare function, but on the other hand the concept of the MCF hardly makes any sense unless one assumes that the government pursues certain objectives. As an example of such an objective I take the case of a utilitarian government which maximizes the sum of utilities:

$$(8) \qquad W=\Sigma v^i.$$

The budget constraint of the government requires that net tax revenue be equal to the resource cost of providing the public good, so that

$$(9) \qquad t\Sigma w^i h^i - na = qz.$$

Here $q$ is the producer price of the public good, which is assumed to be constant. Maximizing (8) subject to (9) with respect to the three policy parameters $t$, $a$ and $z$, yields the first order conditions

$$(10) \qquad -\Sigma\lambda^i w^i h^i + \mu[\Sigma w^i h^i + t\Sigma w^i(\partial h^i/\partial t)] = 0.$$

$$(11) \qquad \Sigma\lambda^i + \mu[t\Sigma w^i(\partial h^i/\partial a) - n] = 0.$$

$$(12) \qquad \Sigma\lambda^i m^i + \mu[t\Sigma w^i(\partial h^i/\partial z) - q] = 0.$$

As a point of reference for the following discussion we note that if individualized lump sum taxes had been feasible - a theoretically interesting although unrealistic case - we would have had $t=0$ and $\lambda^i=\mu$ for all $i$. Equation (12) would then have become

$$(13) \qquad \Sigma m^i = q.$$

which is exactly the Samuelson (1954) condition. The lump sum element in the linear income tax is also non-distortionary, but it is not individualized; consequently, it is not possible to solve the problem of distribution separately from that of efficient resource allocation. This is in fact the main reason for considering the problem of the MCF within the context of a model in which inequality matters.

A problem with defining the private marginal utility of income under second best conditions is obviously that the marginal utility is not equalized among consumers. The non-distortive tax or transfer in this context is the parameter $a$, which is paid equally to everyone. The *average* marginal utility of income is then

$$(14) \quad (1/n)\Sigma\lambda^i=\lambda.$$

By using this definition we can rewrite condition (12) as

$$(15) \quad n\Sigma\lambda^i m^i/\Sigma\lambda^i=\gamma[q-t\Sigma w^i(\partial h^i/\partial z)],$$

where $\gamma=\mu/\lambda$.

In this condition the weighted average formula on the left can be interpreted as $n$ times the marginal willingness to pay of the average or representative consumer[4], the average being computed by using the marginal utilities of income as weights; multiplying this by the number of consumers gives the social marginal willingness to pay. Note that the $\lambda^i$ can also be interpreted as the marginal welfare weights on consumption, since they depend on the particular cardinal representation for preferences that is adopted for the measurement of social welfare. Loosely speaking, the degree of concavity adopted for this representation determines the degree of inequality aversion in the social welfare function.

The main focus of the present paper is on the cost of providing public goods, i.e. on the interpretation of the right-hand side of equation (15). Nevertheless, it is useful to pause for a moment to consider the measurement of benefits. Suppose that instead of the weighted average formula we had simply taken the sum of the benefits $m^i$ and ask whether this would have resulted in a higher estimate of the level of social benefits. Clearly, this would have been the case if and only if

$$n\Sigma\lambda^i m^i/\Sigma\lambda^i<\Sigma m^i.$$

A simple manipulation allows us to rewrite this as

$$(16) \quad Cov(\lambda^i, m^i)<0.$$

[4] This is a *socially* representative consumer in the sense of Deaton (1977), who distinguishes this concept from that of the *market* representative consumer of positive demand theory.

In other words, welfare weighting leads to a lower measure of benefits if the covariance between the marginal utility of income and the willingness to pay is negative (and to a higher measure if it is positive). The underlying parameter that distinguishes between individuals is the wage rate. If we assume that the marginal utility of income declines with the wage rate - an assumption which will be discussed further in Section 5 below - the crucial question becomes whether the marginal willingness to pay increases with the wage rate. If it does, welfare weighting leads to a lower measure of benefits - roughly because the supply of public goods tends to benefit the rich more than the poor.

Turning back to the right-hand side of equation (15), we notice that there are two elements in it which make it differ from the simple resource cost measure $q$. The first is the factor $\gamma$, the nature of which must be assessed from the optimum conditions describing the tax system. The second is a measure of the effect of the increased supply of public goods on tax revenue[5]. An interesting question is now how one should define the MCF. Should it simply be taken as equal to $\gamma$, or should the tax revenue effect also be taken into account?

There is a strong case for settling for the former alternative. First, there is a certain arbitrariness about classifying the tax revenue effect as a part of the cost, since it could equally well have been defined as part of the benefit. If e.g. the tax revenue effect is positive, this can be seen as a benefit that allows the government to increase the transfers to the private sector. Second, and more important, the model can be applied to policy choices in at least two different ways. On the one hand the public good can be interpreted in terms of an aggregate for the whole economy, so that the model is applicable to the choice of the overall balance between the private and public sectors. But if one extends it to the case of many public goods (or publicly supplied private goods), these must be assumed to have quite different degrees of complementarity with labour supply; spending on health and education is very different in this respect from the use of resources on natural parks or institutions of culture and entertainment. Individual agencies doing cost-benefit analyses of their projects would therefore have to be assigned different values for the MCF depending on how completion of their projects would affect tax revenue. This goes against the basic idea of the MCF and provides an additional argument for defining it in terms of $\gamma$.

It has been suggested by Starrett (1988, p. 173) that one ought to concentrate on the case where the tax revenue effect is zero, since «a general project has no obvious net complementarity». If a general project is supposed to mean a general expansion of the whole of the public sector, there may be something to be said for this viewpoint, although even there it does not seem entirely convincing. But if the MCF is conceived of as a parameter to be used by individual public agencies, it clearly is both relevant and realistic to take account of the ability of the project to generate additional revenue. Even if the tax revenue effect should turn out to be of little significance for the aggregate public/private sectoral balance, it might well be of major importance for the cost-benefit ranking of individual projects.

## 4. The MCF with non-distortionary marginal finance.

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[5] Although I shall refer to this as the tax revenue effect, there is a more fundamental explanation of the role that it plays in the analysis. When the provision of public goods has a positive effect on labour supply, this counteracts the substitution or disincentive effect of the marginal tax rate and thus reduces the efficiency cost of taxation.

The Pigovian reasoning suggests that with distortionary taxes the MCF would necessarily be greater then one. However, it should be kept in mind that although the tax system as a whole is distortionary, some elements of it are less distortionary than others. If an increase in the supply of public goods is financed on the margin by an increase of the less distortionary parts of the system, the marginal cost of funds might well be less than one.

This point is easy to illustrate within the present model, since the per capita transfer $a$ is a non-distortionary source of finance. Suppose that the marginal tax rate $t$ is fixed, and that the government chooses the optimal supply of public goods by appropriate adjustment of $a$. Equation (15) continues to hold, having been derived from the first-order condition (12) alone. But (10) does not hold, so that the expression for the MCF must be derived from (11). This yields

$$(17)\qquad \gamma_a=[1-(1/n)t\Sigma w^i(\partial h^i/\partial a)]^{-1}.$$

Here $\gamma_a$ is the MCF when the transfer is the marginal source of finance. Since leisure is a normal good ($\partial h^i/\partial a<0$) this expression is less than one, and it is easy to see the economic intuition behind the result. Because of the presence of a distortionary marginal tax rate there is a negative substitution effect on labour supply. When public expenditure is financed on the margin by a uniform lump sum tax (or by reduction of a transfer payment), the income effect on labour supply is positive, and this should be accounted for as an efficiency gain.

It may seem surprising that the MCF in this case does not reflect distributional concerns at all; after all, this is a regressive form of tax finance. This is a consequence of our utilitarian social welfare function and of the fact that every consumer pays the average amount of tax. But it should be stressed that this does not imply the absence of distributional concerns from the cost-benefit calculation; it simply means that these concerns are only reflected in the evaluation of benefits. This is well illustrated by the special case in which the uniform lump sum tax is not only the marginal but in fact the only source of goverment revenue. Then (17) implies that the MCF=1, and (15) becomes simply

$$(18)\qquad n\Sigma\lambda^i m^i/\Sigma\lambda^i=q.$$

Thus, the social marginal benefit should be set equal to the marginal resource cost. There are now no price distortions in the economy, and the allocation of resources is efficient in the usual Pareto sense. But because of the absence of individualized lump sum transfers, the measure of social benefits must reflect the distributional judgements of the government.

## 5. The case of distortionary marginal finance.

I now reverse the assumption of the previous section. Instead of assuming that public goods can be financed on the margin by a non-distortionary tax, I postulate that the level of the lump sum transfer is fixed, possibly at zero, and that tax revenue must come from the distortionary tax on labour income, $t$. This implies that the MCF must be derived from equation (10), (11) being now irrelevant.

The formula for the covariance implies that

$$(1/n)\Sigma\lambda^i w^i h^i = Cov(\lambda^i, w^i h^i) + (1/n^2)\Sigma\lambda^i \Sigma w^i h^i.$$

Substituting this into (10), we obtain

$$-nCov(\lambda^i, w^i h^i) - (1/n)\Sigma\lambda^i \Sigma w^i h^i + \mu[\Sigma w^i h^i + t\Sigma w^i(\partial h^i/\partial t)] = 0.$$

Dividing through by $\lambda\Sigma w^i h^i$ (remembering the definition (14)), we then obtain

(19) $\gamma_t = (1-\delta)/(1-\theta)$

where

(20) $\theta = -t\Sigma w^i(\partial h^i/\partial t)/\Sigma w^i h^i$

is the elasticity of the tax base with respect to the tax rate and

(21) $\delta = -Cov(\lambda^i, w^i h^i)/\lambda wh.$

is the normalized covariance between the marginal utility of income and labour income. The question is now what we can say on a theoretical basis about the sign and magnitude of $\delta$ and $\theta$.

Going back to equation (10), it is clear that the bracketed expression must be positive at the optimum; the optimal $t$ is at the upward-sloping part of the «Laffer curve». This immediately implies that $\theta<1$. Is it also possible to identify a lower limit for $\theta$? This is more difficult, since it cannot be done on the basis of the optimum conditions alone. But note that (20) can be rewritten as

(22) $\theta = -\Sigma w^i h^i \varepsilon^i / \Sigma w^i h^i,$

which is the weighted average of the uncompensated labour supply elasticities, using labour income as weights. If one tends to think of the supply elasticity as being approximately constant across individuals, then that common elasticity is indeed the appropriate measure of $\theta$.Whatever the variation of the elasticity across consumers, it is clear that individual elasticities can be negative or positive, depending on the relative strengths of the substitution and income effects [6]. Without going into a detailed discussion of empirical estimates at this stage, a cautious generalization would be that most of them result in elasticities that are non-positive, but that the possibility of backward-bending supply curves is a real one.

We now turn to a discussion of the value of $\delta$ and first consider its sign. A natural assumption is that $w^i h^i$ is increasing in $w^i$; this is the *agent monotonicity* assumption in optimum tax theory, whereby the more able individuals reveal their ability through higher incomes. As regards the connection between $\lambda^i$ and $w^i$ one should note first that in a first best optimum $\lambda^i$ would be the same for all, so that the covariance would be zero. In a second best optimum, however, given that the cardinal representation of utility is strictly concave, the rich would have a lower marginal utility of income than the poor. The implication of this is that $\delta > 0$[7]. As regards the upper limit, we can use (21) and the formula for the covariance to write

$$\delta - 1 = -(1/n)\Sigma\lambda^i w^i h^i / \lambda w h < 0,$$

where the inequality sign follows from the fact that the sum is taken over positive numbers. Hence it follows that $\delta < 1$.

It is worth pointing out that while $\delta$ would be zero in a first best optimum, a value close to zero could also emerge if there were an *empirically* weak correlation between labour income and the marginal utility of income. Such a possibility does not emerge explicitly in the present model, but it could be relevant in a context where individuals had other income which could not be taxed, either because of legal restrictions on the concept of taxable income or because of tax avoidance and evasion.

I have now indicated what seems to be reasonable ranges for the parameters $\theta$ and $\delta$. The two measures can be thought of as being indexes of *allocative and redistributional imperfection*, respectively. As regards $\theta$ this interpretation can be motivated as follows. Distortions are due to substitution effects. If substitution effects were zero, i.e. indifference maps were of the Leontief type, the proportional income tax would only create income effects, and the denominator of $\gamma_t$ would be approximately equal[8] to that of $\gamma_a$. Negative substitution effects would raise the value of $\gamma_t$. More generally, income effects tend to repair the damage caused by the substitution effects. If the two effects were to cancel out exactly, the efficiency aspect alone would call for an MCF of unity in spite of the fact that the tax is distortionary[9].

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[6] Most estimates of the elasticity of labour supply are in terms of the wage rate net of tax, i.e. $w(1-t)$. To convert this to a tax elasticity it must be multiplied by $-t/(1-t)$.

[7] The implication also requires that consumption and leisure are normal goods: see Sandmo (1983, p. 320). In Dixit and Sandmo (1977) it was shown in the context of an optimal linear income tax model that a negative covariance is a necessary condition for the marginal tax rate to be positive. The same result is implicit in the present framework.

[8] See the Appendix for a more precise statement of the approximation involved.

[9] Ballard and Fullerton (1992) asked a group of conference participants to assess the MCF for a single-person economy with Cobb-Douglas preferences and a proportional tax on labour income. Most people answered «yes» to the question whether such a tax was distortionary, and this is obviously correct. When asked whether a project

Near the top of the Laffer curve substitution effects become dominant, the revenue elasticity gets close to -1, and $\theta$ approaches 1.

As regards $\delta$, if $\lambda^i$ were constant across consumers, the income tax would in fact mimic the system of perfect lump sum taxes in terms of its distributional effect, and we would have $\delta=0$. As the covariance between the marginal utility of income and the tax base becomes more pronounced negative, indicating the increasing imperfection of labour income as an indicator of individual welfare, $\delta$ increases towards 1. I have as yet barely indicated the most likely empirical values of these parameters. But whatever they are, one condition governing the magnitude of the MCF is at least conceptually very simple; it is greater than one if $\delta<\theta$ and less than one if the reverse inequality holds.

In the first best allocation, we would of course have $\delta=\theta=0$ and an MCF of unity. More generally, it might seem reasonable to argue that a rational policy would indeed be characterized by the condition $\delta=\theta$, since this involves a perfect balancing of distributional gains against allocative losses. Some contributions to the literature which assume a general non-linear optimum tax schedule, such as Kaplow (1996), can be interpreted in this light. But one may doubt whether this framework captures the imperfections of existing tax systems. It seems difficult to offer a general verdict on the likelihood that with only distortionary marginal finance the MCF would be either greater or less than one.

## 6. An optimal marginal income tax.

So far we have assumed that the marginal source of finance is either a non-distortionary or a distortionary tax. It is important also to consider the case where an optimal income tax is determined jointly with the supply of public goods, so that the marginal source of finance is a combination of distortionary and lump sum elements. But if the parameters $t$ and $a$ have both been chosen optimally, it means that at the margin the cost of financing public goods through either of these sources must be the same. In other words, for a small increase in the supply of public goods, *the MCF must be the same, whatever the source of finance.*

This conclusion, which was also pointed out by Wilson (1991) in the context of a somewhat different model, can be seen as an immediate implication of the envelope theorem. Somewhat more laboriously, one may consider the two first-order conditions (10) and (11) and note that both of them can be solved for the value of $\gamma$ ($=\mu/\lambda$). That value must obviously be the same for the two first-order conditions, so that we may write

(23) $\gamma_t=\gamma_a=\gamma$.

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with a benefit-cost ratio only slightly above one should be carried out when financed by a one percent increase in the tax rate, the majority said «no». Ballard and Fullerton argue that the correct answers should be «yes» to both questions, and the present analysis supports their view. In the Cobb-Douglas case the uncompensated elasticity is zero, so that $\theta=0$. Since, by assumption, $\delta=0$ also, it follows that $\gamma_t=1$. However, one may object to a definition of the MCF that makes it equal to one even for distortionary marginal finance; Håkonsen (1997) develops an alternative definition which does not have this feature.

But from (17) it follows that $\gamma_a < 1$, so that

(24) $\gamma < 1$.

With optimal linear income taxation, there is a general marginal cost of funds which is less than one. The implication is, from (19), that $\delta > \theta$. This has an interpretation in terms of the equity-efficiency tradeoff implicit in the choice of parameters in the linear income tax scheme. By increasing the degree of redistribution via the tax system one also increases its distortionary effect, and this is the cost of achieving a more just distribution of income. In a rational equity-efficiency tradeoff, redistribution should be carried to the point where the gain, as measured by $\delta$, exceeds the cost, as measured by $\theta$.

It is clear that this conclusion hinges on the assumed availability of a lump sum transfer. As I have already stressed, the usual objections to ideal lump sum taxes, which are assumed to vary between individuals, do not apply to a uniform tax, which does not require detailed information about individual income earners, and which does not require the revelation of private information on the part of taxpayers. As Wilson (1991) has noted, a uniform transfer is part of the notion of a negative income tax which has been much debated both in the United States and other countries. It is therefore clearly of interest in a normative context. A uniform reduction in the level of social security benefits could also be thought of as coming fairly close to this notion of a uniform lump sum tax.

The central result that the MCF is less than one raises an obvious question with respect to the empirical literature on the subject. Since a number of studies, e.g. Hansson and Stuart (1985) and Browning (1987), have found MCFs well in excess of one, this raises the question of whether the theory is all wrong or whether empirical studies have been misguided. But there is of course no contradiction. Empirical studies are carried out on the basis of an observed state of the economy which is not necessarily characterized by an optimal tax system. This does not make the theory irrelevant, but it underlines the fact that the present analysis so far has derived MCFs that in some way corresponds to an optimum. It is clearly of importance to show the application of the analysis to an initial situation which is not optimal, but where the tax rates take on arbitrary initial values - arbitrary in the sense of not having been derived from an optimum taxation framework.

## 7. The MCF with a non-optimal tax system.

We now wish to compute the welfare effect of an increase in the supply of public goods. The increase in cost must be covered by increases in tax revenue, but the only restrictions on tax rates are now those that follow from the tax revenue constraint. The effect on welfare of a simultaneous change in taxes and expenditure can be written as

$$dW = (\partial W/\partial t)dt + (\partial W/\partial a)da + (\partial W/\partial z)dz.$$

Using the definition (8) of social welfare and substituting from the Roy conditions (5), this can be rewritten as

$$(25)\quad dW=-\Sigma\lambda^i w^i h^i dt+\Sigma\lambda^i da+\Sigma\lambda^i m^i dz.$$

Taking the differential of the government budget constraint (9), we have that

$$[\Sigma w^i h^i+t\Sigma w^i(\partial h^i/\partial t)]dt+[t\Sigma w^i(\partial h^i/\partial a)-n]da=[q-t\Sigma w^i(\partial h^i/\partial z)]dz.$$

We first consider the case of non-distortionary finance. In this case we have $dt=0$; eliminating $da$ from the last equation and substituting into (25) we may then write the condition for a welfare-improving increase in the supply of public goods as

$$dW/dz>0 \leftrightarrow$$

$$(26)\quad n\Sigma\lambda^i m^i/\Sigma\lambda^i>\gamma_a[q-t\Sigma w^i(\partial h^i/\partial z)].$$

Here $\gamma_a$ is as defined by (17), and even though there is no particular reason why its numerical value should be the same as in the second best optimum, we may just as before conclude that it is less than one.

Proceeding in a similar manner, we may compute a corresponding condition for a welfare improving increase in public goods supply with distortionary finance. This becomes

$$dW/dz>0 \leftrightarrow$$

$$(27)\quad n\Sigma\lambda^i m^i/\Sigma\lambda^i>\gamma_t[q-t\Sigma w^i(\partial h^i/\partial z)].$$

Again the formal expression for the MCF is the same as before and given by equation (19). In contrast to the analysis of the previous section, however, we cannot appeal to the first-order conditions for optimal taxation for the assessment of the MCF; in particular, there is no reason for assuming the two concepts of the MCF to be the same. Since there is no optimization with respect to the design of the tax system, we cannot appeal to the envelope theorem, and in general the MCF will differ among sources of tax finance. The magnitude of the MCF for distortionary taxation therefore becomes basically an empirical issue, depending on the values of $\theta$ and $\delta$. As regards $\theta$, most estimates of actual labour supply elasticities are on fairly aggregated data, although with a number of separate studies for men and women. The surveys by Pencavel (1986) and Killingsworth and Heckman (1986) indicate uncompensated wage elasticities for male labour supply that tend to be negative, although close to zero; for female labour supply they tend to be significantly positive. For the whole of the labour force a reasonable guess might be that the elasticity is positive but small; perhaps in the range 0.1-0.2, with tax elasticities being of opposite sign, and numerically perhaps a little lower. This would correspond to values of $\gamma_t$ in the range 1.10-1.20 if $\delta$ is set at zero. A positive value of $\delta$ would modify this conclusion, however, and tend to bring $\gamma_t$ back down towards unity.

These remarks must naturally be taken as illustrative only, and for several reasons. First of all, elasticities are not invariant over time and between countries. Second, while the present paper focuses attention on the labour supply distortion via the marginal income tax rate, there are actually many more distortions relevant to the computation of the MCF. Both commodity taxes and taxes on income from capital would be important for a serious numerical computation. It is not clear, however, that such an extension of the model would necessarily lead to an increase in the estimates of the MCF relative to the pure labour supply model; this would depend on the nature of market interaction - a point well known to all who are familiar with second best models.

In popular discussions about policy one sometimes encounters statements about the MCF that seem to assume that it is necessarily *one* number, which is invariant to the choice of tax instrument for the marginal source of finance. As already shown, this would indeed be the case under an optimal tax system, but otherwise one would have to specify carefully what exactly is the source of tax finance for the contemplated increase in public goods supply. Estimates of a single value of the MCF without the optimality assumption would have to be based on an assessment of the relative importance of different taxes for the finance of the increase in expenditure. Some arbitrariness would perhaps be unavoidable here, and in any case the assumptions to be employed should be made explicit in order to arrive at a clear understanding of any particular measure of the MCF.

## 8. Concluding comments.

The starting point of this paper was the observation that many discussions of the marginal cost of public funds are based on theoretical models where distributional issues have been assumed away. This is somewhat paradoxical, since the distortionary effects of taxation, on which thinking around the MCF is based, can only be justified from a welfare economics point of view by their positive effects on the distribution of income. The present model develops a concept of the MCF which gives a role both to efficiency and distribution. In particular, it develops a formula for the MCF of distortionary taxes which involves a balancing of two indexes, one of allocative and one of redistributive imperfection. On the basis of this formula, there is no obvious theoretical justification for assuming that the MCF is greater than one.

In general, there is no unique number representing the MCF for the tax system as a whole. Instead, there is in principle one number for every tax instrument that might represent the marginal source of finance. However, if all tax instruments are chosen optimally, the MCF must, by the envelope theorem, be the same for all policy instruments. If one of the instruments is a uniform lump sum transfer, its MCF is *less* than one; this MCF is accordingly also the MCF for the tax system as a whole. This result emphasizes the importance of distinguishing between the *optimal* MCF and that which is estimated on the basis of some arbitrary initial state of the system of taxation. Although the analytical form of the MCF - or rather of the different MCF formulae - is the same in the two frameworks of analysis, the empirical counterparts of the formulae may be quite different.

As already noted, the concept of the MCF has in principle two different uses, although the two are related. On the one hand one may look upon the MCF as a convenient conceptual tool for thinking about the overall balance between the private and public sectors. That kind of consideration should presumably be the responsibility of the central government. The other use of the concept is as a parameter to be applied by individual public agencies in their own project evaluations. I have already pointed out that such a procedure must require a definition of the MCF which is not project specific, so that tax revenue effects of public expenditure should be kept out of the definition of the MCF. Apart from this point, recommendations for this kind of use of the MCF seem to presuppose that the MCF is uniquely defined, whereas I have shown that this will only be the case in an optimal tax system. In general, the MCF will differ according to the marginal source of finance, and the choice of this source is in the large majority of cases the responsibility of the central government, not the individual agencies. If the marginal source of funds varies over time, differing from one government budget to another and possibly also within budget periods, it is not obvious that much is gained by asking individual agencies to employ the MCF in their calculations. One pragmatic solution to this problem is perhaps to settle on a value of the MCF which is a rough average of the MCFs for the different sources of funds, assuming also that the weights in the average will be approximately constant over time. However, the MCF must necessarily be sensitive to the design of the tax system; indeed, one of the purposes of a major tax reform is frequently stated to be a reduction of the marginal cost of public funds[10].

Throughout the discussion the emphasis has been on the formulation of the appropriate *rule* for deciding on the optimal provision of public goods in a second best economy. As already emphasized by Atkinson and Stern (1974) and later by Wilson (1991), a comparison of the correct rules in first-best and second-best economies does not automatically imply a comparison between first and second best *levels* of public good provision. The present focus on the decision rule has been motivated by the focus on the second of the two uses of the MCF that were identified above.

Whatever one's attitude to the desirability of using the MCF as part of a policy of decentralized cost-benefit analysis, there would appear to be little justification for letting it reflect the efficiency loss from taxation alone. Just as tax distortions represent the cost of redistribution, a more equal distribution is the gain from distortions. Both need to be taken into account in a balanced evaluation of the marginal cost of public funds.

Finally, it should be pointed out that this conclusion is also very much in line with Pigou's pioneering discussion of this problem. Several authors have referred to the following statement (1947, p. 34):

> «Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure.»

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[10] See e.g. the discussion of the 1991 Swedish tax reform in Agell, Englund and Södersten (1996).

However, in the same chapter (pp. 32-33) Pigou also wrote:

> «...a government may properly engage in larger expenditures (1) the less even is the distribution of income among its citizens and (2) the more progressive is the revenue-raising scheme that it decides to employ.»

The latter statement is clearly a reflection of Pigou's egalitarian attitudes, and must be understood in that light. But it is evident that he had given much thought both to efficiency and equality aspects of the marginal cost of public funds.

**Appendix: Perfect aggregation and the MCF.**

In the text it was stated that in the case of zero substitution effects, the two measures of the MCF, $\gamma_t$ and $\gamma_a$, become approximately equal as far as the efficiency index is concerned. This appendix shows exactly what is involved in this statement.

The Slutsky equations for the effect of the marginal tax rate on labour supply are

$$\text{(A1)} \quad \partial h^i/\partial t = -w^i h^i(\partial h^i/\partial a) + s^i, \qquad (i=1,\ldots,n)$$

where $s^i$ are the substitution effects. Using these equations, we can rewrite $\theta$ from (23) as

$$\text{(A2)} \quad \theta = -t\Sigma w^i[-w^i h^i(\partial h^i/\partial a) + s^i]/\Sigma w^i h^i.$$

When substitution effects are zero, this becomes

$$\text{(A3)} \quad \theta = t\Sigma w^i h^i(w^i \partial h^i/\partial a)/\Sigma w^i h^i.$$

Perfect aggregation involves parallel and linear Engel curves. From the budget constraint (2) it follows that

$$\text{(A4)} \quad \partial c^i/\partial a = (1-t)w^i \partial h^i/\partial a + 1. \qquad (i=1,\ldots,n)$$

If the left hand sides are the same for all i, so are the terms $w^i\partial h^i/\partial a$. Let us write their common value as $\eta$. (17) can then be rewritten as

$$\text{(A5)} \quad \gamma_a = [1-t\eta]^{-1}.$$

Substituting $\eta$ into (A3) yields $\theta = t\eta$, so that (19) becomes

$$\text{(A6)} \quad \gamma_t = (1-\delta)/(1-t\eta).$$

The «efficiency part» of $\gamma_t$ therefore becomes exactly equal to that of $\gamma_a$ in this case. In the general case - relative to the assumption of zero substitution effects - the approximation involved is therefore one that is due to imperfect aggregation

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