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TESTING FOR COMMON CYCLICAL FEATURES IN NONSTATIONARY PANEL DATA MODELS

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Abstract

In this paper we extend the concept of serial correlation common features to panel data models. This analysis is motivated both by the need to develop a methodology to systematically study and test for common structures and comovements in panel data with autocorrelation present and by an increase in efficiency coming from pooling procedures. We propose sequential testing procedures and study their properties in a small scale Monte Carlo analysis. Finally, we apply the framework to the well known permanent income hypothesis for 22 OECD countries, 1950-1992.

Keywords: Panel data, serial correlation common features, permanent income

JEL Classification: C32

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1 Introduction

In economics it is often of interest to test wether a set of time series moves together, that is whether the series are driven by some common factors. The vast literature on cointegration has focussed on long-run comovements for nonstationary time series. More recently, some authors have analyzed the existence of short-run comovements between stationary time series or between first differenced cointegrated-*I(1)* series (see Tiao and Tsay, 1989; Engle and Kozicki, 1993; Gouriéroux and Peaucelle, 1993; Vahid and Engle, 1993; Vahid and Engle, 1997; Ahn, 1997). Among these approaches, the concept of serial correlation common features (SCCF hereafter) introduced by Engle and Kozicki (1993) appeared to be useful. It means that stationary time series move together as there exist linear combinations of these variables that yield white noise processes. These common feature vectors are measures for analyzing short-run relationships between economic variables suggested by economic theory such as relative purchasing power parity (Gouriéroux and Peaucelle, 1993), permanent income hypothesis (Campbell and Mankiw, 1990; Jobert, 1995), cross-country real interest rate differentials (Kugler and Neusser, 1991), real business cycle models (Issler and Vahid, 1996), convergence of economies (Beine and Hecq, 1997a, 1998), Okun's Law (Candelon and Hecq, 1997).

Serial correlation common features implies the existence of a reduced number of common dynamic factors explaining short-run comovements in economic variables. A companion form of the common features models is the common factor representation which has been used in macroeconomics for some decades (see e.g. Engle and Watson, 1981; Geweke, 1977; Lumsdaine and Prasad, 1997; Singleton, 1980). Beyond economic considerations, through the reduced-rank restrictions, the existence of common features leads to a reduction of the number of parameters to be estimated. In general, imposing common cyclical feature restrictions when they are appropriate will induce an increase in estimation efficiency (Lütkepohl, 1991) and accuracy of forecasts (Issler and Vahid, 1999).

Also as for unit roots and cointegration tests, the power of common cyclical feature procedures may be low for small samples (Beine and Hecq, 1999). The power of tests might be increased by relying on panel data instead of using only time series data. Consequently, in this paper we propose to extend these models by testing for serial correlation common features in a panel data framework. In order to avoid confusion, it is worth noticing that standard panel data models with common parameter structures obviously already imply a common feature structure, namely the one which allows to pool the behavior of N individuals. Notice that the assumption of poolability often made in panels may be far too strong. An investigator may want to test which poolability restrictions are supported by the data and which restrictions have to be rejected for the panel data.

We propose to generalize the SCCF approach and apply it to search for common cyclical features in panel data. In particular, we investigate whether there exist linear combinations of the variables for individual or entity i which are white noise for all i, in other words, which weights in the linear combinations are identical across all entities. Developing a methodology to analyze and test common cyclical features in panel data is of theoretical and practical importance since common cyclical feature restrictions are less restrictive than the assumption of identical parameters across individuals usually made in panel data modeling.

Some purists might not speak about panel for this type of analysis. Indeed, in situations we are interested with, N will be relatively small compared to its value in usual panel data and T is assumed large (with $T \to \infty$ asymptotic). Many macroeconomic studies deal with 15 to 50 annuals observations for 20 to 100 countries, regions, industry levels or big firms. In those cases, the border between pure panel analysis ($N \to \infty$) and pure time series analysis ($T \to \infty$) is fuzzy. Far from impoverishing the panel data analysis, taking into account medium or large size time series raises new interesting issues such as testing for unit roots or cointegration in panel data (see *inter alia* Levin and Lin, 1993; Pesaran and Smith, 1995; Evans and Karas, 1996; Kao, 1997; Pedroni, 1997; Phillips and Moon, 1999b, and Phillips and Moon, 1999a, for the asymptotic theory).

The paper is organized as follows. Section 2 provides an example of common features between consumption and income implied by economic theory and likely to be common to data for different countries. In Section 3 we review the concept of serial correlation common features. Section 4 extends it to panel data. As we study differences and similarities in macroeconomic series for different countries, we concentrate our analysis on the fixed effect model (see Hsiao, 1986). Section 5 describes estimation procedures. In Section 6 simulation results are reported. In Section 7 we present an empirical analysis of the liquidity contraint consumption model for 22 OECD countries and the G7. Section 8 concludes.

2 An Example of Common Features in Panel Data

To further motivate this paper, consider the permanent income hypothesis (PIH hereafter) and the heterogeneous consumer model proposed by Campbell and Mankiw (1990, 1991). These authors consider two groups of agents who receive a disposable income Y_{1t} and Y_{2t} in fixed proportions of the total income respectively, such that $Y_{1t} = \lambda Y_t$, $Y_{2t} = (1 - \lambda)Y_t$ and $Y_t = Y_{1t} + Y_{2t}$. Agents in the first group are subject to liquidity constraints. Therefore, they consume their current income while agents in the second group consume their permanent income. We get the following system:

$$\begin{cases} C_{1t} = Y_{1t} = \lambda Y_t \\ C_{2t} = Y_{2t}^P = (1-\lambda) Y_t^P \\ Y_{1t} = Y_{1t}^P + Y_{1t}^T \\ Y_{2t} = Y_{2t}^P + Y_{2t}^T, \end{cases} \tag{1}$$

where C_{it} is the consumption of agent i and Y_{it}^P and Y_{it}^T are the permanent and transitory component of income of the agent i which are assumed to be *I(1)* and *I(0)*, respectively. Aggregating over agents we get $C_t = Y_{1t}^P + Y_{1t}^T + Y_{2t}^P = Y_t^P + Y_{1t}^T$, and thus:

$$\begin{cases} C_t = Y_t^P + \lambda Y_t^T \\ Y_t = Y_t^P + Y_t^T, \end{cases} \tag{2}$$

which shows that aggregate consumption and income share a common trend Y_t^P. Note that because a fraction λ of income accrues to individuals who consume their current income rather than their permanent income, this model has been labelled "λ *model*" by Campbell and Mankiw (1990, 1991). It is also easily seen that if $\lambda = 0$ we get the permanent income model. In order to stress the common cycle component let us take the first difference of aggregate consumption $C_t = C_{1t} + C_{2t}$. By substituting the shares of income in the total income we obtain $C_t = \lambda Y_t + (1-\lambda)Y_t^P$ which in first differences Δ can be written as:

$$\Delta C_t = \lambda \Delta Y_t + (1-\lambda)\Delta Y_t^P. \tag{3}$$

Consequently, assuming that the permanent income is a martingale, the consumption function can be tested by the regression $\Delta C_t = \lambda \Delta Y_t + (1-\lambda)\varepsilon_t$. However, ε_t is a difference martingale which is not orthogonal by construction to ΔY_t. Therefore this equation cannot be consistently estimated by OLS but instrumental variables (IV) estimators are appropriate.

With a few exceptions as Vahid and Engle (1993) and Jobert (1995), most empirical studies do not take the cointegrating vector into account as a valid instrument when testing equation (3) using IV estimates, and therefore may be subject to an omitted variable problem. Vahid and Engle (1993) made the connection with the common feature hypothesis that ε_t is a white noise[1] with $[1 \ -\lambda]$ the associate normalized common features vector. Empirical studies have shown that λ is usually significantly different from zero with coefficients in the range 0.3 to 0.5 for most

[1] Note that Vahid and Engle (1997) have extended their framework to the case where a linear combination is a MA(q) process and not a white noise. They labelled this model non-synchronous common cycle.

countries. Therefore in order to test for the existence of one short-run relationship common to a set of countries and to improve the power of common feature tests, a pooled common features test in panel seems appropriate. The use of the cross-section dimension in the estimation could also give rise to substantial efficiency gains.

3 Common Features in Time Series

In the context of time series analysis, serial correlation common features means that there exist linear combinations of (stationary) economic time series which are white noise processes. Consider a cointegrated VAR of order $p = 2$, written in its VECM form, for consumption and income, for $t = 1 \ldots T$:

$$\begin{bmatrix} \Delta c_t \\ \Delta y_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \lambda\phi_{21} & \lambda\phi_{22} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} \Delta c_{t-1} \\ \Delta y_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 1 \end{bmatrix} [\beta_1 \quad \beta_2] \begin{bmatrix} c_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \tag{4}$$

where μ_1 and μ_2 are constant drift terms, $[\varepsilon_{1t}, \varepsilon_{2t}]'$ is a bivariate white noise process with nonsingular covariance matrix Ω. (β_2/β_1) is the long-run income elasticity if one chooses consumption as normalized variable. The autoregressive coefficients matrix is of reduced rank and the system may be written as:

$$\begin{bmatrix} \Delta c_t \\ \Delta y_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \lambda \\ 1 \end{bmatrix} [\phi_{21} \quad \phi_{22}] \begin{bmatrix} \Delta c_{t-1} \\ \Delta y_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 1 \end{bmatrix} [\beta_1 \quad \beta_2] \begin{bmatrix} c_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}. \tag{5}$$

A distinction could be made at this stage between a strong and a weak form reduced rank structure, as put forward by Hecq, Palm and Urbain (1997a,b). The Strong Form Reduced Rank Structure (SF) is the original formulation proposed by Engle and Kozicki (1993) in which long-run and short-run matrices share the same left null space. It corresponds to $\alpha = \lambda$ in system (5). In this case, there exists a common feature vector $\tilde{\beta}' = [1 \quad -\lambda]$ such that premultiplying expression (5) by $\tilde{\beta}'$ yields a white noise. In the less restrictive model, labelled Weak Form Reduced Rank Structure (WF), $\alpha \neq \lambda$, and a linear combination of first differences in deviation from the long-run equilibrium is a white noise. Consequences in terms of common cycles as well as inference issues are analyzed in Hecq, Palm and Urbain (1997a,b).

Common features relationships give information on short-run comovements. These relationships may come from economic theory (relative purchasing power parity, PIH) or from stylized facts (convergence, RBC models) and give the dynamic common factor within the system, i.e. $\phi_{21}\Delta c_{t-1} + \phi_{22}\Delta y_{t-1}$ in the WF case for instance. The orthogonal complement of the $\tilde{\beta}$, labelled $\tilde{\beta}_\perp$ ($\tilde{\beta}'\tilde{\beta}_\perp = 0_{s\times 2}$), gives the factor loading of the common dynamics in the equations, that is

$\tilde{\beta}_\perp = [\lambda \quad 1]'$ in system (5). Notice that these common dynamic factors should not be confused with common cycles. Common dynamic factors refer in our study to reduced rank in VAR matrices. Common cycles are defined in a specific trend-cycle decomposition as the part of the time series left after removing permanent components. Vahid and Engle (1993) show that the existence of s common feature vectors (of the SCCF or SF type) leads to $n-s$ common cycles in the multivariate Beveridge-Nelson decomposition. Vahid and Engle (1997) extend this definition to nonsynchronous cycles. Hecq, Palm and Urbain (1997b) propose a Beveridge-Nelson decomposition for the WF that allows for a reduced number of common cycles. Notice that the latter weak form reduced rank structure will in the sequel not be explicitly considered as we want to focus on the extension of the standard serial correlation common feature analysis to panel data.

In this simple bivariate model (4)-(5), the serial correlation common feature hypothesis may also be written in terms of moment conditions such as:

$$\mathsf{E}[(\Delta c_t - \mu - \lambda \Delta y_t) \otimes W_t] = 0, \tag{6}$$

where $\mathsf{E}[.]$ is the linear expectation operator and $W_t = \{1, \Delta c_{t-1} \ldots \Delta c_{t-k}, \ \Delta y_{t-1 \ldots} \Delta y_{t-l}, z_{t-1})$ is a set of instruments consisting of a constant term, the lags of both variables and the deviation from the long-run relationship $z_{t-1} \equiv c_{t-1} - \ (\beta_2/\beta_1) y_{t-1}$.

Adopting a two-step approach[2], there are two ways to test for SCCF. The first way is to carry out a canonical correlation analysis between consumption and income on the one hand and the set of instruments on the other hand. The nonsignificant squared canonical correlations reveal the existence of linear combinations which yield white noise processes. Alternatively, one can use generalized method of moments type estimators exploiting the moment condition (6). A test of overidentifying restrictions in (6) is a test of serial correlation common features. The use of canonical correlation estimation has the advantage that results do not rely on the choice of the normalization of the moment conditions. Moreover, it is more convenient when we test for the number of common feature vectors. In this paper we treat the problem in a GMM framework for several reasons. Firstly, we have at most one common feature vector in a bivariate system. Secondly, this framework may be more easily extended to panel data models. Finally, normalization imposed on IV by selecting one variable as having a coefficient equal to one leads

[2] The first step checks for the presence of cointegrating relationships and then, given the estimated cointegration relations, the common feature analysis is carried out in a second step. An alternative is to use an estimation procedure that exploits both the cointegration and common features restrictions using a switching algorithm (Hansen and Johansen, 1998; Hecq, 1999).

to an increase of the power of the test compared with those based on canonical correlations.[3]

4 Extension to Panel Data Models

Frequently, analyses comparing for instance the *PIH* with "λ *model*", concentrate on one country, very often the USA. In order to motivate the generality of the theory, some authors extend their empirical investigation to several countries (Campbell and Mankiw, 1991; Evans and Karas, 1996). However it is difficult to claim that results for different countries are uncorrelated. Since it is not possible to construct a pure time series model with relatively few observations for a large number of individuals, such as a VAR model with $2 \times N$ endogenous variables in a bivariate case, alternatives must be found.

One solution would be to analyze the system under separation in common features (Hecq, Palm and Urbain, 1999), an extension to separation in cointegration (Granger and Haldrup, 1998; Konishi and Granger, 1993). Under separation in common features, the common feature matrix is block-diagonal with blocks corresponding to one individual i only. Treating the issue in the complete system with separation in common features avoids losing efficiency compared to an analysis of the marginal model for individual i since separation does not require block-diagonality of the distrurbance covariance matrix. This solution is however difficult to implement for more then two or three individuals. We illustrate this point via a small Monte Carlo experiment, of which the precise specification will be given in Section 6. Consider a DGP made out of bivariate systems similar to (4), with $\alpha = \lambda$ (SCCF hypothesis), for respectively two and five individuals. The only cross-sectional relations are due to a non-diagonal disturbance variance-covariance matrix. Complete separation in cointegration, in common features as well as absence of bidirectional short-run Granger causality is thus maintained. Using a standard canonical correlation framework (see *inter alia* Hecq, Palm and Urbain, 1998a) we perform a serial correlation common feature analysis in the marginal model for the first individual, ignoring the cross-correlations. Alternatively, under separation in common features, we test the number ($s = 2$ or $s = 5$) of common feature vectors for each individual in the complete system. We then constrain the common feature space to be block-diagonal (see Hecq, Palm and Urbain, 1999) and estimate the vector for the first individual.

INSERT TABLE 1 ABOUT HERE

In Table 1, we report for 5,000 replications the median and the spread (interquantile range) of the bias, χ^2 test statistics for the overidentifying restrictions implied by the presence of

[3] See Anderson and Vahid (1996) for the connection between GMM and canonical correlation estimators.

common features as well as a small sample adjusted version (Hecq, 1999). Although separation in common features holds at the level of the DGP, some efficiency loss, as measured by the spread, is observed in the marginal model compared to the full system from $T = 25$ for $N = 2$ and from $T = 50$ for $N = 5$. However the dispersion is too high for smaller sample size and test statistics reject too often the presence of respectively two and five common feature vectors.

These illustrative Monte Carlo results call for an extension to a (possibly nonstationary) panel common feature analysis.

Let the subscript $i = 1, ..., N$ indicate the different groups/entities/units, $t = 1, ..., T$ denote the sample period and $j = 1, ..., n$ denote the number of variables for each group/entity. We assume that the $n-$dimensional vector of observed $I(1)$ variables for entity i, $X_{i,t}$, is generated by a p_i-th order cointegrated VAR which can be expressed in error-correction form as follows:

$$\Delta X_{i,t} = \mu_i + \gamma_t + \alpha_i \beta_i' X_{i,t-1} + \sum_{j=1}^{p_i-1} \Gamma_{i,j} \Delta X_{i,t-j} + \varepsilon_{i,t}, \qquad i = 1, \ldots N, \quad t = 1, \ldots, T, \qquad (7)$$

where μ_i denotes fixed individual effects, γ_t denotes a vector of deterministic time effects, α_i and β_i are $n \times r_i$ matrices of full column rank with r_i being the cointegrated rank $(r_i < n)$ and $\varepsilon_{i,t}$ is a disturbance. The vector $\varepsilon_t = (\varepsilon_{1,t}', \ldots, \varepsilon_{N,t}')'$ is an $nN \times 1$ dimensional homoskedastic Gaussian mean innovation process relative to $X_{-1} = \{X_{i,t-j},\ i = 1, \ldots, N;\ j < t\}$ with nonsingular contemporaneous covariance matrix Ω, the $(i, j) - th$ block of which being $E(\varepsilon_{i,t}\varepsilon_{j,t}') = \Omega_{i,j}$. Notice that one could allow for random individual effects in expression (7). This would lead to an error-component structure of $\varepsilon_{i,t}$ similar to that used in the panel data literature.

For system (7), we introduce the definition of an homogeneous SCCF panel model:

Definition 1 *A panel model is called an homogeneous panel common feature model if there exists,* $\forall\ i = 1, \ldots, N$, *a* $(n \times s_i)$ *matrix* $\tilde{\beta}_i = \tilde{\beta}_j\ \forall\ i, j = 1, \ldots, N$, *whose columns span the individual cofeature space, such that* $\tilde{\beta}_i' \Delta X_{i,t} = \tilde{\beta}_i' \varepsilon_{i,t}$ *is a* s_i-*dimensional white noise process for each individual.*

This definition applies to the case where the individual cofeature matrices, and hence their column rank s_i, are the same across all individuals. A typical dynamic panel data model with fixed effects μ_i and deterministic time effects γ_t arises as a special case of (7) when the parameters α_i, β_i, $\Gamma_{i,j}$ and Ω_i are the same across entities i (see e.g. Hoogstrate, 1998). In order to clarify the nature of the hypotheses underlying the panel common feature restrictions, in the next subsection, following Groen and Kleibergen (1999) for panel cointegration, we consider a model resulting from sequentially testing and applying restrictions to a high dimensional unrestricted VECM.

4.1 A Panel VECM Representation

We are interested in testing for cointegration and common serial features with respect to n $I(1)$ time series in vector $X_{i,t}$ within a dynamic model for N individuals i. Without loss of generality, we consider a large VECM with one lag in the first differences, e.g. a VAR with two lags in level. The generalization to high order dynamics is immediate by substituting Γ_{ij} by $\Gamma_{ij}(L)$ in (8) but it makes the notation heavy. We consider the model without any time dummies for sake of simplicity. For $t = 1, \dots, T$ we may write the nN-dimensional system as:

$$\Delta X_t = \begin{pmatrix} \Pi_{11} & \dots & \Pi_{1N} \\ \vdots & \ddots & \vdots \\ \Pi_{N1} & \dots & \Pi_{NN} \end{pmatrix} X_{t-1} + \begin{pmatrix} \Gamma_{11} & \dots & \Gamma_{1N} \\ \vdots & \ddots & \vdots \\ \Gamma_{N1} & \dots & \Gamma_{NN} \end{pmatrix} \Delta X_{t-1} + u_t, \tag{8}$$

where $\Delta X_t = (\Delta X'_{1,t} \dots \Delta X'_{N,t})'$, $u_t = (u'_{1,t} \dots u'_{N,t})'$ and $X_{t-1} = (X'_{1,t-1} \dots X'_{N,t-1})'$ are vectors of dimension $nN \times 1$, or more concisely

$$\Delta X_t = \Pi_{ur} X_{t-1} + \Gamma_{ur} \Delta X_{t-1} + u_t, \tag{9}$$

where Π_{ur} and Γ_{ur} are $nN \times nN$ matrices and $u_t = \mu + \varepsilon_t$, $\mu = (\mu'_1, \dots, \mu'_N)$, $\varepsilon_t = (\varepsilon'_{1,t}, \dots, \varepsilon'_{N,t})'$ are $nN \times 1$ vectors with $\varepsilon_t \sim N(0, \Omega)$.

$$\underset{nN \times nN}{\Omega} = \begin{pmatrix} \Omega_{11} & \dots & \Omega_{1N} \\ \vdots & \ddots & \vdots \\ \Omega_{N1} & \dots & \Omega_{NN} \end{pmatrix}. \tag{10}$$

This large unrestricted model (8), without any zero block restrictions, is not estimable in practice. Consequently, restrictions have to be imposed. We first describe cointegrating restrictions before introducing serial correlation common feature restrictions.

4.1.1 Cointegrating Restrictions in a Panel VAR

We first consider restrictions on the long-run matrix Π_{ur} in the unrestricted VECM. Two types (A and B) of sequences of hypotheses naturally arise in panel data. The hypothesis involved in a sequence can be tested either sequentially or jointly.

- **A1**: Absence of long-run Granger-Causality between the individual subgroups, i.e. Π_{ur} is block-diagonal with elements $\Pi_{ij} = 0$ for $i \neq j$.

- **A2**: Cointegration in absence of long-run Granger-causality, i.e. $\Pi_{ii} = \alpha_i\beta_i'$, with α_i and β_i being $n \times r_i$ matrices of rank r_i, $i = 1, ..., N$.

- **A3**: Homogeneous panel cointegration, i.e. $\beta_i = \beta_1$, $i = 1, ..., N$; $r = Nr_1$.

- **B1:** Cointegration, i.e. $\Pi_{ur} = \alpha\beta'$, with α and β being $nN \times r$ matrices of rank r.

- **B2:** Complete separation in cointegration (see Granger and Haldrup, 1997), i.e. α and β are block-diagonal, with typical block $\alpha_i\beta_i'$ as defined in A2, and $r = \Sigma_{i=1}^{N} r_i$.

- **B3:** Homogeneous panel cointegration, i.e. $\beta_i = \beta_1$; $i = 1, ..., N$; $r = Nr_1$

When the first two sets of restrictions in either sequence hold, the following restricted structure arises.

$$\Delta X_t = \begin{pmatrix} \alpha_1\beta_1' & 0... & 0 \\ 0 & \ddots & 0 \\ 0 & 0... & \alpha_N\beta_N' \end{pmatrix} X_{t-1} + \begin{pmatrix} \Gamma_{11} & \dots & \Gamma_{1N} \\ \vdots & \ddots & \vdots \\ \Gamma_{N1} & \dots & \Gamma_{NN} \end{pmatrix} \Delta X_{t-1} + u_t. \tag{11}$$

When it is appropriate to add a restricted trend in the cointegration space, we replace X_{t-1} by $X_{t-1}^* = (X_{t-1}', t)'$. For N fixed, a likelihood ratio statistic for (11) versus (8) can be obtained using the sum of two different conditional likelihood ratio statistics to test the sets of restrictions {A1,A2} or {B1,B2}. Next, homogeneity of panel cointegration can be tested using a likelihood ratio test. A decomposition similar to {A1,A2} is proposed by Groen and Kleibergen (1999). The main problem with this approach is that under A1, that is absence of long-run Granger-causality, the usual tests have an unknown asymptotic distribution, as the possible presence of cointegration interferes with the block-diagonality of Π_{ur}. On the other hand, once the cointegrating rank in the unrestricted VECM has been fixed, a test statistic with separation as the null hypothesis has an χ^2-asymptotic distribution. It is worthwhile to mention that although model (11) looks rather specific, it is less restrictive than the models used in the dynamic panel literature, where quite frequently, in addition to separation in cointegration, the same parameter structure is assumed to hold across individuals (see *inter alia* the overview in Phillips and Moon, 1999b). Occasionally, complete separation is relaxed to requiring β to be block-diagonal leaving α unrestricted. (Larsson and Lyhagen, 1999).

4.1.2 Common Feature Restrictions

Imposing serial correlation common feature restriction, system (11) becomes:

$$\Delta X_t = \begin{pmatrix} \tilde{\beta}_{\perp 1}\Psi_1\beta_1 & 0... & 0 \\ 0 & \ddots & 0 \\ 0 & 0... & \tilde{\beta}_{\perp N}\Psi_N\beta_N \end{pmatrix} X_{t-1} + \begin{pmatrix} \tilde{\beta}_{\perp 1}\Psi_1^* & 0\dots & 0 \\ 0 & \ddots & 0 \\ 0 & 0\dots & \tilde{\beta}_{\perp N}\Psi_N^* \end{pmatrix} \Delta X_{t-1} + u_t. \tag{12}$$

As for cointegrating restrictions, this model may be obtained by considering two of the next three hypotheses under (11).

- **C1**: Serial correlation common features: there exists a $(nN \times s)$ matrix $\overset{\sim}{\beta}$ such that $\overset{\sim}{\beta}'\Delta X_t$ is an s dimensional white noise, with $s = \Sigma_{i=1}^N s_i$.
- **C2**: Absence of short-run Granger-causality between the individual subgroups: Γ_{ur} is block-diagonal, i.e. $\Gamma_{ij} = 0$ for $i \neq j$.
- **C3**: Separation in common features: the matrix $\overset{\sim}{\beta}$ is block-diagonal with the $(s_i \times n)$ matrix $\overset{\sim}{\beta}_i$ being a typical block on the main diagonal, $s = \Sigma_{i=1}^N s_i$.
- **C4:** Homogeneity of common features: $\overset{\sim}{\beta}_i = \overset{\sim}{\beta}_1$; $i = 1, ..., N$; $s = Ns_1$.

Actually the hypothesis C2 is implicit when one stacks VECM. Restriction C3 is developed in Hecq, Palm and Urbain (1999) for the SCCF as well as for the weak form structure. Here again a likelihood ratio for model (12) versus (11) can be obtained as the sum of two conditional likelihood ratio statistics to test of either {C1,C2} or {C2,C3}. This means that we can first test for common cyclical features under the maintained hypothesis of short-run Granger-noncausality C2. Alternatively, we can first test for absence of short-run causality and then test for SCCF since both sequences of restrictions imply separation in common feature. This results derives from Proposition 3.3. in Hecq, Palm and Urbain (1999) which states that under separation in cointegration and block diagonality of this long-run matrix, the presence of common features implies that the cofeature matrix is block-diagonal.

5 GMM Estimation

To test for common features in a time series context, we have the choice between GMM estimators applied to a regression framework and a canonical correlation procedure based on maximum likelihood (ML) estimation. Both methods have their advantages and drawbacks. The ML estimation is fully efficient and likelihood ratio tests are asymptotically most powerful. GMM estimators can be more easily implemented but they are in general not fully efficient. In this Section we present a GMM estimator that will be used in our empirical analysis for a bivariate system for consumption and income for the case where at most one serial correlation common feature vector exists.

For each individual, let us split $X_{i,t} = (y_{i,t}, z_{i,t})'$ and let the bivariate DGP be

$$\Delta y_{i,t} = \mu_i + \tilde{\beta}_i^* \Delta z_{i,t} + \eta_{i,t} \quad (13)$$

$$\Delta z_{i,t} = \alpha_i (y_i - \beta_i^* z_i)_{t-1} + \sum_{k=1}^{p_i-1} \delta_{i,1k} \Delta y_{i,t-1} + \sum_{k=1}^{p_i-1} \delta_{i,2k} \Delta z_{i,t-1} + \varepsilon_{i,t} \quad (14)$$

where the second equation for $\Delta z_{i,t}$ is just one row of the VECM (11), with normalized cointegrating vector $\beta_i' = [1, -\beta_i^*]$. Both the $y's$ and the $x's$ are autocorrelated with $\eta_{i,t}$ a nonlinear function in the parameters of lags in $\Delta y_{i,t}, \Delta z_{i,t}$ and error correction mechanism. Under the null of serial correlation common features for individual i, $\eta_{i,t}$ is a white noise process and the normalized SCCF vector is given by $\tilde{\beta}_i' = \left[1, -\tilde{\beta}_i^*\right]$.

In practice (Vahid and Engle, 1993, 1997), after the cointegration analysis in the first step, the GMM procedure proceeds as follows. Regress the explanatory variables Δz_t on the whole set of instruments (i.e. lags of ΔX_t and cointegrating vectors) in order to obtain the best linear prediction $\Delta \hat{z}_t$. Then regress Δy_t on a constant term and $\Delta \hat{z}_t$. This estimate gives the potential serial correlation common feature vector $\widetilde{\beta}_i$. Finally, one tests for the validity of the overidentifying restrictions using Hansen's (1982) χ^2 test.

5.1 Heterogeneous Independent Case

When the observations on individuals are assumed cross-sectionally independent, a joint test for the existence of one individual-specific (heterogeneous) common feature vector can be obtained by computing the χ^2-statistics for the SCCF restrictions for each individual $[\zeta_i \backsim \chi^2(\nu_i)]$, with the same number of variables for each i but with the possibility of having a proper dynamics and the presence or not of cointegrating vectors. The degrees of freedom are then given by $\nu_i = n(p_i - 1) + r_i - (n - 1)$ since s_i equals one. Using the standard central limit theorem for large N, we then have

$$\frac{\sum_{i=1}^{N}\zeta_i - \nu}{(2\nu)^{1/2}} \overset{a}{\sim} N(0,1) \qquad \text{where } \nu = \sum_{k=1}^{N}\nu_i \tag{15}$$

This procedure is however not appropriate in the presence of cross-correlation, a phenomenon pointed out *inter alia* by O'Connel (1998) in the case of panel unit root tests. The size distortions increase with N and with the cross-correlation. While these distortions are DGP dependent, we observe empirical sizes of about 20 % (nominal size = 5%) for $T = 25$ and $N = 10$ as well as for $T = 50$ and $N = 25$ using a Monte Carlo experiment similar to the one presented in Section 6.[4]

5.2 Homogeneous and Heterogeneous Dependent Case

In most cases disturbances across individuals i will be at least contemporaneously correlated. For instance, when testing for PPP in panel data, contemporaneous disturbance correlation arises because one country must serve as a benchmark. Also, for instance, for a given country consumption and income cannot be assumed independent. One way to deal with this cross-country correlation is to incorporate a common time dummy in the panel. This solution was pursued by Pedroni (1997) in the context of panel cointegration test, but it appears that time dummies do not capture all the correlation, see O'Connel (1998). Another solution we use here is to account for cross-correlation by using GLS or SUR type corrections. These corrections require that $T > N$ and the asymptotic we consider are mainly based on $T \to \infty$ while N is fixed or at least grows at a lower rate than T.

Assuming that all the variables in levels are $I(1)$, we first test for each individual i the existence of a cointegrating relationship using standard time series based procedures. In the case the null hypothesis of no-cointegration can be rejected, the cointegration vector(s) are then considered as known in the subsequent analyses. An alternative to the time series based cointegration analysis is to rely on a testing procedure designed for cointegrated panel models, a procedure which could possibly be more powerful. The asymptotic arguments used in panel cointegration analysis are however mainly based on large $N-$ asymptotic and independence across units while we are here dealing with fixed N cases allowing for dependence across the units. Existing Monte Carlo simulations furthermore reveal (see *inter alia* McCoskey and Kao, 1998, Pedroni, 1997) the existence of some problems when cross-correlation exists. Moreover, the properties of common feature test statistics will be affected by the outcome of the cointegration

[4] Complete results are available upon request.

analysis. Indeed, if one erroneously imposes an identical homogeneous cointegrating matrix β_i^* for all i, while for some j cointegration does not hold or holds with a cointegrating matrix different from β_i^*, the likelihood to reject the SCCF restrictions will tend to increase.

Before presenting the GMM-estimator, we present the model under common features in general terms. Under separation C3, the model (11) can be written as

$$\underset{s\times nN}{\begin{bmatrix} \tilde{\beta}_1' & 0 & \cdots & 0 \\ 0 & \tilde{\beta}_2' & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{\beta}_N' \end{bmatrix}} \underset{nN\times 1}{\begin{bmatrix} \Delta X_{1t} \\ \Delta X_{2t} \\ \vdots \\ \Delta X_{Nt} \end{bmatrix}} = \underset{s\times nN}{\begin{bmatrix} \tilde{\beta}_1' & 0 & \cdots & 0 \\ 0 & \tilde{\beta}_2' & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{\beta}_N' \end{bmatrix}} \underset{nN\times 1}{\begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{Nt} \end{bmatrix}} \tag{16}$$

with $s = \sum_{i=1}^N s_i$ and $u_t = (u_{1t}', u_{2t}', .., u_{Nt}')'$ being $IIN(0, \Omega)$.

Under the homogeneity assumption C4, the model (16) specializes to become

$$(I_N \otimes \tilde{\beta}_1')\Delta X_t = (I_N \otimes \tilde{\beta}_1')u_t. \tag{17}$$

As in (13) and (14), we partition the vector ΔX_{it} as $[\Delta y_{it}', \Delta z_{it}']'$, where Δy_{it} and Δz_{it} are $s_i \times 1$ and $(n - s_i) \times 1$ subvectors. The matrix $\tilde{\beta}_i'$ is normalized (without loss of generality) as follows $\tilde{\beta}_i' = \left[I_{s_i}, -\tilde{\beta}_i^{*\prime}\right]$. Under this normalization, the system (16) can be expressed as

$$\underset{s\times 1}{\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \vdots \\ \Delta y_{Nt} \end{bmatrix}} = \underset{s\times(nN-s)}{\begin{bmatrix} \tilde{\beta}_1^{*\prime} & 0 & \cdots & 0 \\ 0 & \tilde{\beta}_2^{*\prime} & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{\beta}_N^{*\prime} \end{bmatrix}} \underset{(nN-s)\times 1}{\begin{bmatrix} \Delta z_{1t} \\ \Delta z_{2t} \\ \vdots \\ \Delta z_{Nt} \end{bmatrix}} + \tilde{\beta}' u_t \tag{18}$$

or more compactly

$$\Delta y_t = B' \Delta z_t + v_t \tag{19}$$

with $\Delta y_t = [\Delta y_{1t}', ..., \Delta y_{Nt}']'$, $B' = diag(\tilde{\beta}_i^{*\prime})$, $\Delta z_t = [\Delta z_{1t}', ..., \Delta z_{Nt}']', v_t = \tilde{\beta}' u_t$. Transposing (19) and writing the model for a sample of T observations, we get

$$\underset{T\times s}{\Delta Y} = \underset{T\times(nN-s)}{\Delta Z} \underset{(nN-s)\times s}{B} + \underset{T\times s}{V} \tag{20}$$

or in vectorized form

$$\underset{Ts\times 1}{\Delta y^*} = \underset{Ts\times(ns-\Sigma_i s_i^2)}{\Delta Z^*} \underset{(ns-\Sigma_i s_i^2)\times 1}{\delta} + \underset{Ts\times 1}{v^*} \tag{21}$$

with $\Delta y^* = vec(\Delta Y), v^* = vec(V), \Delta Z^* = diag(I_{s_i} \otimes \Delta Z_i)$ with $\Delta Z_i = [\Delta z_{itl}]$, of dimension $T \times (n - s_i)$, with $t = 1, ..., T$, $l = 1, ..., n - s_i$; and δ being a vector with typical i-th subvector being equal to $vec(-\tilde{\beta}_i^*)$. Under the homogeneity assumption C4, $\tilde{\beta}_i^* = \tilde{\beta}_1^*$, $i = 1, ..., N$, $s = Ns_1$, the system (21) specializes to become

$$\Delta y^* = \Delta Z_r^* \delta_r + v^* \tag{22}$$

with the $[TNs_1 \times N(n - s_1)]$ matrix

$$\Delta Z_r^* = \begin{bmatrix} I_{s_1} \otimes \Delta Z_1 \\ I_{s_1} \otimes \Delta Z_2 \\ ... \\ I_{s_1} \otimes \Delta Z_N \end{bmatrix}$$

and the $[s_1(n - s_1) \times 1]$ vector $\delta_r = vec(-\tilde{\beta}_1^*)$.

The vector of parameters δ and δ_r can be estimated by GMM provided we have a $(Ts \times k)$ matrix of instrumental variables W such that $\mathsf{E}W'v^* = 0$ and k is equal to or larger than the number of unknown parameters in δ (or δ_r).

The GMM estimator solving $W'v^* = 0$ using the weighting matrix S is given by

$$\hat{\delta}_{GMM} = \left[\Delta Z^{*\prime} W S^{-1} W' \Delta Z^*\right]^{-1} \Delta Z^{*\prime} W S^{-1} W' \Delta y^*. \tag{23}$$

The optimal weighting matrix is $S = W'\Sigma W$, where $\Sigma = \mathsf{E}v^* v^{*\prime} = I_T \otimes \Sigma_v$, $\Sigma_v = \tilde{\beta}' \Omega \tilde{\beta}$. When Σ is unknown, it will have to be replaced by a consistent estimate. The asymptotic covariance matrix of $\hat{\delta}_{GMM}$ with optimal weighting matrix S is given by

$$\mathsf{Var}(\hat{\delta}_{GMM}) = [\Delta Z^{*\prime} W (W'\Sigma W)^{-1} W' \Delta Z^*]^{-1}. \tag{24}$$

Under homogeneity C4, δ_r can be estimated by expression (23) replacing ΔZ^* by ΔZ_r^*. When the number of instruments k is strictly larger than the number of parameters δ (or δ_r) to be estimated, these overidentifying restrictions can be tests using the well-known criterion

$$\min_{\delta} = (v^{*\prime}W)(W'\Sigma W)^{-1}(W'v^*), \tag{25}$$

which has an asymptotic χ^2 -distribution with the number of degrees of freedom being equal to k minus the number of parameters.

Some remarks on the choice of the instruments have to be made. We can determine the order p_i of the VAR for each country i using for instance information criteria. The lagged first differences of ΔX_{it}, $i = 1, ..., p_i - 1$, and the lagged long run relations can be used to yield $n(p_i - 1) + r_i$, instruments W_i for ΔZ_i in (19) and taking $W = diag(\iota_{Ts_i}, W_i)$ where r_i is the cointegrating rank of individual i. As is well-known, the OLS estimator regressing Δy^* on $\Delta\widehat{Z}^*$, where the $\Delta\widehat{Z}^*$ are the projections of ΔZ^* on W, can be obtained as a GMM estimator by selecting $S = I_{Ts}$ in (23) and taking $W(W'W)^{-1}W'$ as instrument. Similarly, the GLS estimator regressing Δy^* on $\Delta\widehat{Z}^* = W(W'\Sigma^{*-1}W)^{-1}W'\Sigma^{*-1}\Delta Z^*$, with Σ^* being the disturbance covariance matrix of the (multivariate) regression of ΔZ^* on W, can be obtained from (23) by taking $S = \Sigma$ and using as instruments $W(W'\Sigma^{*-1}W)^{-1}W'\Sigma^{*-1}$ instead of W.

In the empirical analysis in Section 7, we consider a fixed effects model because in the macroeconomic application, we study the population and not a sample. Adding fixed effects to the model (21) for the case which we analyze, e.g. for $s_i = 1$, $i = 1, ..., N$ and $n = 2$, yields

$$\Delta y = Z_\mu \mu \; [+Z_\lambda \lambda] + \Delta Z_r^* \delta_r + v^*, \tag{26}$$

where $Z_\mu = \iota_T \otimes I_N$ and $Z_\lambda = I_T \otimes \iota_N$. Let J_N denote a $N \times N$ matrix of ones, so $Z_\lambda Z'_\lambda = I_T \otimes J_N$ and the projection of J_N on Z_λ is $I_T \otimes \bar{J}_N$ with $\bar{J}_N = J_N/N$. This matrix averages over individuals. Also define time means by $Z_\mu Z'_\mu = J_T \otimes I_N$ and the projection of J_T on Z'_μ is $\bar{J}_T \otimes I_N$. It is shown in Baltagi (1995, p28) that

$$\widehat{\delta}_{r,GMM} = (\Delta\widehat{Z}_r^{*\prime} Q\Sigma^{-1} Q\Delta\widehat{Z}_r^*)^{-1}\Delta\widehat{Z}_r^{*\prime} Q\Sigma^{-1} Q\Delta y, \tag{27}$$

where $Q = I_{NT} - \bar{J}_T \otimes I_N$ for model with only individual effects and $Q = I_T \otimes I_N - \bar{J}_T \otimes I_N - I_T \otimes \bar{J}_N + \bar{J}_T \otimes \bar{J}_N$ when time dummies are present. The estimator (26) with $\Delta\widehat{Z}_r^* = W(W'\Sigma^{*-1}W)^{-1}W'\Sigma^{*-1}\Delta Z_r^*$ will be indicated as the GLS-LSDV estimator. When the matrix Σ is replaced by the identity matrix, a less-efficient estimator arises which will be denoted as the LSDV estimator. The asymptotic covariance matrix of $\hat{\delta}_{r,GMM}$ with optimal weighting matrix

S is then given by

$$\mathsf{Var}(\hat{\delta}_{r,GMM}) = [\Delta\widehat{Z}_r^{*\prime}QW(W'\Sigma W)^{-1}W'Q\Delta\widehat{Z}_r^{*}]^{-1}. \tag{28}$$

A test for the validity the overidentifying restrictions is obtained using (25) and is readily seen to be a test for the null hypothesis of **C4,** e.g. for the null of homogeneity of common features: $\widetilde{\beta}_i = \widetilde{\beta}_1;\ i = 1, ..., N$,with $s = Ns_1,\ s_i = s_1 = 1,\ \forall i = 1, ..., N$. In this specific case, the number of degrees of freedom for the overidentifying restrictions test (25) is given by $\sum_{i=1}^{N}[n(p_i - 1) + r_i) - (n-1)] + (n-1)(N-1)$ where n, p_i, r_i are respectively the number of variables, the number of lags and the number of cointegrating relations for each i. Notice that the factor $(n-1)(N-1)$ arises as a consequence of the pooled estimation of the common feature vector. Imposing a common cofeature vector actually decreases by $(n-1)(N-1)$ the number of parameters to be estimated.

More generally, one could naturally extend the analysis (in the case $n > 2$) and consider similar analyses for $s_1 = 1, .., n-1$. Sequentially testing, for $s_1 = 1, .., n-1$, the validity of the underlying overidentifying restrictions with (25), provides a direct way to test the number of common co-features in a GMM set-up, provided we first properly normalize the cofeature matrix as above. A somewhat similar use of GMM for the detection of the dimension of the common feature space, albeit in a pure time series context, is discussed in Vahid and Engle (1997).

In the next section, we evaluate the merits of this analysis (for $s_i = s_1 = 1, \forall i = 1, ..., N$) in a small Monte Carlo experiment.

6 Monte Carlo Simulations

In this section we present some illustrative Monte Carlo evidence on the usefulness of the common feature test statistic (25) presented above for panel data. The data are generated as if there exists a huge VECM with both common feature and cointegrating restrictions. Under the null of reduced rank structures, the bivariate DGP for each of N individuals assumes the existence of one cointegrating vector and of a single common feature vector. It has the form:

$$\begin{aligned}\begin{pmatrix}\Delta y_{i,t}\\ \Delta z_{i,t}\end{pmatrix} &= \begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix} + \begin{pmatrix}.25\\ .5\end{pmatrix}\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}y_{1,t-1}\\ z_{1,t-1}\end{pmatrix} + \\ &\quad \begin{pmatrix}.5\\ 1\end{pmatrix}\begin{pmatrix}.6 & .3\end{pmatrix}\begin{pmatrix}\Delta y_{1,t-1}\\ \Delta z_{1,t-1}\end{pmatrix} + \begin{pmatrix}\varepsilon_{i1,t}\\ \varepsilon_{i2,t}\end{pmatrix},\end{aligned}$$

where the μ's are generated from uniform distributions $\mu_1 \sim U(0,.3)$, $\mu_2 \sim U(-.25,.15)$ so that $E(\mu_1) = .15$ and $E(\mu_2) = -.05$. The normalized common feature vector is $\tilde{\beta} = \begin{bmatrix} 1 & -.5 \end{bmatrix}'$ and the normalized cointegration vector is simply $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}'$. For each individual i, with $[\varepsilon_{i1,t}, \varepsilon_{i2,t}]\prime$ is bivariate Gaussian with variance-covariance matrix Ω_{ii}. The cross-contemporaneous correlation matrices between individual i and j are all equal to Ω_{ij} so that the panel VECM variance-covariance matrix is given by (10) with

$$\Omega_{ii} = \begin{pmatrix} 1 & .8 \\ .8 & 1 \end{pmatrix} \qquad \Omega_{ij} = \begin{pmatrix} .7 & .6 \\ .6 & .75 \end{pmatrix}.$$

We have added an heterogeneous structure increasing with N. From this DGP wee see that under the assumption of reduced rank the short run dynamic matrices (for each i) are simply given by $\begin{pmatrix} 0.30 & 0.15 \\ 0.60 & 0.30 \end{pmatrix}$,while under the alternative we chose to fix arbitrarily one element to zero: $\begin{pmatrix} 0.30 & 0.00 \\ 0.60 & 0.30 \end{pmatrix}$.

We consider three sample sizes, i.e. $T = 10$, 25 & 50, and five cases for the number of individuals, i.e. $N = 1$, 2, 5, 10 and 25. We report the median and the spread (interquantile range) of the bias of the GMM panel estimator. We also report the median of the standard deviation of $\widehat{\delta}_{r,GMM}$. We report the empirical size (nominal being 5%) as well as the empirical size adjusted power for over-identifying restrictions test statistics. Due to the huge computational time required for these simulations, 5,000 replications were used for $N = 1, 2, 3$; 2,000 for $N = 10$ and 1,000 for $N = 25$.

Insert Table 2 about here

The results are presented in Table 2. One can directly observe that the bias is small and decreases when both T and/or N increase. The accuracy of estimates, measured both by the spread and the standard deviation of the estimate, also increases with T and/or N. We interpret these illustrative findings as evidence in favor of the pooling estimator. No substantial size distortions are observed. Remark that the values of N we have retained in these simulations are clearly too small to asses the validity of a central limit theorem based on large N asymptotic.

7 Empirical Analysis

The data we use are taken from the Penn World Tables Mark 5.6 (see Summers and Heston, 1991)[5]. These data, thanks to the homogeneity in their definition, are extremely useful and have been extensively used in empirical literature. This does however certainly not exclude the presence of measurement errors because the price to pay for obtaining long series of homogeneous data for more than 150 countries is the reliance on a set of hypotheses, approximations and interpolations. Because of both the quality of the data as well as the underlying theoretical motivation, we limit our analysis to 22 OECD countries for the sample period 1950-1992 (up to 1991 for Greece and 1990 for Portugal)[6]. The data extracted are $Y =$ *"RGDPL: Real GDP per capita (Laspeyres index) in 1985 international prices"* and $C =$ *"C: Real Consumption share of GDP in 1985 international prices"*$\times Y/100$. This last operation is necessary to get the consumption in level and not in percentage of income[7].

Table 3 reports time series statistics for each country. The first column of Table 3 lists in alphabetical order, the names of the countries as well as the date of joining OECD[8]. Column 2 gives the quality ranking of the data as presented in Summers and Heston (1991). It is seen that for the most part, the quality of the data is reasonable. Columns 3 and 4 give the value of the Augmented Dickey-Fuller unit root test for respectively consumption and income. All the tests are based on both a constant and a trend. The number of lags necessary to whiten the residuals is given in parentheses. Columns 5 and 6 give respectively the value of the Engle-Granger Augmented Dickey-Fuller cointegrating test and the long-run elasticity with consumption as a dependent variable. Column 7 gives the order of the VAR(p_i) in level, where p_i is determined using multivariate Hannan-Quinn (HQC) criteria. These lags, as well as the presence of an error correcting mechanism term, will determine the instruments to be used in common features test

[5] The data may be downloaded via different internet sites such as http://www.nber.org/pwt56.html or http://datacentre.epas.utoronto.ca:5620/pwt.

[6] Because of computation facility, we have balanced the panel in this study and we didn't consider either Greece and Portugal.

[7] We didn't consider here a slightly different model in which real government expenditures are substracted from output. Indeed, as raised by Evans and Karas (1993), the "λ *model*" should be extended to take care of the potential substitutability or complementarily between private and public goods. Without a fine distinction of the components of government expenditures, it might be desirable to extend the model to take into account a third variable. It is also possible to consider a simple alternative model where all the public goods are substitutable to private one by substracting G from Y.

[8] Other countries are now joining OECD. This is the case of the Czech Republic in 1995, Korea in 1996, Poland 1996 and Mexico 1994. We drop them because the ending year is 1992 in our data set. Also note that OECD has its origin in the Organisation for European Economic Cooperation which grouped European Countries. This organisation was charged with administering United States aid, under the Marshall Plan, to reconstruct Europe after the World War II. Consequently, for countries that did not participate at the beginning in this project, homogeneity of cointegration and/or common features might be rejected for that reason.

statistics.

Insert Tables 3-5 about here

In Table 3, a "*" indicates that individual unit root or cointegration test statistics reject the null at a 5% nominal level. It emerges that, except for Portugal, UK and Turkey, we cannot reject the unit root hypothesis for consumption and income. Using Engle-Granger cointegration test, the null hypothesis of non-cointegration is rejected for nine countries with long-run elasticity β_i^* close to 1. Consequently, we will use the cointegrating vectors as instruments in six different versions: four homogeneous cases and two heterogeneous ones. The results are reported in Table 4.

The homogeneous cases refer to a panel estimation of a common cointegrating vector. Because most panel cointegration test statistics assume independence across individuals, we cannot, strictly speaking, rely on panel cointegration test statistics. However because the estimator is still consistent we use them to get estimates for four different cases.

- As Tables 3 shows that even when the absence of cointegration is not rejected, the elasticity is close to one, we first analyze a version in which we assume there exists a homogeneous cointegrating relationship for all the countries with a coefficient β^* equal to one (see upper panel of Table 4). Similar results are obtained using Johansen's MLE based procedure.
- A second panel cointegration test uses the group mean estimator (GM) of Pesaran et al. (1997). This means that we average cointegrating vectors over the 22 individuals.
- A third alternative uses the usual OLS estimator.
- The last one allows for intercept heterogeneity and is the usual LSDV estimator.

Notice that the pooled FM-OLS estimator proposed by Pedroni (1997), which assumes independence across units, gives a point estimate of 0.971 for the 22 OECD countries and 1.021 for the G7 countries, the latter being not significantly different from one. Both results are very close to those obtained with the LSDV and OLS estimators so that the results of the common cyclical feature analysis obtained with Pedroni's FM-OLS estimator are not reported.

For the two heterogeneous cases we impose cointegration for the nine countries for which the Engle-Granger ADF test is significant. We take as an instrument, cointegrating vectors for countries for which we reject the null. Notice that Phillips-Hansen Fully Modified OLS estimation was also used to test formally the assumption of unit long-run elasticity. The null of unit long-run elasticity was formally rejected in all cases of cointegration but for three (Austria, Canada and New-Zealand). Two different cases are considered:

- For the nine countries we take the estimated value of β_i^* given by the long-run regression.
- We fix these values to 1.

The maximum lag length for a country is four, so that $p^* = (p-1) = 0, 1, 2$ or 3 for some

- p^* is fixed uniformly to respectively 1, 2, 3
- p^* is fixed to the estimated value.

Note that over-estimating the lag length will certainly reduce the power of the test statistics (Beine and Hecq, 1999). The results of the two panel common feature statistics are presented. For the heterogeneous cases, the first two columns present the group mean estimates (denoted by $\widehat{\delta}_{r,GM}$) as well as the value of the $Normal$ test statistics (N_{GM}) in (15). The next columns present the value of common feature elasticity for the homogeneous dependent case (denoted by $\widehat{\delta}_{r,GMM}$) as well as the value of the test of the overidentifying restrictions and the associated $p-$values.

It appears that the estimated coefficient $\widehat{\delta}_{r,GMM}$ and $\widehat{\delta}_{r,GM}$ are too high compared with a priori expectations. Moreover we reject the null of a panel common feature model with both test statistics. Table 5 presents the results for the G7. The results are similar to those for the panel of 22 countries. However in several situations we cannot reject the null of one homogeneous common features vector. In these cases, we imposed the unlikely hypotheses of an homogeneous cointegrating vector with a lag order uniformly fixed to $p^* = 3$.

Finally, we want to notice the implications for empirical modeling that follow from a restriction between the number of variables n and the sum of cointegrated vectors and common features vectors. From Vahid and Engle (1993), Theorem 1, it follows that the common feature space and the cointegration space are linearly independent. This means that the sum of the number of common feature vectors (s) and of the number of cointegrating vectors (r) should be less or equal to the number of variables (n): $r + s \leq n$. In a panel context under the absence of Granger long and short-run causality, this has obvious but different implications depending on whether common features vectors and cointegrating vectors are homogeneous or heterogeneous. A misspecification of the number of homogeneous cointegrating vectors may for instance too heavily constrain the dimension of the homogeneous common feature space and lead to flawed inference regarding the existence of common features.

A last remark seems in order. Although we can formally reject the existence of a common homogeneous co-feature relation in this OECD data set, one should be aware our results do not per se imply the absence of SCCF for some of the countries taken individually.

8 Conclusion

In this paper we have proposed to extend the serial correlation common feature analysis to nonstationary panel data models. Concentrating upon the fixed effect model, we defined homogeneous panel common feature models. We give a series of steps allowing to implement these tests. We then apply this framework when investigating the liquidity constraints model for 22 OECD and G7 countries. At a 5% nominal level, we reject the presence of a panel common feature vector.

Our model representation is not stricto sensus a dynamic panel because only a part of the dynamics is common to all individuals. However it does part of the job. Indeed while no size distortions have been noticed in our Monte Carlo results, we can increase the power of test statistics, by going a step further towards dynamic panel data if the null hypothesis of panel common-cyclical feature model is not rejected. In the opposite case, it is not worth imposing further common restrictions if the null is rejected. This is a clue for considering less restrictive models like heterogeneous or group homogeneous models. A bootstrap procedure could certainly be undertaken to find the distribution. This is also perhaps the place to choose more flexible models like the non-synchronous common cycle model (Vahid and Engle, 1997) or the weak form common feature analysis (Hecq, Palm and Urbain, 1997).

References

[1] Ahn, S.K. (1997), Inference of Vector Autoregressive Models with Cointegration and Scalar Components, *Journal of the American Statistical Association*, 92, 350-356.

[2] Anderson, H. and F. Vahid (1996), Testing Multiple Equation Systems for Common Nonlinear Components, Discussion Paper Texas A&M University.

[3] Baltagi, B. (1995), *Econometric Analysis of Panel Data*, (John Wiley: New York).

[4] Beine, M. and A. Hecq (1998), Codependence and Convergence, an Application to the EC Economies, *Journal of Policy Modeling*, 20, 403-426.

[5] Beine, M. and A. Hecq (1999), Inference in Codependence: Some Monte Carlo Results and Applications, *Annales d'Economies et de Statistique*, 54, 69-90.

[6] Beine, M. and A. Hecq (1997), Asymmetric Shocks Inside Future EMU, *Journal of Economic Integration*, 12, 131-140.

[7] Campbell, J.Y. and N.G. Mankiw (1990), Permanent Income, Current Income, and Consumption, *Journal of Business & Economic Statistics*, 3, 265-279.

[8] Campbell, J.Y. and N.G. Mankiw (1990), The Response of Consumption to Income: A Cross-Country Investigation, *European Economic Review,* 35, 723-767.

[9] Candelon, B. and A. Hecq (1997), Stability of the Okun's Law in a Codependent System, University of Maastricht RM/97/027.

[10] Engle, R.F. and S. Kozicki (1993), Testing for Common Features (with comments), *Journal of Business and Economic Statistics*, 11, 369-395.

[11] Engle, R.F. and M.W. Watson (1981), A One-Factor Multivariate Time Series Model of Metropolitan Wages, *Journal of the American Statistical Association*, 76, 545-565.

[12] Evans, P. and G. Karras (1996), Convergence Revisited, *Journal of Monetary Economics,* 37, 249-265.

[13] Evans, P. and G. Karras (1996), Private and Government Consumption with Liquidity Constraints, *Journal of International Money and Finance,* 2, 255-266.

[14] Geweke, J. (1977), The Dynamic Factor Analysis of Economic Time Series, in: Aigner, D.J. and A.S. Goldberger (eds.) *Latent Variables in Socio-Economic Models*, (North-Holland: Amsterdam).

[15] Gouriéroux, C and I. Peaucelle (1993), Séries codépendantes : application à l'hypothèse de parité du pouvoir d'achat, in: *Macroéconomie , Développements Récents*, (Economica: Paris).

[16] Granger, C.W.J. and N. Haldrup (1997), Separation in Cointegrated Systems and P-T Decompositions, *Oxford Bulletin of Economics and Statistics*, 59, 449-463.

[17] Greene, W.H. (1993), *Econometric Analysis* (MacMillan: New York).

[18] Groen, J.J. and F. Kleibergen (1999), Likelihood-Based Cointegration Analysis in Panels of Vector Error Correction Models, Discussion Paper TI 99-055/4, Tinbergen Institute, Erasmus University Rotterdam.

[19] Hamilton, J.D. (1994), *Time Series Analysis* (Princeton University Press: Princeton).

[20] Hansen, L.P. (1982), Large Sample Properties of Generalized Method of Moment Estimators, *Econometrica*, 50, 1029-1054.

[21] Hansen, P.R. and S. Johansen (1998), *Workbook on Cointegration*, (Oxford University Press: Oxford).

[22] Hecq, A. (1999), On the Usefulness of Considering Common Serial Features and Cointegrating Restrictions, University of Maastricht RM/99/017.

[23] Hecq, A, F.C. Palm and J.P. Urbain (1997a, revised 1998), Testing for Common Cycles in VAR Models with Cointegration, University of Maastricht RM/97/031.

[24] Hecq, A, F.C. Palm and J.P. Urbain (1997b, revised 1998), Permanent-Transitory Decomposition in VAR models with Cointegration and Common Cycles, University of Maastricht RM/97/037.

[25] Hecq, A, F.C. Palm and J.P. Urbain (1998), Separation and Weak Exogeneity in Cointegrated VAR Models with Common Features , mimeo, University of Maastricht.

[26] Hoogstrate, A.J. (1997), *Dynamic Panel Data Models: Theory and Macroeconomic Applications*, Ph.D. Thesis, University of Maastricht.

[27] Hsiao, C. (1986), *Analysis of Panel Data* (Cambridge University Press: Cambridge).

[28] Im, K.S., Pessaran, M.H. and Y. Shin (1997), Testing for Unit Roots in Heterogeneous Panels, mimeo University of Cambridge.

[29] Issler, J.V. and F. Vahid (1996), Common Cycles in Macroeconomic Aggregates, mimeo.

[30] Jobert, T. (1995), Tendances et cycles communs à la consommation et au revenu : implications pour le modèle de revenu permanent, *Economie et Prévision*, 121, 19-38.

[31] Johansen, S. (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models* (Oxford Univesity Press: Oxford)

[32] Kugler, P. and K. Neusser (1993), International Real Interest Rate Equalization : A Multivariate Time-Series Approach, *Journal of Applied Econometrics*, 8 , pp.163-174.

[33] Kunst, R. and K. Neusser (1990), Cointegration in Macroeconomic System, *Journal of Applied Econometrics*, 5, 351-365.

[34] Kao, C. (1999), Spurious Regression and Residual-Based Tests for Cointegration in Panel Data, *Journal of Econometrics*, 40, 1-44.

[35] Konishi, T. and C.W.J. GRanger (1993), Separation in Cointegrated Systems, manuscript UCSD.

[36] Levin, A. and C.F. Lin (1993), Unit Root Tests in Panel Data: Asymptotic and Finite Sample Properties, UCSD Discussion Paper.

[37] Larsson, R. and J. Lyhagen (1999) Likelihood-Based Inference in Multivariate Panel Cointegration Models, SSE Working Paper 331, Stockholm School of Economics.

[38] Lumsdaine, R.L. and E. Prasad (1997), Identifying the Common Components in International Economic Fluctuations, NBER Working Paper 5984.

[39] Lütkepohl, H. (1991), *Introduction to Multiple Time Series Models*, (Springer Verlag: Berlin).

[40] McCoskey, S., and C. Kao (1998), A Residual-Based Test of the Null of Cointegration in Panel Data, *Econometric Reviews*, 17, 57-84.

[41] McCoskey, S., and C. Kao (1998), A Monte Carlo Comparison of Tests for Cointegration in Panel Data, mimeo.

[42] O'Connell, P. (1998), The Overvaluation of Purchasing Power Parity, *Journal of International Economics*, 44, 1-19.

[43] Pedroni, P. (1997), Fully Modified OLS for Heterogeneous Cointegrated Panels and the Case of Purchasing Power Parity, Indiana University Working Paper.

[44] Pedroni, P. (1997), Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests with Application to the PPP Hypothesis, Indiana University Working Paper.

[45] Pesaran, M.H. and R.P. Smith (1995), Estimating Long-Run relationships from Dynamic Heterogenous Panels, *Journal of Econometrics*, 68, 79-113.

[46] Pesaran, M.H., Shin, Y and R.P. Smith (1997), Pooled Estimation of Long-Run relationships in Dynamic Heterogenous Panels, University of Cambridge Working Paper.

[47] Phillips, P.C.B. and H. Moon (1999a), Linear Regression Limit Theory for Nonstationary Panel Data, *Econometrica*, 67, 1057-1111.

[48] Phillips, P.C.B. and H. Moon (1999b), Nonstationary Panel Data Analysis: An Overview of Some Recent Developments, mimeo, Yale University.

[49] Singleton, K. (1980), A Latent Time Series Model of the Cyclical Behavior of Interest Rates, *International Economic Review*, 21, 559-575.

[50] Summers, R. and A. Heston (1991), The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988, *Quarterly Journal of Economics*, 327-368.

[51] Tiao, G.C. and R.S. Tsay (1989), Model Specification in Multivariate Time Series, *Journal of Royal Statistical Society (series B)*, 51, 157-213.

[52] Vahid, F. and R.F. Engle (1993), Common Trends and Common Cycles, *Journal of Applied Econometrics*, 8, 341-360.

[53] Vahid, F. and R.F. Engle (1997), Codependent Cycles, *Journal of Econometrics,* 80, 199-221.

[54] Vahid, F. and J.V. Issler (1999), The Importance of Common-Cyclical Features in VAR Analysis: A Monte-Carlo Sudy, paper presented at ESEM99.

Table 1: Monte Carlo Results

(Separated vs. Marginal Systems)

		Marg.				Separ.			
		$bias_{.5}$	$bias_{.75-.25}$	$\chi^2(2)$	$\chi^2_{ss}(2)$	$bias_{.5}$	$bias_{.75-.25}$	$\chi^2(8)$	$\chi^2_{ss}(8)$
$N = 2$	$T = 10$	-.056	.310	14.64	6.22	-.040	.441	70.98	12.8
	$T = 25$	-.026	.155	7.56	5.20	-.027	.138	18.36	7.14
	$T = 50$	-.011	.104	6.30	5.04	-.013	.090	10.16	6.16
	$T = 100$	-.005	.068	4.86	4.42	-.007	.059	6.66	5.14
		Marg.				Separ.			
		$bias_{.5}$	$bias_{.75-.25}$	$\chi^2(2)$	$\chi^2_{ss}(2)$	$bias_{.5}$	$bias_{.75-.25}$	$\chi^2(14)$	$\chi^2_{ss}(14)$
$N = 5$	$T = 10$	-.061	.299	14.14	5.86	–	–	–	–
	$T = 25$	-.025	.152	7.82	5.44	-.019	.241	99.76	35.04
	$T = 50$	-.012	.100	6.30	5.18	-.011	.087	62.88	15.26
	$T = 100$	-.006	.069	5.58	5.04	-.007	.052	25.18	9.38

Table 2: Monte Carlo Results
(GMM estimation and test statistic)

		$bias_{Median}$	$bias_{Q75-Q25}$	$\sigma(\widehat{\delta}_{r,GMM})_{Median}$	χ^2cv	$size$	adj.power
$N=1$	$T=10$	-.0123	.2228	.156	(2)	7.88	9.90
	$T=25$	-.0101	.1387	.098	(2)	5.58	19.78
	$T=50$	-.0067	.0944	.070	(2)	5.54	34.68
$N=2$	$T=10$	-.0136	.1817	.106	(5)	4.98	8.56
	$T=25$	-.0069	.1057	.079	(5)	6.18	16.58
	$T=50$	-.0034	.0726	.057	(5)	5.72	31.52
$N=5$	$T=10$	-.0045	.1409	.067	(14)	3.96	7.26
	$T=25$	-.0044	.0751	.060	(14)	5.68	12.52
	$T=50$	-.0021	.0460	.047	(14)	5.74	24.82
$N=10$	$T=25$	-.0022	.0658	.046	(29)	4.70	11.00
	$T=50$	-.0020	.0377	.038	(29)	4.80	21.55
$N=25$	$T=50$	.0002	.0398	.029	(74)	5.80	13.80

Table 3: Time Series Statistics

(Individual countries)

	Qual.	ADF c_t	ADF y_t	EG	$\widehat{\beta}_i^*$	HQC
Australia (1971)	A-	-1.21(4)	-.93(2)	-1.46(1)	.95	3
Austria (1961)	A-	-.82(0)	-1.25(2)	-3.59(0)*	1.00	1
Belgium (1961)	A	-1.43(1)	-.74(1)	-2.36(0)	.94	1
Canada (1961)	A-	-1.50(1)	-1.80(1)	-3.89(1)*	1.00	1
Denmark (1961)	A-	-.94(0)	-.94(0)	-3.69(0)*	.82	1
Finland (1969)	A-	-2.48(1)	-.20(2)	-1.69(3)	.98	4
France (1961)	A	-.11(2)	-.04(1)	-1.96(0)	.98	2
Germany (1961)	A	-2.18(2)	-3.10(2)	-1.69(2)	1.07	2
Greece (1961)	A-	-.58(0)	.01(0)	-.79(0)	.97	1
Iceland (1961)	B+	-2.64(1)	-2.23(1)	-4.52(0)*	1.04	1
Ireland (1961)	A-	-2.54(1)	-2.82(1)	-3.76(2)*	.81	1
Italy (1961)	A	-.61(1)	-.77(1)	-1.86(1)	1.09	4
Japan (1964)	A	-.91(0)	-.48(1)	-4.75(1)*	.92	4
Luxembourg (1961)	A-	-1.45(1)	-3.32(4)	-2.16(4)	1.34	4
Netherlands (1961)	A	-.71(2)	-.20(2)	-3.07(1)	1.08	4
New Zealand (1973)	A-	-2.26(0)	-1.52(0)	-5.93(0)*	1.02	1
Norway (1961)	A-	-1.29(1)	-1.76(1)	-1.83(1)	.80	1
Portugal (1961)	A-	-3.54(3)*	-2.95(3)	-3.07(3)	.88	3
Spain (1961)	A-	-1.25(0)	-1.34(0)	-2.99(0)	.94	1
Sweden (1961)	A-	-.70(1)	-.30(1)	-3.58(1)*	.81	2
Switzerland (1961)	B+	.03(4)	-1.69(2)	-3.28(0)	.92	2
Turkey (1961)	C	-3.26(2)	-3.48(0)*	-1.73(0)	.85	1
UK (1961)	A	-3.61(1)*	-3.62(1)*	-2.13(0)	1.04	3
USA (1961)	A	-1.75	-2.05(0)	-4.08(0)*	1.15	2

Table 4: Common Features within 22 OECD Countries

		$\widehat{\delta}_{r,GM}$	N_{GM}	$\widehat{\delta}_{r,GMM}$	$\sigma(\widehat{\delta}_{r,GMM})$	Test	cv	p_val
$\beta_i^* = 1$,	$p^* = 1$	.770	3.71	.745	.050	148.98	65	<.001
$(\forall i)$	$p^* = 2$	.769	6.14	.660	.036	173.65	109	<.001
	$p^* = 3$	.770	4.43	.704	.031	211.27	153	.001
	$p^* = p_i^*$	–	–	.718	.036	156.04	93	<.001
$\beta_i^* = \widehat{\beta}_{GM}^* = .979$,	$p^* = 1$	.829	5.36	.768	.051	146.67	65	<.001
$(\forall i)$	$p^* = 2$	.804	6.54	.670	.036	176.61	109	<.001
	$p^* = 3$	.793	4.95	.710	.031	214.06	153	<.001
	$p^* = p_i^*$	–	–	.728	.036	156.92	93	<.001
$\beta_i^* = \widehat{\beta}_{OLS}^* = .939$	$p^* = 1$	.870	5.74	.814	.050	131.96	65	<.001
$(\forall i)$	$p^* = 2$	.837	5.12	.687	.036	170.16	109	<.001
	$p^* = 3$	.822	3.93	.727	.031	206.93	153	.002
	$p^* = p_i^*$	–	–	.738	.036	145.01	93	<.001
$\beta_i^* = \widehat{\beta}_{LSDV} = .968$	$p^* = 1$	.855	6.03	.782	.051	142.93	65	<.001
$(\forall i)$	$p^* = 2$	.821	6.25	.677	.036	175.97	109	<.001
	$p^* = 3$	.804	4.94	.715	.031	213.50	153	.001
	$p^* = p_i^*$	–	–	.733	.036	155.12	93	<.001
$\beta_i^* = \widehat{\beta}_j^*$	$p^* = 1$	.814	6.89	.782	.053	138.45	52	<.001
$(\forall i, j$ with	$p^* = 2$	.726	6.16	.647	.036	158.74	96	<.001
cointegration)	$p^* = 3$	.755	4.46	.696	.031	210.03	140	<.001
	$p^* = p_i^*$	–	–	.707	.037	146.50	80	<.001
$\beta_i^* = 1$	$p^* = 1$	.865	1.59	.810	.056	115.25	52	<.001
$(\forall i$ with	$p^* = 2$	.784	3.89	.682	.037	144.00	96	.001
cointegration)	$p^* = 3$	.775	2.72	.734	.033	192.33	140	.002
	$p^* = p_i^*$	–	–	.750	.040	131.56	80	<.001

Table 5: Common Features within G7 Countries

		$\widehat{\delta}_{r,GM}$	N_{GM}	$\widehat{\delta}_{r,GMM}$	$\sigma(\widehat{\delta}_{r,GMM})$	$Test$	cv	p_value
$\beta_i^* = 1 = \widehat{\beta}^*_{LSDV}$	$p^* = 1$	.866	2.47	1.042	.087	32.83	20	.035
$(\forall i)$	$p^* = 2$	.763	2.37	.856	.060	53.70	34	.017
	$p^* = 3$	.755	1.81	.872	.052	67.05	48	.036
	$p^* = p_i^*$	–	–	.884	.058	50.84	30	.010
$\beta_i^* = \widehat{\beta}^*_{GM} = 1.035$	$p^* = 1$	.893	1.64	1.021	.082	31.51	20	.048
$(\forall i)$	$p^* = 2$	.777	1.815	.857	.060	50.25	34	.036
	$p^* = 3$	.766	1.49	.878	.052	62.75	48	.075*
	$p^* = p_i^*$	–	–	.892	.057	46.22	30	.029
$\beta_i^* = \widehat{\beta}^*_{OLS} = 1.023$	$p^* = 1$	.882	1.75	1.036	.084	32.06	20	.043
$(\forall i)$	$p^* = 2$	.771	1.89	.856	.060	51.11	34	.030
	$p^* = 3$	.762	1.51	.876	.052	63.87	48	.062*
	$p^* = p_i^*$	–	–	.890	.057	47.84	30	.021
$\beta_i^* = \widehat{\beta}^*_j$	$p^* = 1$	.818	6.02	.894	.074	49.07	16	<.001
$(\forall i, j$ with	$p^* = 2$	.710	3.58	.723	.053	52.66	30	.006
cointegration)	$p^* = 3$	.737	2.13	.787	.047	64.46	44	.024
	$p^* = p_i^*$	–	–	.800	.051	46.61	26	.008
$\beta_i^* = \beta_j^* = 1$	$p^* = 1$	.875	2.68	1.029	.089	27.69	16	.034
$(\forall i, j$ with	$p^* = 2$	.753	2.60	.859	.062	47.49	30	.022
cointegration)	$p^* = 3$	.764	1.66	.894	.053	60.14	44	.053*
	$p^* = p_i^*$	–	–	.917	.061	43.97	26	.015