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## TAX-EXEMPT INVESTORS AND THE ASSET ALLOCATION PUZZLE

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### Abstract

Investors frequently hold equity in tax-exempt savings vehicles such as pension plans, despite the prediction of the standard model that they hold only bonds. We provide a new explanation for this empirical puzzle based on differences between pensions and taxable assets in the tax treatment of capital losses. We show how limits on refundability of losses on taxable equities leads to diversity of investors' preferences for corporate leverage on the basis of tax rates. In the simplest equilibrium of the model, tax-exempt savers hold risky, highly leveraged equities, while low-bracket taxable savers hold bonds and high-bracket taxpayers hold relatively safe, unleveraged equities. We discuss the implications of tax-exempts for risk taking and agency costs within the firm.

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# 1 Introduction

In many industrial countries, pension funds and other tax-exempt retirement saving vehicles are major players in the market for corporate equities. In the United States, pension assets totalled \$5.7 trillion at the end of 1997, about 54 per cent of which was held in corporate equities. These assets comprised more than one-third of stock market capitalization in the U.S.,<sup>1</sup> giving pension funds by far the single largest pool of equity capital. Recently, moreover, the bias toward equity in tax-exempt portfolios has been even more pronounced. For example, Bernheim et al. (1997) report that roughly two-thirds of new inflows to 401(k) plans in 1995 were allocated to stock, compared to 24 per cent in 1988.<sup>2</sup> Purchases of bonds fell to 25 per cent from 60 per cent over the same period.

But why is equity held in pensions at all? Conventional models of asset allocation, such as Miller (1977),<sup>3</sup> suggest that tax-exempt investments should be specialized in bonds. Since interest is tax deductible at the corporate level, but are more heavily taxed for taxable investors, bonds pay a premium in equilibrium. Thus the rents associated with the pension's tax exemption are maximized when bonds are held. The puzzle of tax-exempt equity investments remains even if investors wish to hold equity for non-tax (e.g. risk-return) reasons, and even if investors do not wish to save in excess of their pension contribution limits. To see this, suppose an investor can contribute \$2,000 to a pension vehicle and wishes to hold a total of \$1,000 in equities and \$1,000 in bonds. The rational investor then holds \$2,000 in bonds inside the tax-exempt pension and borrows \$1,000 to purchase equities outside the pension. The desired portfolio shares have been achieved, and the advantage of holding tax-exempt debt predicted by the standard model has been maximized.<sup>4</sup>

In this paper, we suggest a new explanation for this empirical anomaly. We argue that investment in risky equity can allow tax-exempts to take advantage of the non-refundability of tax on capital losses for ordinary investors. In countries like the U.S. and Canada, contributions to pensions are initially tax-deductible from ordinary income, accrued income is exempt, and pension balances fully taxable on withdrawal. Pension investments therefore effectively receive a "front-loaded" refund of tax losses,<sup>5</sup> which is taxed back on withdrawal at the investor's ordinary tax rate, despite the absence of any formal system of refundability for pension investors.<sup>6</sup> This yields an advantage for risky equity investments by tax-exempt investors not shared by taxable investors for whom losses are not deductible, and this may more than offset the typical tax advantage which accrues to investments in safer debt.

The mechanism by which such front-loaded tax deductions for pension contributions influence

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<sup>1</sup>The figures are from the Flow of Funds Accounts, and include employer-provided pension plans, but not voluntary individual tax-exempt plans like Individual Retirement Accounts. Poterba and Wise (1996) find similar numbers for IRAs as well.

<sup>2</sup>This excludes purchases of stock in the employee's own company.

<sup>3</sup>For simplicity, we focus on defined-contribution pension plans in what follows, but similar arguments can be applied to defined-benefit plans, along the lines of Black (1980) and Tepper (1981).

<sup>4</sup>The gains to this strategy are reduced, but not eliminated, if interest on loans to finance pension contributions is not deductible.

<sup>5</sup>By "front-loaded," we mean the refund is payable at the time the investment is made rather than at the time the loss accrues to the investor.

<sup>6</sup>Some countries, such as the United Kingdom, follow a different approach for pension saving whereby there is no tax deduction for contributions, income earned by the plan is exempt, and withdrawals are exempt. Our argument regarding losses applies, in that the risk-adjusted return to sheltered saving is exempt from tax, while for taxable saving risk is subject to tax.

asset allocation can perhaps best be seen through a simple example. Consider an individual facing an effective marginal tax rate  $\tau$  on income from taxable assets. The individual can also contribute some amount to a pension; contributions are tax-deductible, and income in the plan accrues tax-free, but assets are fully taxable on withdrawal. Savings can be held in a safe, non-interest-bearing asset, or in stocks, which are worth a stochastic amount  $1 + \rho$  next period per dollar invested. Assume  $1 + \rho \geq 0$  because of limited liability. With full refundability of tax on capital losses, one dollar invested in stocks and held outside the pension returns  $\$1 + (1 - \tau)\rho$  after taxes. In practice, however, tax losses are not refundable,<sup>7</sup> so that the equity investment returns  $\$1 + (1 - \tau)\rho$  if  $\rho \geq 0$  and  $1 + \rho$  otherwise. Thus non-refundability raises the effective tax rate on risky securities, and reduces the desirability of equity investment for any given distribution of pre-tax returns  $\rho$ . The risk premium for non-sheltered investments is taxed because losses are not fully deductible from income. Consider, however, a pension contribution of one dollar, with a fraction  $(1 - \tau)$  held in stocks and  $\tau$  in the safe asset. Since the investment costs only  $\$(1 - \tau)$  in current consumption, the remainder financed with the tax refund  $\$\tau$  for the initial contribution, the rate of return on the initial investment is  $(1 - \tau)[(1 + \rho) + \tau/(1 - \tau)] = 1 + (1 - \tau)\rho$ . Such a strategy therefore pays a return equal to that of a taxable investment under full refundability in all states of the world. It follows that holding equity inside the pension may be desirable, even when bonds are preferred to stocks on an after-tax basis. This reverses the logic of the standard analysis, based on Miller (1977).

In this paper, we formalize this example by extending the Miller model to incorporate tax losses. In the model, firms make a leverage decision in the interests of shareholders, trading off the corporate tax advantage of debt against the risk of personal tax losses for shareholders and the real, agency costs of excessive leverage. Individual investors, who differ in tax rates on debt and equity income, allocate their taxable and tax-exempt savings between bonds and stocks. As the preceding example suggests, equity investors' attitudes to risk depend on tax rates, and shareholders will generally not agree on the optimal financial policy of the firm. We show that investors facing low tax rates—particularly pension funds—prefer to hold highly leveraged equity, if they hold equity at all, since the high risk of capital losses maximizes the tax advantage of equity for this group. High bracket investors, in contrast, prefer to hold safer firms which issue less debt and to which capital losses are less likely to accrue. Unlike in the Miller model, then, leverage is not irrelevant to the value of the firm, as each firm chooses the unique capital structure optimal for its clientele. In equilibrium, the valuation of firms is still independent of capital structure in a more limited sense, however, as firms switch among leverages until security values are equalized.

Our model also has different implications for the segregation of investors into clienteles of bondholders and stockholders. In the Miller model, investors facing low personal tax rates hold only debt, while those in high brackets hold only equity. In our model, this relationship is more complicated, since low-bracket investors have a tax-induced preference for both debt (because of the spread between debt and equity tax rates) and high-leverage equity (because of the non-refundability of tax losses). The relationship between tax rates and asset allocation is therefore complicated in general, and investors in any bracket may hold debt, equity, or both. In a plausible

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<sup>7</sup>Generally, losses can be deducted against gains but not ordinary income, and excess losses can be carried forward and applied to future gains. Since carryforwards do not bear interest, this is costly for taxpayers. Canadian data suggest that non-refundable losses are quite common among taxpayers with investment income. For the 1984-96 period, the number of taxpayers reporting net losses was 49.1 per cent of the number reporting net gains. The dollar value of net losses over this period was 22.6 per cent of the dollar value of net gains.

case, however, the population is segregated into three groups, with tax-exempt plans holding risky equity, taxable saving of individuals in low tax brackets held in bonds, and taxable saving of high-bracket individuals in shares of firms with little leverage.

If the tax-exempt class is interpreted as comprising pension funds and retirement saving, this prediction of the model is broadly consistent with observed asset allocations in pensions and taxable investments. There is considerable, albeit casual, evidence that pension funds invest heavily in very risky equities, such as venture capital funds. This development has been especially pronounced in the U.S. and Canada, where heavily indebted corporations are increasingly being held by pension funds through real estate investment trusts (Sheppard, 1997).

Since it is net capital losses on investors' total portfolio which are not deductible under the current tax system, the advantage of tax-exempt investments we have identified relates only to non-diversifiable market risks, rather than idiosyncratic risks of capital losses on individual assets. This leads us to consider the effects of macroeconomic risk on the portfolio choices of taxable and tax-exempt investors and, in turn, the impact of tax-exempts' asset allocation strategies on the susceptibility of firms to macroeconomic shocks. A common view is that one consequence of corporate tax deductibility of interest is to increase leverage and hence increase agency costs and the vulnerability of firms to macro downturns (Gertler and Hubbard, 1993). Our analysis indicates that non-deductibility of capital losses serves as a "brake" on this tendency. Of greater potential concern, therefore, are investments in highly leveraged firms by tax-exempts, for whom there is no disincentive effect associated with tax losses.

It should be noted that asset allocation of pension funds is likely influenced by considerations other than tax arbitrage, and other explanations have been advanced in the past for their investments in equities. It is sometimes suggested (e.g. Auerbach and King, 1982) that pension funds are less averse to risk or better able to diversify it than ordinary investors, and so invest in stocks to obtain higher expected returns. This argument seems unpersuasive, since pension managers are merely agents of pension beneficiaries or, in the case of defined benefit plans, stockholders of firms providing pensions, and so should act in their interests. As noted above, if beneficiaries "see through the veil" of the pension plan, they can pursue strategies to achieve any desired net portfolio allocation, while still obtaining the arbitrage gains of an all-debt pension. Bodie and Papke (1992) argue that pensions with predictable, nominal liabilities accruing in the future should seek to "immunize" liabilities by holding bonds with similar maturity dates. This view merely compounds the equity anomaly. Finally, because pension surpluses generally belong to the employer and insolvent plans are insured by the Pension Benefit Guaranty Corporation in the U.S., firms effectively have a call option on the assets of their pension funds, which may induce them to pursue high-risk strategies like equity investments. But this cannot explain equity investments by defined contribution plans, which have no such incentive. In short, a number of factors may contribute to equity investments being held by tax-exempt investors, but the phenomenon remains puzzling. Our model is intended to be complementary with previous accounts, rather than a replacement for them.

## 2 Individual portfolio choices

We consider a standard two-period model of portfolio choice and financial structure under uncertainty. In the first period, owners of entrepreneurial, all-equity firms offer debt and equity to investors. In the second period, the value of these claims is realized depending on the state of nature, taxes are levied, and consumption occurs. First we describe the optimal portfolio decisions

of individual investors facing heterogeneous tax rates; the next section addresses optimal capital structure.

Each individual allocates given financial wealth among available securities. Wealth can be held in corporate bonds or equity. Interest on bonds is taxable at the individual's ordinary tax rate  $\tau_p$ ; net returns to equity are taxable at rate  $\tau_e$ . For tax purposes, net capital losses on the investor's equity portfolio cannot be deducted from ordinary income, including interest and labour income. Tax rates are heterogeneous among investors because the income tax is progressive. For simplicity, it is assumed that marginal tax rates are unaffected by portfolio choices. There is a finite number of tax brackets  $\{(\tau_p^i, \tau_e^i)\}_{i=0, \dots, H}$ , where  $\tau_e^i \leq \tau_p^i$  because equity income receives preferential tax treatment. We order brackets such that  $\tau_e^{i+1} \geq \tau_e^i$ . The individual also holds given wealth in a pension. Returns to pension assets are exempt from personal taxation.<sup>8</sup>

Investors are risk neutral and choose portfolios to maximize expected returns after personal taxes. Since all investors have identical preferences, conditional on tax rates, it is possible to treat portfolio decisions for each tax bracket  $i$  as determined by a single representative investor with taxable financial wealth  $A^i$ , equal to the aggregate for investors in the bracket. Similarly, it will be convenient to think of tax-exempt asset allocations as being determined by a single, hypothetical investor with tax rates  $\tau_p^0 = \tau_e^0 = 0$  and total wealth  $A^0$ .

We assume risk neutrality of investors in order to focus on the role of tax losses on equity in determining asset allocations and preferences for corporate leverage. Note, however, that risk aversion by itself is *not* sufficient to explain the asset allocation puzzle for tax-exempts. While risk averse investors will generally hold both debt and equity for purposes of diversification, they should have little difficulty in reshuffling assets to hold debt in the tax-exempt portfolio, while borrowing in taxable form, if necessary, to achieve the desired aggregate portfolio mix. This is a pure tax-arbitrage strategy, with no implications for the risk of the net portfolio held.

Let  $b$  and  $e$  denote bonds and equity held by a typical investor and  $w_f$  the fraction of the equity portfolio held in shares of firm  $f$ . Before personal taxes, bonds earn net return  $r$  with certainty. We do not explicitly consider default on corporate debt, but see the discussion below in Section 3. Returns to equity are stochastic, depending on an aggregate shock  $s \in \mathbf{R}_+$ , a random variable with known c.d.f.  $G(s)$ . A density  $g(s)$  exists for all  $s > 0$ . All firms have identical prospects *ex ante*, but their financial policies may differ. If firm  $f$  has issued bonds  $B_f$ , let  $V(B_f)$  denote the ex ante market value of its equity and  $E(s, B_f)$  the ex post value of equity in the second period. Firms pay no dividends. We assume states can be ordered so that  $E$  is increasing in  $s$  for all  $B$ . Thus all equity returns are positively correlated with the market, and  $s$  can be regarded as representing business cycle risk. Let  $\rho(s, B) = (E(s, B) - V(B))/V(B)$  denote the net return to equity per dollar invested. Since  $V(B_f)$  is the cost basis of equity in firm  $f$ , the expected return after personal taxes for an investor in bracket  $\tau_e$  is

$$h(\gamma, B_f, \tau_e) = \int_0^\gamma \rho(s, B_f) dG(s) + (1 - \tau_e) \int_\gamma \rho_f(s, B_f) dG(s) \quad (1)$$

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<sup>8</sup>Our results would be unchanged if pension contributions were tax deductible and returns fully taxable on withdrawal, as is more typically the case, as long as tax rates are constant over time.

where  $\gamma$ , the state in which the investor's equity portfolio breaks even, is defined implicitly by

$$\sum_f w_f \rho(\gamma, B_f) = 0. \quad (2)$$

For convenience, we also write  $\rho_f(s) = \rho(s, B_f)$  to refer to the return on equity of firm  $f$ , given its leverage decision. Similarly, the after-tax expected return is written  $h_f(\gamma, \tau_e) = h(\gamma, B_f, \tau_e)$ .

To analyze portfolio decisions, it is convenient to decompose the investor's problem into two steps. In the first step, equity portfolio weights are chosen to maximize the after-tax expected rate of return to equity, while in the second, funds are allocated to bonds and equity to maximize the aggregate return on the portfolio. Accordingly, an optimal equity portfolio solves

$$R^*(\tau_e) = \max_{(w, \gamma)} \sum_f w_f h_f(\gamma, \tau_e) \quad (3)$$

$$s.t. \sum_f w_f \rho_f(\gamma) = 0 \quad (4)$$

$$\sum_f w_f \leq 1 \quad (5)$$

$$w_f \geq 0. \quad (6)$$

The non-negativity constraints (6) may require some explanation. Restrictions of some kind on borrowing and short positions are evidently required to eliminate the potential for unlimited tax arbitrage among investors facing different tax rates. These restrictions are inherently plausible for tax-exempt investors, since borrowing and short sales by pension funds are severely limited by law in most countries, and interest on loans that finance pension contributions is generally non-deductible. For taxable investors, these constraints could be replaced by a limit on the investor's aggregate short position, if risk aversion were also modelled. See Auerbach and King (1983) for a discussion.

Let  $(w^*(\tau_e), \gamma^*(\tau_e))$  be the associated optimizers, and let  $\mu$  denote the multiplier associated with (5). Noting that

$$\sum_f w_f \frac{\partial h_f(\gamma, \tau_e)}{\partial \gamma} = \tau_e g(\gamma) \sum_f w_f \rho_f(\gamma) = 0$$

given (4), first-order necessary conditions for an optimum reduce to

$$w_f^* : \quad \mu \geq h_f(\gamma^*, \tau_e) \quad (\text{all } f) \quad (7)$$

with equality in (7) if  $w_f^* > 0$ .

Given the optimal weights for an equity portfolio, the investor then chooses holdings of bonds

and equity to solve

$$W(\tau_p, \tau_e) = \max_{(b,e)} (1 - \tau_p)rb + R^*(\tau_e)e \quad (8)$$

$$s.t. \quad b + e \leq A \quad (9)$$

$$0 \leq b \leq A. \quad (10)$$

Note again that, to eliminate the possibility of unlimited arbitrage in a simple way, investors are constrained not to take short positions in either asset.

Letting  $\lambda$  denote the Lagrange multiplier associated with the budget constraint (9), the first-order conditions can be written

$$b^* : \quad \lambda \geq (1 - \tau_p)r \quad (11)$$

$$e^* : \quad \lambda \geq R^*(\tau_e) \quad (12)$$

with equality in (11) or (12) if  $b^* > 0$  or  $e^* > 0$ , respectively. The optimal portfolio is then characterized as follows.

**Proposition 1** *Assets of investors in bracket  $i$  are held entirely in bonds (equity) if  $(1 - \tau_p^i)r$  is greater than (less than)  $R^*(\tau_e^i)$ . Moreover, investors hold shares only in firms with maximal after-tax expected returns.*

**Proof.** Immediate from (7) and (11)–(12).

Because investors are risk neutral, portfolios are held entirely in assets with maximal after-tax expected returns. This may in principle involve diversification of equity portfolios, however, since non-refundability of equity losses causes taxable investors' objectives to be concave in pre-tax returns and non-linear in portfolio weights  $w$ . By diversifying among stocks with distinct probabilities of accruing capital losses, investors are able to construct “tax shelters,” in which marginal gross losses on one stock are used to shield gross gains on another from taxation. Examining the second-order condition for the problem demonstrates that the potential for diversification is limited, however: investors hold at most one stock which is more or less risky than average for the portfolio. This is stated as follows.

**Proposition 2** *No two stocks held in the optimal portfolio earn gains or losses with strictly greater probability than the aggregate portfolio. Specifically, if  $w_f^*w_j^* > 0$  for two stocks  $f \neq j$  then  $\rho_f(\gamma^*)\rho_j(\gamma^*) \leq 0$ .*

**Proof.** See appendix.

In summary, investors holding equity specialize in shares of firms yielding maximal after-tax returns. Shareholders may strictly prefer holding a portfolio of two or more stocks to specializing in shares of a single firm. However, the potential for diversification is limited. Note that tax-exempt saving is held in stocks if and only if the maximal pre-tax return to shareholder equity exceeds the pre-tax interest rate. To determine when this occurs in equilibrium of the model, we turn next to the corporate decision problem.



### 3 Corporate leverage and investor clientele

Thus far, we have analyzed portfolio decisions of investors, given their tax rates, and given the capital structure decisions of the corporate sector. We now turn to the determination of capital structure and the risk attitudes of investors. At the beginning of the first period, firms are owned by entrepreneurs, who sell debt and equity claims on the firm to maximize its ex ante market value. Gross cash flows  $Y(s)$  to the firm are fixed—that is, the firm has no real investment decisions to make. Thus firms may be regarded as price-takers for their securities, and leverage is chosen to maximize  $V(B) + B$ , where  $V$  is the market's willingness to pay for equity of firms which have issued debt  $B$ . It follows that if  $B_f^*$  maximizes value, then  $V'(B_f^*) = -1$ . In effect, the supply of equity to the market is perfectly elastic at the relative price of unity. It is then convenient to write the equilibrium value of the firm's equity as  $V(B) = \bar{V} - B$ , where  $\bar{V}$  is notionally the value of an unlevered firm.

Since interest payments on debt are deductible from corporate income, net cash flow available to shareholders is  $(1 - \tau_c)Y(s) - (1 + r(1 - \tau_c))B_f$ . In addition, leverage imposes real costs on the firm. Such costs may be associated with deadweight losses occurring in bankruptcy, with output lost due to agency conflicts between shareholders and creditors, or corporate tax losses (DeAngelo and Masulis, 1980). To preserve the generality of the model, we treat such costs in a reduced form fashion, and merely assume that cash flow before corporate taxes is reduced by  $\phi(s, B)$  when the state of the world is  $s$  and the firm has issued debt  $B$ .<sup>9</sup> Assume that the deadweight cost is globally non-decreasing and convex in leverage:  $\phi'_B \geq 0$  and  $\phi''_{BB} \geq 0$  for all  $s$  and  $B$ . It is further assumed that the marginal agency of debt is non-increasing in gross cash flow, so that  $\phi''_{sB} \leq 0$ . This assumption is consistent with many structural models of agency costs, and reflects the notion that agency conflicts are exacerbated when the firm experiences financial distress. Accordingly, shareholders' rate of return on equity in period 2 is

$$\rho(s, B) = \frac{E(s, B) - V(B)}{V(B)} = \frac{(1 - \tau_c)(Y(s) - rB - \phi(s, B)) - \bar{V}}{\bar{V} - B}. \quad (13)$$

Our assumptions regarding the nature of agency costs guarantee that leverage unambiguously increases the risk of equity returns in two senses. First, return functions of firms issuing different amounts of debt are *single crossing* in  $s$ , so that returns to low-leverage firms are less risky in the sense of second-order stochastic dominance. Second, the probability of capital losses on equity is lower for low-leverage firms. These properties are stated in the following lemma and proven in the Appendix.

**Lemma 1** *For all  $B \leq B' \leq \bar{V}$  and  $s \leq s'$ :*

(i)  $\rho(s, B') \geq \rho(s, B)$  implies  $\rho(s', B') \geq \rho(s', B)$

(ii)  $\rho(s, B') \geq 0$  implies  $\rho(s, B) \geq 0$ .

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<sup>9</sup>Thus the safe interest rate  $r$  should be interpreted as the expected return to risky debt. A complete analysis of default in the model would be complicated, since the premium on risky debt decreases an investor's capital gains when they are taxable, and default reduces capital losses. Such effects are however of second-order importance, and we ignore them here.

With these restrictions on the equity return functions, increased corporate leverage can be identified with increased risk in equity returns. It follows that, when net capital losses on equity are non-refundable, investors in high tax brackets are more averse to risk and prefer to hold, on average, shares in firms with lower leverage. Intuitively, increasing the probability of losses on the portfolio requires holding shares in more leveraged firms. Since, given Lemma 1, more leveraged firms are more risky in the sense of second-order stochastic dominance, increasing the leverage of the portfolio is less desirable at higher personal tax rates. Consequently, the probability of losses is non-increasing in  $\tau_e$ , as investors hold less leveraged stocks.<sup>10</sup> This is stated as follows.

**Proposition 3**  $\gamma^*(\tau_e)$  is non-increasing in  $\tau_e$ .

Thus non-deductibility of capital losses on equity leads investors to hold safer stocks, and reduces the quantity of debt issued by firms, despite the preference for debt induced by the corporate tax system. We can examine shareholder preferences for leverage directly by differentiating the value function which solves (8)–(10) with respect to  $B_f$ , given the optimal portfolio allocation and the equilibrium price of equity. Applying the envelope theorem, and using (13) to solve for the marginal after-tax cost of debt, an investor's marginal willingness to pay for leverage can be expressed as

$$\frac{\partial W}{\partial B_f} = \frac{w_f^* e^*}{\bar{V} - B_f} \left[ h_f - (1 - \tau_c) \int_0^{\gamma^*} (r + \phi'_B) dG - (1 - \tau_e)(1 - \tau_c) \int_{\gamma^*} (r + \phi'_B) dG \right]. \quad (14)$$

Let  $\hat{B}_f^i$  be the optimal leverage of firm  $f$  for investors in bracket  $i$ , i.e. that for which  $\partial W / \partial B_f = 0$  for tax rates  $(\tau_p^i, \tau_e^i)$ .

To see more clearly the role of tax losses in generating diversity of investors' preferences for leverage, suppose instead that capital losses are fully tax-deductible. Define  $\bar{\rho}(B) = \int \rho(s, B) dG(s)$  as the expected return to equity and  $\Phi(B) = \int \phi(s, B) dG$  as the expected deadweight cost of debt. With full refundability,  $\gamma = 0$  and  $h_f(\gamma, \tau_e) = (1 - \tau_e) \bar{\rho}_f$ . The first-order condition (14) for optimal leverage then reduces to

$$\frac{\partial W}{\partial B_f} = (1 - \tau_e) \frac{w_f^* e^*}{\bar{V} - B_f} [\bar{\rho}(B_f) - (1 - \tau_c)[r + \Phi'(B_f)]] = 0. \quad (15)$$

In this case, investors' indifference curves in  $(B, V)$ -space coincide, and optimal leverage is independent of  $\tau_e$ . Shareholders agree unanimously on the financial policy of the firm, and segregation into leverage clienteles need not occur.

The risk of tax losses also has implications for leverage neutrality. In the Miller model, leverage is irrelevant to shareholders, as security prices adjust until the marginal shareholder is indifferent between bonds and equity, and corporate and personal borrowing are perfect substitutes for all shareholders. With non-deductibility of losses, the model exhibits a form of leverage neutrality, but of a far weaker nature than the Miller result. In the initial stage of the model, entrepreneur-owned firms adjust their offerings of securities to maximize value. The market value of the firm is equalized at any capital structure offered in equilibrium. For a given tax clientele of investors,

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<sup>10</sup>The formal proof of the result follows this intuition very closely. Since  $\gamma^*(\tau_e)$  cannot be guaranteed to be a continuous function, the standard approach to comparative statics based on the Implicit Function Theorem cannot be employed. Instead, we adopt an ordinal approach similar to that of Milgrom and Shannon (1994).

however, leverage is not irrelevant, since it entails non-diversifiable risk of tax losses. If a firm were to attempt to deviate from its initial capital structure after shares traded, it would face opposition from shareholders and see a decline in its share price.

## 4 The asset allocation puzzle

Our analysis was motivated by the observation that equities are often held in tax-exempt form by pension funds and individual investors, despite the equilibrium advantage to bond ownership for tax-exempts predicted by the Miller model. We have shown that the deductibility of initial contributions to tax-exempt plans leads to implicit refundability of tax losses, generating an advantage to risky equity investments for tax-exempt investors relative to taxable investors. It remains to be determined, however, whether the refundability advantage for tax-exempts can dominate the conventional advantage to debt holdings in the model.

Previously it was demonstrated that an investor in bracket  $i$  holds equity if  $R^*(\tau_e^i) > (1 - \tau_p^i)r$  at equilibrium prices of securities. The equilibrium return to equity for each bracket  $i$  is a complicated function, depending on the distribution of the population among tax brackets, the corporate tax rate, and the agency cost function; and in general returns to the two assets may cross multiple times, making asset allocation a non-monotone function of tax rates. In some cases, however, a simple pattern of asset allocations emerge, in which tax brackets can be partitioned into two or three intervals, each associated with holdings of bonds or particular classes of stocks.

To determine how the two return functions may cross, it is convenient to treat personal tax rates as continuous variables. First, we establish that the optimal return to equity is decreasing in the investor's tax rate, but at a decreasing rate, as investors in higher brackets switch to safer, less indebted firms and reduce the risk of tax losses. In the appendix, we prove the following result.

**Lemma 2** *The optimal after-tax return to equity  $R^*(\tau_e)$  is a decreasing, convex function of  $\tau_e$ .*

To simplify the analysis of equilibrium, assume additionally that  $\tau_p$  is an increasing, convex function of  $\tau_e$ . This subsumes the standard case in which the personal tax advantage of equity is proportional, i.e.  $\tau_p = \theta\tau_e$  for  $\theta > 1$ . If the tax advantage is not proportional, then it is most reasonable to assume taxes on capital gains are more easily avoided in higher tax brackets (e.g. if average deferral periods are longer), which implies  $\tau_p$  is strictly convex in  $\tau_e$ . With this assumption, the after-tax return to bonds is a decreasing, concave function of  $\tau_e$ . It follows that the return functions for bonds and equity cross at most twice. This motivates the following characterization of equilibrium asset allocation rules in the model.

**Proposition 4** *Suppose that  $\tau_p$  is an increasing, convex function of  $\tau_e$ . If there exists an equilibrium with an interior aggregate debt–equity ratio for the economy, then there exist tax rates  $\hat{\tau}_1, \hat{\tau}_2$ , with  $0 \leq \hat{\tau}_1 < \hat{\tau}_2 \leq \tau_e^H$ , such that investors in bracket  $i$  hold bonds if and only if  $\hat{\tau}_1 < \tau_e^i < \hat{\tau}_2$ .*

An equilibrium with two crossings is depicted in Figure 1, where it is assumed the equity tax advantage is proportional. In this case, the return on equity for tax-exempt and low-bracket taxable investments exceeds that of bonds, so that these savings are held in equities of highly leveraged firms. Taxable investments by investors in middle brackets are held in bonds. Beyond the second intersection of the two curves, stocks again dominate bonds on an after-tax basis, but these high-bracket investors choose to hold relatively safe, unlevered equity, in order to avoid the risk of tax losses.

(INSERT FIGURE 1 ABOUT HERE.)

Our principal interest lies in equilibria in which  $\hat{\tau}_1 > 0$ , so that tax-exempt investors hold equity in their portfolios. Since the equilibrium return to equity for tax-exempt investors solves, given (14),

$$R^*(0) = (1 - \tau_c) \left( r + \Phi'(\hat{B}^0) \right),$$

equity is preferred to debt if  $R^*(0) > r$  or equivalently  $\tau_c r < (1 - \tau_c)\Phi'(\hat{B}^0)$ . This gives rise to the following condition that is necessary for tax-exempts to hold equity in equilibrium.

**Proposition 5** *If tax-exempt investors hold equity in equilibrium, then the marginal deadweight cost, evaluated at the optimal leverage, equals or exceeds the marginal corporate tax advantage of debt finance.*

Intuitively, when the cost of debt finance is high, then high-risk equity earns a premium, which may offset the conventional advantage of debt to tax-exempt investors. More generally, the result obtains only if: (i) capital losses on taxable equity portfolios are non-deductible; and (ii) marginal deadweight costs of debt are large, relative to its corporate tax advantage.

To see the role of tax losses, we suppose in contrast that losses are fully refundable and adopt the standard assumption that

$$\frac{1 - \tau_p^i}{1 - \tau_e^i} \text{ is decreasing in } i \tag{16}$$

so that debt is more tax-disadvantaged for investors in higher brackets. In this case, preferences for leverage are given by (15), all investors prefer the same financial policy  $\hat{B}$ , and the after-tax return to equity for bracket  $i$  is

$$R^*(\tau_e^i) = (1 - \tau_e^i)\bar{\rho}(\hat{B}).$$

Investors then prefer debt to equity if and only if  $\bar{\rho}(\hat{B}) < r(1 - \tau_p^i)/(1 - \tau_e^i)$ . Since the right-hand side of the inequality is decreasing in  $i$ , it follows that low-bracket investors, including tax-exempts, hold bonds in equilibrium.

**Proposition 6** *Suppose tax losses are refundable and that (16) holds. In an equilibrium with an interior aggregate debt–equity ratio, tax-exempt investors hold only bonds.*

Similarly, suppose that tax losses are non-refundable, but that there are no deadweight costs associated with leverage, so that  $\phi(s, B) \equiv 0$ . Then the optimal leverage condition for tax-exempts reduces to

$$\frac{\partial W}{\partial B_f} = \frac{w_f^* e^*}{\bar{V} - \hat{B}^0} \left[ \bar{\rho}(\hat{B}^0) - (1 - \tau_c)r \right] = 0.$$

Hence  $R^*(0) = (1 - \tau_c)r < r$  and tax-exempts prefer to hold bonds. In the absence of agency costs, the corporate tax advantage of debt drives the maximal pre-tax return to equity below that of

bonds. Consequently, as in the Miller model, only investors facing relatively high taxes on interest income can be induced to purchase equity. This establishes the following.

**Proposition 7** *Suppose that the marginal deadweight cost of debt is zero. Then tax-exempt investors hold bonds in equilibrium.*

When the conditions of the two preceding propositions hold simultaneously, then the model reduces to that of Miller (1977). The optimal leverage condition becomes

$$\frac{\partial W}{\partial B_f} = \frac{w_f^* e^*}{\bar{V} - \hat{B}} (1 - \tau_e) \left[ \bar{\rho}(\hat{B}) - (1 - \tau_c)r \right] = 0.$$

In this case, shareholders are indifferent to corporate leverage only if  $\bar{\rho}(\hat{B}) = (1 - \tau_c)r$ . At an interior equilibrium, there exists a tax bracket  $m$  at which investors are indifferent between debt and equity, so that  $1 - \tau_p^m = (1 - \tau_e^m)(1 - \tau_c)$ , and investors in lower brackets hold bonds. Firms choose financial policy to equate aggregate supply and demand for debt and equity in equilibrium.

## 5 Conclusion

In this paper, we have extended the conventional model of capital structure to incorporate non-deductibility of capital losses from taxable income, focussing in particular on the differences in tax treatment of losses for taxable and tax-exempt assets. Non-refundability of tax losses makes risky equity investments less attractive to taxable but not to tax-exempt investors, and this difference may dominate the conventional tax disadvantage of debt, when the risk of losses is large. Our model, we believe, does a better job than the conventional framework (which ignores personal tax losses) in explaining two key stylized facts of asset allocation—tax-exempt investors such as pension funds tend to hold significant quantities of corporate stock in their portfolios and, when they do so, these stocks can be riskier and more highly leveraged on average than those held in taxable form.

These predictions of the model are consistent with a number of recent transactions in the U.S. and Canada, in which corporations have sought financing from pension plans, while increasing leverage substantially in order to pay returns in a manner that is tax-deductible for the corporation (McDonnell, 1997; Sheppard, 1997). Investment by pensions in high-leverage equity has prompted concern among policy-makers, inasmuch as it permits business income to escape taxation at both corporate and personal levels. The Internal Revenue Service in particular has recently promulgated a number of rules aimed at discouraging such transactions. Our analysis suggests another reason for concern about such investments, based on efficiency considerations rather than tax avoidance. In the standard view, debt finance reduces the firm's corporate tax liabilities but also entails real costs—those associated with agency conflicts between bondholders and stockholders and the attendant risk of financial distress in macroeconomic downturns. Thus investors choose the firm's financial policy in a way which trades off the marginal tax advantages of debt and its real costs. Such a policy, while optimal for the firm per se, is inefficient for the economy as a whole, as fiscal gains to taxpayers balance government revenue losses, leaving only real agency costs. While such adverse incentives are clearly present in the tax system, we conclude that they are attenuated by non-refundability of personal tax losses, which makes taxable investors reluctant to put the firm at risk through excessive leverage. But this deterrent is absent for tax-exempts, and they may well

be lead by the tax system to excessive and costly risk taking. Our model can shed no light on the empirical magnitude of such effects. We suggest, however, that it is an issue of increasing concern as the size of the tax-exempt sector grows.

## Appendix

### Proof of Proposition 2.

Fix  $\tau_e > 0$ , and suppose there exist two equity securities 1, 2 with  $w_1^* > 0$  and  $w_2^* > 0$  and distinct probabilities of capital losses, i.e.  $\rho_1(\gamma^*) \neq \rho_2(\gamma^*)$ . Recall the break-even state of the portfolio  $\gamma^*$  is implicitly defined by

$$\sum_f w_f^* \rho_f(\gamma^*) = 0.$$

Thus, applying the implicit function theorem,

$$\frac{\partial \gamma^*}{\partial w_j} = - \frac{\rho_j(\gamma^*)}{\sum_f w_f \partial \rho_f(\gamma^*) / \partial \gamma}.$$

Consider a marginal change in portfolio weights holding the cost basis of the aggregate portfolio constant, i.e.  $-dw_2 = dw_1 > 0$ . The associated change in marginal return to security 1 is

$$\begin{aligned} \frac{dh_1(\gamma^*, \tau_e)}{dw_1} &= \frac{\partial h_1(\gamma^*, \tau_e)}{\partial \gamma} \left( \frac{\partial \gamma^*}{\partial w_1} - \frac{\partial \gamma^*}{\partial w_2} \right) \\ &= -\tau_e g(\gamma^*) \rho_1(\gamma^*) \left( \frac{\rho_1(\gamma^*) - \rho_2(\gamma^*)}{\sum_f w_f^* \partial \rho_f(\gamma^*) / \partial \gamma} \right) \end{aligned}$$

Since  $w^*$  is optimal, the second-order condition implies  $dh_1(\gamma^*, \tau_e)/dw_1 \leq 0$ , which reduces to

$$\rho_1(\gamma^*) [\rho_1(\gamma^*) - \rho_2(\gamma^*)] \geq 0.$$

Hence  $\rho_1(\gamma^*) > 0$  implies  $\rho_2(\gamma^*) \leq 0$  and conversely.  $\square$

**Proof of Lemma 1.** To prove part (i), suppose  $\rho(s, B') \geq \rho(s, B)$  for  $B' \geq B$ . Note that, since  $\phi''_{sB} \leq 0$  by assumption,

$$\begin{aligned} \frac{\partial \rho(s, B)}{\partial s} &= \frac{(1 - \tau_c)(Y'(s) - \phi'_s(s, B))}{\bar{V} - B} \\ &\leq \frac{(1 - \tau_c)(Y'(s) - \phi'_s(s, B'))}{\bar{V} - B'} = \frac{\partial \rho(s, B')}{\partial s}. \end{aligned}$$

Integration then yields  $\rho(s', B') \geq \rho(s', B)$ .

For part (ii), since  $\phi$  is non-decreasing in  $B$ ,  $\rho(s, B') \geq 0$  implies

$$(1 - \tau_c)(Y(s) - rB - \phi(s, B)) - \bar{V} \geq (1 - \tau_c)(Y(s) - rB' - \phi(s, B')) - \bar{V} \geq 0$$

so that  $\phi(s, B) \geq 0$ .  $\square$

**Proof of Proposition 3.** Consider two equity tax rates  $\tau^b > \tau^a$ . Suppose  $\gamma^a = \gamma^*(\tau^a) < \gamma^*(\tau^b) = \gamma^b$ . Let  $i$  be the safest stock held in the optimal portfolio for  $\tau^a$ , i.e.  $i = \arg \max_f \{\rho_f(\gamma^a) : w_f^*(\tau^a) > 0\}$ . By definition,

$$\sum_f w_f \rho_f(\gamma) = 0$$

so that  $\rho_i(\gamma^a) \geq 0$ . Since  $\rho_i$  is increasing in  $\gamma$ ,  $\gamma^b > \gamma^a$  implies  $\rho_i(\gamma^b) > 0$  and, by part (ii) of Lemma 1,  $\rho_f(\gamma^b) > 0$  for all  $f$  such that  $B_f \leq B_i$ . It follows that the optimal equity portfolio for  $\tau^b$  contains a positive quantity of a stock  $j$  with  $B_j > B_i$  and  $\rho_j(\gamma^b) \leq 0$ . From the first-order conditions (7),  $w_i^*(\tau^a) > 0$  implies

$$h_i(\gamma^a, \tau^a) - h_j(\gamma^a, \tau^a) = \int_0^{\gamma^a} (\rho_i - \rho_j) dG + (1 - \tau^a) \int_{\gamma^a} (\rho_i - \rho_j) dG \geq 0. \quad (17)$$

Similarly,  $w_j^*(\tau^b) > 0$  implies

$$h_i(\gamma^b, \tau^b) - h_j(\gamma^b, \tau^b) = \int_0^{\gamma^b} (\rho_i - \rho_j) dG + (1 - \tau^b) \int_{\gamma^b} (\rho_i - \rho_j) dG \leq 0. \quad (18)$$

Substituting (17) into (18) yields

$$\begin{aligned} h_i(\gamma^b, \tau^b) - h_j(\gamma^b, \tau^b) &= [h_i(\gamma^a, \tau^a) - h_j(\gamma^a, \tau^a)] \\ &\quad + \tau^a \int_{\gamma^a}^{\gamma^b} (\rho_i - \rho_j) dG + (\tau^a - \tau^b) \int_{\gamma^b} (\rho_i - \rho_j) dG. \end{aligned} \quad (19)$$

The first term in (19) is non-negative by (17). Since  $\rho_i(\gamma^a) > 0 \geq \rho_j(\gamma^b)$  by construction, part (ii) of Lemma 1 implies

$$\rho_i(s) > \rho_j(s) \quad \text{for all } s \leq \gamma^b. \quad (20)$$

Hence

$$\int_{\gamma^a}^{\gamma^b} (\rho_i - \rho_j) dG > 0$$

and the second term in (19) is non-negative. Moreover, (18) and (20) imply

$$\int_{\gamma^b} (\rho_i - \rho_j) dG \leq -\frac{1}{1 - \tau^b} \left[ \int_0^{\gamma^b} (\rho_i - \rho_j) dG \right] < 0$$

so that the third term in (19) is also positive, since  $\tau^a < \tau^b$ . Collecting these observations,  $\gamma^a < \gamma^b$  implies

$$h_i(\gamma^b, \tau^b) - h_j(\gamma^b, \tau^b) > 0$$

which contradicts (18). It follows  $\gamma^a \geq \gamma^b$ .  $\square$

**Proof of Lemma 2.** Monotonicity is obvious. To show convexity, choose any two equity tax rates  $\tau^a, \tau^b$ , and let  $\tau^c = \lambda\tau^a + (1 - \lambda)\tau^b$  for  $\lambda \in (0, 1)$ . Let  $(w^m, \gamma^m)$  solve (3)–(6) for  $\tau^m$ ,  $m = a, b, c$ . By the definition of a maximum, since  $(w^c, \gamma^c)$  is feasible at any tax rate,

$$R^*(\tau^m) \geq \sum_f w_f^c \left[ \int_0^{\gamma^c} \rho_f dG - (1 - \tau^m) \int_{\gamma^c} \rho_f dG \right] \quad (m = a, b)$$

Multiplying and summing the inequalities yields

$$\begin{aligned} \lambda R^*(\tau^a) + (1 - \lambda) R^*(\tau^b) &\geq \sum_f w_f^c \int_0^{\gamma^c} \rho_f dG - [1 - (\lambda\tau^a + (1 - \lambda)\tau^b)] \sum_f w_f^c \int_{\gamma^c} \rho_f dG \\ &= R^*(\tau^c). \quad \square \end{aligned}$$



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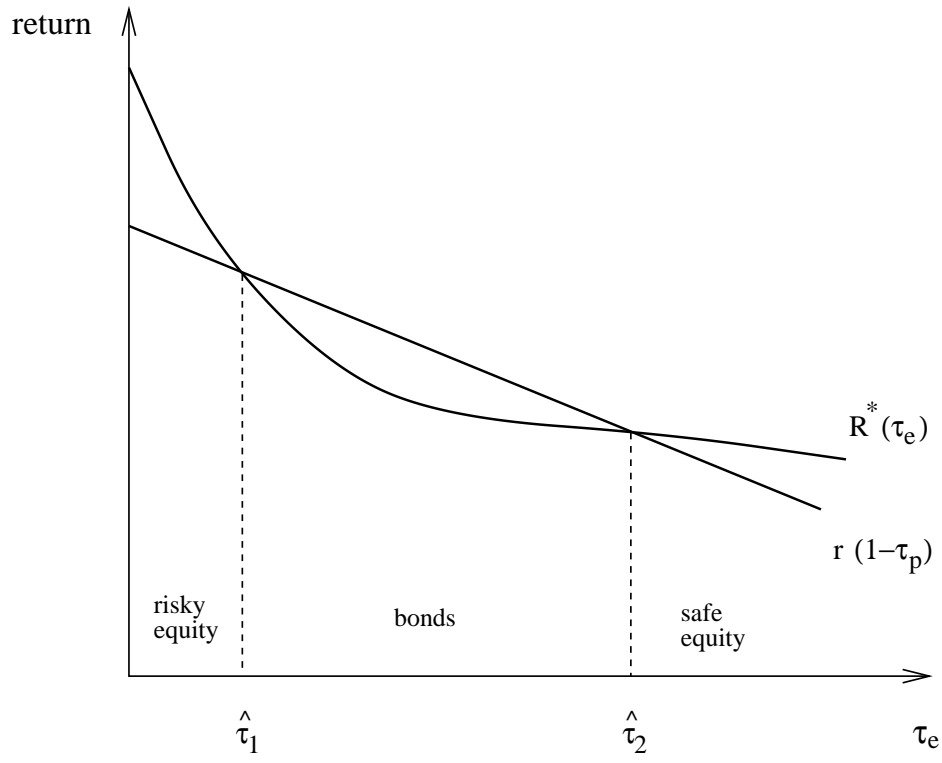


Figure 1: Equilibrium returns to debt and equity by tax bracket.