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FUNDED AND UNFUNDED PENSION SCHEMES: RISK, RETURN AND WELFARE

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Abstract

This paper uses stochastic simulations on calibrated models to assess the optimal degree of reliance on funded pensions and on a particular type of unfunded (PAYG) pension. Surprisingly little is known about the optimal split between funded and unfunded systems when there are sources of uninsurable risk that are allocated in different ways by different types of pension system. This paper calculates the expected welfare of agents in different economies where in the steady state the importance of PAYG pensions differs. We estimate how the optimal level of unfunded, state pensions depends on rate of return and income risks and also upon the actuarial fairness of annuity contracts.

Keywords: Pensions, annuities, risk-sharing

JEL Classification: H55, D91, G22, J14

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Funded and Unfunded Pension Schemes: Risk, Return and Welfare

Introduction:

Demographic changes across the developed world will put strain on unfunded, pay-as-you-go (PAYG) pension systems. The scale of the problems is large and has prompted a growing literature on the reform of state pension systems (see, for example, Feldstein (1996), Feldstein and Samwick (1998), OECD (1996), Mitchell and Zeldes (1996), Disney (1996), Kotlikoff (1996), Miles and Timmerman (1999) and Sinn (1999)). Advocates of a switch away from PAYG towards fully funded systems have stressed two relevant factors. First, in dynamically efficient economies the average rate of return on assets will exceed GDP growth. Since the tax base for unfunded state pensions is likely to move closely in line with GDP a funded system will tend to generate higher returns so that in the long run funded systems can pay higher pensions for a given level of contributions (or generate the same level of pensions for lower contributions). Second, if contributions to funded pension systems are accumulated in a personal fund, and the value of pensions paid is proportional to that fund, then distortions to labour supply are likely to be lower than with most existing PAYG systems where there is a weak link between marginal contributions and increments to the expected present value of future pensions.

Neither of these factors provides a compelling reason to favor a wholesale switch to a funded system. It is well known that in general there is no Pareto improving way of making a transition from an unfunded system to a funded system no matter how high the average rate of return on assets is relative to sustainable GDP growth. On the transition path funds need to be accumulated while pensions must still be paid to those who are in, or near, retirement and who have accumulated rights to receive (unfunded) pensions. Somebody, somewhere needs to pay for funds to be accumulated. (See Breyer (1989) for the original contribution. For generalisations of his result see Homburg (1990) and Fenge (1996)). Only if we combine some other benefits from a switch to funding – for example lower distortions to labor supply – to the potential long run gains from funded pensions can we engineer a transition where there are no losers. But those extra benefits, if they exist, are not really a benefit of funding per se. Labour distortions *may* be lower with funded schemes but they need not be. In principle we can envisage a funded scheme with only weak linkage between marginal contributions and extra (future) pension receipts. Likewise we can imagine a wholly unfunded system where, at the margin, there is a tight link between extra contributions and higher future pensions.

If we do assume that funded pension systems are ones where individuals accumulate their own fund and there is no redistribution across individuals (preserving a tight link between contributions and pensions received) then benefits to labor supply decisions are likely to come at the expense of gains from risk sharing. This paper focuses on the risk issues. PAYG systems do typically reduce exposure to some forms of risk because there is often substantial redistribution from those with high average labor income over their working lives to those with low average earnings. Redistribution within cohorts is common with PAYG systems. In some countries the scale of redistribution, and consequent reduction in risk exposure to bad earnings shocks, is very substantial. In the UK, for example, the basic state pension is paid at a flat rate and pension receipts are to a large extent independent of earnings and contribution history. Contributions paid, however, are much more closely linked to labor income[2]. Flat rate state pensions are also paid in Canada, the Netherlands and New Zealand. State pensions are also highly redistributive in the US (see Gruber and Wise 1977).

With risk averse agents, and given uncertainty about both rates of return on assets and about future labor income, the ways in which funded and unfunded systems redistribute risk is clearly important. It is a mistake when considering changes to the funded/unfunded mix in pension provision to count any gains in terms of lower labor supply distortions without considering the potential for losses from less implicit insurance against risk in a world where not all risk can be insured.

Even if neither funded nor unfunded systems provided any redistribution of risks *across* agents their risk characteristics would still be very different. An unfunded system with zero redistribution within a cohort would be one where the pension received was proportional to the value of contributions paid. In a balanced system with a constant overall contribution rate the factor of proportionality (which implicitly

[2] The UK system is not entirely a flat rate one because the number of years in the labor force – either employed or unemployed – is relevant to pension entitlement. Contributions paid are a proportion of income between lower and upper earnings limits. In practice the system has been highly redistributive.

defines the rate of return on contributions) would reflect the growth in the aggregate wage bill over time. For two agents of the same cohort their pensions would differ in a way which reflected the difference in their overall earnings while in the labour force. In this system there are two sources of risk relevant to an agent's pension: individual earnings history risk and aggregate risk that affects the rate of growth of the aggregate wage bill over time (the latter reflects demographic and productivity shocks). A funded system with no redistribution exposes individuals to pension uncertainty from two sources: individual labor income risk (which affects the scale of contributions into the fund) and rate of return risk (which affects the growth of the fund). Aggregate wage growth risk is not directly relevant.

The aim of this paper is to try to work out, with stylised, calibrated models, what the optimal split between a specific type of funded pension and a particular type of unfunded (PAYG) pension might be. Surprisingly little is known about the optimal split between funded and unfunded systems when there are sources of uninsurable risk that affect risk averse agents and where those risks are allocated in different ways by different types of pension system. Merton (1983) addressed the issue and showed that in general a mixed system had benefits on standard portfolio allocation grounds. Feldstein and Ranguelova (1998) and Feldstein, Ranguelova and Samwick (1999) consider uncertainty about rates of return and how it affects the transition to funded systems. But they do not consider idiosyncratic risks that are important in practice and which individuals find it hard to insure against. Bohn (1999) analysed the impact of uncertainty about future demographic structure, but did not consider either rate of return or labor income uncertainty.

This paper uses model simulations to calculate the expected welfare of agents in different economies where in the steady state the importance of PAYG pensions differs. We focus on steady states where demographic structure is unchanging and where funded schemes are in balance. We do not address transition issues. Logically, analysing the characteristics of systems in steady states should be done prior to facing transition problems; we need to know what the optimal steady state generosity of unfunded pensions is before we turn to problems of how to get there from wherever we start.

To be more specific the question we address is this. What is the optimal size of flat rate, unfunded pensions that act as a safety net in a system of funded individual retirement accounts? Advocates of funded pensions who point to the labor market benefits of linking individual pensions to the value of a personal fund are advocating personal retirement accounts, which we will refer to as personal pensions. Personal funded pensions may allow people to insure perfectly against some risks – if annuities are available at actuarially fair rates then length of life risk can be avoided. But personal pensions mean that labor income risk from working years, which will have an impact on the contributions to a personal pension fund, have lasting effects upon pension income; such pensions obviously also generate rate of return risk. Given this we consider what role might be played by unfunded, "safety net" pensions that give insurance against labor income risk and are not dependent on rate of return risk. We ask what the optimal size of these safety net pensions might be.

To bring out the issues in a stark way we assume that the unfunded pensions have no risk and are highly redistributive. In practice unfunded state pension systems are not flat rate in most countries and do have risk. Nonetheless there is a large element of redistribution in most state run, unfunded systems. (For details of the redistibutive nature of state pensions across developed countries see the country chapters in Gruber and Wise (1999)). And while there has been a lot of volatility in the effective returns from state pension systems to different generations this is not an intrinsic feature of such systems. Governments have changed the rules on pension systems over time, often as a result of a pressure from lobby groups as well as failure to see in advance the implications of slow moving changes in demographics.

We take a model with lots of idiosyncratic labor income risk but where there is no aggregate labor income risk. We assume that the distribution of rates of returns on assets is known and is invariant to the relative importance of the unfunded, safety net pensions.[3] The simplifying assumptions mean that unfunded, flat rate pensions offer insurance against all three types of risk that exist in the model and that matter to

[3] A more general model would allow for systematic, aggregate risk which would affect labor productivity and make the contribution rate needed to finance a PAYG system, or else the generosity of unfunded pensions, stochastic. The covariance between returns on financial assets and these aggregate productivity shocks would obviously be a key factor. What limited empirical evidence as exists suggests this correlation is small (see Palacios (1998)).

individuals: longevity risk, labor income risk and rate of return risk. Personal pensions offer insurance against only one of these risks (uncertainty about length of life), and then only if annuities markets work well. Whether or not there is a role for safety net pensions, and how great that role should be, will depend in a complicated way on the scale of risks to labor income, the risks to asset returns, the average rate of return on assets, the growth of labor productivity (the key factor behind the return on safety net pensions), the degrees of risk aversion and of time preference of agents, the importance of borrowing constraints and on whether annuity markets for personal pension wealth work well.

In the stylised model unfunded pensions are financed from a proportional tax on labor income. We will assume that contributions to personal pensions are flexible – contributions can be as high or low as people wish (in fact we allow negative contributions - ie. dissaving from the pension pot). We also assume that throughout their lives agents face borrowing constraints: financial assets must be non-negative. These assumptions have an important bearing on the advantages of different types of pension which are financed in different ways. The borrowing constraints and the different types of financing for funded and unfunded pensions means that the pattern of labor income over the life cycle is relevant.

We are not able to find analytic solutions to even the highly stylised models used in this paper. The standard optimisation problem that the agents in this model solve – maximising the expectation of an additively separable lifetime utility function in the presence of multiple sources of risk – is one for which no analytic results are available, at least for the (standard in the literature) assumptions we make about preferences. So we solve dynamic optimisation problems by numerical methods and perform simulations with a large number of agents. We can then calculate the expected utility at the start of life of agents given the parameters of the pensions regime. We evaluate pension regimes by reference to the ex-ante life-time expected utility of someone who was behind a veil of ignorance – they know their preferences but not whether they will be a productive or less productive worker or whether they will be lucky or unfortunate in portfolio selection.

The model:

We assume an economy where a given (large) number of agents are born each period and where mortality rates (probabilities of surviving to given ages) are unchanging. Such an economy will ultimately generate an unchanging demographic structure. We focus on steady state population structures.

Given the stochastic processes for future labor income and for future rates of return (and conditional on pensions arrangements and mortality rates) agents choose consumption and saving in each period to maximise expected lifetime utility. We assume an additively separable form of the agent's lifetime utility function. We also assume a constant coefficient of risk aversion, the inverse of the intertemporal substitution elasticity. Agents are assumed to know the probabilities of surviving to given ages. Agent k who is aged j at time t maximises:

$$U_k = E_t \left[\sum_{i=0}^{i=T-j} s_{ij} \{ [c_{kt+i}]^{1-\zeta} / (1-\zeta) \} / (1+\rho)^i \right] \quad (1)$$

where T is the maximum length of life possible (120 years of age) and the probability of surviving i more periods conditional on reaching age j is $s_{ij.}$ ($s_{0j}=1$). ρ is the rate of pure time preference; c_{kt+i} is consumption of the agent in period t+i.

ζ is the coefficient of relative risk aversion.

Agents face two constraints:

First there is a budget constraint governing the evolution of financial assets taken from one period to the next.

$$W_{k\,t+1} = [W_{k\,t} + \exp(y_{kt}).(1-\tau) - c_{kt} + b_{kt}].\exp(r_{kt}) \quad (2)$$

$W_{k\,t}$ is the stock of wealth of agent k in period t

y_{kt} is the log of gross labour income

τ is the tax rate on labour income. Tax paid is simply a proportion of gross income

b_{kt} is the level of the unfunded, state pension received by an agent, this pension is zero until age 65.

r_{kt} is the one period (log) rate of return on financial wealth between period t and period t+1. It has both a time subscript and an agent subscript. We will describe how rates of return on financial investments are determined shortly.

Agents also face a borrowing constraint, wealth cannot be negative:

$W_{kt} \geq 0$ for all k and t.

This constraint may bind in various periods. Whether it does so depends in a complex way upon the profile of the deterministic component of labor income, the realisations of income and rate of return shocks, the degree of risk aversion and the volatility of shocks. It also depends on the tax rate and the generosity of "safety net" state pensions.

Agents work from age 20 to the end of their 64th year (if they survive that long) and are retired thereafter. We assume that the profile of gross of tax labor income reflects three factors. First, there is a time-related rise in general labor productivity. This is set at 2% per year. Second, there is an age-related element to the growth of labor income over an agent's life. This is modelled as a quadratic in age.
The age-specific part of the log of labour income is:

$$\alpha + \gamma age - \phi age^2 \qquad (3)$$

We set γ and ϕ so that the age-income profile matches typical patterns from developed economies. We set $\gamma = 0.01657$and $\phi= -0.000376$ so that in the absence of time-related productivity, or of stochastic elements to income, earnings power would peak at about 22 years after agents enter the labor force. This corresponds to peak earnings typically coming in the early 40's. The implied life cycle pattern of efficiency units of labor is very similar to that used by Rios-Rull (1996) based on Hansen's (1993) estimates using US labor market data; the Hansen data imply that efficiency units of labor peak at about 45 years of age and fall substantially by age 60. The parameters chosen are also very close to those used in Miles (1999) which are the weighted average of the coefficients from sector specific age-earnings regressions based on UK Family Expenditure Survey data. (The sector specific age-income regressions are reported in Miles (1997)). Using these coefficients, and with aggregate (time-related) productivity of around 2% a year, earnings, on average, would not fall with age until very close to retirement – only then would the decline of productivity with age more than offset exogenous (time-related) productivity growth. This is consistent with the results of Meghir and Whitehouse (1996) who find that hourly wages for cohorts continue to rise until age 56.

There is also an idiosyncratic (agent specific) stochastic element of labor income. The log of labor income for an agent is the sum of the age-related element, the time related element and the additive income shock. The income shock is assumed to be normally distributed.

Denoting the log of gross labor income of agent k who is aged j in period t as y_{kt} we have:

$$y_{kt} = \alpha + gt + \gamma.j - \phi.j^2 + e_{kt} \qquad (4)$$

where $e \sim N(0, \sigma_e)$

α is a constant.

g is the rate of growth of labor productivity over time.

Figure 1 shows the pattern of labor income over the life cycle where there is 2% general labor productivity growth and for an agent who experiences no income shocks ($e_{kt} = 0$ for all t); here we set the state pension at 20% of pre-retirement average earnings. Pensions grow in line with aggregate labor productivity. This pattern is similar to the profiles used by Cubeddu (1998) in an analysis of the redistributive effects of unfunded pension programs in the US.

The assumption that labor income shocks are serially uncorrelated is unrealistic. It is plausible that for the majority of agents a large part of the shocks to income are persistent. But our focus in this paper is on the welfare implications of different pension regimes and how *overall* labor income risk over the working life affects retirement resources. In comparing personal retirement accounts with redistributive, unfunded state pensions the relevant risk factors are to do with uncertainty over rates of return and over the pattern of contributions to pensions. With funded, personal pensions we will allow agents complete flexibility over the pattern of contributions over their working lives; so what really matters for the overall rate of contributions is the overall level of income over a working life. And we model the unfunded state pension system as one where contributions are a constant proportion of gross income, so what determines the overall level of contributions to the state pension system is the overall level of income over the working life rather than the pattern from year to year.

Computationally there is a huge advantage to assuming that shocks to log income are identically and independently distributed (iid); by reducing the dimension of the state space the computer time needed to solve the model falls dramatically. By setting the variance of the iid shocks to log income appropriately we can match the typical degree of variability in incomes across agents seen in developed countries.

We assume that rates of return on financial wealth vary across periods because there are random shocks that hit stock and bond markets. We assume that rates of return at a particular time differ between individuals because financial institutions take into account the probabilities of death of agents and offer actuarially fair investment products. More specifically, financial institutions offer the following contracts. For every $ invested in period t the investor receives the stochastic market return adjusted for a probability of survival to the next period. If the agent dies the institution keeps the funds. With no bequest motives agents will always chose these contracts over ones which just pay the market rate of return. If the insurance element of this contract is offered on actuarially fair terms the ex-post rate of return on a $ invested in period t by an agent k who is aged j and who survives to the next period is given by:

$$\exp(r_{kt}) = \exp(r + v_t) / s_{1j} \quad (5)$$

r is the mean rate of return on assets

v_t is the random element of the rate of return on assets in period t.

We assume v is iid and normal: $v \sim N(0, \sigma_r)$

s_{1j} is the probability of surviving one more year conditional on reaching age j

We can write (5):

$$r_{kt} = r + v_t - \ln(s_{1j}) \qquad (6)$$

This financial contract can be offered at no risk by financial institutions because they pass on all the rate of return risk to investors and are assumed to be able to take advantage of the law of large numbers and face no uncertainty about the proportion of agents who will survive. It seems natural to assume that financial firms will offer insurance against risks that are idiosyncratic (individual length of life risk) but not offer insurance against systematic risk (rate of return risk). The contracts offered by financial institutions can be thought of as highly flexible personal pension schemes. Effectively agents have their own pot of assets into which they pay contributions and make deductions. Contribution rates and drawdowns from the fund are subject only to the constraint that the pot of assets can never fall below zero. The average rates of return on the fund increase with age since survival probabilities decline with age. Just as standard flat annuities available for a given sum rise with age, so the average rate of return offered by financial institutions increases with age.

In effect we are assuming that financial institutions offer one period annuities. These are the vehicles through which agents save for retirement. Agents are able to draw down such accounts in a flexible way in retirement. Individuals may decide to mimic the payments from standard flat annuities by having the "pot" size (ie. W) decline with age at a rate that is offset by rising average rates of return[4].

To show how the one period contracts we assume are available allow agents to create standard annuities – should they so wish – it is easier to start with the simple case of non-stochastic rates of return (assumed constant ar rate r). As before we focus on an agent aged j at time t who has wealth W.

[4] A simple example shows how such assets can be used to mimic annuities. Suppose the probability of death is invariant with respect to age and is at a constant rate p. Assume a non-stochastic, constant rate of return r. With an initial stock of wealth at retirement of W an agent could buy a standard annuity from a firm offering an actuarially fair deal which pays W(r+p)/(1+r) each period until death. We assume here that the first of these level payments is made immediately and then come at the start of each subsequent period so long as the agent is alive. One period savings contracts of the sort we envisage pay a return per dollar invested of (1+r)/(1-p) if the agent survives and nothing otherwise. An agent starting with wealth of W could immediately take W(r+p)/(1+r) out and reinvest the rest for one period. If they survive their wealth at the start of the next period is: W [1 – (r+p)/(1+r)].{(1+r)/(1-p)} = W. Obviously this policy can be sustained indefinitely. Thus the standard annuity contract can be replicated exactly by rolling forward one period contracts.

A standard, actuarially-fair annuity contract would promise to pay to a j year old an annual amount of A (until death) in exchange for a lump sum of W, where A satisfies:

$$W = A\,[\,1 + s_{1j.}\, e^{-r} + s_{2j.}\, e^{-2r} + \ldots\ldots\ldots\ldots. + s_{nj.}\, e^{-nr}\,] \qquad n \to \infty \qquad (7)$$

Here we are assuming payments are made at the start of each period and the first payment is made immediately. Let:

$$\phi \equiv [\,1 + s_{1j.}\, e^{-r} + s_{2j.}\, e^{-2r} + \ldots\ldots\ldots\ldots. + s_{nj.}\, e^{-nr}\,] \qquad n \to \infty \qquad (8)$$

So $\quad A = W / \phi$

Note the link between one period survival probabilities at different ages:

$$s_{2j} = s_{1j}\, s_{1j+1}$$

and more generally:

$$s_{nj} = s_{1j}\, s_{1j+1}\, s_{1j+2} \ldots\ldots\ldots.\, s_{1j+n-1} \qquad (9)$$

If a j year old agent has wealth W in a fund and withdraws an amount $A= W/\phi$ and reinvests the remainder with our one-period contracts, their wealth at the start of the next peirod is:

$$W[\,1 - 1/\phi\,]\, e^{r} / s_{1j}$$

Withdrawing the same amount in the second period, when the agent is aged j+1, generates a fund at the start of the third period of:

$$W[\{(1 - 1/\phi).\, e^{r} /s_{1j} - 1/\phi\}.\, e^{r} / s_{1j+1}\,] = W[(1 - 1/\phi)\, e^{2r} /s_{2j} - (1/\phi)\, e^{r} /s_{1j+1}]$$

Assuming a constant per period withdrawal rate of W/ϕ the level of funds at the start of period n+1 is:

$$W[\,(1 - 1/\phi)\, e^{nr} /s_{nj} - (1/\phi)\{\, e^{(n-1)r} /s_{n-1\, j+1} + e^{(n-2)r} /s_{n-2\, j+2} + e^{(n-3)r} /s_{n-3\, j+3} + \ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots + e^{r} /s_{1\, j+n-1}\,\}\,] \qquad (10)$$

Which we can write:

$W(e^{nr} / s_{nj})\ [1 – (1/\phi).\{1 + s_{1j.}\ e^{-r}\ +\ s_{2j.}\ e^{-2r}\ + ...\ +\ s_{n-1\,j.}\ e^{-(n-1)r}\ \}]$ (11)

From (8) we have that for finite n

$0\ <\ (1/\phi).\{1 + s_{1j.}\ e^{-r}\ +\ s_{2j.}\ e^{-2r}\ + ... + s_{n-1\,j.}\ e^{-(n-1)r}\ \}\ <\ 1$

So (11) is always positive. This proves that the one period annuity contracts allow agents to mimic the returns from standard (open-ended) annuity contracts and satisfy budget constraints.

By assuming the availability of one period annuity contracts we are giving agents more options on lifetime accumulation and decumulation of assets than with standard annuities. Agents will value flexibility in annuitising and are unlikely to want a flat drawdown of their accumulated fund. The optimal rate of accumulation and decumulation of funds over time is a complicated function of all the parameters in the model and depends on the realisation of shocks; it can only be ascertained by simulations.

But in assuming that agents are offered these savings vehicles we are making a strong assumption that factors that seem to be important in the real world, and that make rates of return implicit in annuities contracts tend to be less than actuarially fair, are absent. (See, for example, Friedman and Warshawsky (1988); Mitchell, Poterba and Warshawsky (1997) and Brown, Mitchell and Poterba (1999)). It is important to allow for problems that make annuities less than fair. We introduce a measure of the efficiency of annuities markets. When this measure, β , is 1 the annuities market work perfectly. When $\beta = 0$ annuities are, effectively, not offered. The survival probability implicit in the contract offered by a financial institution is a weighted average of the true survival probability, s_{1j} , and the rate when no annuity is offered, an effective survival probability of unity. β is the weight placed on the actuarially fair survival probability

The rate of return paid on one period savings for an agent aged j at time t becomes:
$\exp[r_{kt}]\ =\ \exp[r + v_t] / [\beta s_{1j} + (1-\beta)]$ (12)

This way of modelling the efficiency of annuity contracts allows the departure from actuarially fair contracts to vary with age. The greater is age, the lower the probability of surviving and for all $\beta < 1$ the greater is the departure from actuarially fair contracts. Recent empirical evidence from the US suggests that annuity rates do become increasingly less favorable with age. Mitchell, Poterba and Warshawsky (1997) estimate that the average US annuity in 1995 delivered payouts with expected present value of between 80% and 85% of each $ annuity premium for 65 year olds; but the payout ratio was less for older people. A payout ratio of 80% of the actuarially fair value for a 65 year old corresponds[5] to a value of β of about 0.44 if the rate of return on assets is a flat 6%. Friedman and Warshawsky (1988) report US payout ratios from the 1970's and 1980's of around 75% which corresponds to a β of about 0.23. Brown, Mitchell and Poterba (1999) provide some evidence that in the UK annuities average about 90% of the actuarially fair rates. This corresponds to a β of around 0.7. In both the US and the UK there is strong evidence of substantial variability in annuity rates across companies.

State pensions

The state pays a flat rate pension to all retired people. The system is financed by the proportional tax on labor income levied on all those working. The demographic structure is stable so the ratio of workers to pensioners is constant and is determined by the survival probabilities used. Since we assume all shocks to labor income are idiosyncratic, and are independent of the rate of return shocks, there is no uncertainty about the aggregate wage bill or about aggregate tax revenue. The tax rate is set to balance the unfunded state pension system in every period. We define the replacement rate of the state pension as the ratio between the pension paid in period t and the average gross income of those in the last year of their working life at period t-1. For a given replacement rate there is a constant tax rate which balances the system and allows the state pension to rise at the rate of aggregate labor productivity growth (g). Thus pensioners continue to benefit from aggregate labor productivity growth after leaving the work force. In this simple, constant population model the average (across all agents) implicit rate of return on contributions made to the unfunded (PAYG) system is g.

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[5] using our mortality rates

The tax rate to finance state pensions of a given generosity is proportional to the replacement rate of the unfunded system. The factor of proportionality reflects the support ratio which in turn reflects mortality rates and life expectancy. We use data on mortality rates from the UK (as reported by the UK Government Actuary in 1998) which for males imply a life expectancy at birth of around 76. Mortality rates are fairly similar across the developed world and the UK life table figures are representative. Using the life tables the conditional life expectancy at different ages is shown in figure 2 and the attrition rate of pensioners from age 65 is shown in figure 3.

The steady state population structure of adults is easily calculated from the one period survival probabilities given in the life tables. Given the life table data we use, the steady state support ratio (the ratio between those aged 65 and more to the population aged 20 to 64) is around 0.3. We have defined the replacement rate as the ratio between the pension of a just retired person and the average gross income of those in their last year of work. We assume pension income is untaxed. The equilibrium tax rates for different gross (and net) replacement rates are shown in table 1:

Table 1

Replacement Rates and Contribution Rates to State, Unfunded Pension System (%)

Gross replacement rate♣	5	10	20	30	50
Net replacement rate♦	5.1	10.3	21	32.3	57
Balanced contribution rate♠	1.23	2.46	4.91	7.37	12.28

♣ ratio of age 65 pension to average gross earnings of age 64 agent one period earlier

♦ ratio of age 65 pension to average net earnings of age 64 agent one period earlier

♠ contribution rate to balance unfunded, state pension scheme

Solving the Model:

The set of first order conditions from individual k's optimisation problem are:

if $c_{kt} < [W_{kt} + \exp(y_{kt}).(1-\tau) + b_{kt}]$

then (13)

$$U`(c_{kt}) = E_t\, [s_{1j}\, \{ U`[c_{kt+i}].\, \exp[r + v_t] / [\beta s_{1j} + (1-\beta)] \} / (1+\rho)]$$

else:

$c_{kt} = [W_{kt} + \exp(y_{kt}).(1\text{-}\tau) + b_{kt}]$

and (14)

$U`(c_{kt}) \geq E_t\ [s_{1j}\ \{ U`[c_{kt+i}].\ \exp[r + v_t] / [\beta s_{1j} + (1\text{-}\beta)]\ \} / (1+\rho)]$

where $U`(c_{kt})$ is $\partial U_k / \partial c_{kt}$

(13) holds when the borrowing constraint is not binding. When the constraint binds complementary slackness implies that (14) holds.

Although characterising optimal plans is easy enough solving explicitly for optimal consumption and for the optimal accumulation path for funds is not possible. Instead we have to turn to numerical methods. We solve the problem backwards in a now standard way (see Deaton (1990), Zeldes (1989) and Skinner, Hubbard and Zeldes(1995)). The resources available for consumption in any period are what Deaton calls cash in hand (given the borrowing constraint this is an upper limit on consumption). Cash in hand at time t for agent k is given by

$$[W_{kt} + \exp(y_{kt}).(1\text{-}\tau) + b_{kt}] \qquad (15)$$

In the final possible period that an agent could survive to (age 120) consumption of all resources is optimal since we assume no bequest motive. We take a set of possible values for cash in hand available in the penultimate period to when the agent might be alive. For each of those values we solve a constrained optimisation problem. We seek a value of consumption to satisfy the first order conditions (13) and (14) given the current state variable (cash in hand). To solve this we need to calculate the right hand side of (13) or (14) for a given level of consumption today at today's cash in hand. The right hand side of (13) and (14) is the expected product of marginal utility in the next period and the rate of return (adjusted for probabilities of death and for the rate of pure time preference). To do this calculation we need to consider the probability distribution of different values for cash in hand tomorrow *and* estimate what optimal consumption (and so marginal utility) will be at each realisation of cash in hand tomorrow. In general this involves integrating in two dimensions because there are two random components – a shock to rates of return and to labor income. (In fact after age 65 labor income is zero so then there is only one source of uncertainty). In the final period all cash in hand is consumed so finding the optimal level of consumption in the penultimate period – the value that solves the first order conditions – is relatively easy. We use the constrained optimisation routine in GAUSS to solve this problem.

Now we have a set of estimated values for optimal consumption at a finite set of cash in hand values in the penultimate period. We proceed to the second from last possible period and consider another set of possible cash in hand values. At each cash in hand value we again seek a level of consumption which satisfies the first order condition. To estimate what optimal consumption in the next period would be for a given a realisation for cash in hand we use the solved values at the finite set of cash in hand points for the penultimate period and interpolate between them.

We proceed backwards in this way storing solved values for optimal consumption for a given set of cash in hand values at each age. We continue back to the first period. At that stage we have a complete grid of optimal consumption values at various cash in hand values for every age an agent might reach.

What makes the process intensive of computer time is the need to perform many evaluations of the expected value of the product of marginal utility and the random rate of return. At each cash in hand value at every possible age we need to work out this expectation for many possible values of current consumption until we find one that satisfies the first order conditions. If it takes 5 iterations to find optimal consumption at a given cash in hand value for a given age, then with 101 periods (ages from age 20 to a maximum life of 120) and 80 cash in hand values we need over 40,000 calls to numerical integration in two dimensions.

We use 10 point Gauss Hermite numerical integration which is the natural choice with normally distributed errors. With two sources of uncertainty this requires 100 function evaluations for the calculation of each numerical integral (at least until we reach retirement when integration in a single variable is sufficient). Each of these 100 evaluations requires interpolation using the cash in hand / consumption pairs from the estimated solution grid in the next period. For the interpolation we estimate a 4th order polynomial through the cash in hand / consumption points for each age.

A good deal of trial and error is needed to find the size of the grid that is required to get accurate solutions. We found that a grid that was fine at relatively low levels of cash in hand (regions where agents spend a good deal of time fairly early in life and when constraints are often binding) and becomes much coarser at high cash in hand values worked well. We found that 80 values for cash in hand were required to get accurate solutions. (Solutions were checked using simulations with large numbers of agents to ensure that first order conditions held).

To work out a complete grid for a given set of parameters took about 1 1/2 hours of computer time using GAUSS on a PC running at 400 Mhz. Once we have a grid of vales for a given set of parameters running simulations is relatively easy. We draw 7000 paths of normally distributed random shocks to incomes and to rates of return over a life that could last until age 120. For each of the 7000 individuals (or sample lives) we use the first element of their vector of shocks to log income to calculate initial period income. For each agent we then use the calculated grid of optimal consumption values for various initial values of cash in hand to estimate optimal first period consumption. This involves interpolating between the cash in hand points on the grid. Since the relation between cash in hand and optimal consumption is smooth the interpolation does not pose any problems.

We now have a set of initial values for consumption from which we calculate saving. Second period wealth for each agent is initial saving multiplied by the random rate of return (adjusted for the common survival probability). We then take another draw for the income shock using a random number generator. After deducting tax we have second period net labor income which is added to the realised value for financial wealth (which depends on the shock to rates of return) to give second period cash in hand. We then use the grid solutions to calculate consumption at these new cash in hand values. The simulation proceeds in this manner.

What we end up with is 7000 profiles of consumption. Each profile shows what consumption would have been for each agent facing a particular set of shocks and *if* they survived until the longest possible age. From this set of profiles we calculate expected utility of someone about to start their life. This calculation is straightforward and proceeds in stages. First at each age we calculate utility for the 7000 agents given their consumption. Second we average these utilities across the 7000 agents at each age. Third we take a weighted sum of these period averages using the overall discount factors as perceived at age 20 as weights. Thus the weight on the average utility for age 90 is the probability of surviving to age 90, conditional on a current age of 20, multiplied by a factor of $1/(1+\rho)^{90}$, which is a very small number. The weights attached to future average utilities (conditional on living) fall away very fast after age 85.

Key parameters:

We set the discount rate equal to 0.015 and the coefficient of relative risk aversion (ζ) equal to 2, which implies an intertemporal elasticity of substitution of 0.5. The discount rate assumption is the same as that used by Auerbach and Kotlikoff (1987) and is consistent with the empirical evidence of Rust and Phelan (1997) who suggested a rate somewhat in excess of 1%. The intertermporal elasticity of substitution is more controversial; Cooley and Prescott (1995) use unity for their simulations whereas Auerbach and Kotlikoff (1987) use a coefficient of relative risk aversion of 4, implying the elasticity is only 0.25. Empirical work by Hansen and Singleton (1983) and Mankiw, Rotemberg and Summers (1985) suggest values a little over unity for intertemporal substitutability suggesting, in our framework, a coefficient of relative risk aversion a little under unity. Grossman and Shiller (1981), Mankiw (1985) and Hall (1980) found values between 0 and 0.4. for the intertemporal elasticity suggesting coefficients of risk aversion in excess of 2. Blundell et al (1994) present evidence consistent with an intertemporal elasticity of about 0.75 for the intertemporal elasticity, implying a value of about 1.3 for risk aversion. Hubbard, Skinner and Zeldes (195) use a relative risk aversion of 3 in their simulations. Zeldes (1989) estimated the risk aversion coefficient as 2.3. We consider a value of 2 for the risk aversion coefficient is a central estimate but clearly the evidence makes it hard to be confident about what a plausible figure is.

Setting the volatility of the shock to labor income is particularly important for the simulations. As noted above, a significant part of the shocks to individual incomes appear to be persistent. Our strategy is to set the variance of the iid shocks so as to generate an overall degree of income uncertainty which is typical, even though the serial correlation of the random component of income will not be. Hubbard, Skinner and Zeldes (1995) use a model of income dynamics to simulate the impact of social security which is based on characteristics of US household income data. Their model is similar to that used here except that they allow for both transitory and persistent shocks. Their model for the log income of household k at time t is:

$$y_{kt} = f(age_{kt}) + u_{kt} + w_{kt}$$

$$u_{kt} = \rho\, u_{kt-1} + v_{kt}$$

where v and w are iid shocks that are not correlated and $f(age_{kt})$ is a deterministic function.

A measure of the unconditional volatility of log income is:

$$\sigma^2_w + \sigma^2_v / (1 - \rho^2)$$

Typical values for ρ, σ^2_w and σ^2_v used by Hubbard et al are 0.955, 0.025 and 0.025. These imply that some income shocks are highly persistent With these values our measure of the unconditional variance of the shock to log income is 0.31 and the standard deviation of the shock is 0.56[6]. To generate the same sort of uncertainty as is implied by these figures we set the standard deviation of the iid shocks to log income at 0.60.

If the shocks to labor income have a standard deviation of 0.6 the cross section distribution of incomes generated share many of the properties of real world data. Figure 4, for example, illustrates that with a standard deviation of 0.6 for iid shocks to log income the implied distribution of the levels of income is very close to the actual 1998 distribution of UK household income. (For this comparison we adjust the intercept in our process for log income to match the mean of UK household income in 1998).

[6] In fact Hubbard et al set different values of ρ, σ^2_w and σ^2_v for those with no high school, high school and College education. The implied unconditional standard deviation of the shocks to log income for these three groups are 0.64, 0.51 and 0.44 respectively.

For these reasons we think of 0.6 as a sensible figure for the volatility of the type of income shocks we consider.

When it comes to rate of return distributions a number of questions arise. If we are thinking about portfolios of stocks an average real return (before any deductions for charges) of about 6% a year has been typical in many developed countries in the past. Variability is high with annual standard deviations of around 17.5% being common. (See Miles and Timmerman (1999) for details). Bond portfolios are less volatile. Miles and Timermann suggest that a mixed bond and stock portfolio in developed countries would have generated a lower average real return than an equity portfolio and with a significantly lower annual volatility. We take a combination of a 4% average real return and a 10% annual standard deviation as reasonable for a more conservative investment strategy. These figures are for gross returns; net of charges annual returns to individuals are likely to be lower by at least 50 basis points. For this reason we think of net returns with means of either 6% or 4% as relatively optimistic. We also consider a pessimistic scenario with average net returns as low as 2% with a 10% standard deviation. We also consider results from a high return scenario with a mean return of 8% and a standard deviation of 17.5%.

Results:

We show results for various combinations of assumptions about the distribution of rates of return and pension generosity. We focus first on results when we assume a relatively high average rate of return on assets. Table 2 summarises the results for a 6% mean annual rate of return on assets with a standard deviation of 17.5%. If financial markets offer actuarially fair contracts (perfect annuity markets) expected utility is maximised when there are no unfunded state pensions. The third row in the table ("% gain over no PAYG") shows by how much average labor income in a world with no state pensions would have to be different so as to generate the same expected utility as in a world with unfunded pensions of given generosity. If state pensions were set at a very low rate such that someone with average income at the end of their working life only got a state "safety net" pension at age 65 of 10% of their last labor income, expected lifetime utility would be significantly lower. Table 2 shows that this would reduce utility by the equivalent of a 0.88% cut in labor incomes. An unfunded flat rate pension of 10% would also reduce the average amount saved by retirement significantly. With no state pensions the average pension pot at age 65 is 20.58 (average labor income over the work life is unity so this represents a very large stock of saving for retirement). With 10% safety net pensions average retirement savings falls to 17.61 – on average people save about 3 years worth of income less.

But if annuities are not available (β=0) then not only are agents much less well off but there is a welfare enhancing role for state (flat rate) pensions, even though the rate of return on assets (with a mean of 6%) is, on average, three times the implicit rate of return on contributions to unfunded pensions (2%). The bottom panel of Table 2 shows that a flat rate, unfunded pension which generates a 20% replacement rate at retirement increases expected utility by the equivalent of a 1.81% increase in labor productivity at all ages. It also generates a very much smaller stock of retirement assets than if no state pensions were paid: average private savings at retirement would be 23.15 with no state pensions (significantly higher than if annuities markets are perfect, itself a reflection of risk aversion) but only 14.42 with flat rate pensions. Having state pensions at around their optimal level (which is close to a 20% replacement rate) reduces retirement savings by, on average, almost 9 years worth of income.

With no state pensions agents would never chose to run down assets in retirement to zero, regardless of the efficiency of one period annuity contracts. But with 20% replacement rate pensions a significant proportion of those who live longer than average would optimally run out of private savings. Table 3 shows the proportion of those of various ages who would chose to consume all their cash in hand (and are therefore credit constrained) at various ages and assuming various degrees of efficiency in annuities markets. Whatever the efficiency of annuity markets a substantial proportion of agents are credit constrained very early in life. The less perfect are annuity markets, the more people are credit constrained as they get older.

With no state pensions people never run out of savings as they age (though there remain substantial numbers of credit constrained among the young). With a 6% average rate of return and pensions worth only 10% of average income at retirement about 11% of those who survived to age 90 would become credit constrained if β = 0. If pensions are worth 20% this proportion rises to almost 40%. With a lower rate of return the proportion credit constrained is somewhat higher.

Table 3

% of Population Credit Constrained by Age:

A: average rate of return = 6%; pension worth 10% of average income at retirement

	Age				
	20	25	35	80	90
β = 1	13.1	1.9	0.6	0	0
β = 0.5	13.3	2.2	0.6	0	0
β = 0	13.5	2.3	0.7	0	11

B: average rate of return = 4%; pension worth 20% of average income at retirement

	Age				
	20	25	35	80	90
β = 1	14.2	1.9	1.0	0	0
β = 0.5	14.1	1.8	1.1	0	3.8
β = 0	14.0	1.1	1.1	0	42

The pattern of average savings rates over the life cycle with different generosities of flat rate state pensions, and various degrees of efficiency of annuity contracts, is illustrated in figures 5 and 6. The profile of saving at the start of working life displays a pattern typical when precautionary motives are important and where income uncertainty is significant. In the first few periods in work agents save a substantial fraction of wealth to establish a small stock of assets as a buffer to protect consumption against future labor income shocks. On average agents then dissave mildly for several years before starting to build up a stock of assets for retirement. When savings starts to become significant, and how great the stock of wealth at retirement is, are sensitive to both the generosity of state pensions and to the efficiency of annuity markets. With no state pensions and perfect annuity markets saving typically becomes significant from the mid 30's. With a state pension worth 30% of average labor income at retirement and completely imperfect annuity markets savings is typically not significant (after the initial establishment of a precautionary buffer at the start of work) until the late 40's.

With average rates of return below 6%, and lower volatility of rates of return, the optimal level of safety net, unfunded pensions is larger. Table 4 shows welfare levels (that is expected utility at birth) for various combinations of annuity efficiency and state pension generosity. Here we use a 4% average annual real rate of return and 10% annual standard deviation. It remains the case that if annuity markets work perfectly there is no welfare enhancing role for unfunded pensions. (We keep the assumption of 2% aggregate annual labor productivity growth). With completely imperfect annuity markets (β=0) unfunded pensions worth about 20% of average retirement incomes are optimal and generate a welfare gain worth the equivalent of just over a 1.75% addition to lifetime labor incomes.

With even lower returns (averaging 2%) the benefits of unfunded pensions when there are imperfect annuity markets are larger still. When the net return on assets is no larger than productivity there are advantages of having (small) unfunded pensions even if annuity markets are perfect. Table 5 illustrates. If annuity markets are half way between being perfect and totally imperfect (so β = 0.5) unfunded state pensions worth about 25% of average retirement income are optimal and generate welfare gains worth the equivalent of about 2.5% higher labor productivity. With no annuities, unfunded pensions should be set at about 30% of average retirement earnings and give gains equivalent to labor productivity being permanently almost 8% higher.

For all rates of return the greater are unfunded pensions the lower are average savings rates and the scale of the impact is large. Unfunded pensions being worth 20% of average retirement wages reduce the average stock of savings held at retirement by between 25% and 40% (depending on the efficiency of annuities contracts as well as average rates of return and asset price volatility).

Conclusions:

Two factors emerge as crucial to the design of pension systems with an optimal degree of reliance on personal retirement accounts relative to unfunded pensions that are highly redistrbutive. First, the nature of the distribution of rates of return. Second, how efficient are annuities contracts. The optimal size of unfunded pensions is highly sensitive to both factors. If average rates of return on funds are as high as 8% then even if volatility in returns is high (17.5% annual standard deviation) and annuity markets highly imperfect there is no useful role for unfunded pensions. If rates of return have a lower mean and volatility the situation is very different. With 4% mean returns and a 10% annual standard deviation then with wholly imperfect annuity markets unfunded safety net pensions worth about 20% of average gross labor income at retirement are optimal. But with perfect annuity markets there is again no welfare-enhancing role for pensions.

We have assumed that rates of return are independently (and identically) distributed over time. This means that investment risk can, to a significant extent, be smoothed over time. There is evidence that rates of return are not independent over time; mean reversion at long horizons may exist (Poterba and Summers (1988)). This can have an important impact on the risk of funded pensions where investment horizons of thirty or more years is relevant. It can also mean that the iid assumption can underestimate the risk of funded pensions (see Miles and Timermann (1999) for illustrative calculations). An important area for future work is the modelling of optimal pension arrangements when asset markets are subject to sustained periods of above or below average returns; if there are bull and bear markets funded pensions are likely to be riskier than if returns come from the same distribution each period.

Perhaps the key policy implication in this paper is that governments need to carefully consider the efficiency of annuity contracts before embarking on pension reform. The results reported here suggest that how efficient annuity contracts are, why deviations from actuarially fair contracts might exist, and how reform might change this are crucial. The welfare gains from making annuity contracts more efficient are substantial. Table 6 shows the equivalent permanent increase in labor incomes which generates the same welfare gain as a given increase in the efficiency, or degree of actuarial fairness, of financial contracts. In each case we consider the gain which arises when β increases and when government responds by setting the optimal replacement rate of state pensions. So if β rises from 0 (completely imperfect annuity contracts) to1 (perfect contracts) when the rate of return is 4% we take account of the optimal reduction in state pensions from being worth about 20% of wages to about zero. The gain from this change is the equivalent of a permanent rise in productivity of just over 6%. Improving annuity contracts from being "semi-perfect" (β=0.5) to perfect (β=1) generates welfare gains worth the equivalent of a permanent addition of between 2% and 3.25% to productivity, depending on rates of return.

The results suggest that understanding why annuity contracts may not be actuarially fair is a major policy question.

Table 6

Gains in welfare from improvements in annuity rates: equivalent rise in labor productivity

average rate of return	$\beta = 0.5 \rightarrow \beta = 1$	$\beta = 0 \rightarrow \beta = 1.0$
2%	2.1%	3.1%
4%	3.25%	6.1%
6%	3.0%	5.7%

Notes: for average rates of return of 2% and 4% we assume the standard deviation of the stochastic component of returns is 10%; for an average rate of return of 6% we use a standard deviation of 17.5%

References:

Auerbach, A. and .Kotlikoff, L. (1987). *Dynamic Fiscal Policy*, Cambridge University Press.

Bohn, H (1999): "Social Security and Demographic Uncertainty: the Risk Sharing Properties of Alternative Policies", working paper of the University of California at Santa Barbara.

Breyer, F. (1989), "On the Intergenerational Pareto Efficiency of Pay-As-You-Go Financed Pension Schemes" **Journal of Institutional and Theoretical Economics,** 145, pp 643-658**.**

Brown , J. Mitchell, O. and Poterba, J. (1999) "The Role of Real Annuities and Indexed Bonds in an Individual Accounts Retirement Program", NBER working paper no 7005.

Chand, S. and Jaeger, A. (1997), "Aging Populations and Public Pension Schemes", IMF Occasional Paper no 147, Washington.

Cooley, T. and Prescott, E. (1995), "Economic Growth and Business Cycles", in *Frontiers of Business Cycle Research*, edited Cooley, T. Princeton, Princeton University Press.

Cubeddu, L (1998) "The Intragenerational Redistributive Effects of Unfunded Pension Programmes" IMF Woking paper, December.

Deaton, A. (1991) "Saving and Liquidity Constraints" **Econometrica**, 59, pp 1221-48.

Disney, R. (1996), *Can We Afford to Grow Older*, MIT Press.

Feldstein, M. (1996), "The missing piece in policy analysis: social security reform*",* **American Economic Review**, May 1996, pp. 1-14.

Feldstein, M. Ranguelova, E. and Samwick, A. (1999) "The Transition to Investment-based Social Security When Portfolio Returns and Capital Profitability are Uncertain"*,* NBER working paper no 7016.

Feldstein, M. and Ranguelova, E. (1998) "Individual Risk Sharing and Intergenerational Risk Sharing in an Investment Based Social Security System"*,* NBER working paper no 6839.

Feldstein, M. and Samwick, A (1998), "The transition path in privatizing social security", in *Privatizing Social Security,* ed Martin Feldstein, Chicago, University of Chicago Press.

Fenge, R (1996) "Pareto Efficiency of the Pay-as-you-go pension System with Inergenerational Fairness", **Finanzarchiv**, 52, pp357-363.

Friedman and Warshawsky (1988) "Annuity Prices and Saving Behaviour in the US" in *Pensions in the US Economy*, edited by Bodie, Shoven and Wise, Chicago University Pres, Chicago.

Grosssman, S and Shiller, R (1981), "The Determinants of the Variability of Stock Market Prices", **American Economic Review**, 71, no 2.

Gruber, J. and Wise, D. (1997), "Social Security Programs and Retirement Around the World", National Bureau of Economic Research Working Paper no 6134.

Gruber, J. and Wise, D. (1999), *Social Security and Retirement Around the World*, Chicago University Press, Chicago.

Hall, R. (1980), "Intertemporal Substitution in Consumption", NBER Working Paper No 720, July.

Hansen, G. (1993), "The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked", **Journal of Applied Econometrics**, 8, pp 71-80.

Hansen, P and Singleton, K (1983), “Stochastic Consumption, Risk Aversion and the Temporal Behaviour of Asset Returns”, **Journal of Political Economy**, 91, no 2.

Homburg, S. (1990) “The Efficiency of Unfunded Pension Schemes” **Journal of Institutional and Theoretical Economics**, 146, pp 640-47.

Hubbard, G. Skinner, J. and Zeldes, S (1995) “Precautionary Saving and Social Insurance”, **Journal of Political Economy**, vol 103, pp 360-399.

Kotlikoff, L.J. (1996), “Privatizing social security at home and abroad”, **American Economic Review**, May 1996, pp. 368-372.

Kotlikoff, L. and Summers, L. (1981), “The Role of Intergenerational Transfers in Aggregate Capital Accumulation”, **Journal of Political Economy**, 89, no 4, pp 706-32.

Mankiw, G. (1985), “Consumer Durables and the Real Interest Rate”, **Review of Economics and Statistics**, August.

Mankiw, G., Rotemberg, J. and Summers, L (1985), “Intertemporal Substitution in Macroeconomics”, **Quarterly Journal of Economics**, February.

Masson, P., T. Bayoumi and H.Samiei (1996), “International Evidence on the Determinants of Private Saving,” CEPR Discussion Paper No.1368.

Meghir,C. and Whitehouse, E. (1996) “The Evolution of Wages in the UK: Evidence from Micro Data”, **Journal of Labor Economics**, vol 14, no 1, pp 1-25.

Merton, R (1983) “On the Role of Social Security as a Means for Efficient Risk Sharing in an Economy where Human Capital is not Tradable” in Bodie, Z. and Shoven J. (eds) *Financial Aspects of the United States Pension System*, pp 325-358. Chicago, University of Chicago Press.

Miles, D. (1997), “A Household Level Study of the Determinants of Incomes and Consumption”, **The Economic Journal,** vol 107, No 440, pp 1-26.

Miles, D. (1999), “Modelling the Impact of Demographic Change Upon the Economy”, **The Economic Journal,** vol 109, no 452.pp 1-36.

Miles, D. and Timmermann, A. (1999) “Risk Sharing and Transition Costs in the Reform of Pension Systems in Europe”, **Economic Policy**, October 1999.

Mitchell, O. Poterba, J. and Warshawsky, M (1997) “New Evidence on the Money’s Worth of Individual Annuities”, NBER working paper no 6002.

Mitchell, O.S. and Zeldes, S. (1996), “Social Security Privatization: a structure for analysis”, **American Economic Review**, May 1996, pp. 363-367.

OECD (1996) “Ageing Populations, Pension Systems and Government Budgets: Simulations for 20 OECD Countries”, by Roseveare, D. Leibfritz, W. Fore, D. and Wurzel, E. Economics Department Working Paper no 168, OECD, Paris.

Palacios, R (1998) “A note on Diversification Between Funded and PAYG Schemes”, mimeo, The World Bank.

Poterba, J. and Summers, L. (1988) “Mean Reversion in Stock Returns: Evidence and Implications” **Journal of Financial Economics**, vol 22, 27-60.

Rios-Rull, J. (1996), “Life-Cycle Economies and Aggregate Fluctuations”, **Review of Economic Studies**, 63, pp 465-489.

Rust, J. and Phelan, C. (1997) "How Social Security and Medicare Affect Retirement Behaviour in a World of Incomplete Markets", **Econometrica**, vol 65, (4) p 781-831.

Sinn, H (1999) "Pension Reform and Demographic Crisis: Why a Funded System is Needed and Why it is Not Needed", Centre for Economic Studies at Munich University, Working Paper no. 195.

Skinner, J. (1988), "Risky Income, Life Cycle Consumption and Precautionary Savings", **Journal of Monetary Economics**, Vol 22, pp 237-55.

Zeldes, S. (1989), "Optimal Consumption with Stochastic Income", **Quarterly Journal of Economics**, Vol 104, 275-98.

Zeldes, S. (1990), "Consumption and Liquidity Constraints: An Empirical Invsetigation", **Journal of Political Economy**, Vol 97, 305-346.

Table 2: Mean return 6% Standard deviation 17.5%

		Replacement rate of state pension				
		0%	5%	10%	20%	30%
β=1	Expected Utility	-47.09	-47.28	-47.51	-48.08	-48.75
perfect	Average wealth at 65	20.58	19.08	17.61	15. 20	13.07
annuities	Equivalent prod gain		-0.40	-0.88	-2.05	-3.40
β=0.5	Expected Utility	-48.56	-48.52	-48.60	-48.92	-49.42
semi perfect	Average wealth 65	21.60	19.57	17.89	14.90	12.47
annuities	Equivalent prod. gain		+0.07	-0.07	-0.73	-1.73
β=0	Expected Utility	-50.67	-50.02	-49.81	-49.77	-50.04
imperfect	Average wealth 65	23.15	19.86	17.81	14.42	11.63
annuities	Equivalent prod gain		+1.29	+1.73	+1.81	+1.27

Notes:

Average Wealth at 65 is the mean level of cash in hand at retirement (age 65)
Equivalent prod gain is the percent rise in labor productivity needed to generate the same expected utility in a world with no state pensions.

Table 4: Mean return 4% Standard deviation 10.0%

		Replacement rate of state pension				
		0%	5%	10%	20%	30%
β=1	Expected Utility	-48.34	-48.58	-48.84	-49.41	-50.06
perfect	Average wealth at 65	15.80	14.79	13.81	11.95	10.19
annuities	Equivalent prod gain		-0.50	-1.03	-2.17	-3.43
β=0.5	Expected Utility	-49.91	-49.96	-50.06	-50.35	-50.76
semi perfect	Average wealth 65	16.48	15.16	13.92	11.64	9.59
annuities	Equivalent prod. gain		-0.10	-0.29	-0.88	-1.68
β=0	Expected Utility	-52.17	-51.66	-51.42	-51.26	-51.38
imperfect	Average wealth 65	17.53	15.43	13.78	11.04	8.78
annuities	Equivalent prod gain		+0.98	+1.46	+1.77	+1.54

Notes:

Average Wealth at 65 is the mean level of cash in hand at retirement (age 65)
Equivalent prod gain is the percent rise in labor productivity needed to generate the same expected utility in a world with no state pensions.

Table 4: Mean return 2% Standard deviation 10.0%

		Replacement rate of state pension					
		0%	5%	10%	20%	30%	50%
β=1	Expected Utility	-51.51	-51.48	-51.50	-51.66	-51.95	-52.93
perfect	Average wealth at 65	13.47	12.41	11.40	9.52	7.82	4.94
annuities	Equivalent prod gain		+0.05	+0.02	-0.28	-0.85	-2.68
β=0.5	Expected Utility	-53.75	-53.29	-52.97	-52.65	-52.57	-53.17
semi perfect	Average wealth 65	14.06	12.61	11.31	9.05	7.13	4.29
annuities	Equivalent prod. gain		+0.86	+1.46	+2.13	+2.24	+1.08
β=0	Expected Utility	-57.11	-55.40	-54.49	-53.47	-53.06	-53.35
imperfect	Average wealth 65	14.98	12.63	10.97	8.40	6.44	3.80
annuities	Equivalent prod gain		+3.10	+4.81	+6.81	+7.64	+7.05

Notes:

Average Wealth 65 is the mean level of cash in hand at retirement (age 65)
Equivalent prod gain is the percent rise in labor productivity needed to generate the same expected utility in a world with no state pensions.

Figure 1: profile of average income over life with state pension worth 20% of retirment income

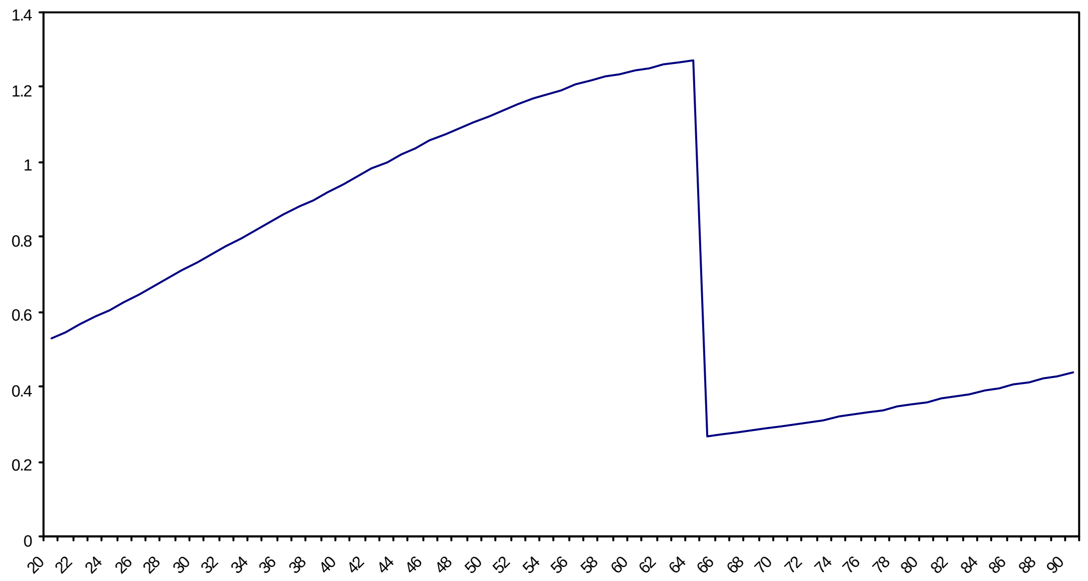

Figure 2: Conditional Life Expectancy

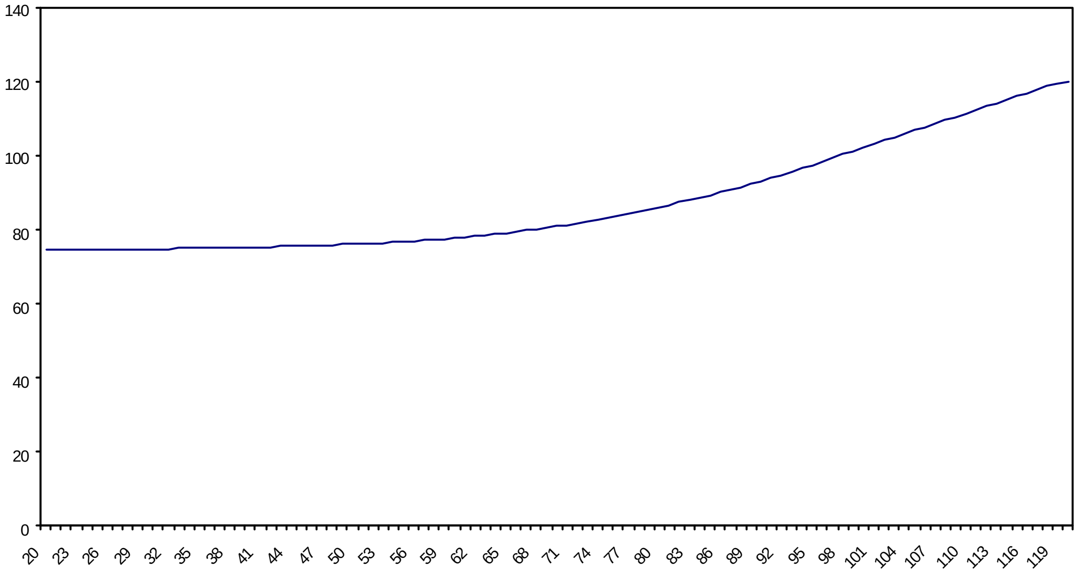

Figure 3: Attrition rate for pensioners - proportion surviving of those alive at 65

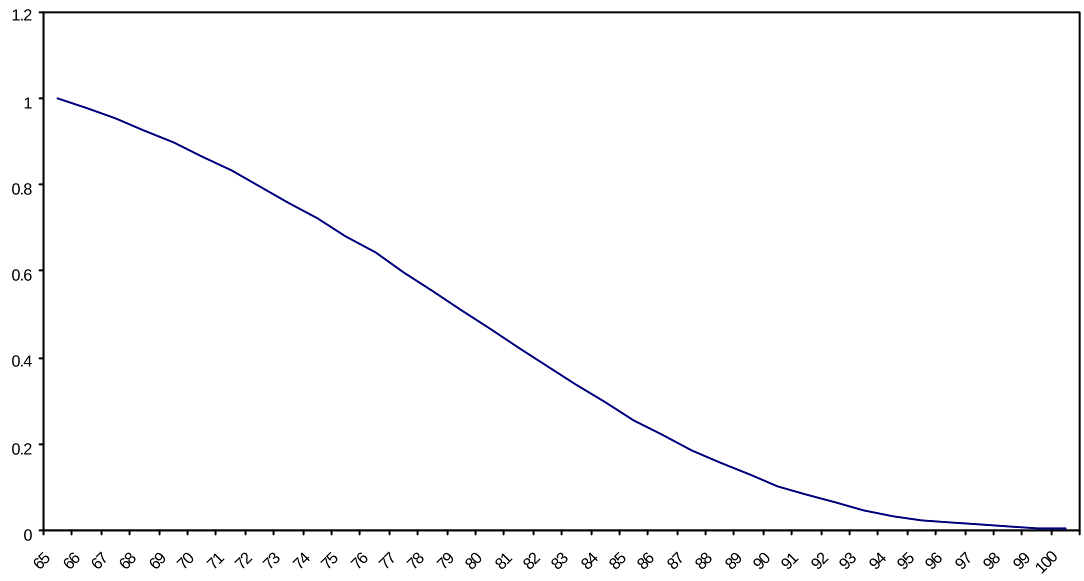

Figure 4: Actual 1998 UK Income Distribution and Log Normal Approximation

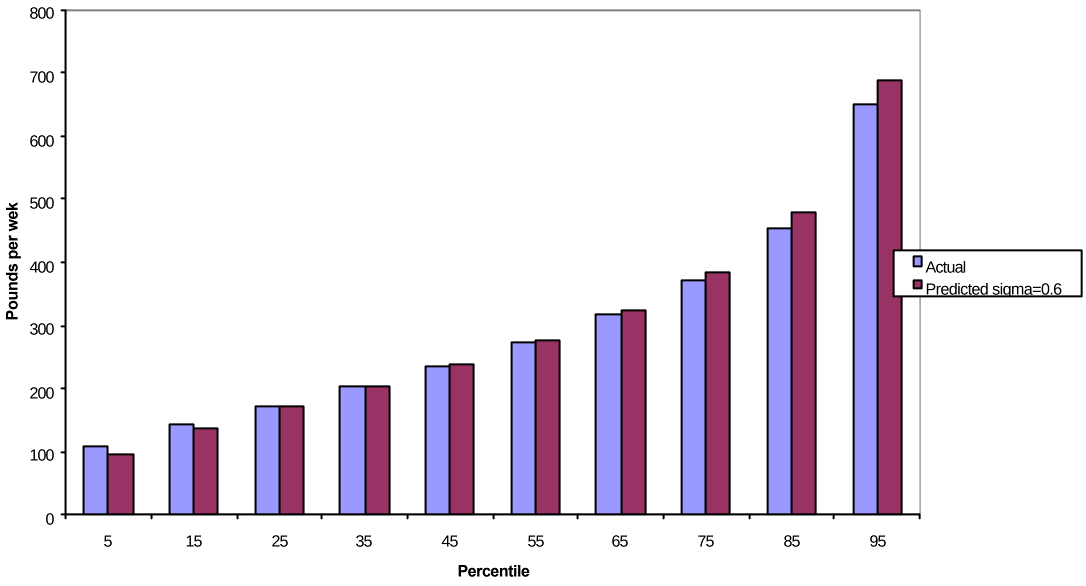

Figure 5: Average Savings Rates by Age: pension = 10% and average rate of return = 6%

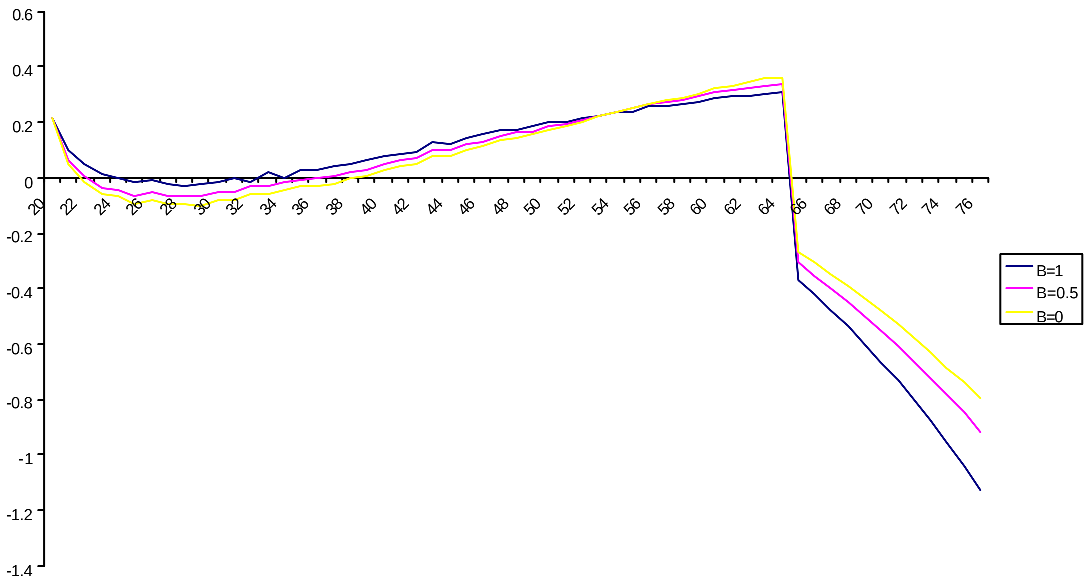

Figure 6: Average Savings Rates by Age: Pension = 20% and average return = 4%