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ESTIMATING THE OUTPUT GAP USING  
BUSINESS SURVEY DATA  
A BIVARIATE STRUCTURAL TIME SERIES  
MODEL FOR THE GERMAN ECONOMY

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Abstract

This paper deals with the estimation of the output gap. We use uni- and bivariate unobserved components models in order to decompose the observed German GDP-series into trend, cycle and seasonal components. The results show that using the ifo business assessment variable as an indicator for the cycle the estimation of the output gap is much more precise and out-of-sample forecasts exhibit smaller prediction errors.

Keywords: Output gap, unobserved component models, survey data

JEL Classification: C22, E17, E32

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## 1 Introduction

Many business cycle researchers and policy makers in central banks and government consider the output gap as the single most important and comprehensive measure of the cyclical state of the economy. The output gap is defined as the proportional deviation of realized output from potential or full employment output which can be maintained without running into a pressure of inflation. Like the “natural rate of unemployment“ the concept of potential output needs a careful operationalization in order to get reasonable and reliable estimates in empirical work. From an econometric viewpoint the task entails a decomposition of the observed output series into a non-stationary trend component and a stationary cycle. Unfortunately, this decomposition is only unique within a given statistical framework where assumptions are not fully testable. For the task of measuring the output gap this would not be a serious practical problem if the different approaches lead to the same results. But as has been shown e.g. by Canova (1998) estimated output gaps differ dramatically between different detrending methods.

In this paper we start from a structural time series model which decomposes the observed German GDP-series into trend, cycle and a seasonal part (Harvey/Jaeger 1993, Kuttner 1994). Structural time series models are specified explicitly in terms of unobserved components which have a direct interpretation (see Harvey 1989). This approach has a number of advantages:

1. Although each individual component is specified in a simple and intuitively interpretable way the reduced form can capture quite complex dynamic properties of the observed time series.
2. The model allows for stochastic trends, growth rates and seasonal components without the need of pre-testing for the existence of one or more unit roots (for some arguments why unit root tests may be misleading see Harvey 1997). Deterministic components as a limiting case can be handled in an easy way by setting a variance term to zero.
3. In contrast to the popular Hodrick-Prescott filter the structural time series approach defines a sensible and plausible model whose parameters are not set a priori but are estimated in an efficient way by well understood econometric methods. This avoids the great danger of the Hodrick-Prescott procedure of generating spurious cycles (for a discussion of the limitations of the HP-

filter see Cogley/Nason 1995 and Harvey/Jaeger 1993; for a similar criticism on methods based on moving averages see Osborn 1995).

4. Structural time series models can easily be extended to a multivariate framework. This opens the way to analyze common trends and/or cycles or to apply some variant of a dynamic factor analysis (see Harvey 1989). The multivariate approach was used by Gerlach/Smets (1998) who estimated a model where the output is a key determinant of inflation (for a three-equation model in the same spirit see Apel/Jansson 1999). In this paper we use results from the ifo-business-survey as an indicator for the state of the business cycle. We define a two-equation system for real GDP and the variable “business assessment“ where both variable share the same cycle component. Since we have two observed variables that depend on the unobserved cycle component we expect a more precise estimation of the output gap.

The paper is organized as follows. In the following we discuss the univariate unobserved components model as well as its bivariate extension. Section 3 presents the data and the empirical results for the German economy from 1969 to 1999. We conclude in section 4.

## 2 Uni- and Bivariate Unobserved Components Models

The basic univariate unobserved components model decomposes a single time series  $y_t$  into unobserved components, e.g. into the sum of a trend or permanent component  $y_t^P$ , a cycle  $c_t$  and a seasonal component  $y_t^S$ . In case of decomposing GDP the trend may be interpreted as potential output and the cycle as the output gap.

$$y_t = y_t^P + c_t + y_t^S + \mathbf{e}_{1t} \quad \text{var}(\mathbf{e}_{1t}) = \mathbf{s}^2(y) \quad (1)$$

In context of structural time series models equation (1) is called the measurement equation. Equations (2) to (5) specify the evolution through time of the unobserved components or state variables.

$$y_t^P = y_{t-1}^P + \mathbf{m}_{t-1} + \mathbf{d}_1 d_{1t} + u_{1t} \quad \text{var}(u_{1t}) = \mathbf{S}^2(y^P)$$

(2)

$$\mathbf{m}_t = \mathbf{m}_{t-1} + u_{2t} \quad \text{var}(u_{2t}) = \mathbf{S}^2(\mathbf{m}) \quad (3)$$

$$c_t = \mathbf{f}_1 c_{t-1} + \mathbf{f}_2 c_{t-2} + u_{3t} \quad \text{var}(u_{3t}) = \mathbf{S}^2(c) \quad (4)$$

$$y_t^S = -(y_{t-1}^S + y_{t-2}^S + y_{t-3}^S) + u_{4t} \quad \text{var}(u_{4t}) = \mathbf{S}^2(y^S) \quad (5)$$

The trend or potential output  $y_t^P$  is usually modeled as a random walk. For reasons of flexibility we allow the drift term  $\mathbf{m}$  in equation (3) also to vary over time and to follow a random walk. Since our data refer to West Germany up to the fourth quarter of 1990 and to unified Germany afterwards we have a permanent break in the level of GDP. We model this event by a level intervention dummy  $d_{1t}$  in the potential output equation with  $d_{1t} = 1$  in 1991:1 and zero otherwise. The output gap  $c_t$  in equation (4) is modeled as an AR(2)-process. This is the simplest possibility to produce cyclical behavior. Apart from a random disturbance the seasonal component is assumed to average to zero over the course of the year. Equation (5) states this idea for quarterly data.<sup>1</sup> The error terms are assumed to follow a normally distributed white noise process. Applying the Kalman filter and using maximum likelihood procedures to the system of equations (1) to (5) delivers estimates for the state variables  $y_t^P$ ,  $\mathbf{m}$ ,  $c_t$  and  $y_t^S$  as well as of the model parameters. Identification requires some parameter restrictions (see Watson 1986). The usual proceeding in the literature also adopted here is to restrict the contemporaneous correlations of the error terms of the unobserved components to zero.

The above model uses the information of only a single time series. A natural extension that potentially improves forecasts and the identification of the output gap is to use a further series as an indicator for the output gap. The assessment by firms of their actual business situation reflects demand fluctuations and could serve as a coincident indicator of the output gap. This extends the univariate model to a bivariate one. We now have a second measurement equation namely for business assessment  $ba_t$ , which is also decomposed into a permanent component  $ba_t^P$ , a cycle and a seasonal component  $ba_t^S$  (equation (6)). The idea of business assessment being an indicator for the output gap can be

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<sup>1</sup> Unlike most empirical studies in the literature we do not use seasonally adjusted data because the joint modeling of all components is more reasonable way to treat seasonality (Harvey 1989, Maravall 1997).

modeled by assuming that both series share the same cyclical component. Hence, besides the measurement equation for business assessment the bivariate model contains equations describing the evolution of the two additional unobserved components  $ba_t^P$  and  $ba_t^S$ .

$$ba_t = ba_t^P + \mathbf{b}_1 c_t + \mathbf{b}_2 c_{t-1} + ba_t^S + \mathbf{d}_2 d_{2t} + \mathbf{e}_{2t} \quad \text{var}(\mathbf{e}_{21t}) = \mathbf{S}^2(ba) \quad (6)$$

$$ba_t^P = ba_{t-1}^P \quad (7)$$

$$ba_t^S = -(ba_{t-1}^S + ba_{t-2}^S + ba_{t-3}^S) + u_{5t} \quad \text{var}(u_{5t}) = \mathbf{S}^2(ba^S) \quad (8)$$

The impulse dummy variable  $d_{2t}$  in equation (6) controls for a strike in the manufacturing sector in 1984:2. We expect the permanent component of business assessment  $ba_t^P$  to be more or less constant over time. We opted to model it as a state variable without own disturbance term (7) because it allows the estimated permanent component to change slightly through time reflecting possibly changing ideas of the answering firms of what is a good or a bad business situation. Alternatively, we also specified it as a constant parameter without changing the results. Analogously to GDP the seasonal component of business assessment in equation (8) is assumed to sum up to zero over the year.

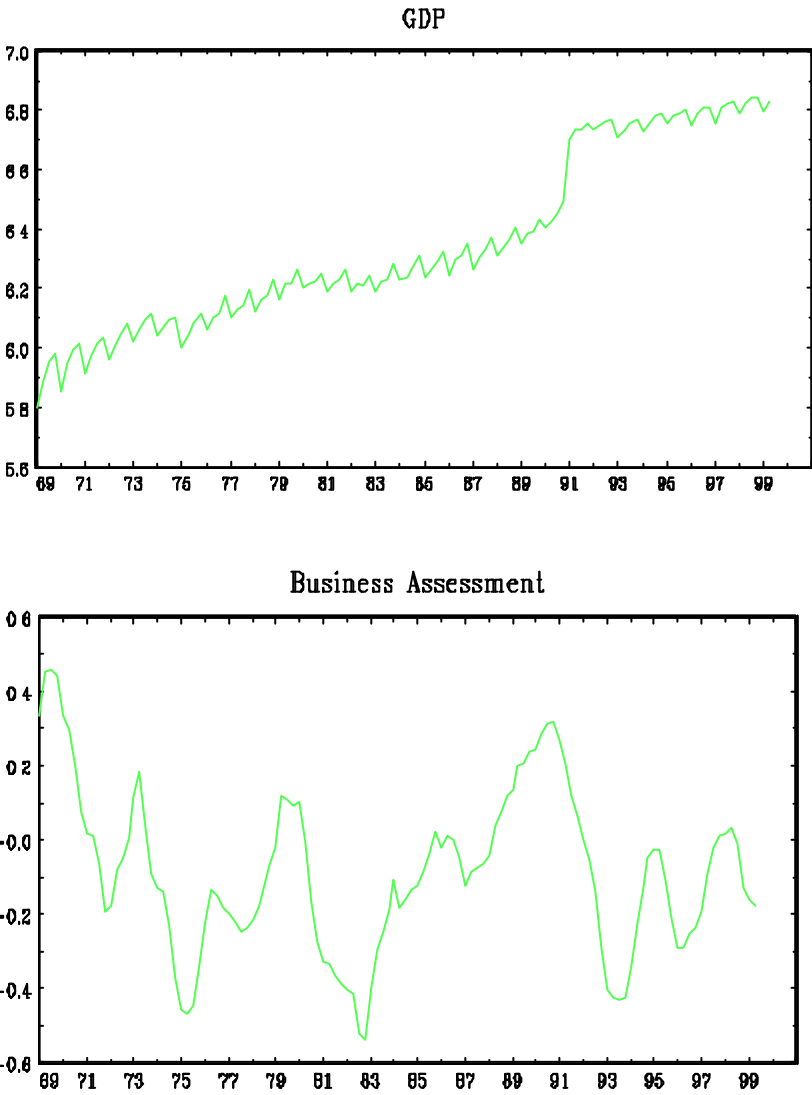
### 3 Empirical Analysis

#### 3.1 Data

In our empirical analysis we use quarterly data for the German GDP and the variable “business assessment“ from 1969:1 to 1999:2. The two series are plotted in figure 1. The GDP series is represented in logs. It shows a break in 1991:1 attributable to three different reasons. Until 1990:4 GDP refers to West Germany, is measured in prices of 1991 and is defined according to the old System of National Accounts. From 1991:1 on GDP covers unified Germany, is in prices of 1995 and defined according to the new European System of Accounts (see Strohm et al. 1999). The level intervention  $d_{1t}$  in equation (2) absorbs all three effects. Business assessment has been constructed from aggregated business survey data collected by the ifo institute (for more information on ifo survey data see Oppenlaender/Poser 1989). Firms report monthly whether they assess their actual business situation as being good, satisfactory or bad. The series we use contains the quarterly means of the balance of

positive and negative answers (divided by 100). The survey covers a considerable part of the economy namely the manufacturing, the construction, the wholesale and the retail sectors. Although the business assessment series refers only to West Germany we think that it is justified to be used as an indicator for the output gap of the whole economy because the East German economy still has a relatively small weight.

Figure 1



3.2 Results

Following a suggestion by Harvey (1989) we used the unconditional mean and the unconditional covariance matrix to initialize the Kalman filter for the stationary component (i.e. the cycle) and diffuse priors for the non-stationary components. The first eight observations are used to initialize the Kalman filter and do not enter the likelihood function so that the number of observations used for estimation reduces to 114. In order to allow comparisons with the bivariate model and as a preliminary data exploration table 1 shows the parameter estimates of the univariate models for GDP and for business assessment. GDP is modeled according to equations (1) to (5). Business assessment is decomposed into a permanent, a cyclical and a seasonal component analogously to equation (1). The cycle (denoted  $ba_t^C$  with variance  $s^2(ba^C)$ ) is autoregressive as in equation (4) and the permanent component is modeled according to equation (7).

**Table 1**

**Parameter Estimates of the Univariate Models**

	<b>GPD</b>		<b>Business Assessment</b>	
	<b>Parameter</b>	<b>t-value</b>	<b>Parameter</b>	<b>t-value</b>
$f_1$	1.65	12.24	1.61	20.65
$f_2$	-0.69	-5.01	-0.70	-9.22
$d_1$	0.24	15.70	-	-
$d_2$	-	-	-0.08	-3.18
$s^2(y)$	$2.74 * 10^{-5}$	2.20	-	-
$s^2(m)$	$2.34 * 10^{-8}$	0.55	-	-
$s^2(c)$	$1.80 * 10^{-5}$	1.90	-	-
$s^2(y^S)$	$1.90 * 10^{-5}$	3.32	-	-
$s^2(ba^C)$	-	-	$1.95 * 10^{-3}$	7.20

Table 2 presents the results for the bivariate model.

**Table 2**



### Parameter Estimates of the Bivariate Model

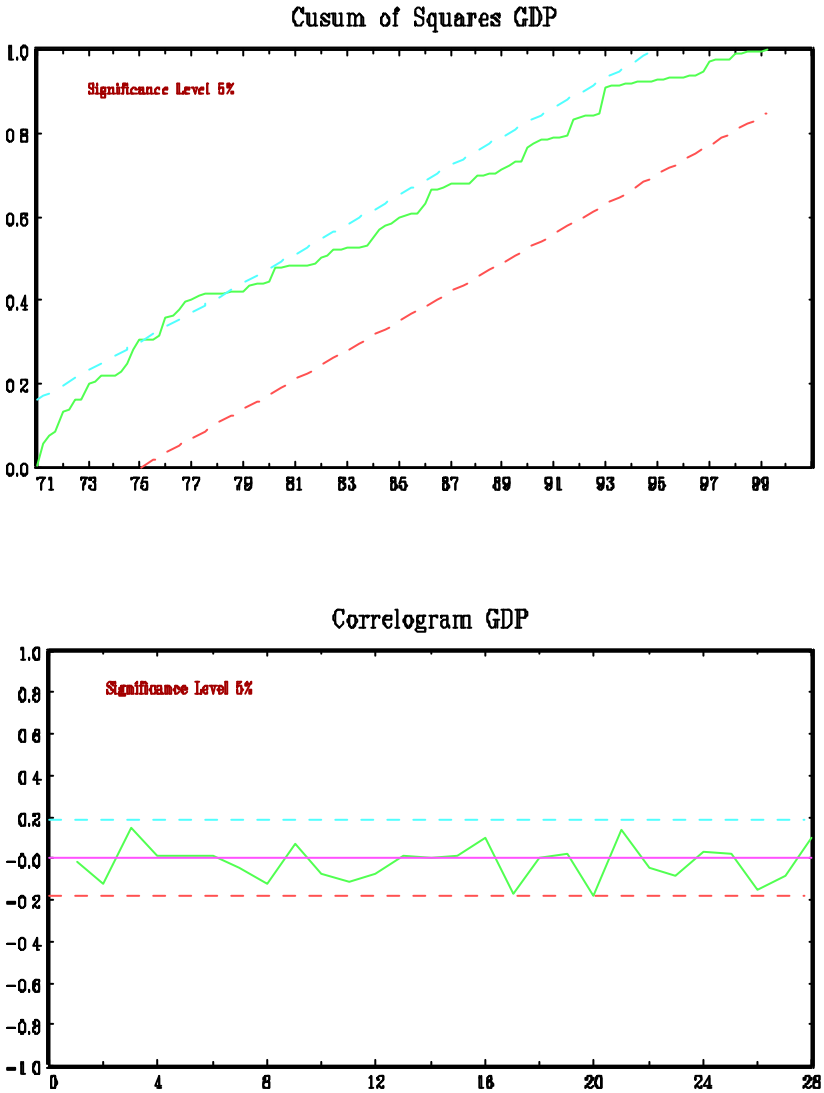
	Parameter	t-value
$f_1$	1.44	12.55
$f_2$	-0.52	-4.44
$b_1$	10.07	6.06
$b_2$	3.85	3.50
$d_1$	0.25	24.39
$d_2$	-0.07	2.87
$s^2(y)$	$2.86 * 10^{-5}$	4.21
$s^2(m)$	$4.02 * 10^{-6}$	3.26
$s^2(c)$	$1.91 * 10^{-5}$	4.66
$s^2(y^S)$	$1.34 * 10^{-5}$	4.48

The sum of the autoregressive parameters of the cyclical components  $f_1$  and  $f_2$  is below one in all three estimations, so in all cases the cycle is stationary and it exhibits a quasi-cyclical behavior. For GDP preliminary regressions showed that in both the univariate and the bivariate model the variance of the disturbance term of permanent output  $s^2(y^P)$  in equation (2) was zero. This leads to a smooth trend model. The variance of the disturbance of business assessment  $s^2(ba)$  in equation (6) and of its seasonal component  $s^2(ba^S)$  in equation (8) turned out to be zero as well. These parameters have been restricted to zero in the final regressions and therefore do not show up in tables 1 and 2.

Figure 2 presents some model diagnostics which are restricted to the bivariate model due to space limitations. The Cusum of squares test indicates that there could be a stability problem around 1978 for GDP. However, as the Cusum of squares leaves the 2-sigma band relatively early the reason therefore lies probably in imprecise estimates for the state variables of the first few periods due to the initialization of the Kalman filter. Because of the accumulation of the relatively large residuals around 1978 Cusum of squares stays near to the upper band. If we start to calculate the Cusum of squares in 1973:1 instead of 1971:1 it remains within the 2-sigma band and lies more to the center of the interval. The correlogram of the recursive residuals of GDP shows no significant autocorrelation. For

business assessment the Cusum of squares stays within in the 2-sigma band and the correlogram does not reveal serious autocorrelation of recursive residuals. The Jarque-Bera test statistic is 0.05 for GDP and 3.71 for business assessment, so normality is not rejected at the 5% level (critical value 5.99) for both equations. The inspection of the Kernel densities of the residuals does not reveal any signs of misspecification. The fact that business assessment does not seem to be misspecified may be interpreted as indirect evidence for the idea of a common cycle.

Figure 2



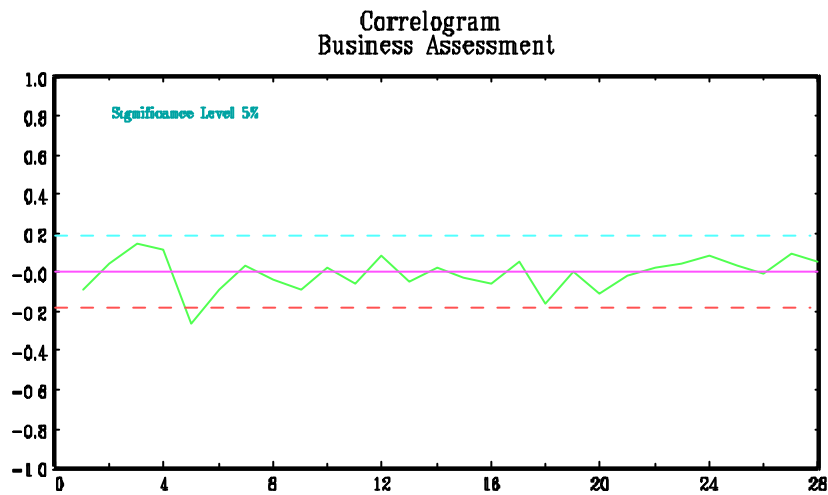
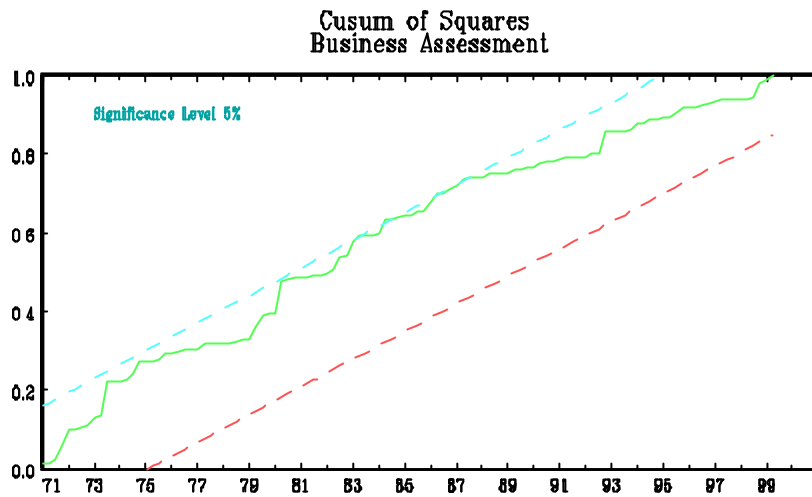
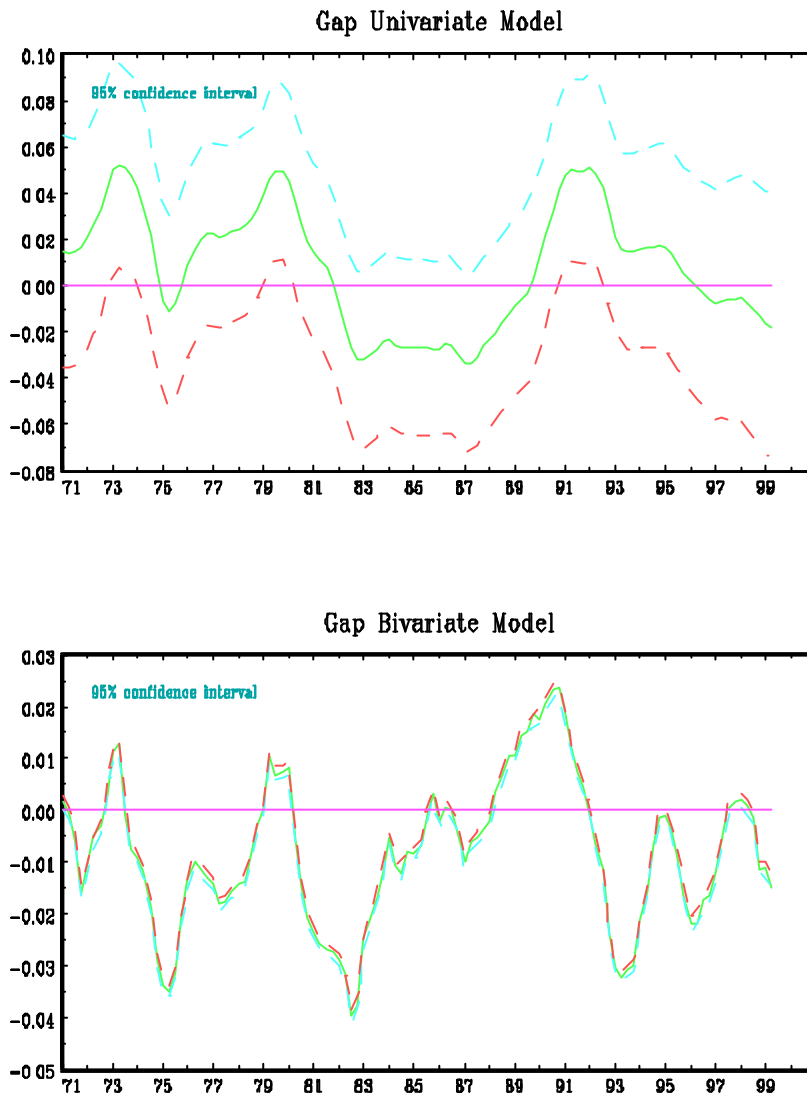


Figure 3 shows the two-sided filtered or “smoothed“ output gap according to the univariate and the bivariate model together with the corresponding 2-sigma bands. In contrast to one-sided filtered series that only contain the information available up to every point in time smoothed series always use all information available.

Figure 3



Different models or decomposition methods almost inevitably lead to different series for the components of a time series or as Canova (1998) puts it: “different detrending methods are alternative windows which look at series from different perspectives“ (p. 477). Hence, it is generally not possible to say which method or specification -if any at all- delivers “the correct“ output gap. As we assumed business assessment to reflect demand fluctuations the gap from the bivariate model looks quite similar to the business assessment series. It has both lower mean and lower variance than the gap derived from the univariate model. Generally the movements of the two series are similar but for the gap from the bivariate model the ups and downs are more pronounced. This becomes especially apparent for the time from the economic peak in 1991 until now. According to the bivariate model

the recession of 1993 was as severe as the one of 1975 and was followed by two minor peaks in 1995 and 1998. In contrast the univariate model shows a long slowdown since 1993 which was only delayed in 1995 and in 1998. Although the gap looks different in the univariate and the bivariate model the turning points of the major up- and downswings are roughly at the same point in time in both models. Sometimes the gap derived from the bivariate model seems to lead the gap from the univariate model by one quarter. We therefore tested whether business assessment is a leading rather than a coincident indicator but this hypothesis was rejected by the data.<sup>2</sup> The perhaps most striking observation is the difference in uncertainty of the estimated gap between the two models. While the 2-sigma bands lie widely apart in the univariate model they are very small and hardly distinguishable in the bivariate model.

Besides of improving the precision of identification of the output gap the use of an indicator may improve forecasting of GDP. We performed forecasts and compared the forecasting errors of the univariate and of the bivariate model for different forecasting horizons in order to assess if forecasting accuracy is improved by using business assessment as a coincident indicator of the output gap. The forecasting errors were computed the following way: We first restricted the estimation period to 1969:1 to 1993:4 and performed forecasts for up to six quarters starting in 1994:1. Then we compared the predicted values and the actual ones for each forecasting horizon. In the next step we re-estimated the models prolonging the estimation period by one quarter and again calculated the forecasting errors. We repeated this exercise as long as we had real data to compute forecasting errors (16 times) and then averaged the forecasting errors for each forecasting horizon. Table 3 and table 4 present the mean and the mean absolute forecasting errors for business assessment and for GDP, respectively.

### **Table 3**

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<sup>2</sup> The test proceeded in the following way: We decomposed business assessment into trend, cycle and season and specified the output gap as a function of the contemporaneous and lagged cyclical component of business assessment. If business assessment was leading GDP the parameters of the lagged cycle should be significant. However, this was not the case.

### Forecasting Errors for Business Assessment

forecast horizon	Univariate Model		Bivariate Model	
	mean error	mean abs. error	mean error	mean abs. error
1 - step	0.0024	0.0300	0.0013	0.0276
2 - step	0.0032	0.0682	-0.0007	0.0642
3 - step	0.0025	0.0988	-0.0072	0.0950
4 - step	-0.0068	0.1201	-0.0237	0.1234
5 - step	-0.0196	0.1334	-0.0448	0.1432
6 - step	-0.0327	0.1407	-0.0670	0.1580

**Table 4**

### Forecasting Errors for GDP

forecast horizon	Univariate Model		Bivariate Model	
	mean error	mean abs. error	mean error	mean abs. error
1 - step	-0.0030	0.0081	-0.0001	0.0079
2 - step	-0.0060	0.0100	-0.0002	0.0078
3 - step	-0.0090	0.0110	0.0005	0.0086
4 - step	-0.0116	0.0146	0.0008	0.0108
5 - step	-0.0167	0.0184	0.0007	0.0137
6 - step	-0.0212	0.0226	0.0008	0.0164

For business assessment results are mixed. While the bivariate model gives better forecasts for short forecasting horizons the basic univariate model slightly outperforms the bivariate model in terms of forecasting accuracy for four quarters and more. But the ultimate goal is to predict GDP rather than business assessment and here the performance of the bivariate model is clearly better at all forecasting horizons. It gives both smaller mean errors as well as smaller mean absolute errors. Moreover, as can be seen from the negative sign of the mean errors the univariate model overestimates GDP on average at every forecasting horizon while for the bivariate model forecasting errors seem less sys-

tematic. Summarizing, the evidence from the forecasting exercise suggests that using business assessment as an indicator of the output gap improves forecasting accuracy substantially.

#### **4 Concluding Remarks**

Decomposing the observed GDP time series into the structural components trend, cycle and season seems to be a natural and promising approach for business cycle research. A problem often encountered in empirical work are the high variances of the estimated components. In this paper we try to improve the precision by specifying a bivariate model where the business assessment variable provided by the ifo business survey serves as an additional indicator of the cyclical state of the economy. The results show that this reduces uncertainty of the estimated output gap considerably and leads to better prediction properties.

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