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Abstract

We analyse the workings of a simple non-linear exchange rate model in which agents hold different beliefs about the underlying model. We distinguish between 'chartists' and 'fundamentalists'. The non-linearities in the model originate from transactions costs and from the existence of non-linear adjustment dynamics in the goods market. We find, first, that the simple non-linear structure of the model is capable of generating a very complex exchange rate dynamics. Second, our model is capable of explaining some empirical puzzles concerning exchange rate behaviour, i.e. the 'disconnect' puzzle which says that the exchange rate is disconnected form its underlying fundamentals most of the time and the excess volatility puzzle.

JEL Classification: F31, F37.

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1. Introduction

In the 1970s economists developed exchange rate models, which explain exchange rates changes by *news* in fundamental economic variables. These models led to the propositions, first, that exchange rate changes can only occur because of unexpected movements (news) in the underlying fundamental economic variables (inflation growth of output, interest rates, etc.), and, second, that the link between exchange rates and fundamentals is a stable one. Well-known examples of these models are the monetary model, the Dornbusch model (Dornbusch(1976)) and the portfolio balance model. Although these models continue to be popular and maintain a prominent place in textbooks, they have failed empirically. The most notorious empirical rejection was made by Meese and Rogoff at beginning of the 1980s (Meese and Rogoff(1983)). This led to a large empirical literature that uncovered a number of empirical puzzles concerning the behaviour of the exchange rate, which could not be explained by the 'news' models.

A first empirical puzzle is the "excess volatility" of the exchange rate, i.e. the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables. Baxter and Stockman (1989) and Flood and Rose (1995) found that while the movements from fixed to flexible exchange rates led to a dramatic increase in the volatility of the exchange rate no such increase could be detected in the volatility of the underlying economic variables. This contradicted the 'news' models that predicted that the volatility of the exchange rate can only increase when the variability of the underlying fundamental variables increases.

A second empirical puzzle can be called the " disconnect puzzle", i.e. the movements of the exchange rate appear to be disconnected from movements in the underlying fundamentals. Goodhart (1989) and Goodhart and Figlioli (1991) found that most of the changes in the exchange rates occur when there is no observable news in the fundamental economic variables. Again, this contradicted the theoretical models which imply that the exchange rate can only move when there is news in the fundamentals.

A third puzzle relates to PPP and is closely related to the previous one. Many researches have found that the deviations from PPP are large and sustained (Rogoff (1995), Obstfeld & Rogoff (2000), Cheung & Lai (2000)). The half-life of the PPP deviations has been estimated to be of the order of 4 to 5 years. Some researchers have found even longer half-lives (Lothian & Taylor (1998), Engel(2000), O' Connell (1998)). Other researchers (Dumas (1992)) have stressed that the long time needed to adjust to PPP might be due to the existence of transaction costs. The transaction cost hypothesis implies a non-linearity in the adjustment process. This hypothesis has been confirmed by the empirical evidence based on time series analysis (Michael, Nobay, Peel (1997), Kilian & Taylor (2001)).

The empirical failure of the exchange rate models of the 1970s has led to new attempts to model the exchange rate. These attempts have led to three different modelling approaches. The first one uses the Obstfeld–Rogoff framework of dynamic utility optimisation of a representative agent. The models that came out from this approach have a high content of intellectual excitement. However, up to now they have led to few testable propositions.

A second approach starts from the analysis of the microstructure of the foreign exchange market. This approach has led to new insights into the way information is aggregated and is important for the understanding of the very short-term behaviour of the exchange rate.

Finally, a third approach recognises that heterogeneous agents have different beliefs about the behaviour of the exchange rate. These different beliefs introduce non-linear features in the dynamics of the exchange rate. In this paper we present a simple model of the exchange rate, which incorporates these non-linear features and we analyse their implications for the dynamics of the exchange rate. In addition, we will make use of the recent empirical evidence, which strongly suggests that the adjustment towards PPP is not linear in nature. It will be shown that our simple non-linear model is capable of solving the empirical puzzles about the exchange rate behaviour.

2. A Simple non-linear exchange rate model

In this section we develop a simple non-linear exchange rate model. We start with a popular model of the exchange rate, which is often used in the literature. We then introduce heterogeneous agents who use this model as a benchmark to define their beliefs about the future exchange rate.

We start from the determination of the exchange rate as follows:

$$s_t = f_t + \alpha\left[E_t s_{t+1} - s_t\right] \tag{1}$$

where f_t represents the fundamentals in period t, s_t is the exchange rate in period t, s_{t+1} is the exchange rate in period t+1, E is the expectations operator. Underlying the fundamental one could specify a whole model of the economy, e.g. a monetary model, or a more elaborate one like the

Obsfeld-Rogoff new open economy macro model (Obstfeld&Rogoff(1996)). We leave this for further research. Here we concentrate on the simplest possible exchange rate modelling. For the sake of simplicity, we assume that the fundamentals are determined exogenously.

Equation (1) can be rewritten as follows:

$$s_t = \frac{1}{1+\alpha} f_t + \frac{\alpha}{1+\alpha}\left[E_t s_{t+1}\right] \tag{2}$$

We use this model to define the fundamental equilibrium exchange rate. This is the rational expectations solution of equation (2). It will be used as a benchmark against which the beliefs of different agents are measured.

In the absence of bubbles the fundamental solution to (2) is given by

$$s_t^* = \frac{1}{1+a} \sum_{i=0}^{\infty} \left(\frac{a}{1+a} \right)^i E_t f_{t+i} \tag{3}$$

For the sake of simplicity we will assume that f_t follows a random walk process without drift. We then find the following fundamental solution of the exchange rate :

$$s_t^* = f_t \tag{4}$$

In some applications we will assume that f_t is a constant

We now introduce the assumption that the agents have heterogeneous beliefs and we classify them according to their beliefs. Let us assume that there are N_h individuals of type h belief (where $\Sigma N_h = N$). We can then characterize the beliefs of type h agents as follows[1]:

[1] See Brock and Hommes(1998) for such a formulation

$$E_{h,t}s_{t+1} = s_t^* + g_h(s_{t-1}, s_{t-2}, ...) \quad (5)$$

where $E_{h,t}$ represents the expectations operator of type h agent at time t. Thus agents' beliefs can be classified depending on how they view the process by which the market price will grope towards the fundamental exchange rate s^*_t. They all use information on past exchange rates to forecast these future developments.

The market expectation can then be written as follows:

$$E_t s_{t+1} = \sum_{h=1}^{H} n_h E_{h,t}[s_{t+1}] = s_t^* + \sum_{h=1}^{H} n_h g_h(s_{t-1}, s_{t-2}, ...) \quad (6)$$

Note that $n_h = N_h/N$, so that n_h can be interpreted as the weight of agents of type h in the market.

The efficient market assumption then allows us to write that the realised market rate in period t+1 equals the market forecast made at time t plus some white noise error (i.e. the news that could not be predicted at time t) :

$$s_{t+1} = s_t^* + \sum_{h=1}^{H} n_h g_h(s_{t-1}, s_{t-2}, ...) + \varepsilon_{t+1} \quad (7)$$

In the previous discussion the nature of the beliefs of agents was specified in very general terms. We further simplify the model by assuming that there are only two types of agents in the foreign exchange market, which we will call *chartists* and *fundamentalists*[2].

[2] This way of modelling the foreign exchange market was first proposed by Frankel and Froot (1988). It was further extended by De Long et al. (1990) and De Grauwe et al.(1993) and more recently Kilian and Taylor (2001). For evidence about the use of chartism see Allen and Taylor (1989).

The *fundamentalists* base their forecasts on a rule like in equation (5), i.e. they compare the past market exchange rates with the fundamental rate and they forecast the future market rate to move towards the fundamental rate. In this sense they follow a *negative feedback* rule[3]. We will make the additional assumption that they expect the speed with which the market rate returns to the fundamental rate to be determined by the speed of adjustment in the goods market.

As pointed out earlier, there is an increasing amount of empirical evidence indicating that the speed of adjustment in the goods market follows a non-linear dynamics, i.e. the speed with which prices adjust towards equilibrium depends positively on the size of the deviation from equilibrium. We will assume that this adjustment process is quadratic in nature[4]. Fundamentalists take this non-linear dynamic adjustment into account in making their forecast. This leads us to specify the following rule for the fundamentalists:

$$E_{f,t}(\Delta s_{t+1}) = \theta\left(s_{t-1} - s^*_{t-1}\right)^2 \tag{8}$$

where $E_{f,t}$ is the forecast made in period t by fundamentalists and

$\theta < 0$ when $s_{t-1} - s^*_{t-1} > 0$

$\theta > 0$ when $s_{t-1} - s^*_{t-1} < 0$

Thus when the size of the deviation from equilibrium is large the fundamentalists expect a faster speed of adjustment towards the fundamentals rate than when the size of the deviation is small. The economics behind this non-linear specification is that in order to profit from arbitrage opportunities in the goods market, some fixed investment

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[3] This is also the approach taken in the popular Dornbsuch model.

must be made, e.g. trucks must be bought, planes be chartered, etc. These investments become profitable with sufficiently large deviations from the fundamental exchange rate.

The *chartists* are assumed to follow a *positive feedback* rule, i.e. they extrapolate past movements of the exchange rate into the future. Their forecast is written as:

$$E_{c,t}\left(\Delta s_{t+1}\right) = \beta \sum_{i=1}^{T} \alpha_i \Delta s_{t-i} \tag{9}$$

where $E_{c,t}$ is the forecast made by chartists using information up to time t; Δs_t is the change in exchange rate.

As can be seen, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β. Note that in contrast to the general rule as given by equation (5) (and also in contrast to fundamentalists) they do not take into account information concerning the fundamental exchange rate. In this sense they can be considered to be pure *noise traders*[5].

In a similar logic as in equation (7) the market exchange rate can now be written as

$$\Delta s_{t+1} = n_{ft}\theta\,(s_{t-1} - s^*_{t-1}\;)^2 + n_{ct}\beta \sum_{i=1}^{T} a_i \Delta s_{t-i} + \varepsilon_{t+1}$$

In the following we will assume that the weights n_{ft} and n_{ct} are constant. We set them equal to 0.5, not because we think this is realistic but to see how far the simplest possible model goes in explaining the exchange rate dynamics. At a later stage we will make the weights given to

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[4] See Kilian and Taylor(2001). See also De Grauwe and Grimaldi(2001) in which we showed that a quadratic specification fits the data rather well.

fundamentalists and chartists react endogenously to the profitability of these forecasting rules.

3. The model with transactions costs

As stressed in the empirical and theoretical literature, transaction costs are important to explain the dynamics of the adjustment. Therefore, we will develop a version of the previous model represented by equations (1)-(9) in which the transaction costs play a role.

We take the view that if transaction costs exist, the fundamentalists will take this information into account. Therefore, if the exchange rate is within the transaction costs band the fundamentalists will behave differently than if the exchange rate moves outside the transaction costs band.

Consider the first case, when the exchange rate deviation from its fundamental value is larger than the transaction costs *C* (assumed to be of the 'iceberg' type). Then the fundamentalists follow the same forecasting rule as in equation (8). More formally,

when $\left| s_{t-i} - s^*_{t-i} \right| > C$ holds, then equation (8) applies.

In the second case, when the exchange rate deviations from the equilibrium value are smaller than the transaction costs, then the fundamentalists know that arbitrage in the goods market does not apply. As a result, they expect the changes in the exchange rate to follow a white noise process ε_t. The best they can do is to forecast no change. More formally,

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5 See De Long et al. (1990)

when $\left|s_{t-1} - s^{*}_{t-1}\right| < C$, then $E^{F}_{t-1}(\Delta s_t) = 0$.

4. Solution of the model

The non-linear structure of our model does not allow us to derive analytic solutions. Therefore we provide results with simulation techniques using plausible values of the parameters. We will also analyse how sensitive the results are with respect to these parameter values.

In the first step we look at the deterministic part of the model, i.e. we eliminate all stochastic variables. In the second step we introduce the stochastic variables (*news*).

4.1 Solution of the deterministic model

In order to better understand the implications from the deterministic model we first analyse a simplified version where we set all the lags in the exchange rate beyond 2 periods equal to zero. This yields the following simplified version of the model:

$$s_t = \left(1 + \frac{\beta}{2}\right)s_{t-1} - \frac{\beta}{2}s_{t-2} + \frac{\theta}{2}s_{t-1}^{2} \qquad (11)$$

This simplified version of our model has a logistic structure[6], i.e. for a given value of s_{t-2} we obtain a logistic equation. We first represent the logistic curve for θ =-0.3 and β =5 in figure 1.

Figure 1 shows the logistic structure of the model, which is the result of the different behaviour of the chartists and fundamentalists. When the exchange rate is close to its equilibrium value, i.e. zero, the exchange rate movements are driven by the chartists who extrapolate the past into

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[6] See Baumol and Benhabib (1989)

the future. Therefore the curve is upward-sloping. When the exchange rate deviates more and more from its equilibrium value the fundamentalists become more important and overwhelm the chartists. Therefore the curve becomes downward-sloping. The exact shape of the logistic curve depends on the value of the parameters. In figure 2 we show the logistic curve for the given value of β =5 and different values of θ. When the speed of adjustment increases the downward action of the fundamentalists occurs sooner and the intersection point with the 45° line also occurs sooner.

Figure 1

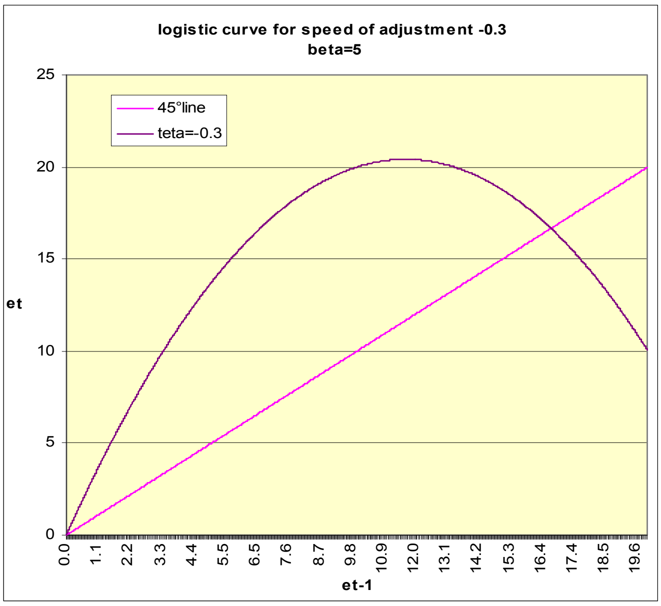
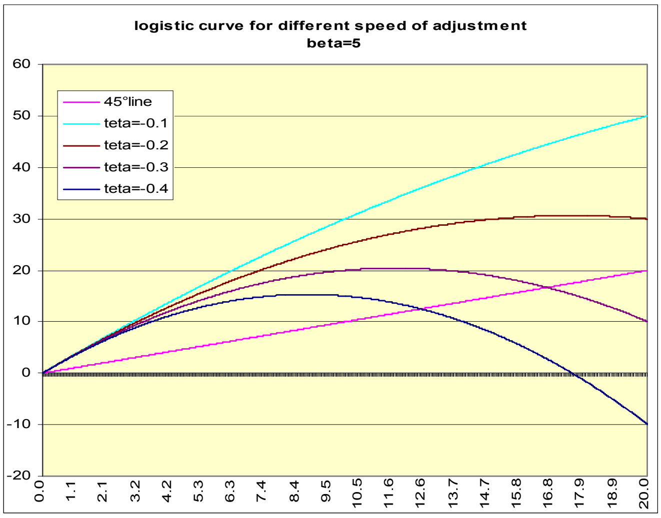


Figure 2



We now return to the full model represented by equations (1)-(9) and we simulate it using different combinations of parameters. Our main result is that the model, which is extremely simple, is capable of generating very complex exchange rate behaviour. In appendix 1 we produce a table where we present the nature of the solution for different combinations of parameters. It can be seen that for some combinations we obtain a fixed-point solution, for other combinations we have periodic solutions, and other combinations give chaotic solutions. In fact we find that the exchange rate follows a chaotic pattern for a relatively broad range of parameter values. We show some examples of chaotic dynamics in figures 3 and 4. Figure 3 presents results when we assume transactions costs (C=5) and figure 4 shows results in the absence of transactions costs (C=0). In the panels a of figures 3 and 4 we show the strange attractors in the phase space. In panels b we show the results of performing a sensitivity analysis, which consisted in increasing slightly (0.01) the size of the shocks in the initial exchange rate. Note that we have normalised the equilibrium value of the exchange rate to be equal to zero.

The strange attractor's panels in both figures 3 and 4 show that our model has a potential of creating a chaotic structure, i.e. for certain combinations of parameters the exchange rate follows a chaotic path designed by the shape of the strange attractor. The tests of sensitivity to initial conditions confirm the intrinsic chaotic nature of the model.

Figure 3a

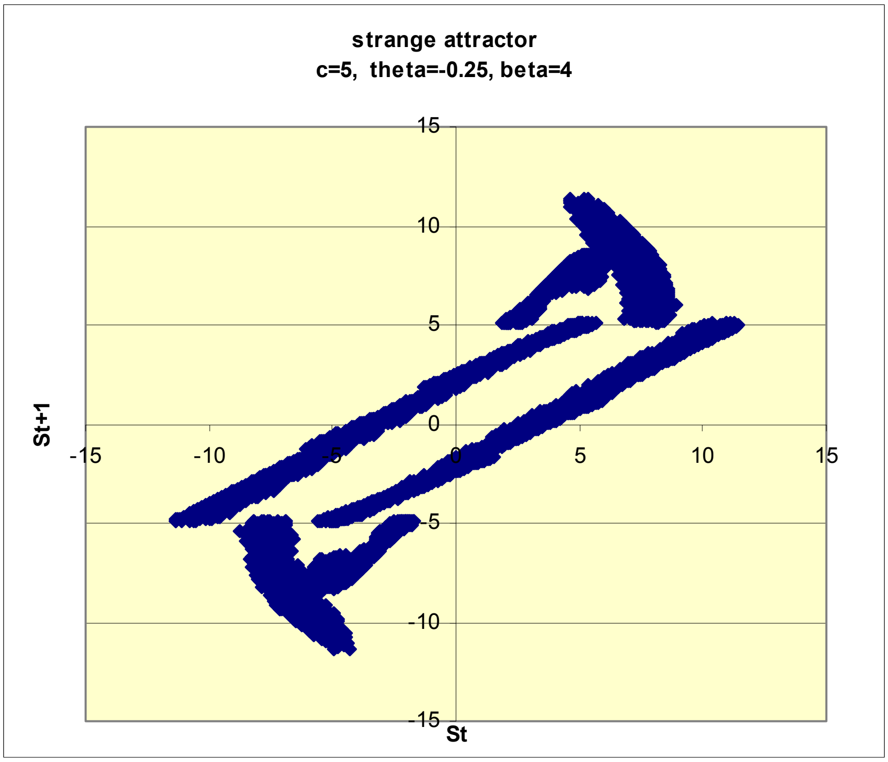


Figure 3b

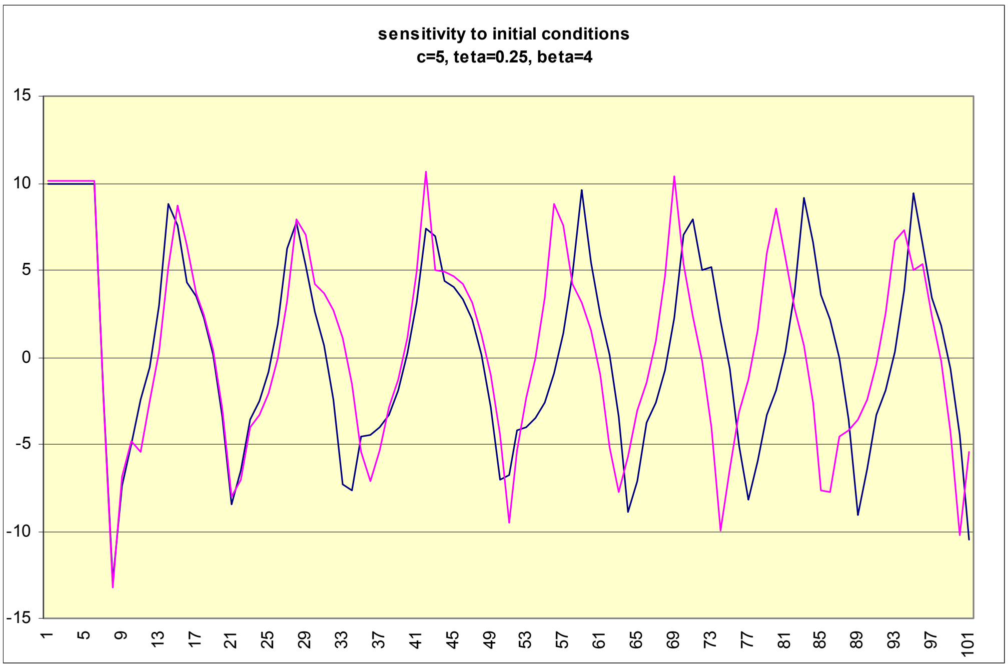


Figure 4a

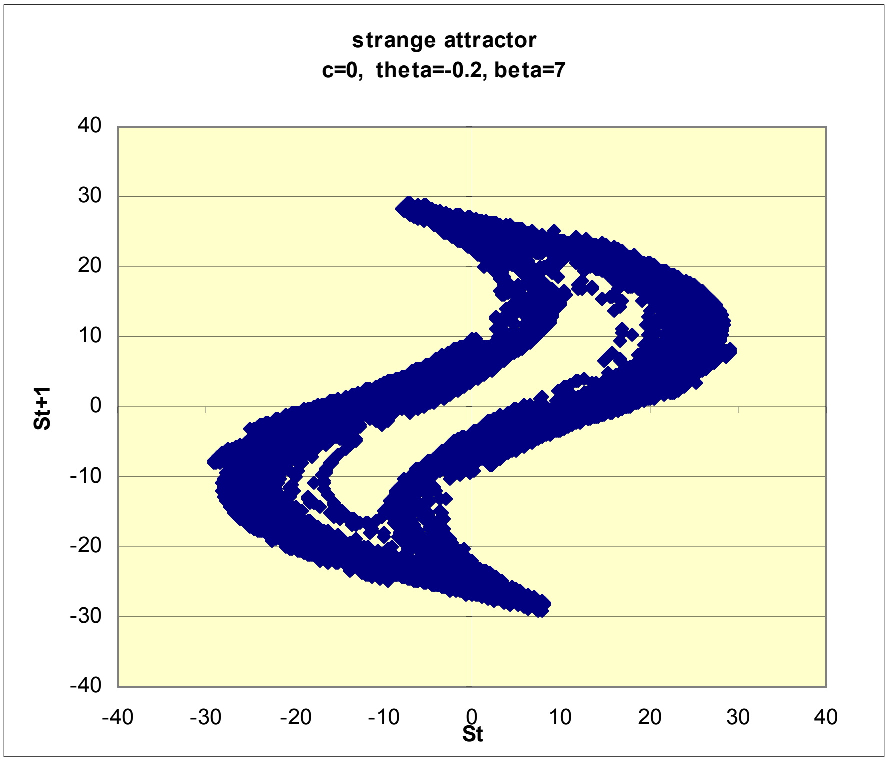
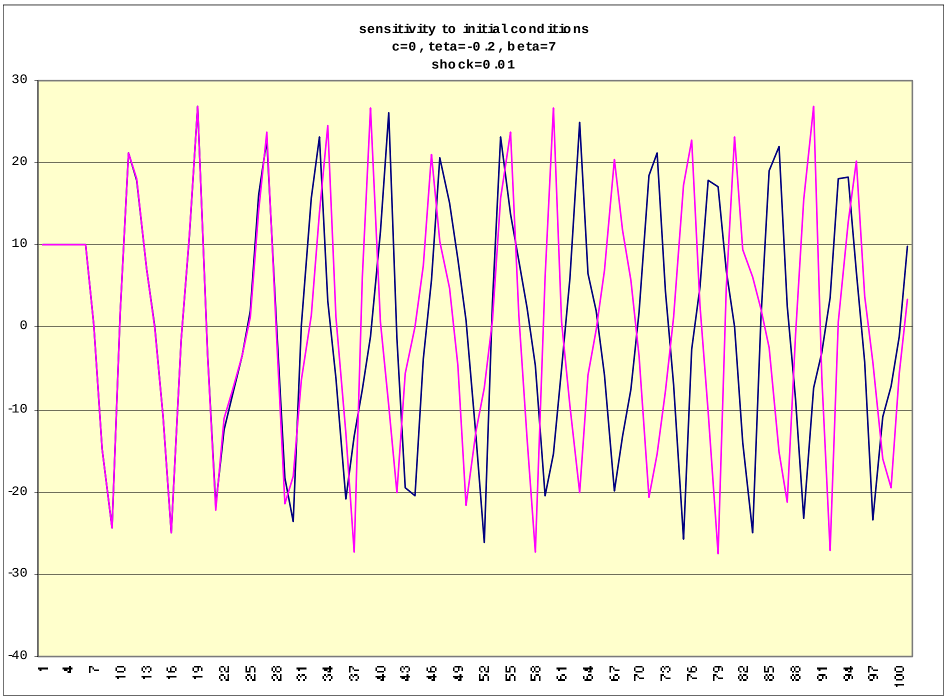


Figure 4b

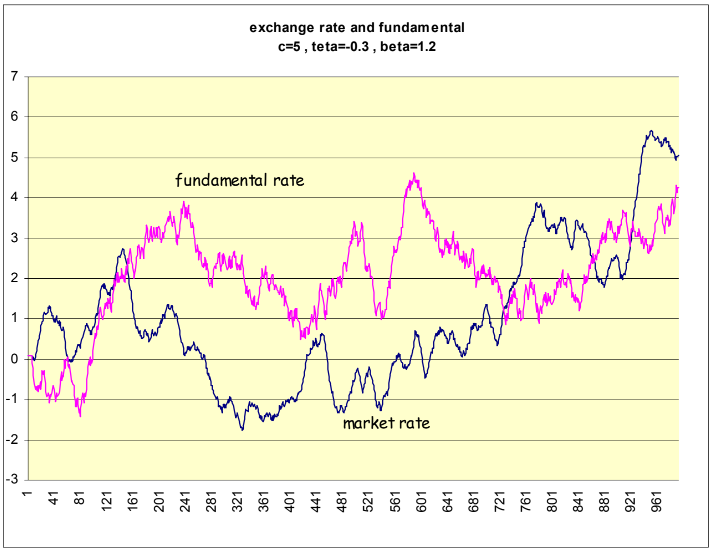


4.2 Solution of the model with random shocks (news)

In this section we investigate the solution of the model when random shocks in the equilibrium exchange rate occur. We will not restrict the analysis to the cases where the deterministic part of the model produces a chaotic dynamics. Thus, our results have a general character.

The first question we analyse is how the market exchange rate behaves relative to the fundamental exchange rate. In figure 5 we show the two variables, for a combination of parameters that does not produce deterministic chaos.

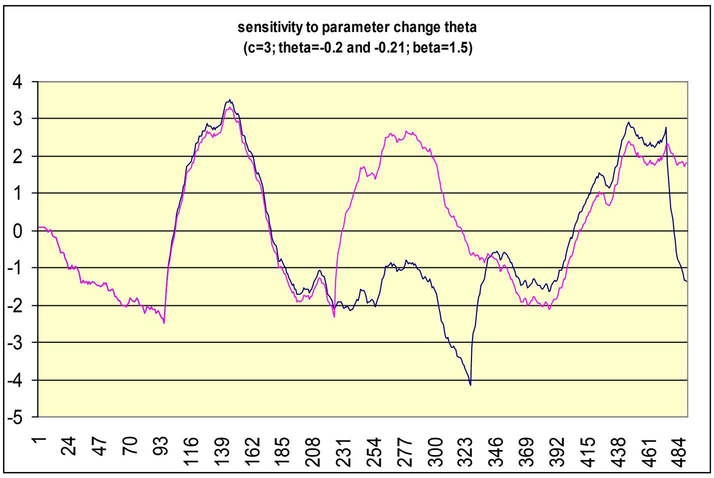
Figure 5



We observe that the market rate can deviate from the fundamental value substantially and in a persistent way. Moreover, it appears that the exchange rate movements are often disconnected from the movements of the underlying fundamental. In fact, they often move in opposite directions. Thus, the model is able to generate empirical regularities (the 'disconnect' puzzle) that have also been observed in reality[7]. We will return to this result later to analyse how sensitive it is to particular parameter values, like speed of adjustment and transaction costs.

Sensitivity to initial conditions is a crucial feature of the complex dynamics. We illustrate its power showing how a very small change in the speed of adjustment in the goods market can affect the market exchange rate. In Figure 10 we show the market exchange rate dynamics under two assumptions about the speed of adjustment. The first case is our standard case where θ is equal to –0.2, in the second case the value of θ is equal to -0.19. A way to interpret this simulation is to think of the case where the fundamentalists make a small error (0.01) in estimating the speed of adjustment, and thus in their forecast of the future exchange rate. As figure 6 shows, this small error will lead after some time to a different time-path of the exchange rate, producing the appearance of large structural breaks. It should be noted that we obtain this result, even though we have a parameter combination that does not produce chaos in the deterministic part of the model.

Figure 6



[7] See Obsteld and Rogoff(2000). See also De Grauwe(2000) for a survey of the empirical evidence. In De Grauwe and Vansteenkiste(2001) we present additional empirical evidence.

4.3 Additional results of the sensitivity analysis

We now investigate the sensitivity of our results to changes in the value of certain parameters. We first analyse the effect of assuming different *speeds of adjustment* in the goods market. The results are presented in the following figures 7a and 7b, which compare the movements of the market exchange rate under two assumptions about the speed of adjustment.

Figure 7a

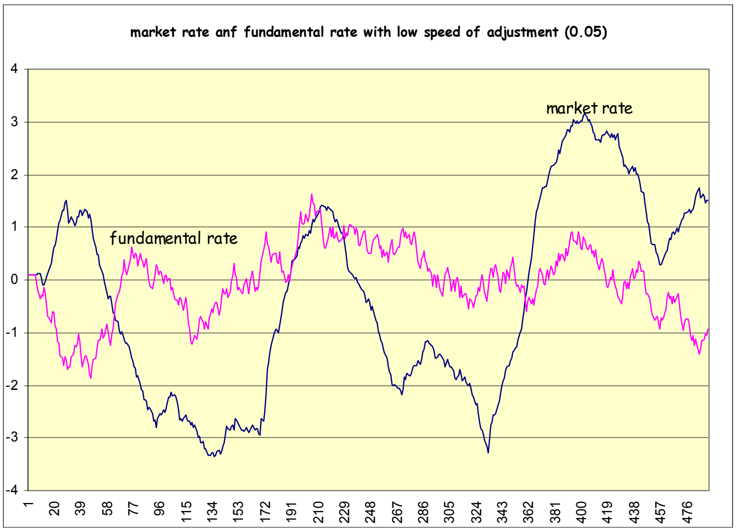
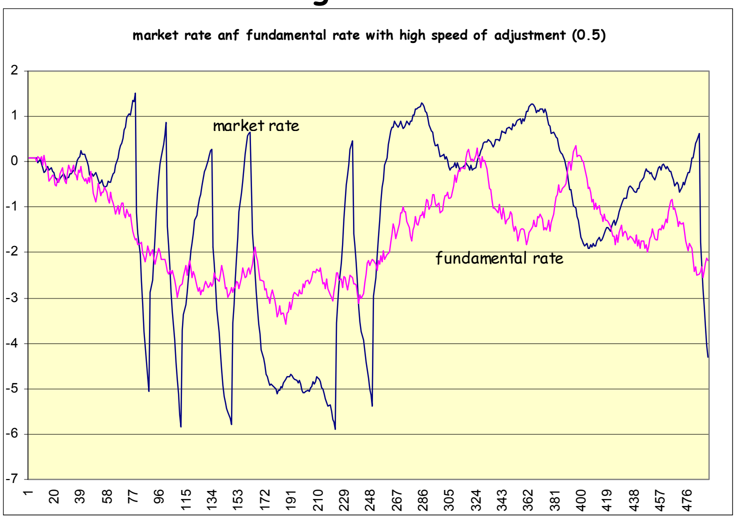


Figure 7b



The comparison of panels a and b in figure 7 allow us to conclude the following. When the speed of adjustment is low (θ= -0.05) the deviations of the exchange rate from its fundamental value show persistence for a long time. In contrast, when the speed of adjustment is high (θ= -0.5) the deviations from the equilibrium value are short-lived, and we also obtain substantial short-term volatility in the market exchange rate. Thus, the contrast between a world of low and high speeds of adjustment in the goods market is that in the former case the exchange rate can deviate from the fundamentals for a long time, while in the case of speedy adjustments these sustained misalignments are less likely, but short-term turbulence becomes more prevalent.

In the next figure 8 we compare the movements of the market exchange rate under two assumptions about *transaction costs*. In panel a we assume that transaction costs are zero and in panel b we assume them to be high, i.e. equal to 5.

Figure 8a

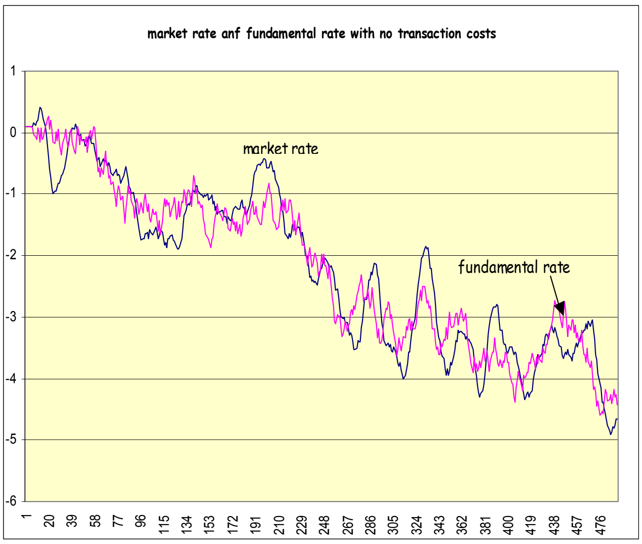
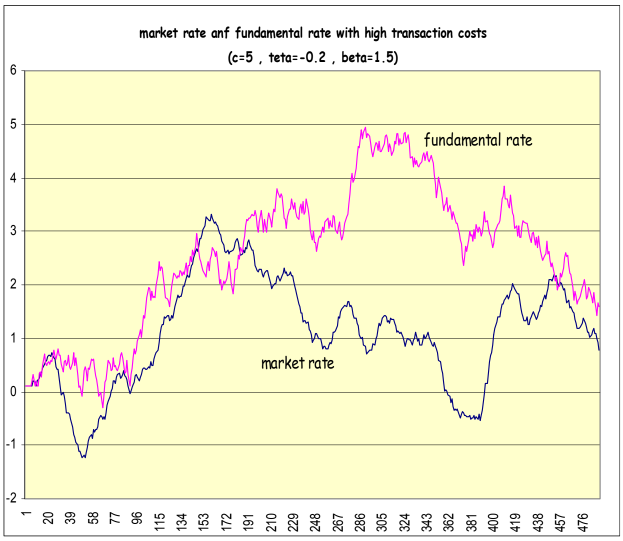


Figure 8b



The contrast between the two panels is striking. When transaction costs are zero, the market exchange rate does not deviate substantially for a long period of time from its fundamental value. In contrast, when transaction costs are present the deviations of the market exchange rate from its equilibrium value are large and persistent. Thus transactions costs have similar effects on the dynamics of the exchange rate as the speed of adjustment.

Transaction costs have also other important implications for the dynamics of the exchange rate. We show this in figure 9, where we introduce a negative and permanent shock (-0.01) in the fundamental exchange rate change. Thus, over time the "new" fundamental exchange rate progressively but slowly departs from the "old" one.

Figure 9

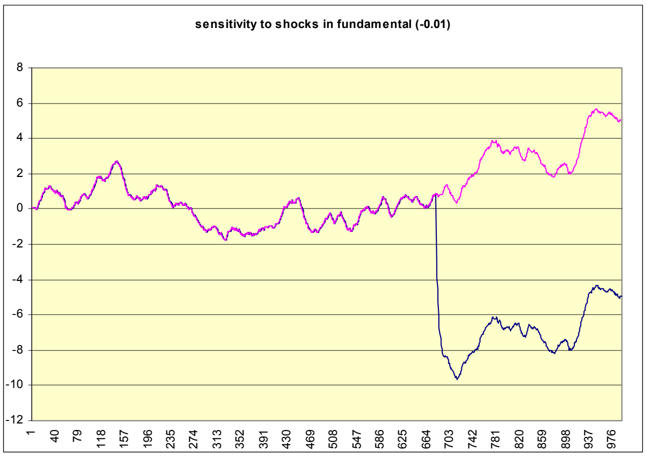
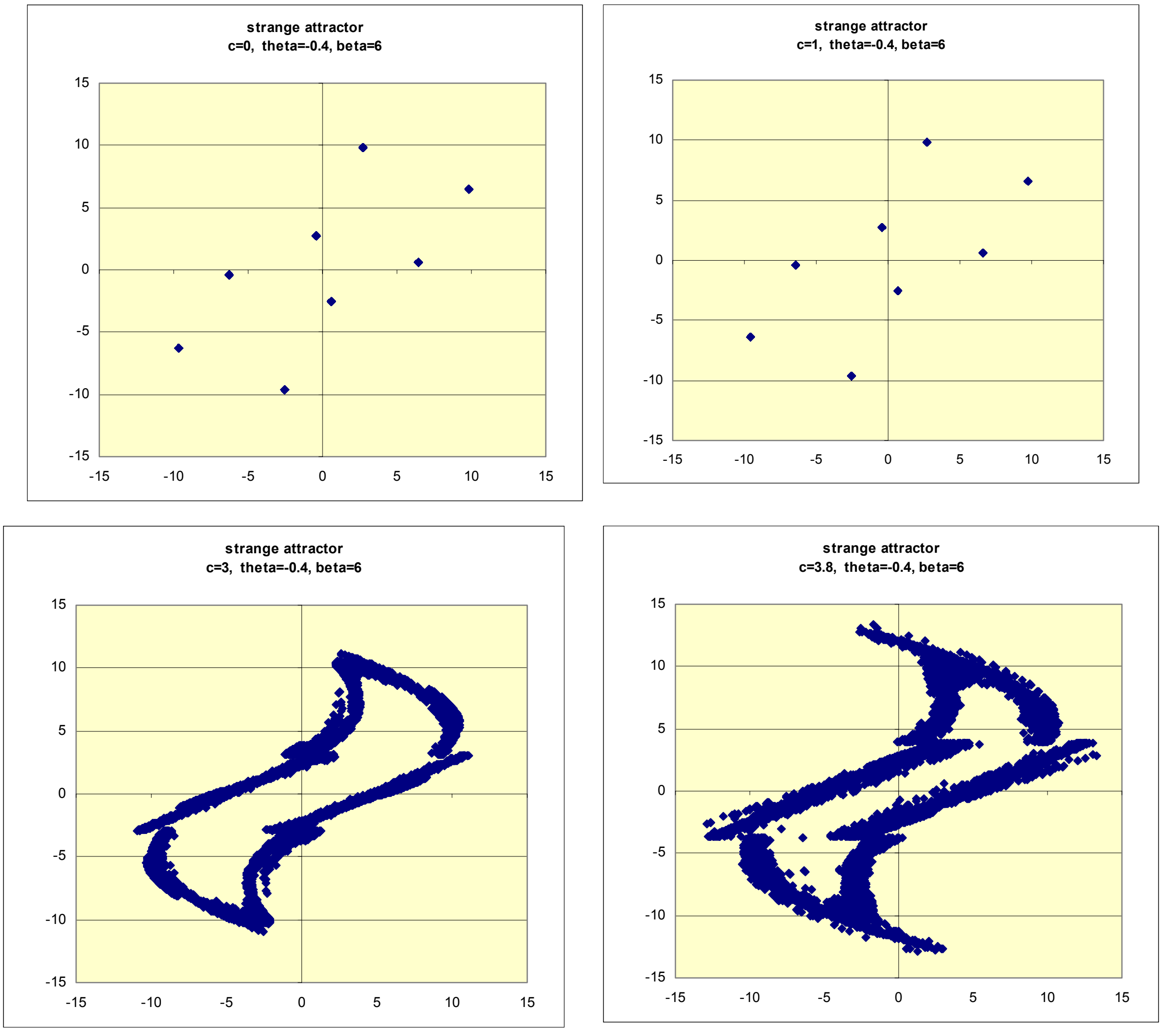


Figure 9 allows us to see how this accumulating small change in the equilibrium value of the exchange rate, which occurs in period 1, leads to a large jump in the exchange rate many periods later. This change has the appearance of a regime shift in spite of the fact that the change in the fundamental exchange rate is very small and continuous. This feature is much related to the existence of transaction costs, which implies that the effect of the accumulated changes in the fundamental exchange rate will be visible only when it will overcome the transaction costs band.

The existence of a transaction costs band has another remarkable implication, i.e. it affects the nature of the dynamics. This is shown in figure 10, which presents the strange attractor for different values of the transaction costs band (given the value of the other parameters).

From figure 10 it can be seen that for low transaction costs we obtain an eight-period cycle. When the transaction costs band increases to three and beyond we obtain a strange attractor, which increases in complexity when the transaction costs increase. Thus, increasing transaction costs leads to an increasing complexity into the dynamics of the exchange rate.

Figure 10



5. "Excess volatility"

The model discussed in the previous sections is driven by exogenous news in the fundamentals and by the noise produced by the non-linear speculative dynamics embedded in the model. As a result, the non-linear dynamics is capable of producing "excess volatility" in the exchange rate, i.e. volatility that exceeds the volatility of the underlying fundamental. In this section we analyse the sources of this excess volatility. We do this by computing the noise to signal ratio in the simulated exchange rate. We define this noise to signal ratio as follows:

$$\text{var(s)} = \text{var(f)} + \text{var(n)} \qquad (12)$$

where var(s) is the variance of the simulated exchange rate, var(f) is the variance of the fundamental and var(n) is the residual variance (noise) produced by the non-linear speculative dynamics which is assumed to be uncorrelated with var(f). Rewriting (12) we obtain

$$\frac{\text{var}(n)}{\text{var}(f)} = \frac{\text{var}(s)}{\text{var}(f)} - 1$$

The ratio var(n)/var(f) can be interpreted as the noise to signal ratio. It gives a measure of how large the noise produced by the non-linear dynamics is with respect to the exogenous volatility of the fundamental exchange rate. We simulate this noise to signal ratio for different values of the parameters of the model. We show the results in figures 11-13. We observe that the noise to signal ratio is very sensitive to the extrapolation parameter of the chartists and to a lesser degree to the transactions costs. We observe that the noise produced by the non-linear dynamics can become very large relative to the volatility of the fundamental exchange rate when the extrapolation parameter is large and when transactions costs are large. The noise to signal ratio is much less sensitive to the speed of adjustment in the goods market. Overall our model is capable of generating volatility of the exchange rate that is much in excess of the volatility of the underlying fundamental (Goodhart (1989) and Goodhart and Figlioli (1991)).

Figure 11

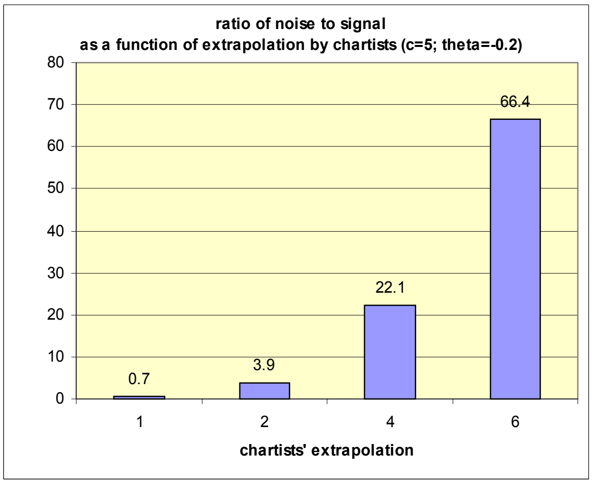


Figure 12

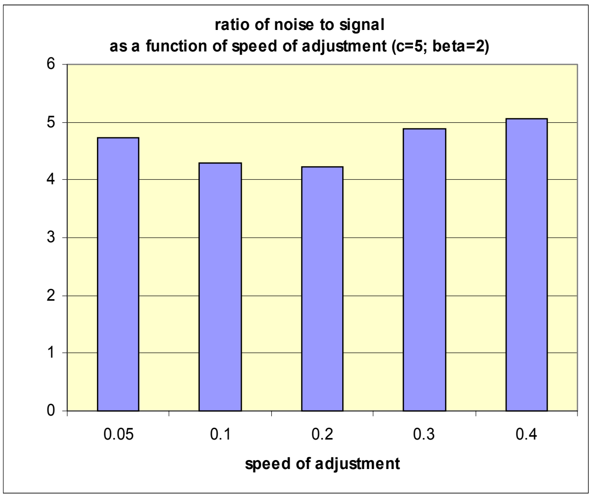
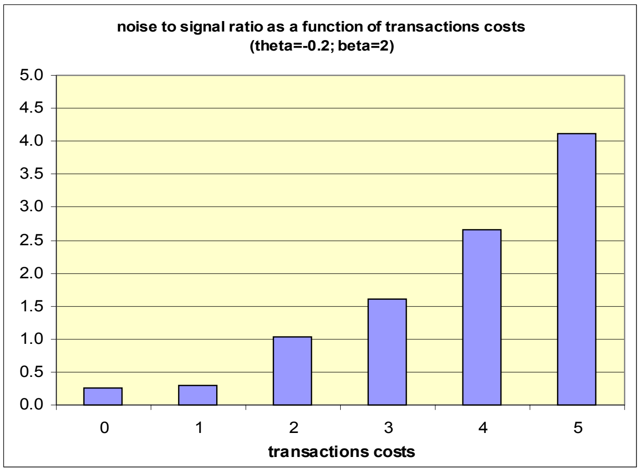


Figure 13



6. Small and large shocks and the dynamics of the exchange rate

In linear models the size of the shocks does not affect the nature of the dynamics. In non-linear models things are different. The size of the shocks matters. This is also the case in our exchange rate model. In order to illustrate this we simulated the model under two different assumptions about the variance of the shocks in the fundamental exchange rate. In the first case we assume low variance of these shocks, in the second case we assume a high variance (ten times higher). The results of our simulations are presented in figures 14-15. (The simulations shown here are representative for a wide range of parameter values).

Figure 14: low variance of equilibrium exchange rate

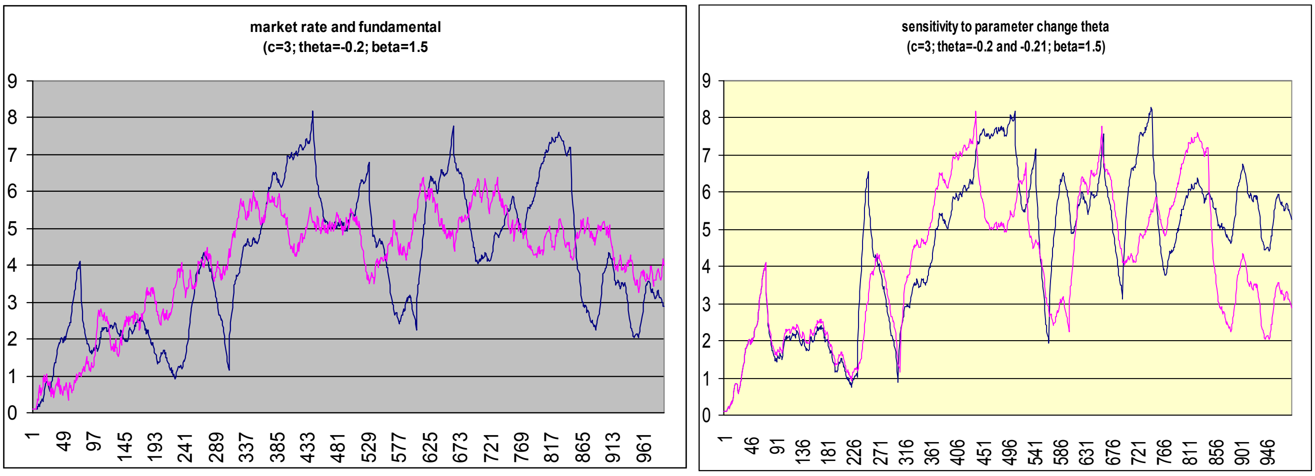
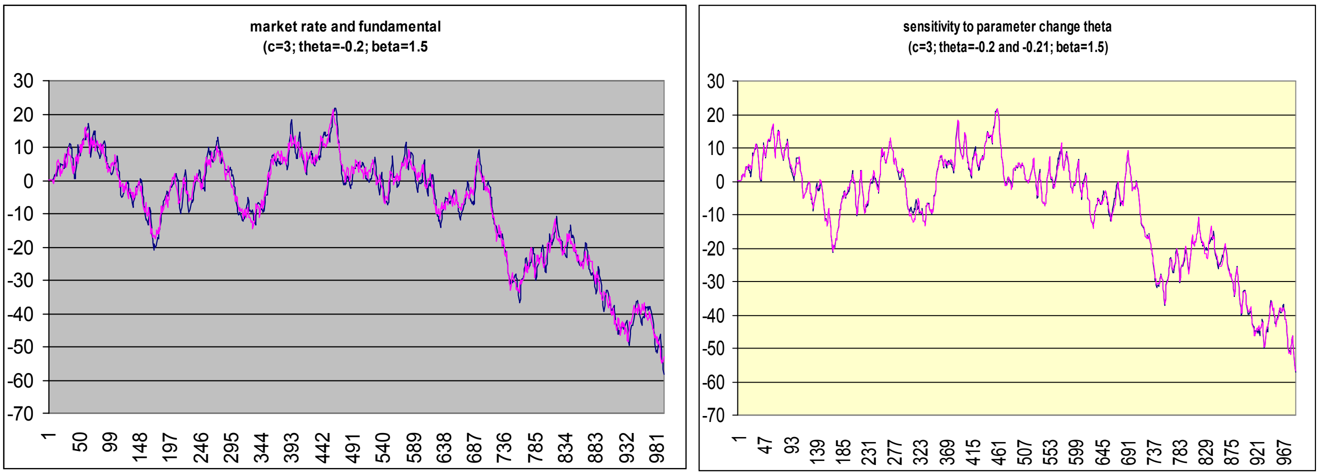


Figure 15: high variance of equilibrium exchange rate

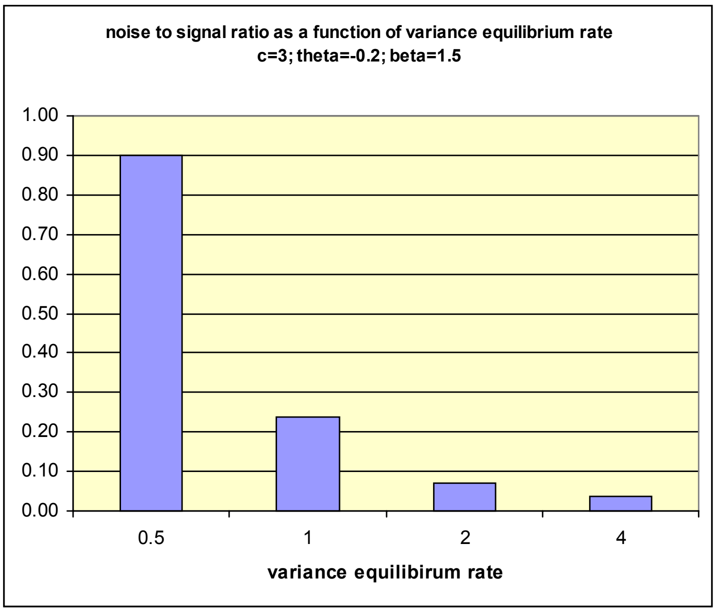


Two conclusions follow from a comparison of the low and high variance cases. First, in the low variance case we observe sustained deviations from the equilibrium exchange rate; this is not the case when the equilibrium exchange rate is subject to large shocks (see left-hand panels of figure 14 and 15). Second, the sensitivity to small changes in parameters is clearly visible when the variance of the exchange rate is low (see right-hand panel of figure 14). When this variance is high, no such sensitivity can be observed (right-hand panel of figure 15). It is important to stress that the transactions cost band is the same in both cases. Thus, when the shocks are small relative to the given band of transactions costs the movements of the exchange rate show more complexity than when the shocks are large.

This feature is also evident from a comparison of the noise to signal ratio for different variances of the fundamental exchange rate. We show this in figure 16. We observe that when the variance of the equilibrium exchange rate is low, a large part of the volatility of the exchange rate is produced by the noise from the non-linear dynamics. For high variance the noise is very small, implying that the exchange rate follows the fundamental rate very closely[8].

[8] It should be stressed that the total variability of the exchange rate in the high variance scenario is much larger than the total variability of the exchange rate in the low variance scenario. The point is that in the high variance scenario almost all of the variability of the exchange rate is explained by the (much higher) variability of the fundamental. This is not the case in the low variance scenario where a large part of the variability of the exchange rate cannot be related to the variability of the underlying fundamental.

Figure 16



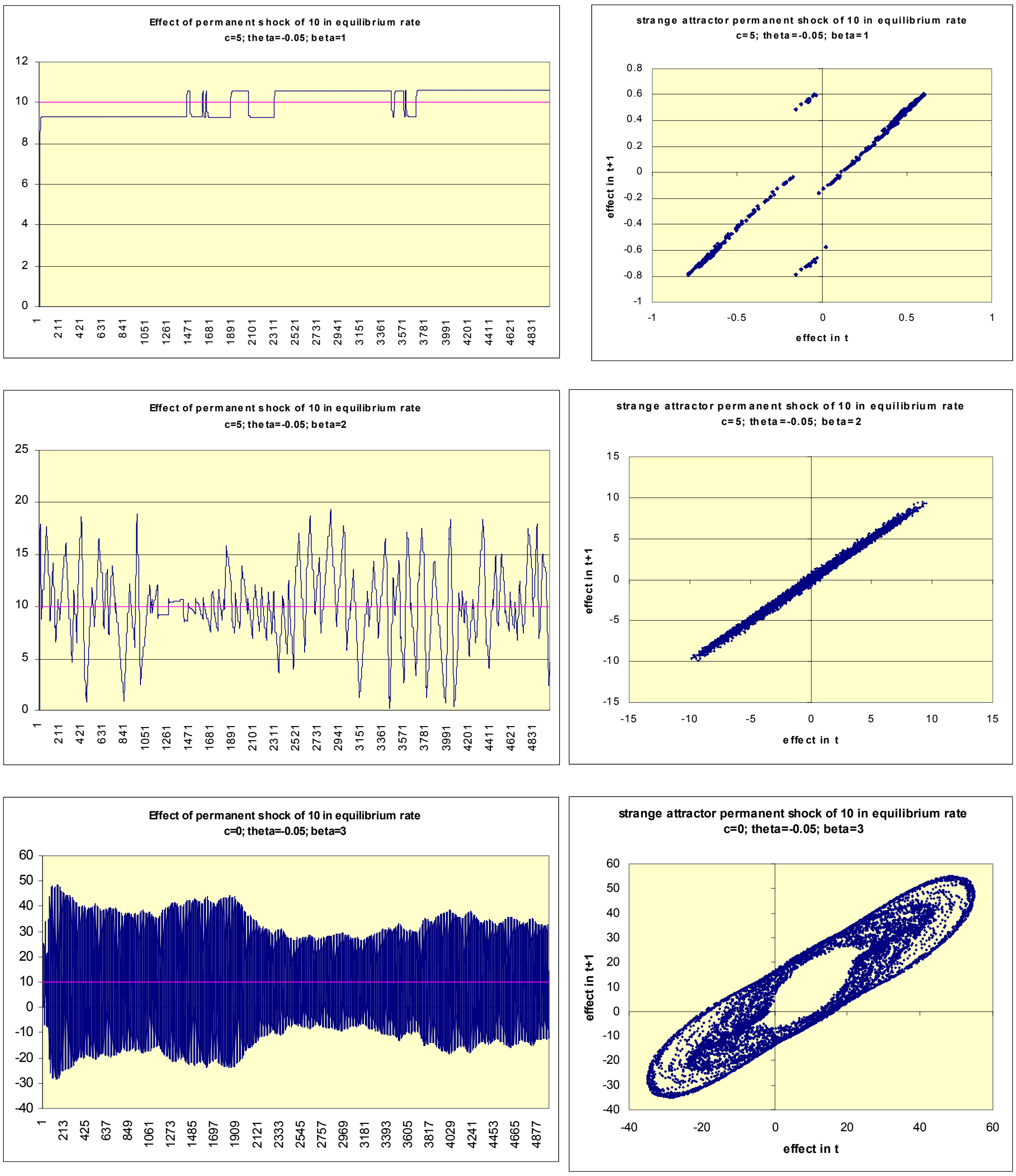
The intuition of this result is that when the fundamental shocks are small the exchange rate regularly switches from the dynamics inherent in the band to the one prevalent outside the band. This non-linearity produces a lot of noise and complexity in the dynamics of the exchange rate. When the shocks are large relative to transactions cost band the dynamics outside the band mostly prevails, leading to a tighter link between the exchange rate and the fundamental. This feature has also been found to hold empirically (See De Grauwe an Vansteenkiste(2001)). These results of our model are also consistent with the empirical evidence suggesting that the link between inflation differentials and exchange rate changes of low inflation countries is weak, if non-existent (see De Grauwe and Grimaldi(2001), De Grauwe and Polan(2001)).

7. On the transmission of permanent shocks

In linear models a permanent shock in the fundamental has a predictable effect on the exchange rate, i.e. the coefficient that measures the effect of the shock in the fundamental on the exchange rate converges after some time to a fixed number. Things are very different in our non-linear model. We illustrate this by showing how a permanent increase in the fundamental is transmitted to the exchange rate. We assumed that the fundamental rate increases by 10, and we computed the effect on the exchange rate by taking the difference between the exchange rate with the shock and the exchange rate without the shock. In a linear model we would find that in the long run the exchange rate increases by 10. This is not the case in our model. We present the evidence in figure 17 where we show the effect of the same permanent shock of 10 in the fundamental rate on the exchange rate. We do this for three different values of the extrapolation parameter. The simulations are done using the deterministic part of the model. Thus, there is no exogenous noise in the model that could blur the transmission process from the fundamental rate to the exchange rate.

The most striking feature of these results is that the effect of the permanent shock does not converge to a fixed number. In fact it follows a very complex pattern. The complexity of this effect is shown in the strange attractors of the effects of the shock (right hand panels). Thus, in a non-linear world it is very difficult to predict what the effect will be of a given shock in the fundamental, even in the long run. Such predictions can only be made in a statistical sense, i.e. our model tells is that *on average* the effect of a shock of 10 in the fundamental will be to increase the exchange rate by 10. In any given period, however, the effect could deviate substantially from this average prediction.

Figure 17



These results have the following interpretation. When the fundamental rate is shocked permanently, this has two effects. First, it shifts the strange attractor permanently. Second, it changes the initial conditions of the new 'history of the exchange' rate. The combination of the two produces the complexity in the transmission of the initial shock. As an illustration, we show the displacement of the strange attractor after the shock in the fundamental rate in figure 18.

Figure 18

Our results help to explain why it appears so difficult to predict the effects of changes in the fundamental exchange rate on the market rate, and why these effects seem to be very different when applied in different periods.

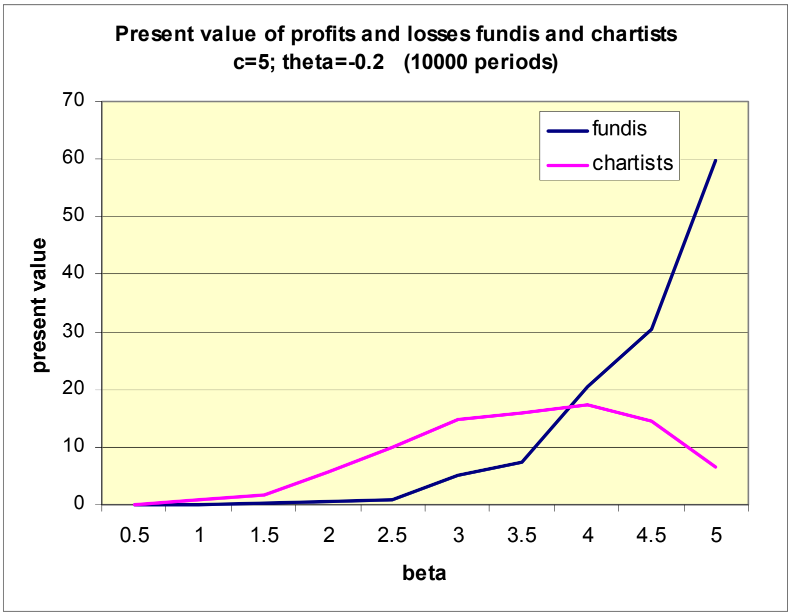
8. Is chartism profitable?

In this section we analyse how profitable forecasting based on chartism is in relation with fundamentalism. This analysis is important because particular forecasting rules will only survive if they are profitable. If chartism turns out to be unprofitable, fewer and fewer agents will use this technique, and it will disappear.

In order to analyse this issue we simulated the model and asked the question how the profit and loss accounts of chartists and fundamentalists evolve over time. We assumed that each of them started with an initial capital of €1. When they expect the exchange rate to increase (decrease) they buy (sell), and hold for one period. They repeat this operation each period.

We calculated the net present value of these profits and losses using a discount rate of 4%. Results are shown in figure 19 where the present value of profits and losses are related to different values of beta.

Figure 19



We observe the following. First, the cumulative profits of both chartists and fundamentalists are positive. Second, for values of beta lower than 4 the chartists make profits higher than the fundamentalists. However, for high values of beta the chartists' rule looses its profitability and the fundamentalists' rule becomes much more profitable. This implies that we are unlikely to observe chartists to use large extrapolation parameter values in their forecasting.

The next step was to analyse profits and losses under two different stochastic regimes. The first one has a low variance of noise (same as in previous simulations). The second regime has a variance ten times higher. Results are shown in figures 20 and 21. We see that chartism becomes less profitable in a regime of high variance, while fundamentalism then becomes more profitable. It is worthwhile to note that this result is consistent with the results obtained in the previous section, where we showed that in a high variance regime the link between fundamentals and the market exchange rate is tighter than in the low variance regime. Thus, it is not surprising that in a high variance regime the fundamentalists forecasting rule is relatively profitable. This result also implies that we should observe more chartists in the low variance currency markets than in the high variance markets. We leave it for further research to verify the empirical validity of this hypothesis.

Figure 20

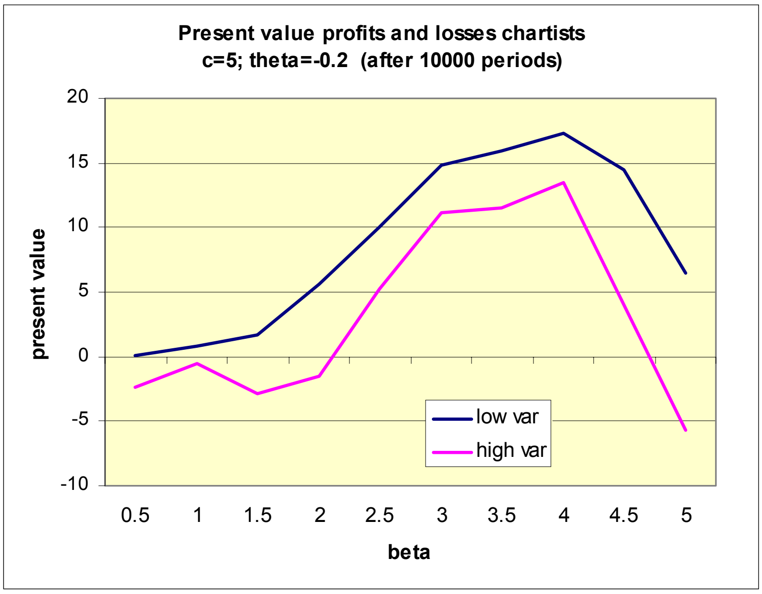
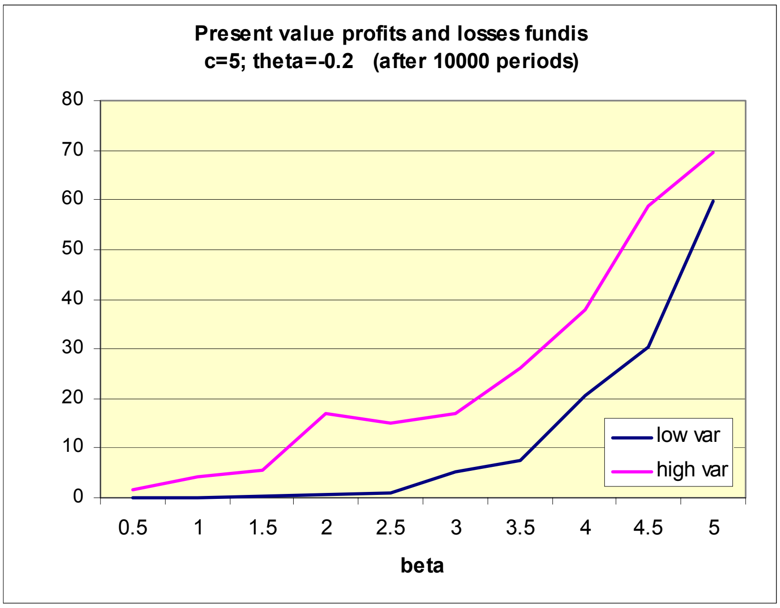


Figure 21



Finally we computed the present value of profits and losses with and without transactions costs (figure 22 and 23). Not surprisingly, when transactions costs are zero chartism becomes less profitable while fundamentalisms increases in profitability. The reason is that in the absence of transaction costs the mean-reverting behaviour of fundamentalists is stronger implying that the exchange rate remains close its fundamental value.

Figure 22

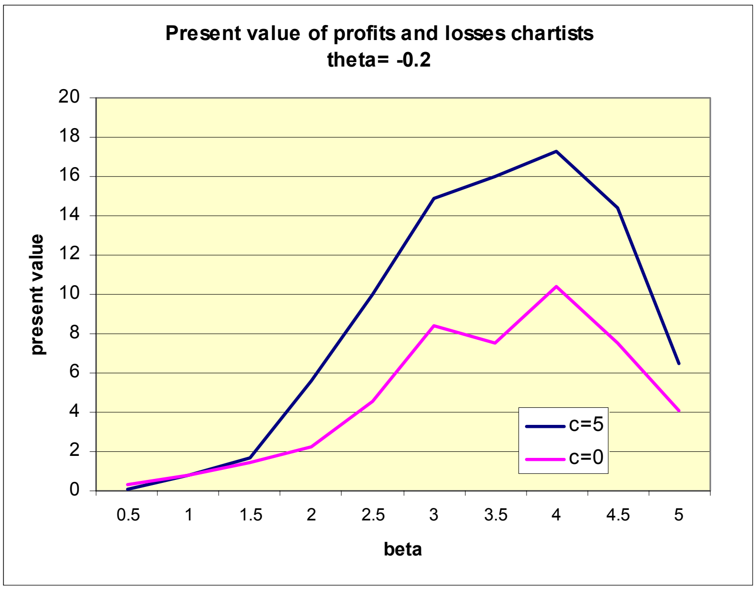
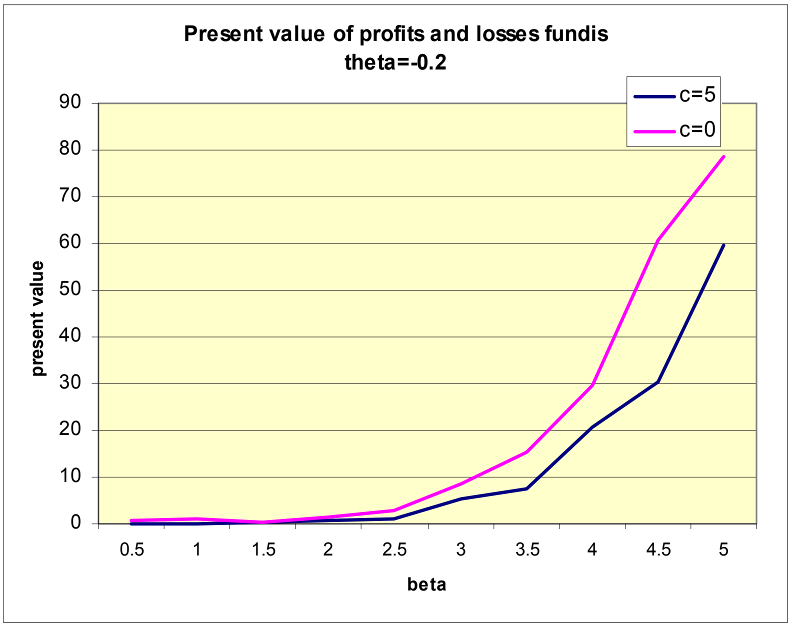


Figure 23



9. Conclusion

In this paper we analysed the workings of a simple non-linear exchange rate model in which agents hold different beliefs about the underlying model. We distinguished between chartists and fundamentalists, where the chartists apply a *positive feedback rule* and the fundamentalists a *negative feedback rule*. The non-linearities in the model originate from transactions costs and from the existence of non-linear adjustment dynamics in the goods market.

Our main results can be summarised as follows. First, the simple non-linear structure of the model is capable of generating a very complex exchange rate dynamics. We found that for plausible parameter values this complex dynamics can be chaotic. This implies that small shocks in the equilibrium exchange rate lead to very different time-paths of the exchange rate.

Second, our model is capable of explaining some empirical puzzles. One puzzle is that the market exchange rate can deviate substantially and for relatively long periods of time from its fundamental value ("disconnect puzzle"). We showed that such disconnections are a natural outcome of the non-linear dynamics in our simple model. There is no need to invoke exogenous events and special factors to explain why exchange rates deviate from their fundamental values. It should also be noted that our model generates these disconnections even in the absence of deterministic chaos. In other words we do not need to invoke chaos to explain disconnections.

Another empirical puzzle observed in exchange rate economics is the frequent occurrence of "regime shifts", i.e. structural breaks in the relation between the exchange rate and the fundamentals. This phenomenon has first been noted in the celebrated studies of Meese and Rogoff(1982)[9]. It is now customary to explain these structural breaks by changes in the policy regime. Our model provides an alternative explanation. The non-linear dynamics embedded in the model produces endogenous regime shifts that change the link between the exchange rate and its fundamentals. These structural breaks can be triggered by very small changes in parameters, or by small errors in the estimates of these parameters by agents who forecast the future exchange rate. Thus, in a non-linear world, structural breaks in the link between the exchange rate and its fundamentals occur naturally even when no changes occur in the policy regime.

Third, we found that our simple non-linear dynamic model can generate "excess volatility" in the exchange rate. The size of this excess volatility crucially depends on the degree of extrapolation applied by chartists and on the size of the transactions cost band.

Fourth, we found that the size of shocks to the underlying fundamental exchange rate matters for the dynamics of the exchange rate. More specifically, we found that when these shocks are small relative to the size of the transactions cost band, the phenomena just described will tend to be prevalent. That is, in regimes of low shocks relative to the transactions cost band, the exchange rate movements are complex, and can even be chaotic. In such a regime exchange rates deviate substantially from the underlying fundamentals and frequent structural breaks in the link between the fundamentals and the exchange rate are observed. The latter occur in the absence of changes in the policy regime.

[9] For more recent evidence see De Grauwe and Vansteenkiste(2001).

Finally, we checked whether the forecasting rules used by chartists and fundamentalists are profitable. We found that for a broad range of parameter values both rules are profitable.

Some implications of these findings are the following. The exchange rates of the major currencies are subject to relatively small shocks in the underlying fundamentals (e.g. inflation differentials are almost zero). Compared to these shocks the transactions costs can be said to be relatively large (see Obstfeld & Rogoff(2000) on this), i.e. a large part of goods and services are non-traded (or difficult to trade) because the cost of shipping them across borders is quite high. Thus the regime confronted by the exchange rates of the major industrialised countries comes close to the regime we have identified to be the one producing complexity, speculative noise, and structural breaks between exchange rates and underlying fundamentals. Put differently, the movements of the exchange rates of the industrialised countries are likely to be clouded by a non-linear speculative dynamics that makes it difficult if not impossible to explain this or that movement of these exchange rates. In contrast the exchange rates of high inflation countries experience large shocks in the fundamentals. As a result, the movements of the exchange rates of these countries can be explained much better by movements in underlying fundamentals (e.g. inflation differentials).

The results of our paper make it easier to understand why it will remain difficult, if not impossible to find (fundamental) logic in the movements of the exchange rates of major currencies that are subject to relatively low nominal disturbances. However, our inability to understand why, say, the dollar moved up against the euro during the 1999-2000 does not prevent analysts from developing exotic theories explaining these movements. Probably this has to do with the fact that the human mind abhors the emptiness created by its inability to understand. It is no surprise therefore that new explanations based on fundamentals are created, and will continue to be created for each and every new turn of the exchange rate.

Appendix 1: Sensitivity of dynamics to parameter values

theta / beta	-0.1	-0.2	-0.3	-0.4	-0.5
1.9	FPM	FPM	FPM	FPM	U
2	FPC	FPC	FPC	FPC	U
2.1	*C*	*C*	*C*	*C*	U
2.2	*C*	*C*	*C*	*C*	U
2.3	*C*	*C*	*C*	*C*	U
2.4	*C*	*C*	*C*	*C*	U
2.5	*C*	*C*	*C*	*C*	U
2.6	*C*	*C*	*C*	*C*	U
2.7	*C*	*C*	*C*	*C*	U
2.8	*C*	*C*	*C*	*C*	U
2.9	*C*	*C*	*C*	*C*	U
3	*C*	*C*	*C*	*C*	U
3.1	*C*	*C*	*C*	*C*	U
3.2	*C*	*C*	*C*	*C*	U
3.3	*C*	*C*	*C*	*C*	U
3.4	*C*	*C*	*C*	*C*	U
3.5	P (12)	P (12)	P (12)	P (12)	U
3.6	*C*	*C*	*C*	*C*	U
3.7	*C*	*C*	*C*	*C*	U
3.8	*C*	*C*	*C*	*C*	U
3.9	*C*	*C*	*C*	*C*	U
4	*C*	*C*	*C*	*C*	U
4.1	*C*	*C*	*C*	*C*	U
4.2	*C*	*C*	*C*	*C*	U
4.3	*C*	*C*	*C*	*C*	U
4.4	*C*	*C*	*C*	*C*	U
4.5	P (10)	P (10)	P (10)	P (10)	U
4.6	*C*	*C*	*C*	*C*	U
4.7	*C*	*C*	*C*	*C*	U
4.8	*C*	*C*	*C*	*C*	U
4.9	*C*	*C*	*C*	*C*	U

Note: *C* = chaos
P (N) = N-period cycles
U = unstable
FPM = fixed point reached monotonically
FPC = fixed point reached cyclically

Table continued

theta / beta	-0.1	-0.2	-0.3	-0.4	-0.5
5	*C*	*C*	*C*	*C*	*C*
5.1	*C*	*C*	*C*	*C*	*C*
5.2	*C*	*C*	*C*	*C*	*C*
5.3	*C*	*C*	*C*	*C*	*C*
5.4	P (8)	P (9)	P (9)	P (9)	P (9)
5.5	P (9)	P (8)	P (8)	P (9)	P (9)
5.6	*C*	*C*	*C*	*C*	*C*
5.7	P (23)	P (26)	P (26)	P (26)	P (26)
5.8	P (34)	P (17)	P (17)	P (17)	P (17)
5.9	P (42)	P (42)	P (42)	P (42)	P (42)
6	P (24)	P (8)	P (12)	P (8)	P (8)
6.1	P (8)	P (8)	P (8)	P (8)	P (8)
6.2	P (18)	P (17)	P (17)	P (17)	P (17)
6.3	P (34)	P (34)	P (34)	P (34)	P (34)
6.4	*C*	*C*	*C*	U	*C*
6.5	*C*	*C*	*C*	U	*C*
6.6	*C*	*C*	*C*	U	*C*
6.7	*C*	*C*	*C*	U	*C*
6.8	*C*	*C*	*C*	U	*C*
6.9	*C*	*C*	*C*	U	P (16)
7	*C*	*C*	*C*	U	*C*
7.1	U	U	U	U	U

Note: *C* = chaos
P (N) = N-period cycles
U = unstable
FPM = fixed point reached monotonically
FPC = fixed point reached cyclically

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